Relating Classical Strings and Gravitons in AdS/CFT Jet Quenching

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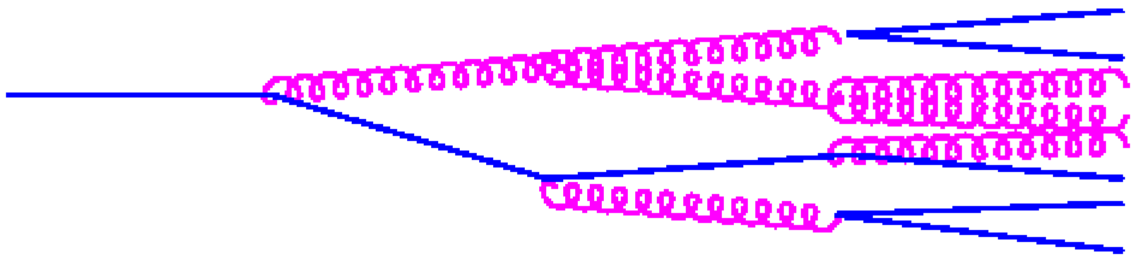
Stopping Distance of High Momentum Excitations

strongly-coupled QCD-like plasmas: $\ell_{\mathrm{stop}}^{\mathrm{max}} \propto E^{1/3}$

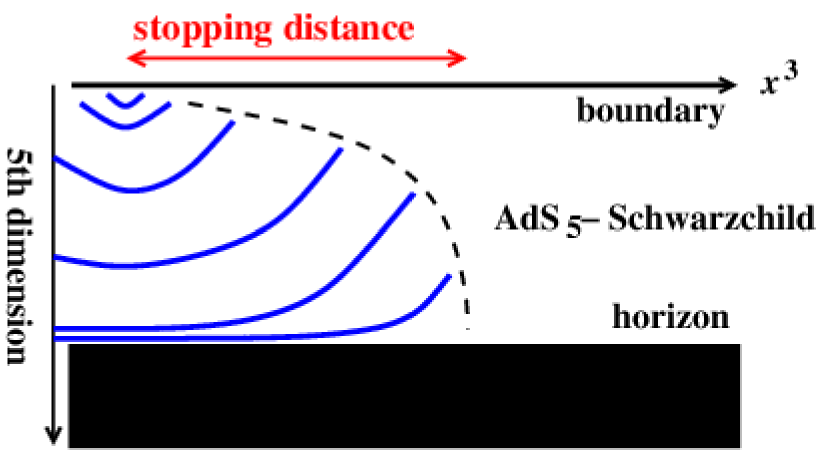
weakly-coupled QCD and QCD-like plasmas: $\ell_{\mathrm{stop}}^{\mathrm{max}} \propto E^{1/2}$



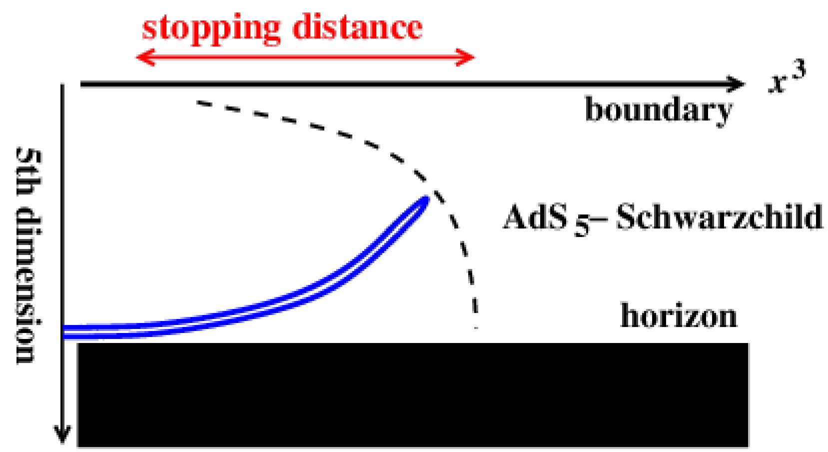
The difference inspires people to experiment with different models for energy loss in phenomenology.

A Discrepancy in AdS/CFT treatments

Stopping of high-momentum states dual to classical strings:

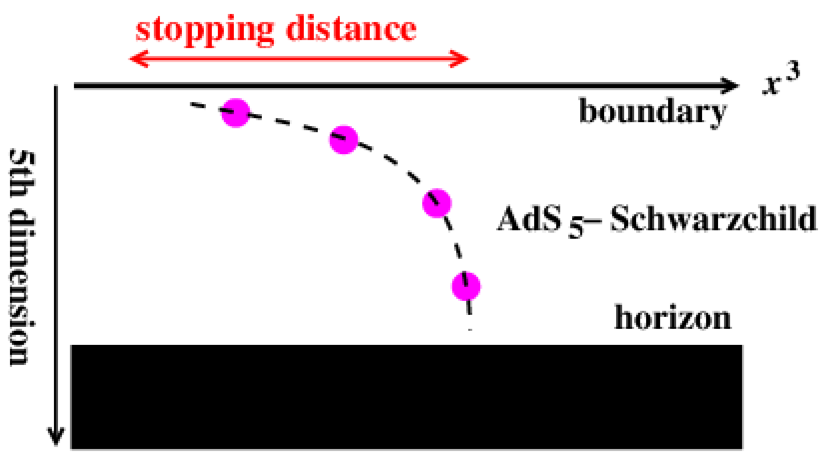


[Chesler, Jensen, Karch, Yaffe (2008)]



[Gubser, Gulotta, Pufu, Rocha (2008)]

and those that are not:



[Hatta, Iancu, Mueller (2008)]

A Discrepancy in AdS/CFT treatments

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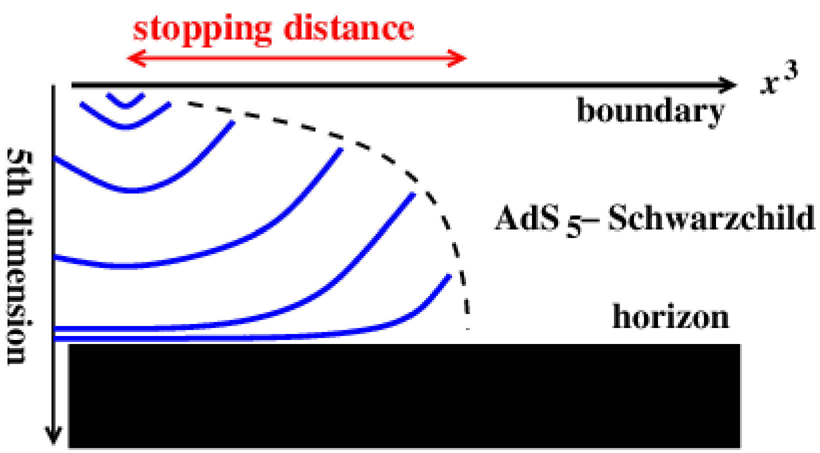
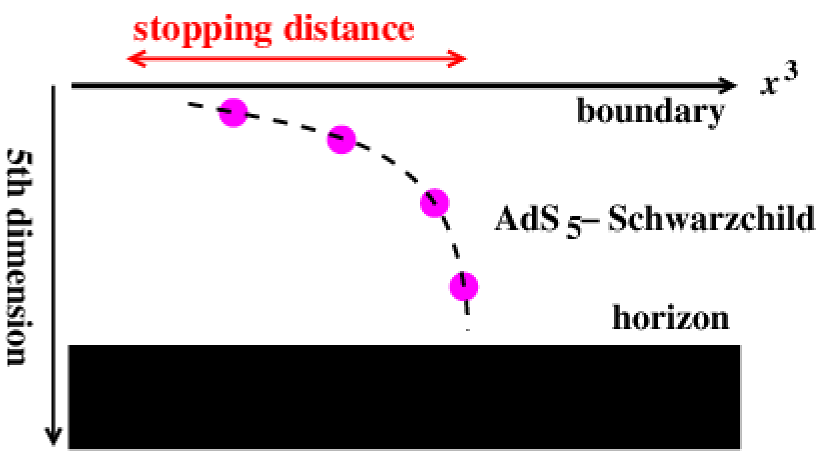


$$\ell_{\text{stop}} \lesssim \ell_{\text{stop}}^{\max} \sim T^{-4/3}\left(\frac{E}{\sqrt{\lambda}}\right)^{1/3}$$

and those that are not:



$$\ell_{\text{stop}} \lesssim \ell_{\text{stop}}^{\max} \sim T^{-4/3} E^{1/3}$$

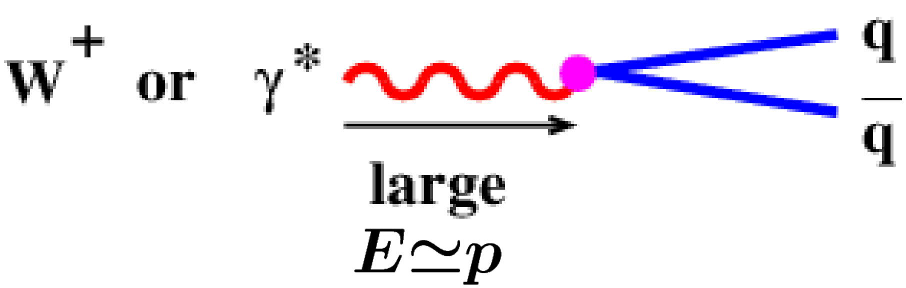
(in $\lambda \equiv N_c g^2 \to \infty$ limit)

Slightly dissatisfying:

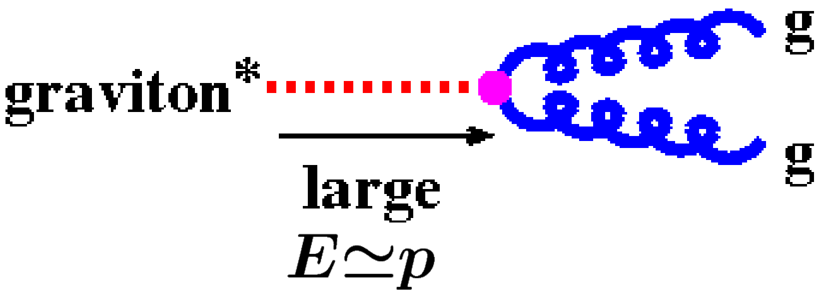
Original investigations set up initial state in gravity dual, not directly in the QFT.

Gedankin expts that specify a well-defined QFT problem:

External field method: Analogous to considering the hadronic decay of some very high-momentum, unstable particle in a QCD plasma. [Arnold & Vaman (2010)]



or



virtuality (or W mass) determines how far the "jet" goes up to the maximum.

Synchrotron method: Drag a heavy test quark around in a circle to make a beam of gluon synchrotron radiation. [used by Chesler, Ho, Rajagopal (2011)]

Stringy Corrections to Gravity

Usual Lore

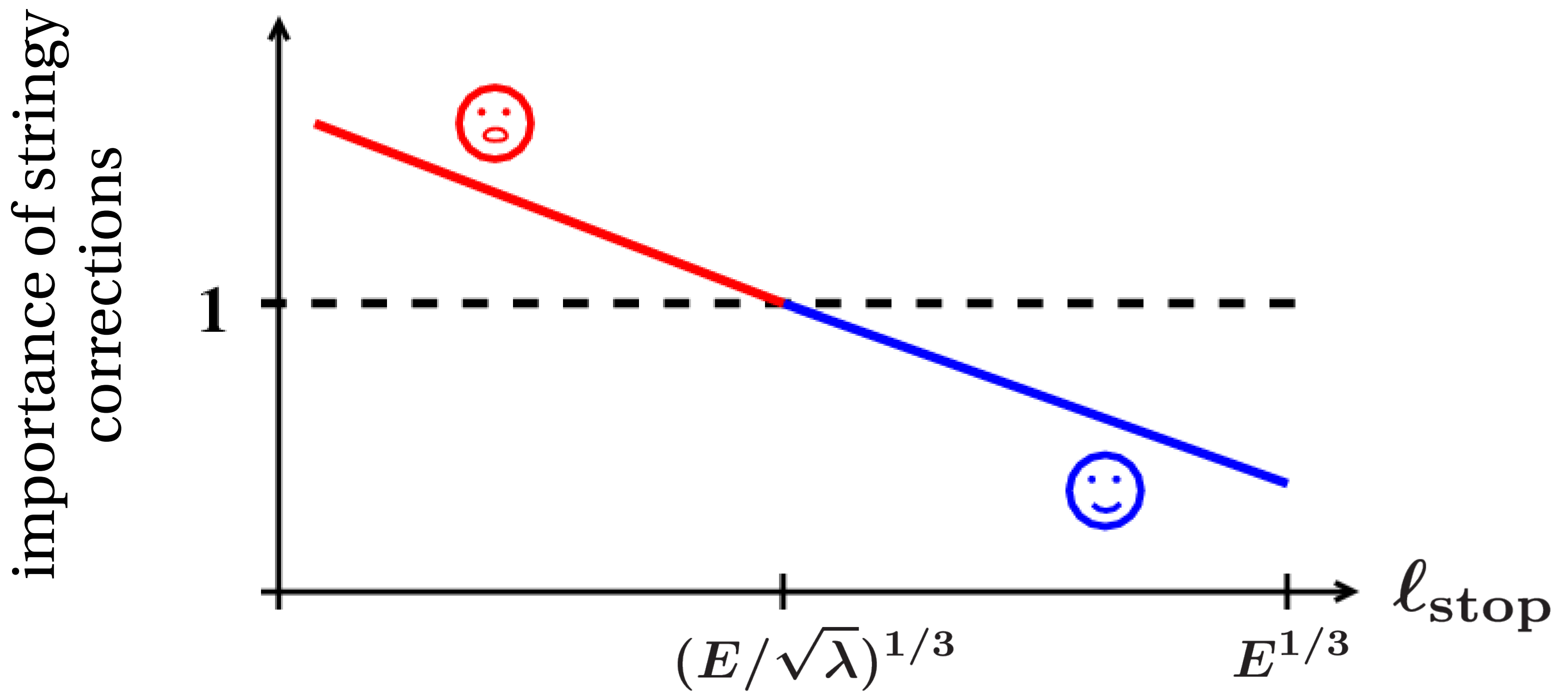
Stringy corrections to gravity theory suppressed by $\frac{1}{\sqrt{\lambda}}$

$\rightarrow$ small when λ is large!

Worry

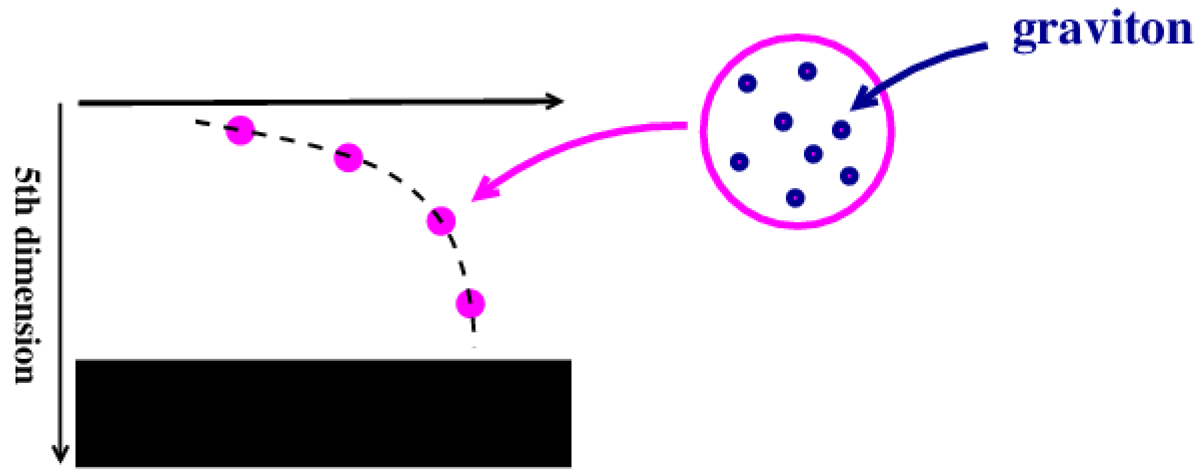
In this two-scale problem, stringy corrections might be, e.g., $\frac{1}{\sqrt{\lambda}}\frac{E}{T}$

Worry Realized



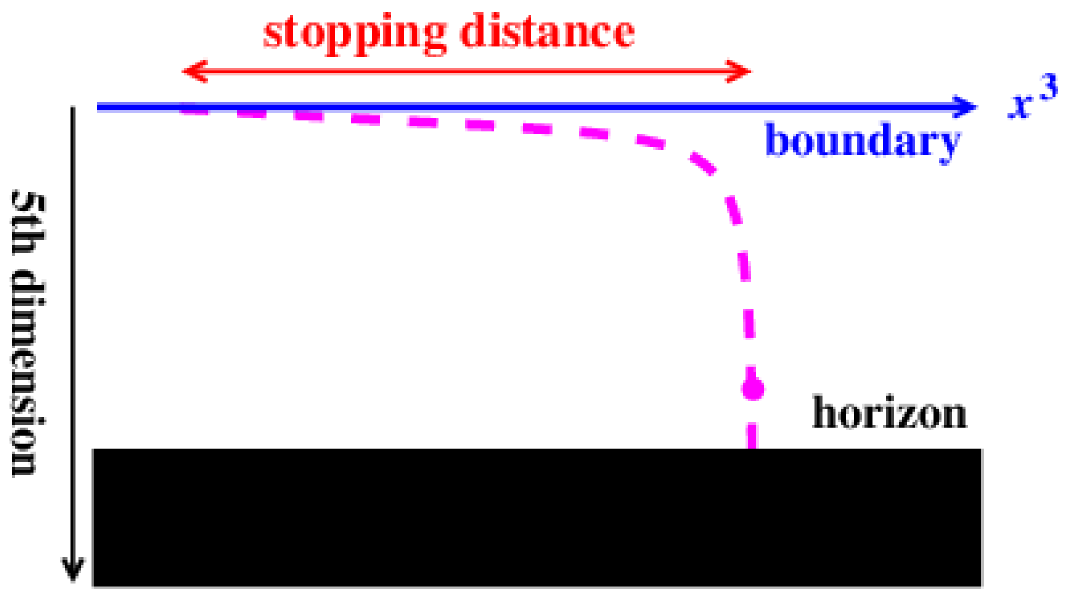
What now?

Think of excitation in gravity dual as made up of gravitons (or gauge bosons or whatever)

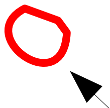


A fun problem in gravity

A high-momentum graviton is launched from the boundary of AdS_5-Schwarzschild. What happens to it?

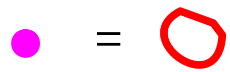


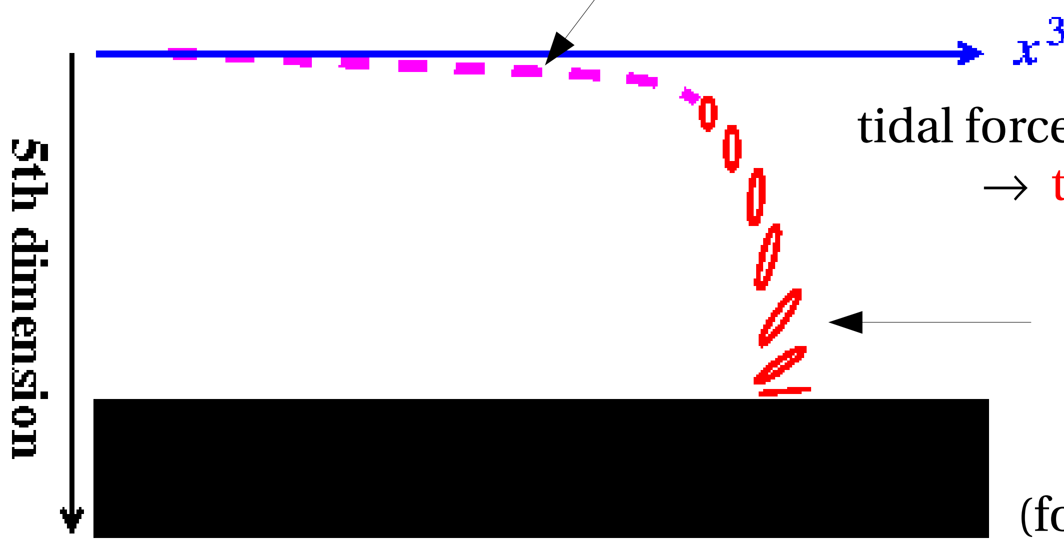
But really

 =  w/ internal degrees of freedom in ground state

proper size ~ (string tension)$^{-1/2}$ ~ $(\alpha')^{1/2}$

But really





Q: Is it possible to quantitatively calculate the late-time probability distribution of classical string configurations?

A: Yes, for a certain range of parameters.

Uses Penrose limit and quantization of strings in pp-wave backgrounds

AdS/CFT dictionary

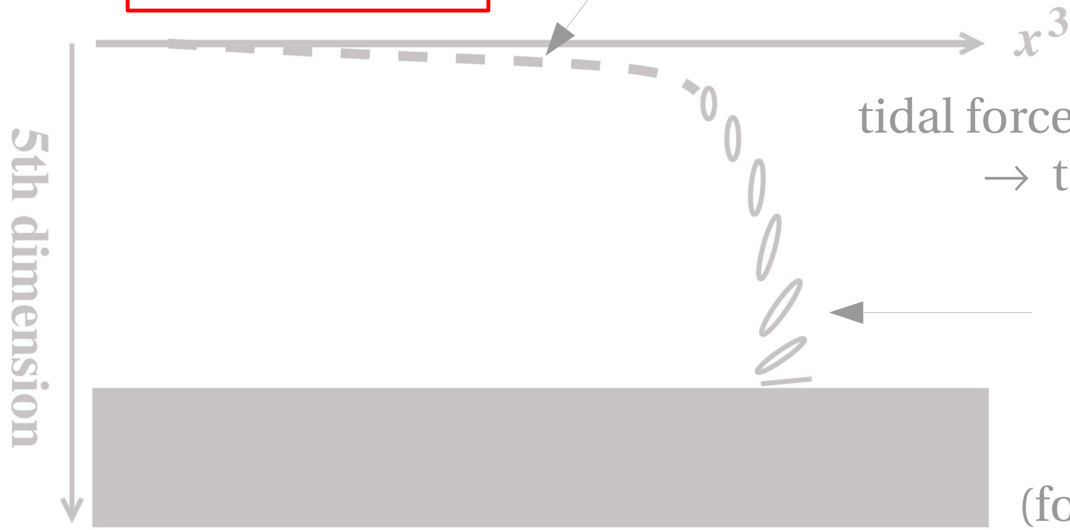
But really

● = ○ w/ internal degrees of freedom in ground state

proper size ~ (string tension)$^{-1/2}$ ~ $(\alpha')^{1/2}$

tension $\propto \sqrt{\lambda}$

graviton is a quantum closed string



tidal forces eventually exceed string tension
→ tidal stretching of string!

now a classical closed string

(for large enough momentum q_3)

(also for $g_{\text{string}} \to 0$, else string can break)

$N_c \to \infty$

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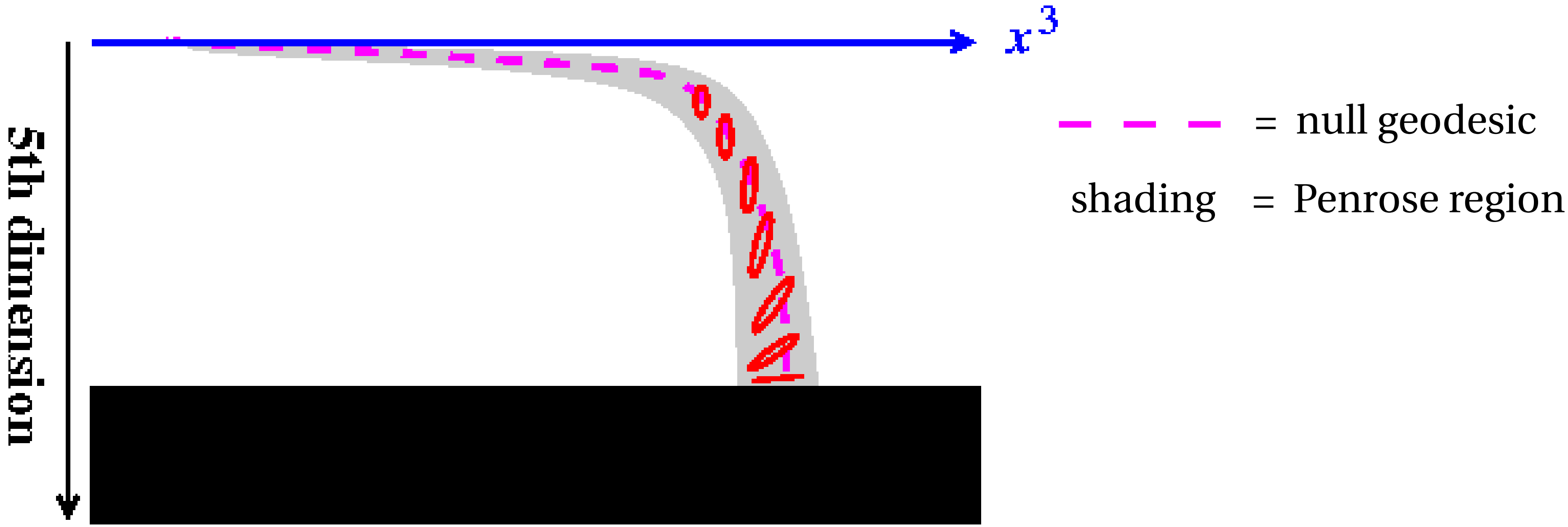
Uses Penrose limit and quantization of strings in pp-wave backgrounds

The Penrose Limit



In neighborhood of a null geodesic, can approximate the metric in a form (a pp-wave metric) for which practical calculations involving string quantization are possible!

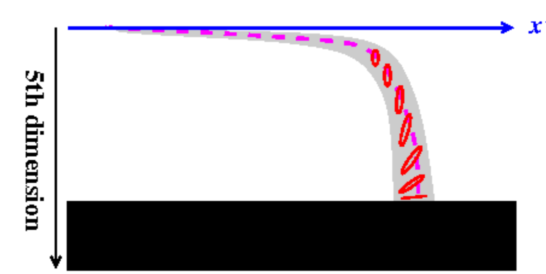
How do we know if string will stay close enough to reference geodesic?
Assume it does, calculate answer, and then check.

String Quantization in Flat Space

Work with light cone time (and in light cone gauge)
 → string decomposes into decoupled oscillators for each harmonic:

$$\omega_n^2 = \left(n\,\frac{2\pi\ \text{tension}}{p^+}\right)^2$$

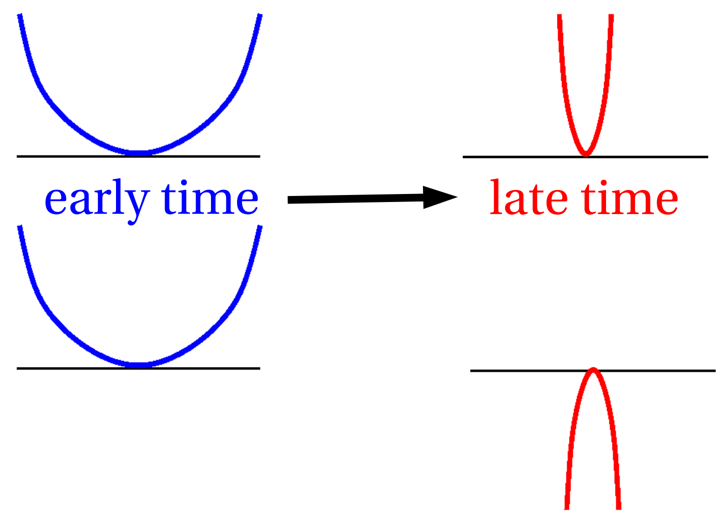
String Quantization in Penrose Limit



Work with time along a null geodesic
 → as before, but HO's pick up tidal force terms from curvature of space-time:

$$\omega_{1,n}^2 = \omega_{2,n}^2 = \left(n\,\frac{2\pi\ \text{tension}}{E}\right)^2 + \frac{1}{2}\,G(\tau)$$

$$\omega_{3,n}^2 = \left(n\,\frac{2\pi\ \text{tension}}{E}\right)^2 - G(\tau)$$



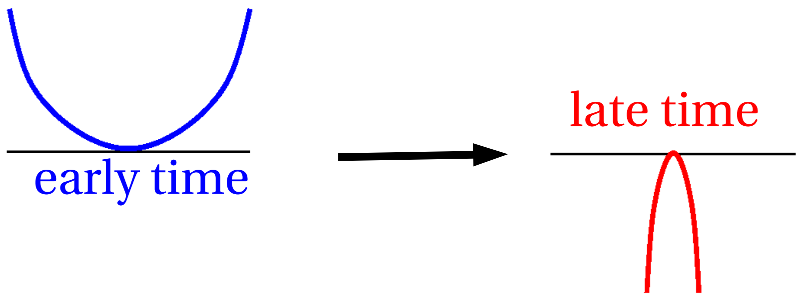
STRING TENSION EFFECTS

TIDAL FORCES

$G(\tau)$ grows as one moves away from the boundary.

Now just a QM problem:

Need QM solution to a time-dependent harmonic oscillator that starts in its ground state.

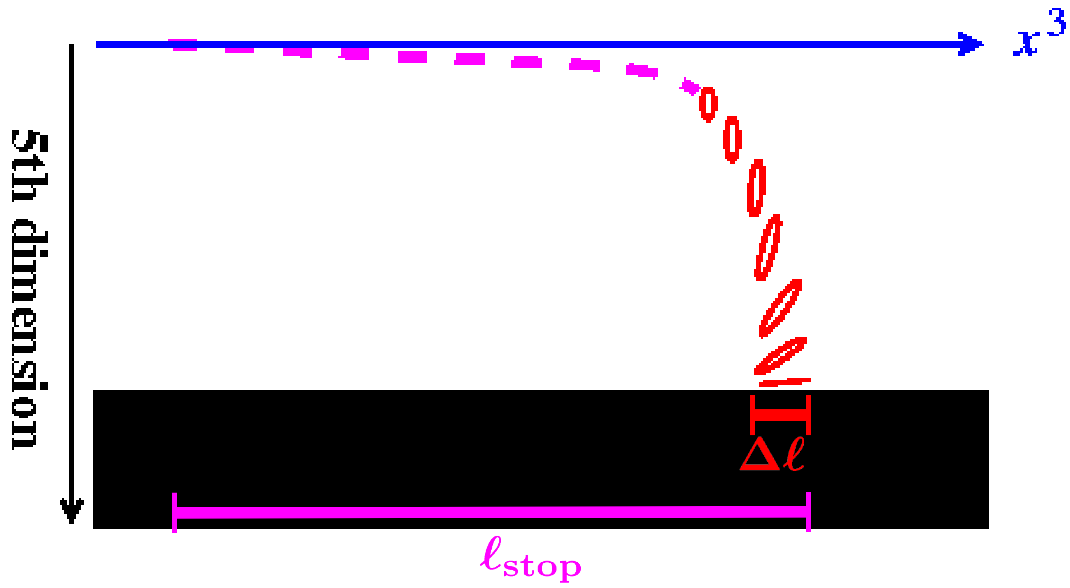


At late times, dynamics become classical.

Can calculate late-time probability distribution for each oscillator (i.e. for the amplitude of each string harmonic).

$\rightarrow$ Can calculate the late-time size of the classical string.

Results



For case $\ell_{\text{stop}} \lesssim \lambda^{-1/6}\ell_{\text{max}}$ where string excitation is important,

$$\boxed{(\Delta\ell)_{\text{rms}} \simeq 0.8660\,\lambda^{-1/4}\ell_{\text{stop}}\ln^{1/2} n_*}$$

where

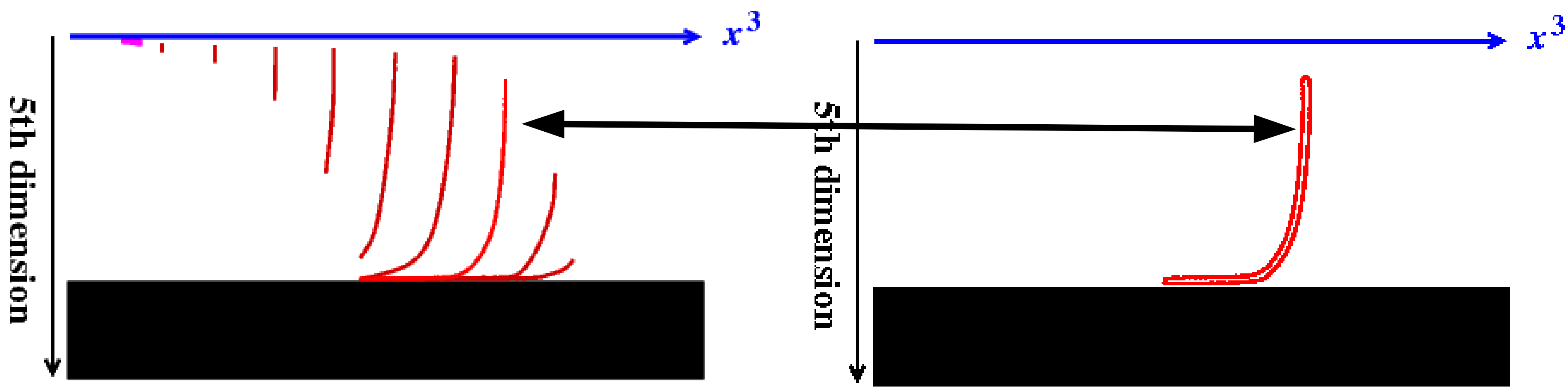
$$n_* \sim \#\text{ harmonics excited } \sim \frac{\lambda^{-1/6}\ell_{\text{max}}}{\ell_{\text{stop}}}$$

Moral: Stretching of string has negligible impact on jet stopping unless n_* is exponentially large !

And what if it is? ...

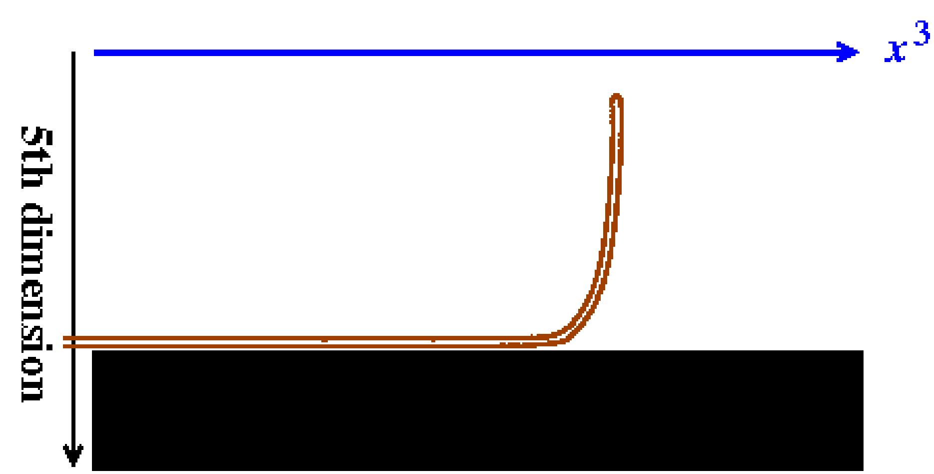
Large $\ln(n_*)$

$$\lambda^{-1/4}\ln^{1/2} n_* \sim 1$$



Note: Penrose limit breaks down

qualitatively similar to



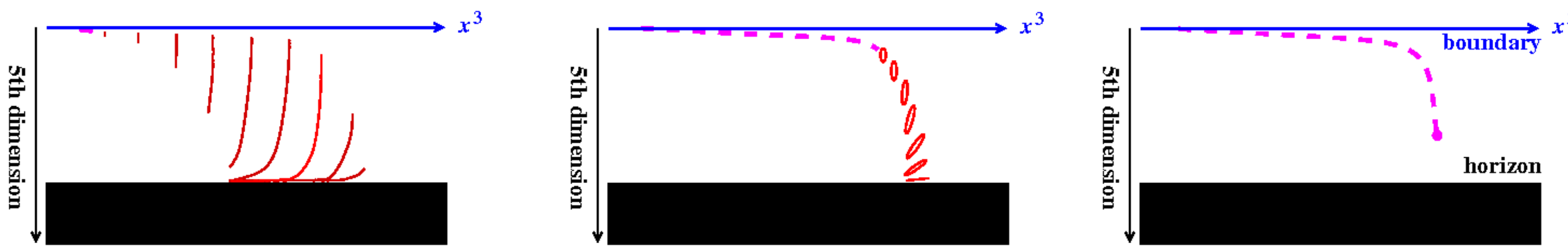
Gubser, Gulotta, Pufu, Rocha (2008)

Summary

At very high energy $E \gg \sqrt{\lambda}\, e^{\#\sqrt{\lambda}} T$

[units T=1] $\quad \left(\frac{E}{\sqrt{\lambda}}\right)^{1/3} e^{-\#\sqrt{\lambda}} \quad \left(\frac{E}{\sqrt{\lambda}}\right)^{1/3} \quad \ell_{\max} \sim E^{1/3}$

ℓ_{stop}



$\Delta\ell/\ell_{\mathrm{stop}} \sim 1 \qquad \Delta\ell/\ell_{\mathrm{stop}} \ll 1$

$-q^2$

$(\lambda E)^{2/3} e^{\#\sqrt{\lambda}} \quad (\lambda E)^{2/3} \quad E^{2/3}$

<u>What happens as λ decreases?</u>

All the above scales coalesce as λ→1.