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Electro-weak stability in the presence of higher dimension operators in the Higgs sector.

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Electro-weak stability

- As the standard model couplings are run to very high energy the Higgs self-interaction λ turns negative at some point.
- Since the Higgs mass decreases with decreasing λ this happens earlier for lighter masses.
- This gives a lower bound on the Higgs mass as a function of energy.

Electro-weak stability

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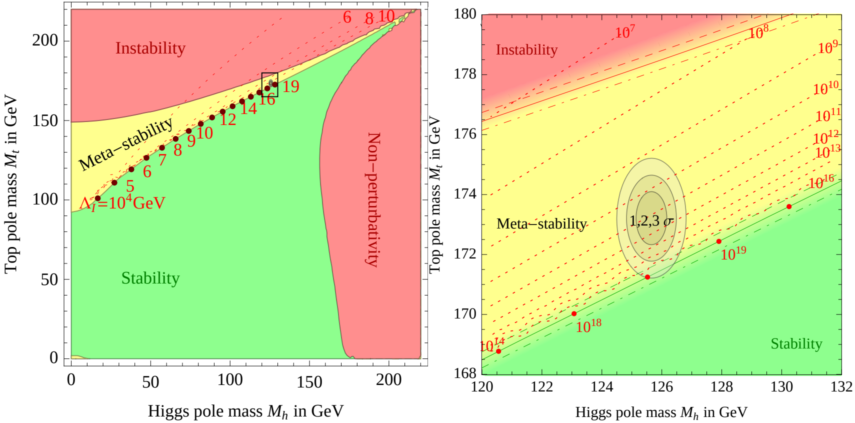
- Since the Higgs mass decreases with decreasing λ this happens earlier for lighter masses.
- This gives a lower bound on the Higgs mass as a function of energy.
- The meaning of this energy is to give a mass scale of higher dimension BSM operators.
- The Higgs mass measured at CERN indicates that λ turns negative at $\sim 10^{10\ldots14}$GeV.
- Calculations have shown that it may be possible to postpone BSM physics to the Planck scale and still have a meta-stable EW vacuum (lifetime $\gg$ age of the universe).

Electroweak stability

G. DeGrassi et al. *JHEP* 1202 098 (2012), *JHEP* 1312 089 (2013)

The Higgs mass is very near the limit of stability of the SM.
Is this the result of some fine-tuning?



Is the Higgs mass lower bound relevant?

- Yes and no.
- Clearly BSM physics must exist but

the bound derived within the SM will be altered by the BSM physics.

- With higher dimension operators to stabilize the vacuum, a negative λ does not imply instability.
- It is thinkable that the lower bound is very low, i.e. regardless of the typical energy scale of the BSM physics, it might be perfectly viable to have an almost massless Higgs boson.

Goals of the presentation:

- To show that by adding higher dimension operators to the Higgs potential (in a simplified version of the SM) we can significantly alter the lower mass bound.

- To investigate the finite temperature transition in the presence of a ϕ^6 operator. In particular, can a strong first order transition be generated to make EW baryogenesis viable?

Our model: The Higgs-Yukawa model

- The SM Higgs sector is dominated by the Higgs field ϕ and the top quark.
- We neglect gauge degrees of freedom and use a model with ϕ and fermions.
- Poor man's version of the Standard Model which nonetheless captures the nonperturbative Higgs-top interaction.
- The Higgs part of the Lagrangian is given by:

$$\mathcal{L}_{\mathrm{H}} = |\partial_\mu \phi|^2 + m_0^2\,|\phi|^2 + \lambda\,|\phi|^4 + V_{\mathrm{BSM}}(|\phi|)$$

Fermion content

- Let $\Psi_t = (t, b)^\intercal = (t_\mathrm{L}, t_\mathrm{R}, b_\mathrm{L}, b_\mathrm{R})^\intercal$ and $\Psi_{t,\mathrm{L}} = (t_\mathrm{L}, b_\mathrm{L})^\intercal$. The top-bottom Yukawa part of the Lagrangian is then:

$$\mathcal{L}_\mathrm{tb} = \overline{\Psi}_t \partial\!\!\!/ \Psi_t + y_b \overline{\Psi}_{t,\mathrm{L}} \phi b_\mathrm{R} + y_t \overline{\Psi}_{t,\mathrm{L}} \widetilde{\phi} t_\mathrm{R} + \mathrm{h.c.} \qquad \widetilde{\phi} = i\tau_2 \phi^\dagger$$

- The other weak SU(2) doublets contribute analogously to the Lagrangian and are all included in our model.

Using the symmetries

- Anticipating spontaneous symmetry breaking (SSB) we parametrize the Higgs field as:

$$\phi(x) = \begin{pmatrix} g_2(x) + i g_1(x) \\ v + h(x) - i g_3(x) \end{pmatrix} \qquad \langle \phi \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}$$

- Because of SU(2) invariance the fermion determinant can only depend on $|\phi(x)|^2 = (v + h(x))^2 + g_1(x)^2 + g_2(x)^2 + g_3(x)^2$:

$$\mathcal{L}_\mathrm{F} = \sum_{f,f'} \overline{\Psi}_f M\left(|\phi|^2\right)_{ff'} \Psi_{f'} \Rightarrow \mathrm{LogDet}\left(M\left(|\phi|^2\right)\right)$$

Lattice action

- We first discretize the Higgs field:

$$\mathcal{L}_{\mathrm{Latt}} = -\kappa \sum_{\mu} \hat{\phi}_x^{\dagger} \hat{\phi}_{x+\hat{\mu}} + \mathrm{h.c.} + \left|\hat{\phi}_x\right|^2 + \hat{\lambda}\left(\left|\hat{\phi}_x\right|^2 - 1\right)^2 + V_{\mathrm{BSM}}(\left|\hat{\phi}_x\right|) + \mathrm{LogDet}\left(M\left(\left|\hat{\phi}\right|^2\right)\right)$$

with dimensionless variables

$$a\phi(x) = \sqrt{2\kappa}\hat{\phi}_x, \ (am_0)^2 = \frac{1-2\hat{\lambda}}{\kappa} - 8, \ \hat{\lambda} = 4\kappa^2\lambda, \ \hat{v} = \frac{av}{\sqrt{2\kappa}}$$

BSM potential

- In general one has an infinite sum of higher dimension operators.
- The simplest choice is to keep only the ϕ^6 operator.
- Assuming that the typical scale of the BSM physics is $M_{\rm BSM}$, ϕ^6 will be suppressed by a scale separation $\sim (v/M_{\rm BSM})^2$ or $\sim (E/M_{\rm BSM})^2$.
- On the computer we rescale the Higgs field with the lattice spacing a leading to:

$$a^4 V_{\rm BSM}\left(\left|\hat{\phi}\right|\right) = (2\kappa)^3 \frac{1}{(aM_{\rm BSM})^2} \left|\hat{\phi}\right|^6 \equiv (2\kappa)^3 \lambda_6 \left|\hat{\phi}\right|^6$$

Implications of the new potential

- The ϕ^6 operator ensures that the action is bounded for any value of the quartic coupling λ_4.
- In particular, λ_4 can be negative which might lead to a lower Higgs boson mass.
- The model is no longer renormalizable. We choose a cutoff a^{-1} above all other energy scales, ie. $(aM_{\mathrm{BSM}}) = 1/3$. Results change little upon decreasing the lattice spacing further.

Setting the scale

- On the lattice we measure all observables in units of the lattice spacing a.
- We determine a by demanding: $v = \frac{\hat{v}}{a\sqrt{2\kappa}} = 246$ GeV.
- We also fix the Yukawa couplings to the fermion tree level masses: $y_f = \frac{m_f}{246\,\mathrm{GeV}}$.

Extended Mean Field Theory (EMFT)

OA, P. de Forcrand, P. Werner and A. Georges *Phys. Rev. D* 88 125006 (2013) [arXiv:1305.7136]

OA, P. de Forcrand, P. Werner and A. Georges [arXiv:1405.6613]

- We solve the model approximately by using an extended version of Mean Field Theory which takes also quadratic fluctuations into account.

- The original 4*d* problem reduces to a 0*d* problem with some self-consistency conditions which can be solved at a very low computational cost.

- EMFT can be formulated in any finite or infinite box which gives access to finite volume and finite temperature effects.

Setup

- EMFT yields an implicit action where expectation value and propagation need to be solved for self-consistently.

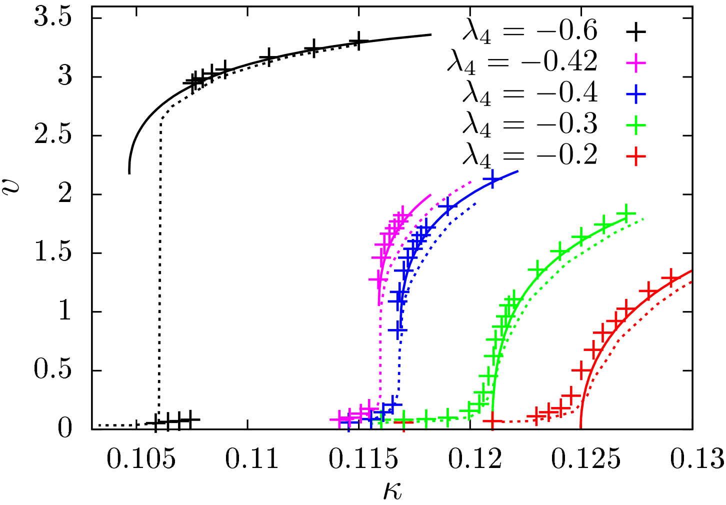
- We use a root solver to find fixed points where the input, ϕ_{ext}, equals the output, $\langle\phi\rangle$.

Bench-marking the approximation

- EMFT has already proved to be very accurate on ϕ^4 models.
- To check that the fermions are treated correctly we compare to full Monte Carlo simulations of the Higgs-Yukawa model [1].
- The results are very encouraging (see next slide).
- Due to the large scale separation $M_{\mathrm{BSM}} \gg v$ and the Goldstone bosons, Monte Carlo simulations suffers from prohibitive finite size effects. EMFT does not have this problem.
- Note that the Monte Carlo simulation suffers from a "sign problem" unless the fermions are mass-degenerate whereas EMFT can handle the physical case.

[1] P. Hegde, K. Jansen, C. -J. D. Lin and A. Nagy *PoS LATT13* [arXiv:1310.6260]

$\lambda_6 = 0.1$, "perturbative"



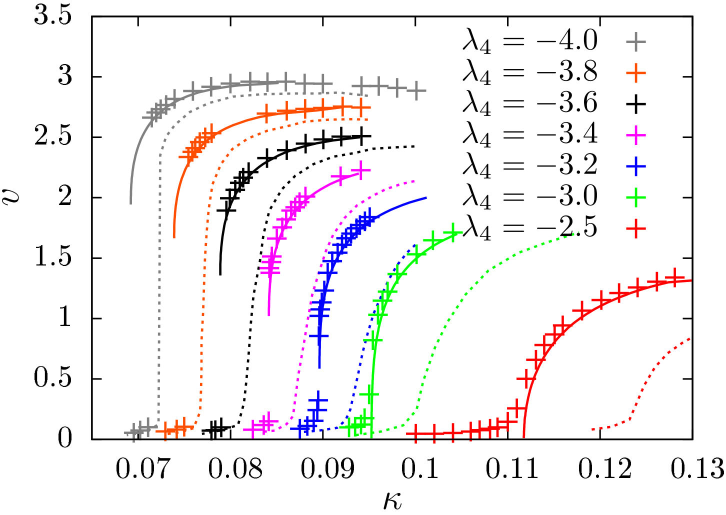
—: EMFT

+ : Monte Carlo

– –: Constraint Effective Potential (1 loop)

K. Jansen et al. (2013)

$\lambda_6 = 1$, “non-perturbative”



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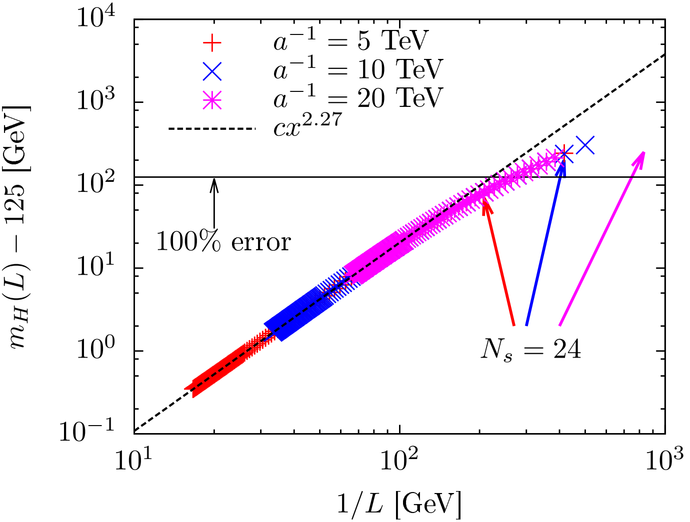
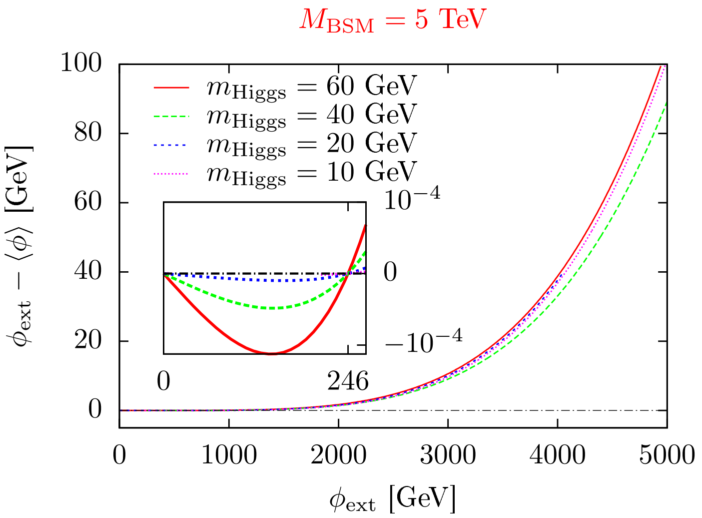
K. Jansen et al. (2013)

Because of massless Goldstone bosons, the finite volume corrections are power like → major problem for Monte Carlo simulations.



Lowering the Higgs mass bound



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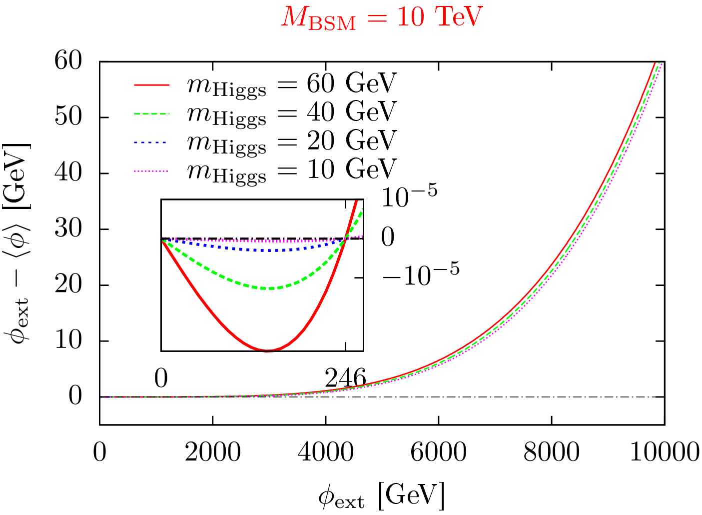
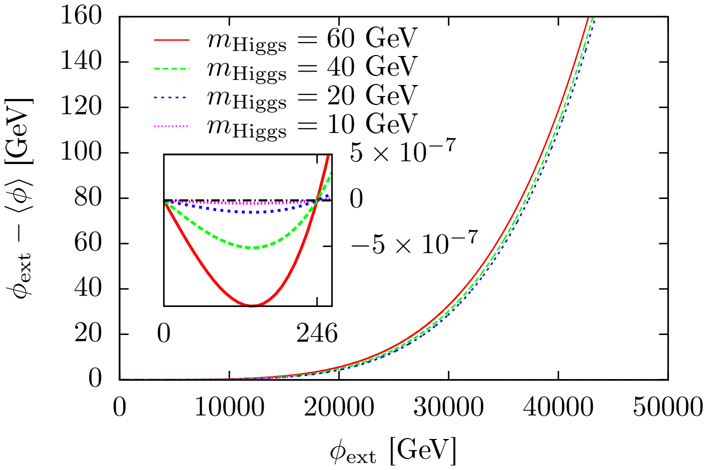
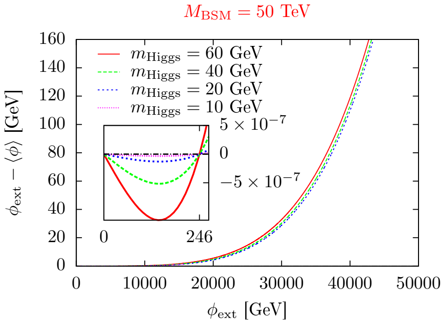


Lowering the Higgs mass bound

$M_{\rm BSM} = 50$ TeV

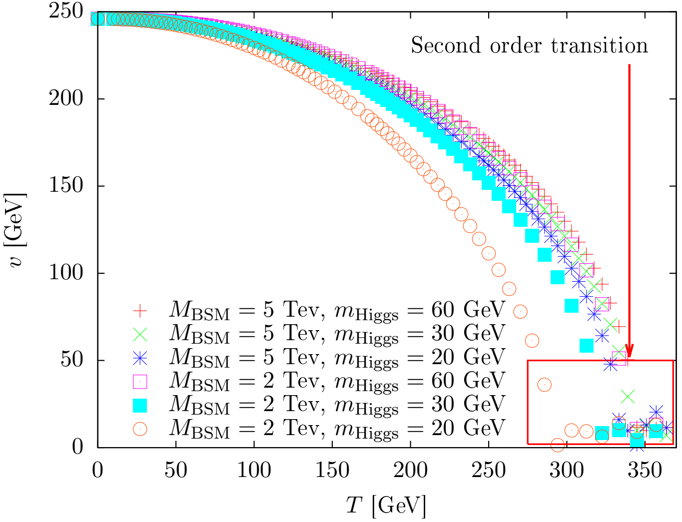


Lowering the Higgs mass bound



Even when $M_{\rm BSM}$ is as heavy as 50 TeV, $m_{\rm Higgs}$ can be as small as 10 GeV

Finite temperature transition



Conclusions

- Higher dimension operators can stabilize the Higgs potential even at negative quartic couplings.
- This allows for a lowering of the stability bound of the Higgs boson mass, presumably all the way down to zero.
- A small Higgs mass does not imply a small BSM scale.
- Within a generic BSM model, the Higgs mass of 125 GeV does not appear as fine-tuned.
- The addition of the ϕ^6 operator is not enough to make the finite temperature phase transition first order in our model but in the full SM - with gauge fields- it can [1].

[1] C. Grojean, G. Servant, J. Wells *Phys. Rev. D* **71** 036001 (2005)

Thank you for your attention!

Extended Mean Field Theory (EMFT)

- We assume small fluctuations around the vacuum expectation value (vev).

$$\Phi_x = (\hat{h}_x, \hat{g}_{1,x}, \hat{g}_{2,x}, \hat{g}_{3,x})^\intercal + (\hat{v}, 0, 0, 0)^\intercal \equiv \delta\Phi_x^\intercal + \langle\Phi\rangle^\intercal ,$$

- The hopping term can be expressed as:

$$\Delta S = -2\kappa \sum_{\pm\mu} \delta\Phi_0^\intercal \delta\Phi_\mu - 4d\kappa\hat{v}\hat{h}_0 .$$

- We can integrate out all fields except Φ_0 at the cost of new couplings, $c_p \Phi_0^p, \ p \geq 2$.

- Truncating at second order is enough to capture most of the dynamics, cf. mass renormalization.

- The effective action becomes:

$$S_{\mathrm{EMFT}} = \Phi_0^{\mathrm{T}}(\boldsymbol{I}_4 - \boldsymbol{\Delta})\Phi_0 + \hat{\lambda}\left(\|\Phi_0\|^2 - 1\right)^2 - 2\hat{v}(\hat{v} + \hat{h})(2d\kappa - \Delta_1) + \mathrm{TrLog}\left(M\left(\|\Phi_0\|\right)\right) + V_{\mathrm{BSM}}\left(\|\Phi_0\|\right)$$

- Where $\boldsymbol{\Delta} = \mathrm{diag}\left(\Delta_1, \Delta_2, \Delta_2, \Delta_2\right)$ emulates propagation in the effective bath.

Self-consistency equations

- We have introduced three unknowns in the action so we need three self-consistency conditions:

1. $$\left\langle \Phi_0^\mathsf{T} \right\rangle = (\hat{v}, 0, 0, 0)^\mathsf{T}$$

2. $$2\left\langle \hat{h}_0^2 \right\rangle_c = \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \frac{1}{\frac{1}{2\left\langle \hat{h}_0^2 \right\rangle_c} + \Delta_1 - 2\kappa \sum_\mu \cos(p_\mu)}$$

3. $$2\left\langle \hat{g}_{i,0}^2 \right\rangle = \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \frac{1}{\frac{1}{2\left\langle \hat{g}_{i,0}^2 \right\rangle} + \Delta_2 - 2\kappa \sum_\mu \cos(p_\mu)}$$

The fermion determinant

- In the EMFT approximation the fermions see the uniform field $\|\Phi_0\| = \sqrt{(\hat{v}+\hat{h}_0)^2 + \hat{g}_{1,0}^2 + \hat{g}_{2,0}^2 + \hat{g}_{3,0}^2} \Rightarrow$ the fermion matrix is diagonal in Fourier space.

- We can choose a basis for the determinant where the different flavors decouple:

$$M\left(\|\Phi_0\|\right)_{ff'} \to \left(\not{\partial} + y_f\sqrt{2\kappa}\|\Phi_0\|\right)\delta_{ff'}$$

- Fermions discretized using the Neuberger overlap operator which respects chiral symmetry up to $\mathcal{O}(a^2)$ corrections.

- The fermion matrix becomes:

$$M_f^{(\mathrm{ov})} = D^{(\mathrm{ov})} + y_f \sqrt{2\kappa} \|\Phi\| \left(I_4 - \frac{1}{2} D^{(\mathrm{ov})} \right) .$$

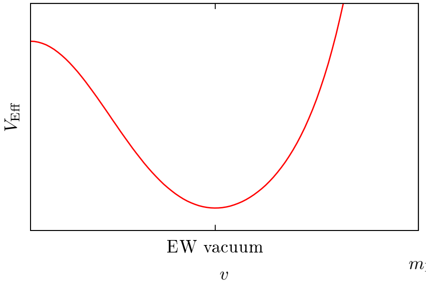
- Since $\|\Phi\|$ is constant we can calculate the TrLog efficiently in Fourier space:

$$\mathrm{TrLog}\left(M_f^{(\mathrm{ov})}\right) = 2 \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \log \left| \nu(p) + y_f \sqrt{2\kappa} \|\Phi_0\| \left(1 - \frac{\nu(p)}{2} \right) \right|^2$$

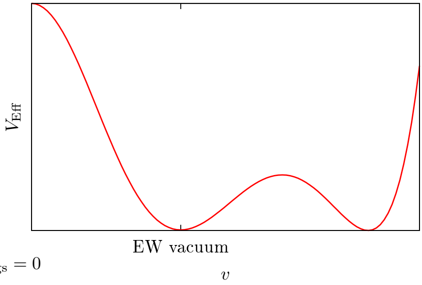
$$\nu(p) = 1 + \frac{i\sqrt{\tilde{p}^2} + \frac{1}{2}\hat{p}^2 - 1}{\sqrt{\tilde{p}^2 + \left(\frac{1}{2}\hat{p}^2 - 1\right)^2}}$$

$$\tilde{p}^2 = \sum_\mu \sin^2(p_\mu), \; \hat{p}^2 = 4 \sum_\mu \sin^2\left(\frac{p_\mu}{2}\right)$$

$m_{\text{Higgs}} > m_c$



$m_{\text{Higgs}} = m_c$



$m_{\text{Higgs}} = 0$