Three-loop HTL perturbation theory at finite temperature and chemical potential

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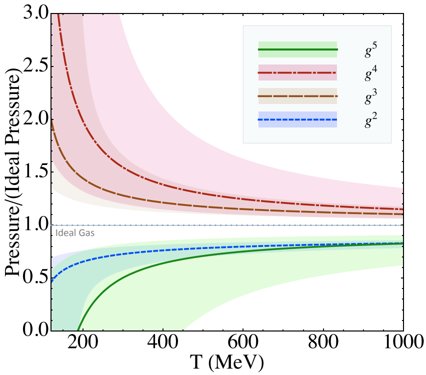
[1] In collaboration with Najmul Haque, Sylvain Mogliacci, Munshi Mustafa, Michael Strickland, Nan Su, and Aleksi Vuorinen, PRD 87 074003 (2013) JHEP 13 055 (2013) PRD 89 01701 (2014) JHEP 05, 27 (2014)

Introduction



- The weak-coupling expansion of the free energy of QCD has been calculated to order $\alpha_s^3 \log \alpha_s$ [a].
- Very poorly convergent. Generic problem in hot field theories (scalar field theory, QED).
- Goal: gauge-invariant framework with better convergence properties+able to describe dynamical properties+easy to generalize to finite μ_i.

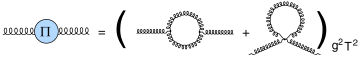
[a] Arnold and Zhai, '94/'95, Kastening and Zhai '95, Braaten and Nieto '95, Kajantie, Laine, Rummukainen, and Schröder '02, Vuorinen '03.

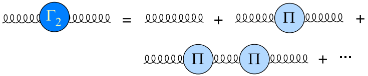
Hard-thermal-loop perturbation theory

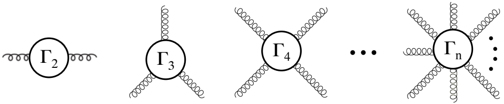
- QCD Lagrangian:

$$\mathcal{L}_{\rm QCD} = -\frac{1}{2}{\rm Tr}[G_{\mu\nu}G^{\mu\nu}] + i\bar{\psi}\gamma^{\mu}D_{\mu}\psi + \mathcal{L}_{gh} + \mathcal{L}_{gf} + \Delta\mathcal{L}_{\rm QCD}\,,$$

- For soft momenta gT, one needs effective propagators (2PI effective action: Blaizot, Iancu, and Rebhan '99, '00).





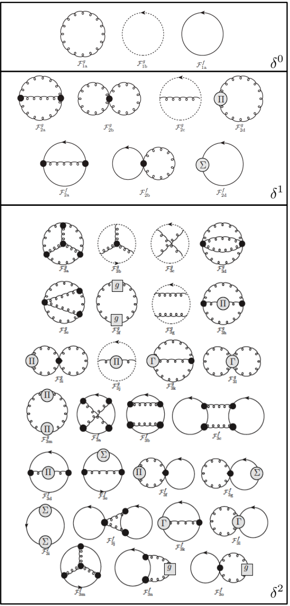


- Expansion point to gas of massive quasiparticles by adding HTL Lagrangian.

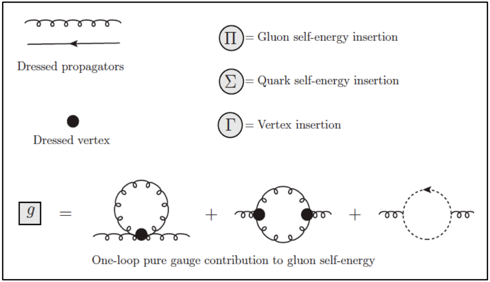
$$
\begin{aligned}
\mathcal{L}_{\rm HTL} &= -\frac{1}{2}(1-\delta)m_D^2 {\rm Tr}\left(G_{\mu\alpha}\left\langle \frac{y^\alpha y_\beta}{(y\cdot D)^2}\right\rangle_{\hat{\mathbf{y}}} G^{\mu\beta}\right) \\
&\quad (1-\delta) i m_q^2 \bar{\psi}\gamma^\mu \left\langle \frac{y_\mu}{y\cdot D}\right\rangle_{\hat{\mathbf{y}}} \psi\ , \\
\mathcal{L} &= \left(\mathcal{L}_{\rm QCD}+\mathcal{L}_{\rm HTL}\right)\big|_{g\to\sqrt{\delta}g} + \Delta\mathcal{L}_{\rm HTL}\ ,
\end{aligned}
$$

- HTLpt formal expansion parameter δ and $\delta = 1$ at the end.
- Expansion generates effective propagators and vertices.

Three-loop calculation



- Compute all Feynman diagram through 3 loops using HTL propagators and vertices

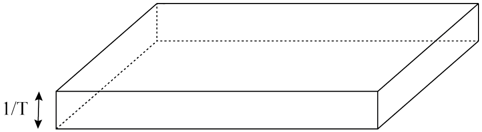


- Give a prescription for mass parameters m_D and m_q.

Thermodynamic potential

$$
\begin{aligned}
\frac{\Omega_{\rm NNLO}}{\Omega_0} = {} & \frac{7}{4}\frac{d_F}{d_A}\left(1+\frac{120}{7}\hat{\mu}^2+\frac{240}{7}\hat{\mu}^4\right)+\frac{s_F\alpha_s}{\pi}\left[-\frac{5}{8}\left(1+12\hat{\mu}^2\right)\left(5+12\hat{\mu}^2\right)+\frac{15}{2}\left(1+12\hat{\mu}^2\right)\hat{m}_D\right.\\
& \left.+\frac{15}{2}\left(2\ln\frac{\hat{\Lambda}_q}{2}-1-\aleph(z)\right)\hat{m}_D^3-90\hat{m}_q^2\hat{m}_D\right]+s_{2F}\left(\frac{\alpha_s}{\pi}\right)^2\left[\frac{15}{64}\left\{35-32\left(1-12\hat{\mu}^2\right)\frac{\zeta'(-1)}{\zeta(-1)}+472\hat{\mu}^2\right.\right.\\
& \left.\left.+1328\hat{\mu}^4+64\Big(-36i\hat{\mu}\aleph(2,z)+6(1+8\hat{\mu}^2)\aleph(1,z)+3i\hat{\mu}(1+4\hat{\mu}^2)\aleph(0,z)\Big)\right\}-\frac{45}{2}\hat{m}_D\left(1+12\hat{\mu}^2\right)\right]\\
& +\left(\frac{s_F\alpha_s}{\pi}\right)^2\left[\frac{5}{4\hat{m}_D}\left(1+12\hat{\mu}^2\right)^2+30\left(1+12\hat{\mu}^2\right)\frac{\hat{m}_q^2}{\hat{m}_D}+\frac{25}{12}\left\{\left(1+\frac{72}{5}\hat{\mu}^2+\frac{144}{5}\hat{\mu}^4\right)\ln\frac{\hat{\Lambda}_q}{2}\right.\right.\\
& +\frac{1}{20}\left(1+168\hat{\mu}^2+2064\hat{\mu}^4\right)+\frac{3}{5}\left(1+12\hat{\mu}^2\right)^2\gamma_E-\frac{8}{5}\left(1+12\hat{\mu}^2\right)\frac{\zeta'(-1)}{\zeta(-1)}-\frac{34}{25}\frac{\zeta'(-3)}{\zeta(-3)}\\
& -\frac{72}{5}\Big[8\aleph(3,z)+3\aleph(3,2z)-12\hat{\mu}^2\aleph(1,2z)+12i\hat{\mu}\left(\aleph(2,z)+\aleph(2,2z)\right)-i\hat{\mu}\left(1+12\hat{\mu}^2\right)\aleph(0,z)\\
& \left.\left.-2(1+8\hat{\mu}^2)\aleph(1,z)\Big]\right\}-\frac{15}{2}\left(1+12\hat{\mu}^2\right)\left(2\ln\frac{\hat{\Lambda}_q}{2}-1-\aleph(z)\right)\hat{m}_D\right]\\
& +\left(\frac{c_A\alpha_s}{3\pi}\right)\left(\frac{s_F\alpha_s}{\pi}\right)\left[\frac{15}{2\hat{m}_D}\left(1+12\hat{\mu}^2\right)-\frac{235}{16}\left\{\left(1+\frac{792}{47}\hat{\mu}^2+\frac{1584}{47}\hat{\mu}^4\right)\ln\frac{\hat{\Lambda}_q}{2}-\frac{144}{47}\left(1+12\hat{\mu}^2\right)\ln\hat{m}_D\right.\right.\\
& +\frac{319}{940}\left(1+\frac{2040}{319}\hat{\mu}^2+\frac{38640}{319}\hat{\mu}^4\right)-\frac{24\gamma_E}{47}\left(1+12\hat{\mu}^2\right)-\frac{44}{47}\left(1+\frac{156}{11}\hat{\mu}^2\right)\frac{\zeta'(-1)}{\zeta(-1)}-\frac{268}{235}\frac{\zeta'(-3)}{\zeta(-3)}\\
& \left.-\frac{72}{47}\Big[4i\hat{\mu}\aleph(0,z)+\left(5-92\hat{\mu}^2\right)\aleph(1,z)+144i\hat{\mu}\aleph(2,z)+52\aleph(3,z)\Big]\right\}+90\frac{\hat{m}_q^2}{\hat{m}_D}+\frac{315}{4}\left\{\left(1+\frac{132}{7}\hat{\mu}^2\right)\ln\frac{\hat{\Lambda}_q}{2}\right.\\
& \left.\left.+\frac{11}{7}\left(1+12\hat{\mu}^2\right)\gamma_E+\frac{9}{14}\left(1+\frac{132}{9}\hat{\mu}^2\right)+\frac{2}{7}\aleph(z)\right\}\hat{m}_D\right]+\frac{\Omega_{\rm NNLO}^{\rm YM}(\Lambda_g)}{\Omega_0}\,.
\end{aligned}
$$

- Hard scale T, soft scale gT, and supersoft scale g^2T.
- Integrate out hard scale T perturbatively to obtain effective three-dimensional theory (EQCD) (talks by Rummukainen/ Schroder+poster by Ghisoiu). [2]



- Soft scale gT contributions from calculations using EQCD.
- As in HTLpt, keep full g-dependence of m_D to resum.

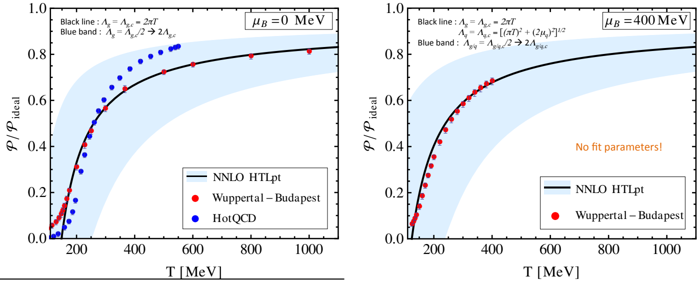
$$\mathcal{L}_{\rm eff} = \frac{1}{2}{\rm Tr}[G_{ij}^2] + {\rm Tr}[(D_iA_0)^2] + m_D^2{\rm Tr}[A_0^2] + \ldots$$

[2] Braaten and Nieto '96, Kajantie, Laine, K. Rummukainen, and Shaposhnikov '96, Vuorinen '03.

Results - pressure $\mu_B = 0$ and $\mu_B = 400$ MeV

- Use one-loop running with $\alpha_s(1.5\ \mathrm{GeV}) = 0.326$ (Bazavov et al 2012)
- Mass prescription: use $m_f = 0$ and m_D from EQCD [3]

$$\hat m_D^2 = \frac{\alpha_s}{3\pi}\left\{c_A + \frac{c_A^2\alpha_s}{12\pi}\left(5+22\gamma_E+22\ln\frac{\hat\Lambda_g}{2}\right) + \frac{1}{N_f}\sum_f\left[s_F\left(1+12\hat\mu_f^2\right) + \frac{c_A s_F\alpha_s}{12\pi}\left(\left(9+132\hat\mu_f^2\right)+22\left(1+12\hat\mu_f^2\right)\gamma_E + 2\left(7+132\hat\mu_f^2\right)+4\aleph(z_f)\right) + \frac{s_F^2\alpha_s}{3\pi}\left(1+12\hat\mu_f^2\right)\left(1-2+\aleph(z_f)\right) - \frac{3}{2}\frac{s_{2F}\alpha_s}{\pi}\left(1+12\hat\mu_f^2\right)\right]\right\}$$



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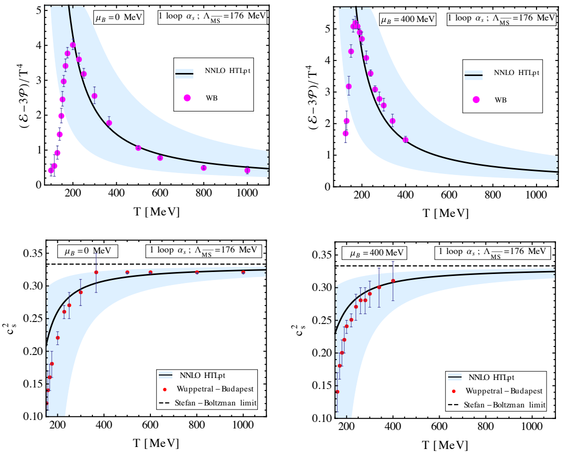
[3] Vuorinen '03.

[4] Borsanyi et al '10 and '12, Bazavov et al '09.

Results - interaction measure and speed of sound

$$c_s^2 = \frac{\partial \mathcal{P}}{\partial \mathcal{E}} .$$



Susceptibilities

- Pressure

$$\frac{\mathcal{P}}{T^4} = \sum_{ijk} \frac{1}{i!j!k!} \chi_{ijk} \left(\frac{\mu_u}{T}\right)^i \left(\frac{\mu_d}{T}\right)^j \left(\frac{\mu_s}{T}\right)^k$$

- Results - quark - and baryon-number susceptibilities

$$\chi_{ijk\ldots} = \left. \frac{\partial^{i+j+k+\ldots} \mathcal{P}(T, \boldsymbol{\mu})}{\partial\mu_u^i \partial\mu_d^j \partial\mu_s^k} \right|_{\boldsymbol{\mu}=0} .$$

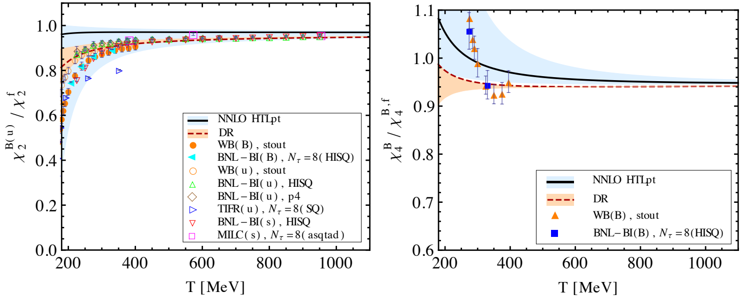
$$\chi_n^B = \left. \frac{\partial^n \mathcal{P}}{\partial\mu_B^n} \right|_{\mu_B=0} .$$

$$\chi_2^B = \frac{1}{9}\left[\chi_2^{uu} + \chi_2^{dd} + \chi_2^{ss} + 2\chi_2^{ud} + 2\chi_2^{us} + 2\chi_2^{ds}\right] ,$$

$$\chi_2^{uu} = \chi_{200} , \qquad \text{etc.}$$

(Talk by Borsanyi)

Results - susceptibilities



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5 Borsanyi et al '10 and '12, Bazavov et al '13.

Summary and Outlook

- Hard-thermal-loop perturbation theory represents a gauge-invariant reorganization of the perturbative series.
- Analytic result for the three-loop QCD thermodynamic potential at finite T and μ.
- Agreement with lattice data for a number of variables is good, in particular considering that there are no fit parameters.
- HTLpt is formulated in Minkowski space and can therefore be applied to real-time quantities as well.
- Resummed DR also in good agreement with lattice for fourth-order