# Exotic mesons in a holographic approach to QCD

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QCD describes strong interactions among quarks as processes with colored self-interacting gluons exchanged. This picture brings to the prediction of hybrid bound states with gluons appearing as constituents. Hybrid mesons are configurations composed by a quark, an antiquark and an excited gluon, which accounts for either ordinary or exotic $J^{PC}$ quantum numbers. Mesons with exotic quantum numbers cannot be described as simple quark-antiquark pairs, so their detection would demonstrate the existence of non-standard structures comprising gluons as constituents. Several QCD models indicate the hybrid meson with quantum numbers $J^{PC}=1^{-+}$ as the lowest-lying exotic state. In the light quark sector there are at least three quite well established hybrid candidates: the $\pi_1(1400)$ and the $\pi_1(1600)$, observed in diffractive $\pi^- N$ reactions and $\bar{p}N$ annihilation, and the $\pi_1(2015)$, seen only in diffraction.

## Soft-Wall AdS/QCD

**SUPER-YANG-MILLS (SYM) theory on Minkowski space $\mathcal{M}_4$**

- Coupling constant $g_{YM}$
- N =4 SUSY generators
- Gauge group $SU(N)_{\text{color}}$

Strong-coupling limit

$N\to\infty,\ \lambda=g^2_{YM}N\to\infty,\ g^2_{YM}\to 0$

**Maldacena**

**TYPE IIB STRING theory on $AdS_5(R)\times\mathcal{S}^5(R)$ space**

- Coupling constant $g_s$
- $R$ curvature radius
- $\sqrt{\alpha'}$ length of the string

Supergravity limit

$g_s\to 0$ e $R^2/\alpha'\to\infty$

**SYM/SUGRA DICTIONARY** *(Gubser, Klebanov, Polyakov, Witten)*

*Gauge-invariant p-form with conformal dimension* $\Delta$ ⟷ *Bulk* field with mass $m_5$ given by $m_5^2R^2=(\Delta-p)(\Delta+p-4)$

To describe hybrid $J^{PC}=1^{-+}$ mesons,we use the QCD local operator

$$J^a_\mu=\bar{q}T^aG_{\mu\nu}\gamma^\nu q \qquad \mu,\nu=0,1,2,3$$

with $G_{\mu\nu}$ the gluon field strength and $T^a$ flavour matrices normalised to $Tr[T^aT^b]=\delta^{ab}/2$. The dual field is a *1*-form

$$H_M=H^a_MT^a \qquad M=0,1,2,3,4$$

whose dynamics is described by the action

$$S_{SUGRA}=\frac{1}{k}\int d^4x\int_0^\infty dz\,\sqrt{|g|}e^{-c^2z^2}Tr\left[-\frac{1}{4}F^{MN}F_{MN}+\frac{1}{2}m_5^2H_MH^M\right]$$

with $F_{MN}=\partial_MH_N-\partial_NH_M$ and $g$ determinant of the Poincarè metric

$$ds^2\big|_{AdS_5}=\frac{R^2}{z^2}\left(dt^2-d\vec{x}^2-dz^2\right), \qquad z>0$$

The Soft-Wall factor $e^{-c^2z^2}$ gives the infrared conformal symmetry breaking through the mass scale *c*.

$$Z_{SYM}[H_0] = Z_{SUGRA}[H_0]$$

$H_0^{a\mu}(x)$ is interpreted as the source of the operator $J^a_\mu(x)$

$Z_{SUGRA}\simeq\exp\left[iS_{SUGRA}[H]\right],\quad H^{a\mu}(x,0)=H_0^{a\mu}(x)$ with $H^{a\mu}(x,z)$ solution of the Euler-Lagrange equations on $AdS_5(R)$

**Two-point correlation function** $\Pi^{ab}_{\mu\nu}(x-y)=\langle J^a_\mu(x)J^b_\nu(y)\rangle=$

$$=\frac{1}{Z_{SYM}[0]}\left(-i\frac{\delta}{\delta H_0^{a\mu}(x)}\right)\left(-i\frac{\delta}{\delta H_0^{b\nu}(y)}\right)Z_{SYM}[H_0]\Bigg|_{H_0=0} = \frac{\delta^2(S_{SUGRA})_{\text{on shell}}}{\delta H_0^{a\mu}(x)\,\delta H_0^{b\nu}(y)}\Bigg|_{H_0=0}$$

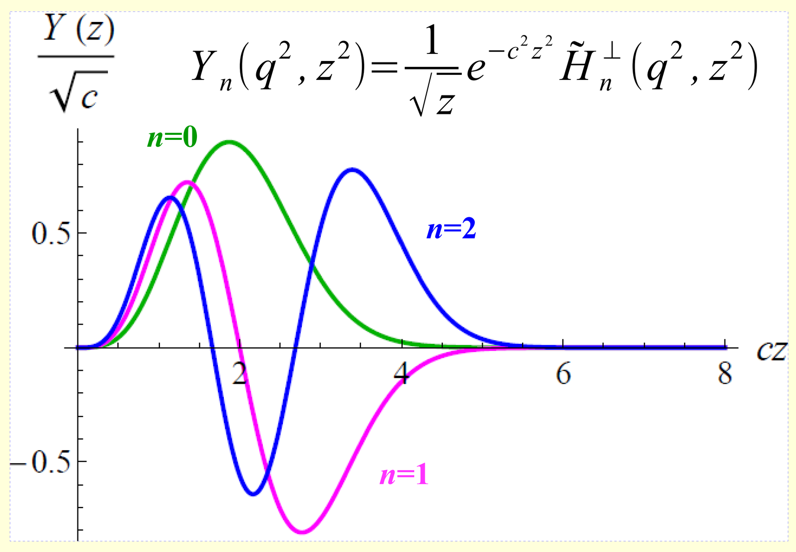
## Mass Spectrum and Decay Constants

In the Fourier space the $\tilde{H}_\mu$ field can be decomposed in a longitudinal $\tilde{H}^{||}_\mu$, and in a transverse $\tilde{H}^\perp_\mu$ component; the latter satisfies the condition $q^\mu\tilde{H}^\perp_\mu=0$ and can be used to describe the $1^{-+}$ mesons.

The equation of motion for $\tilde{H}^\perp_\mu$ is

$$\partial_z\left[\frac{1}{z}e^{-c^2z^2}\partial_z\tilde{H}^\perp_{i\mu}\right]+\frac{1}{z}e^{-c^2z^2}q^2\tilde{H}^\perp_\mu-\frac{8}{z^3}e^{-c^2z^2}\tilde{H}^\perp_\mu=0$$

EIGENFUNCTIONS



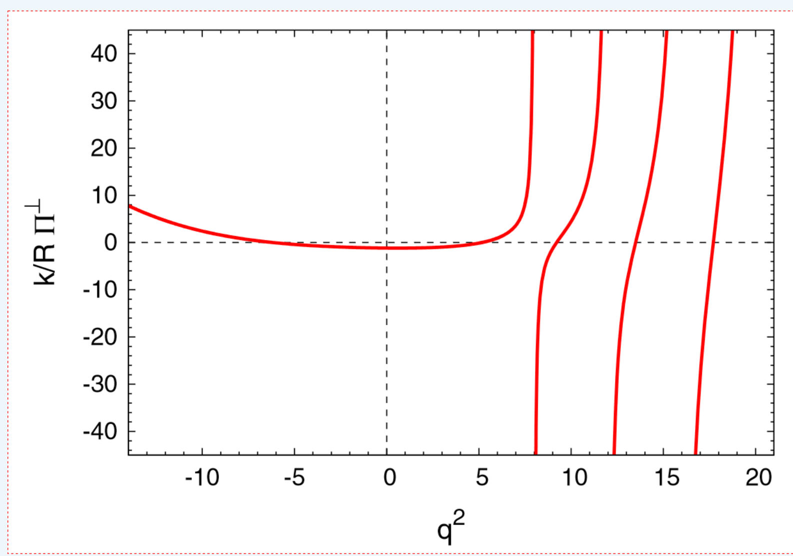

The Soft Wall model produces a $1^{-+}$ **spectrum** with a linear Regge trajectory $M_n^2\approx n$ having the same slope as for other bound states with different quantum numbers and quark content.

| $J^{PC}$ | $M_n^2$ |
|---|---|
| $1^{--}(\bar{q}q)$ | $c^2(4n+4)$ |
| $0^{++}(\bar{q}q)$ | $c^2(4n+6)$ |
| $0^{++}(glueball)$ | $c^2(4n+8)$ |
| $1^{-+}(\bar{q}Gq)$ | $c^2(4n+8)$ |

For the lowest-lying state we find $M_0\approx 1.1-1.3$ GeV, depending on the criterion chosen for fixing the mass scale *c*. The mass of the radial excitations grows more slowly than in the Hard Wall model, where $M_n^2\approx n^2$.

The two-point correlation function in the Fourier space can be expressed as the sum of a transverse and a longitudinal contribution:

$$\Pi^{ab}_{\mu\nu}(q)=-\left(\eta_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right)\frac{\delta^{ab}}{2}\Pi^\perp(q^2)+\frac{q_\mu q_\nu}{q^2}\frac{\delta^{ab}}{2}\Pi^{||}(q^2)$$




The poles of $\Pi^\perp(q^2)$ reproduce the mass spectrum $M_n^2=c^2(4n+8)$ with residues

$$F_n^2=\frac{2Rc^8}{3k}(n+3)(n+2)(n+1)$$

The ratio $R/k=2/(5\pi^4)$ is fixed by matching the leading-order perturbative QCD expression for $\Pi^\perp(q^2)$ at $q^2\to\infty$.

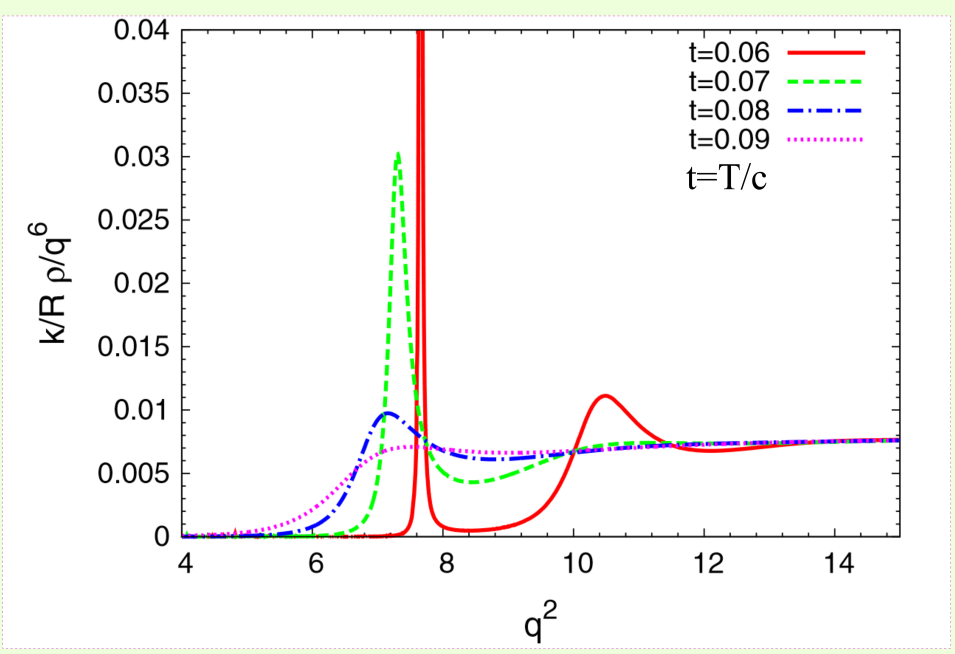
## Stability against thermal effects

Temperature can be incorporated in the holographic models introducing a black hole in the $AdS_5(R)$ space

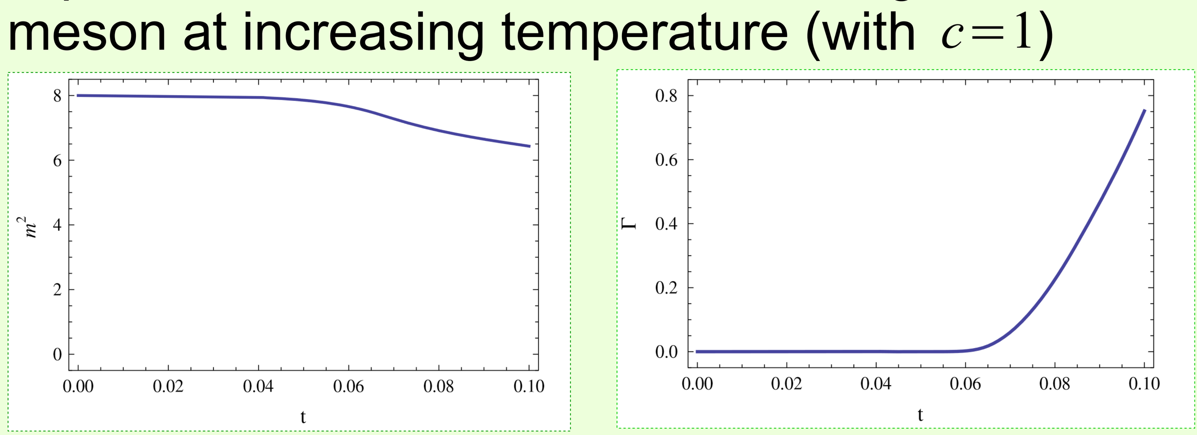
$$ds^2=\frac{R^2}{z^2}\left(f(z)dt^2-d\vec{x}^2-\frac{dz^2}{f(z)}\right), \qquad f(z)=1-\frac{z^4}{z_h^4}$$

with the horizon position related to the inverse temperature by $z_h=1/(\pi T)$.

**Spectral function** $\rho(q^2)=\text{Im}\,\Pi^\perp(q^2)$



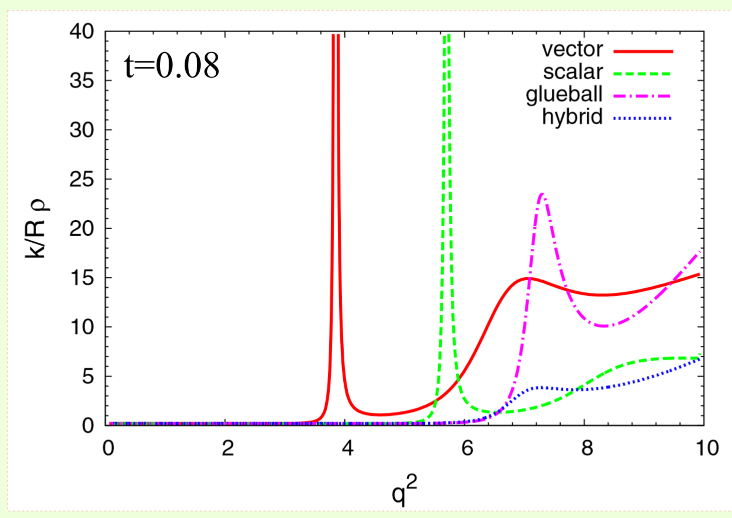

Squared mass $m^2$ and width $\Gamma$ of the lightest $1^{-+}$ meson at increasing temperature (with $c=1$)



At the ***melting temperature*** the peak in the spectral function is reduced by a factor $\simeq 20$ with respect to the point where $\Gamma$ starts to broaden.

**RESPONSE TO TEMPERATURE OF BOUND STATES**

| $J^{PC}$ | $t_{melting}$ |
|---|---|
| $1^{--}(\bar{q}q)$ | 0.23 |
| $0^{++}(\bar{q}q)$ | 0.18 |
| $0^{++}(glueball)$ | 0.12 |
| $1^{-+}(\bar{q}Gq)$ | 0.10 |




**HYBRID MESONS MELT AT A LOWER TEMPERATURE!**

The ***melting temperature*** of the hybrid mesons can also be determined computing the ***binding potential*** in the Schrödinger-like equation for the transverse field $\tilde{H}^\perp_i(z,q)$

$$V(z)=\frac{f(z)}{z^2}\left(c^4z^4f(z)+\frac{4c^2z^6}{z_h^4}+\frac{35}{4}+\frac{5}{4}\frac{z^4}{z_h^4}\right), \qquad \partial_r=-f(z)\partial_z$$

depending on the temperature through the horizon position. Below the melting temperature, the *r*-dependence of the potential becomes monotonous, such that no quasi-bound states can be formed.