Jets in medium

Strong and Electroweak Matter
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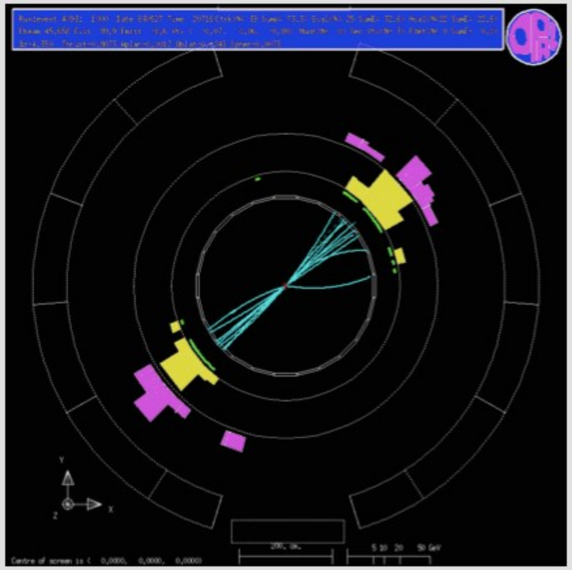
Outline

- Jets in vacuum
- Jets as 'colored' hard probes. What can we learn ? For QCD, for the physics of the QGP ?
- Jets in medium: loss of coherence, medium-induced gluon branching
- The jet quenching parameter
- The in-medium QCD cascade, and its characteristic features (energy flow, angular structure)
- Summary

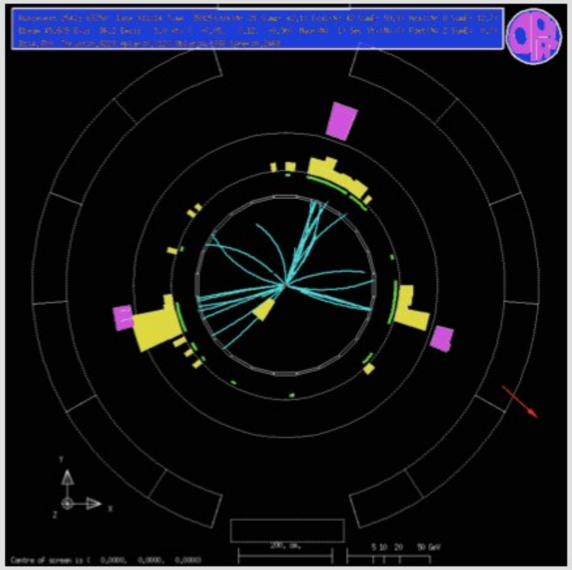
Thanks to F. Dominguez, E. Iancu and Y. Mehtar-Tani

Jets in 'vacuum'

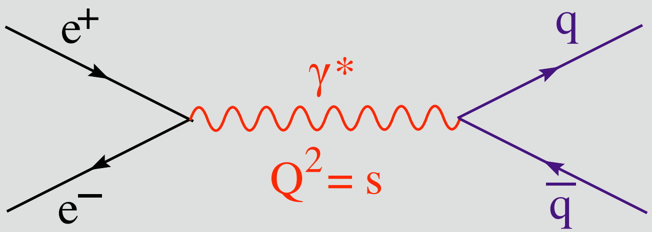
Examples of 2 and 3 jet events at LEP

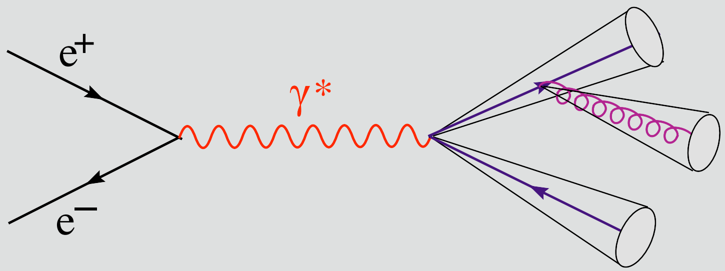


2 jets



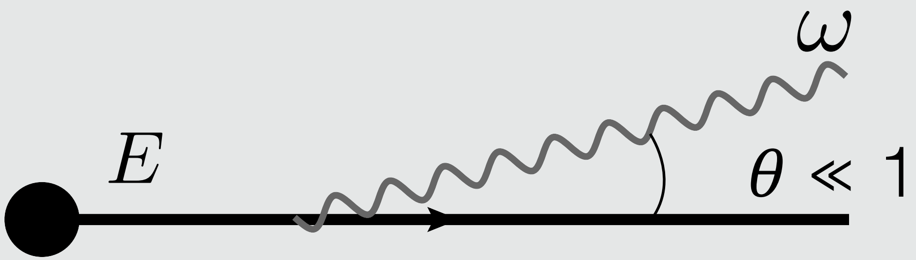
3 jets





Jets in vacuum

- Jets originate from energetic partons that successively branch into additional partons
- Elementary branching process is enhanced in the collinear region $\Longrightarrow$ Collimated jets

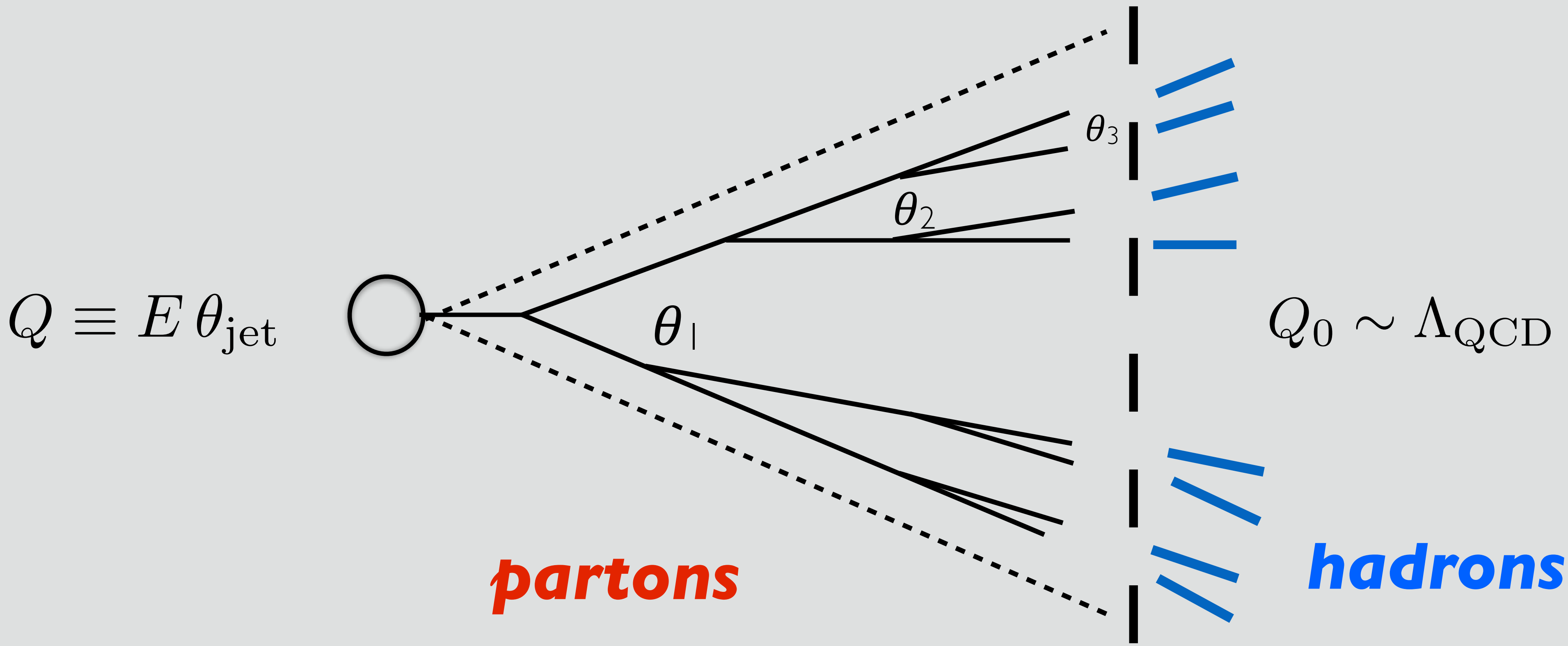


Branching probability

$$dP \sim \alpha_s \, C_R \, \frac{d\theta}{\theta} \frac{d\omega}{\omega}$$

Jets in vacuum

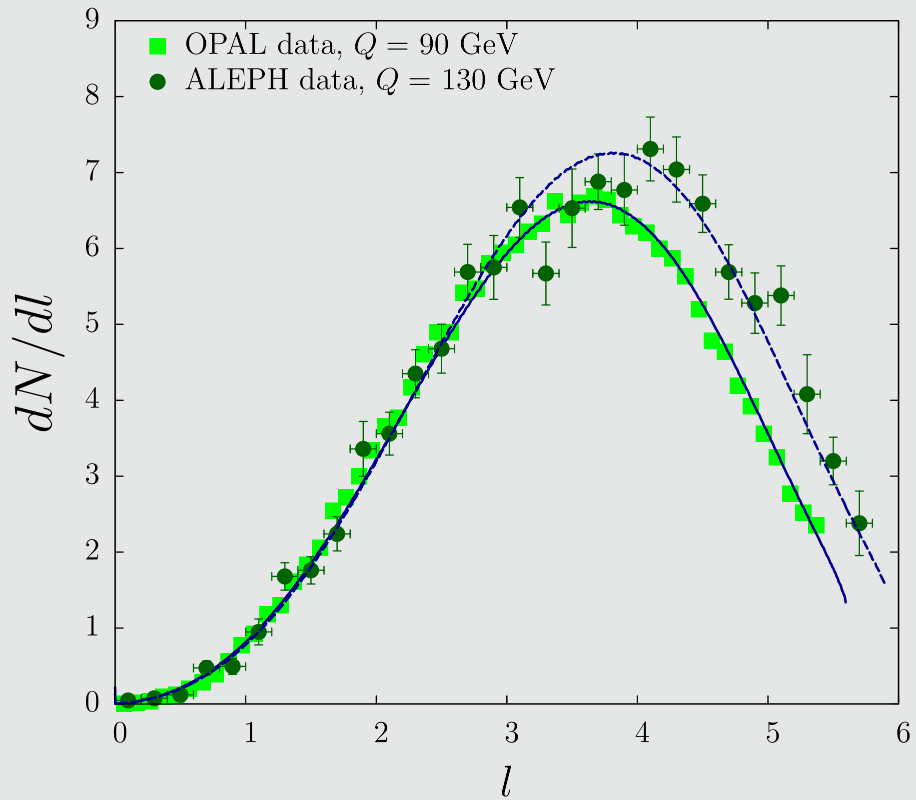
- The jet is a coherent object, successive branchings are ordered from larger to smaller angles $\theta_1 > \theta_2 > \ldots > \theta_n$



- large separation of scales (QCD-factorization) $Q \gg \Lambda_{QCD}$

Jets in vacuum

Fragmentation Function

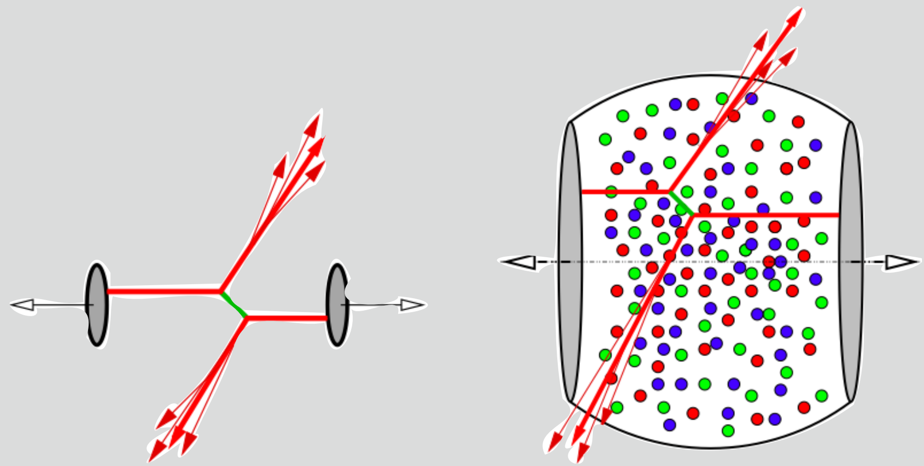


$$l = \ln(E_{\rm jet}/E_{\rm h})$$

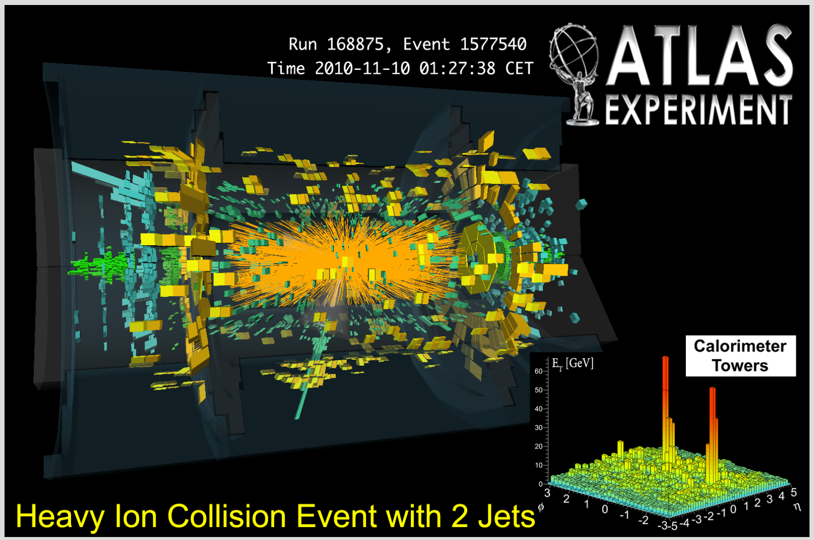
- **Perturbative QCD** prediction for the distribution of hadrons in a jet
- **2 scales:** $Q_0 = \Lambda_{QCD}$ $Q = E\,\theta_{jet}$
- **Angular Ordering:** soft gluon emissions (large l) are suppressed

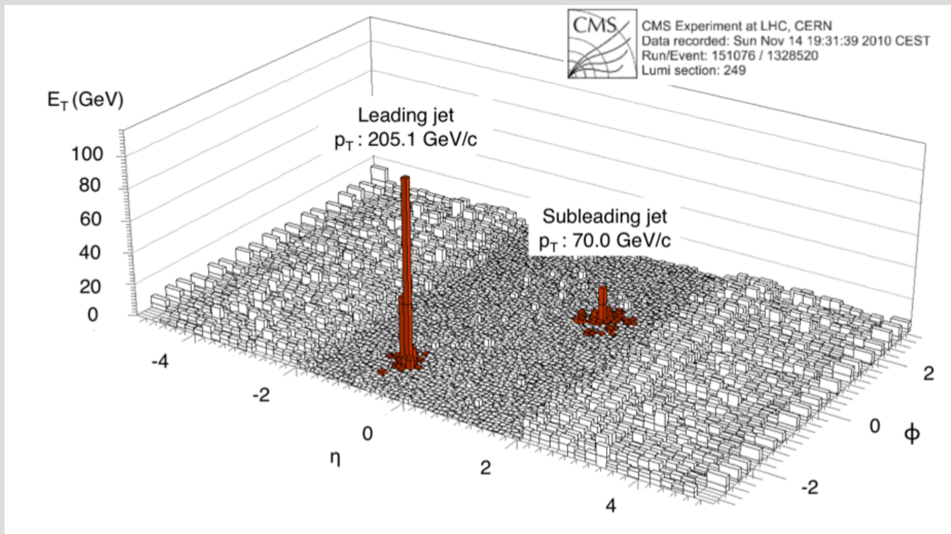
[Dokshitzer, Khoze, Mueller, Troyan, Kuraev, Fong, Webber…80']

Jets in a quark-gluon plasma



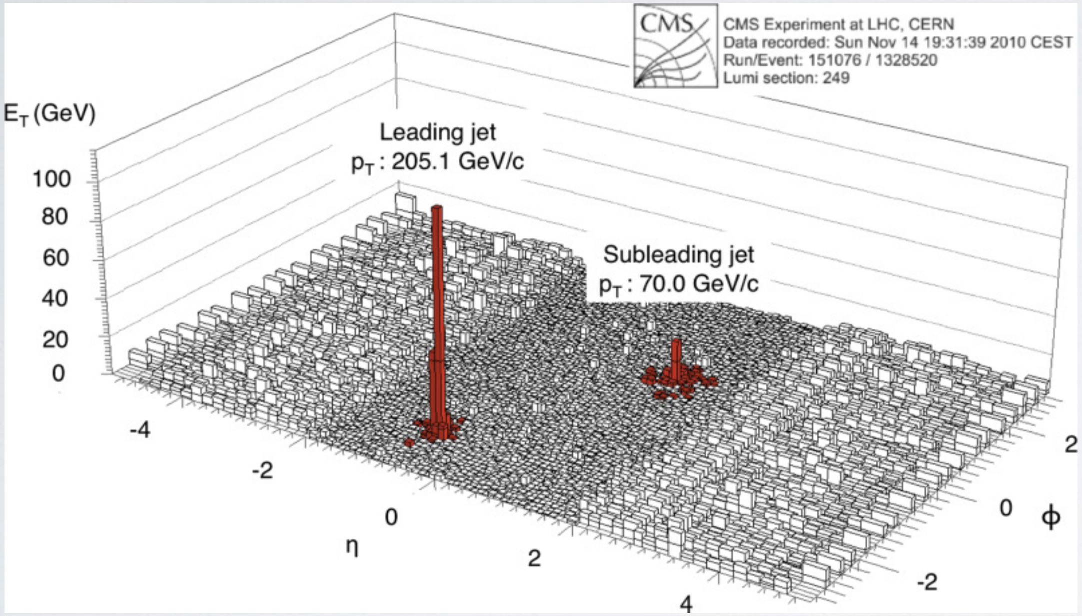
Jets are quenched due to interactions with the QGP

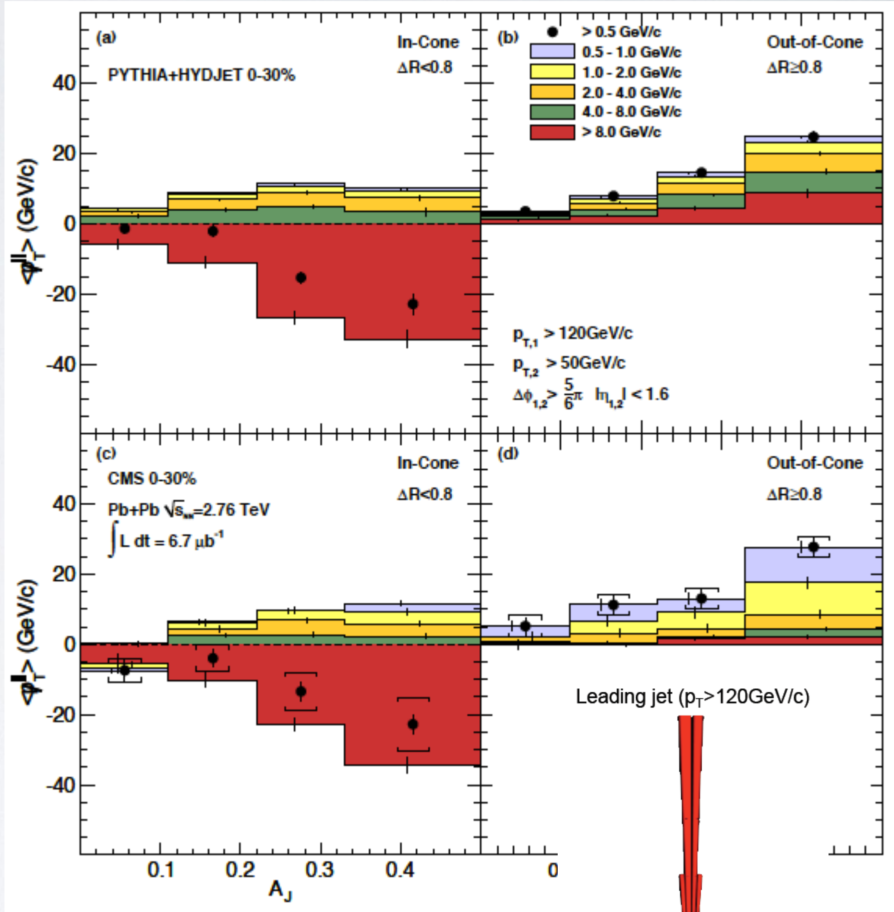


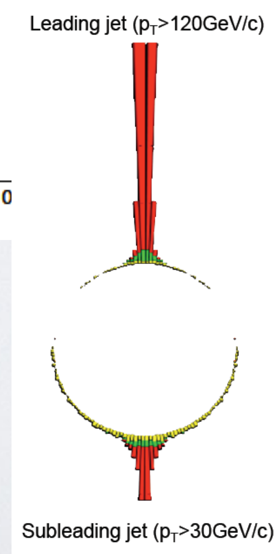


Di-jet asymmetry

there is more to it than just ‘quenching’...







Missing energy is associated with additional **radiation of many soft quanta at large angles**

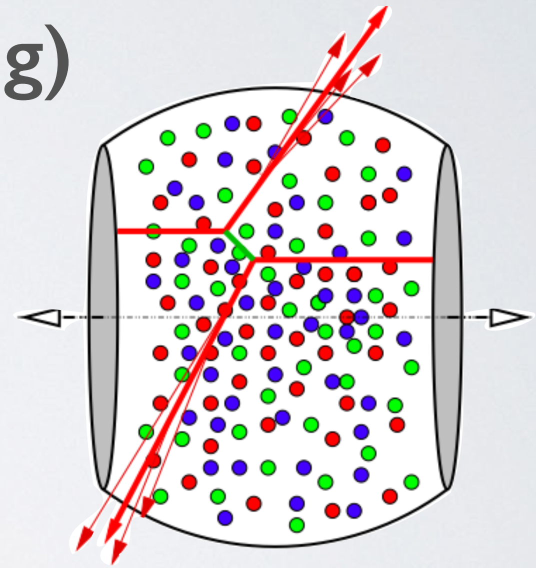
$$A_J = \frac{p_{T,1} - p_{T,2}}{p_{T,1} + p_{T,2}}$$

This reflects a **genuine feature of the in-medium QCD cascade.**

Jets are complicated objects

Extra complications when 'in medium'

- the initial parton 'evolves' (branching)
- color coherence
- hadronization
- geometry, expansion, triggger
- jet definition
- need space-time picture
- etc



Some idealization is needed

Theorist's idealization

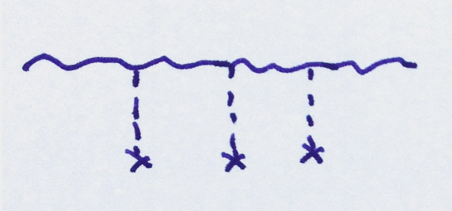
- Hard parton (gluon or quark) propagating in a QGP
- Ignore vacuum radiation and its interference with medium induced radiation

Questions:

- Can we build a probabilistic picture?
- If yes, what are its characteristic features?

Propagation of a hard parton in a medium

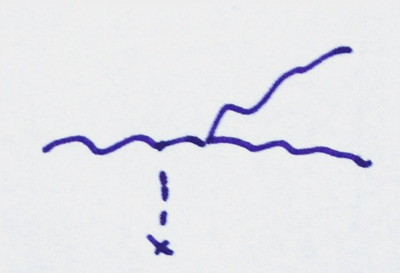
Momentum broadening

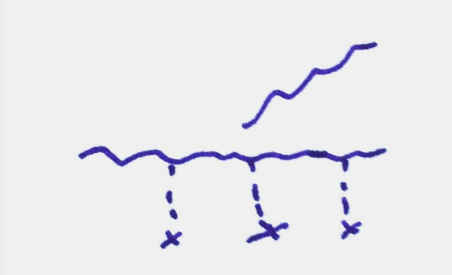


$$\Delta\, k_\perp^2 \;\simeq\; \hat{q}\, \Delta t$$

jet quenching parameter

Induced radiation, LPM effect, Energy loss





NB. Weak coupling, high energy (nearly eikonal)

Momentum broadening

Probabilistic equation

$$\begin{aligned}\mathcal{P}(\boldsymbol{p}_1, t_1|\boldsymbol{p}_0, t_0) &= (2\pi)^2\delta(\boldsymbol{p}_1 - \boldsymbol{p}_0) \\ &+ \int_{t_0}^{t_1} \mathrm{d}t \int_{\boldsymbol{q}} w(\boldsymbol{q})\mathcal{P}(\boldsymbol{p}_1 - \boldsymbol{q}, t|\boldsymbol{p}_0, t_0) - \int_{t_0}^{t_1} \mathrm{d}t\, \mathcal{P}(\boldsymbol{p}_1, t|\boldsymbol{p}_0, t_0) \int_{\boldsymbol{q}} w(\boldsymbol{q})\end{aligned}$$

$$w(q) = \frac{nN_c g^4}{q^4} \sim \frac{\mathrm{d}^2\sigma_{\mathrm{el}}}{\mathrm{d}^2 q}$$

Differential version - Diffusion approximation

$$\frac{\partial}{\partial t}\mathcal{P}(\boldsymbol{p}, t) = -\frac{N_c}{2} n \int_{\boldsymbol{q}} \sigma(\boldsymbol{q})\mathcal{P}(\boldsymbol{p} - \boldsymbol{q}, t)$$

$$\frac{\partial}{\partial t}\mathcal{P}(\boldsymbol{p}, t) = \frac{1}{4}\left(\frac{\partial}{\partial \boldsymbol{p}}\right)^2 \left[\hat{q}(\boldsymbol{p}^2)\, \mathcal{P}(\boldsymbol{p}, t)\right]$$

$$-\frac{N_c}{2} n\sigma(\boldsymbol{q}) \equiv w(\boldsymbol{q}) - (2\pi)^2\delta(\boldsymbol{q}) \int_{\boldsymbol{q}'} w(\boldsymbol{q}')$$

(« dipole cross-section »)

Jet quenching parameter

$$\hat{q}(\boldsymbol{p}^2) = \int_{\boldsymbol{q}} \boldsymbol{q}^2 w(\boldsymbol{q}) = -\frac{N_c}{2} n \int_{\boldsymbol{q}} \boldsymbol{q}^2 \sigma(\boldsymbol{q}) \simeq 4\pi\alpha_s^2 N_c\, n\, \ln\frac{\boldsymbol{p}^2}{m_D^2}$$

$$\frac{N_c}{2} n\sigma(\boldsymbol{v}) \simeq \frac{1}{4}\hat{q}(\boldsymbol{v}^{-2})\, \boldsymbol{v}^2$$

(harmonic approximation)

Momentum broadening

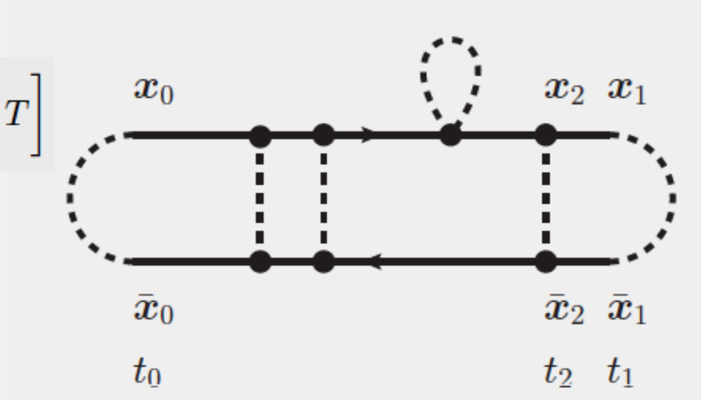
Momentum distribution and dipole forward amplitude

$$\frac{dN}{d^2p_\perp} = \int \frac{d^2x_\perp}{(2\pi)^2} e^{-ip_\perp x_\perp} S(x_\perp)$$

(For validity beyond leading order, see A.H. Mueller and S. Munier , 2012)

Dipoles, Wilson lines

- Almost eikonal propagation $\mathrm{T}\exp\left[ig\int_{t_0}^{t_1}\mathrm{d}t\, A^-(t,\boldsymbol{r}(t))\cdot T\right]$
- Interaction with medium (collisions) treated as interaction with a random color field



$$\langle A_a^-(\boldsymbol{x},t)A_b^-(\boldsymbol{y},t')\rangle = \delta_{ab}\, n\, \delta(t-t')\gamma(\boldsymbol{x}-\boldsymbol{y})$$

$$\gamma(\boldsymbol{x}) = g^2\int_q \frac{e^{i\boldsymbol{q}\cdot\boldsymbol{x}}}{\boldsymbol{q}^4}$$

Exponentiation (multiple scattering)

$$(X_1|S_{\rm eik}^{(2)}(t_1,t_0)|X_0) = \delta(X_1-X_0)\exp\left[-\frac{N_c}{2}n\,(t_1-t_0)\,\sigma(\boldsymbol{v})\right]$$

$$\sigma(\boldsymbol{x}) = 2g^2\left[\gamma(\boldsymbol{0})-\gamma(\boldsymbol{x})\right]$$

(dipole cross-section)

Corrections to the jet quenching parameter

- Perturbative corrections (Arnold, Xiao, 2008)

- Euclidean correlators near the light cone (Caron-Huot, 2009)

- Lattice calculations (Panero, Rummukainen, Schaëfer, 2013)

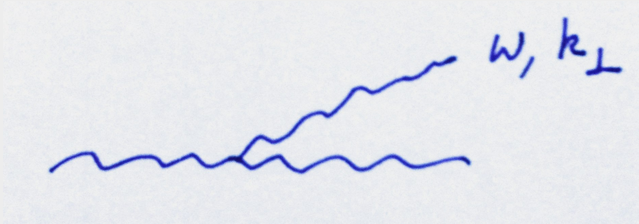
- Radiative corrections
  (Liou, Mueller, Wu, 2013
  Mehtar-Tani, 2013
  JPB, Mehtar-Tani, 2014
  Iancu, 2014)

The BDMPSZ mechanism

[Baier, Dokshitzer, Mueller, Peigné, Schiff (1995-2000) Zakharov (1996)]

Gluon emission is linked to momentum broadening

Matching of $\tau \sim \frac{2\omega}{k_\perp^2}$ **and** $k_\perp^2 \sim \hat{q}\tau$



defines time scale for the branching process

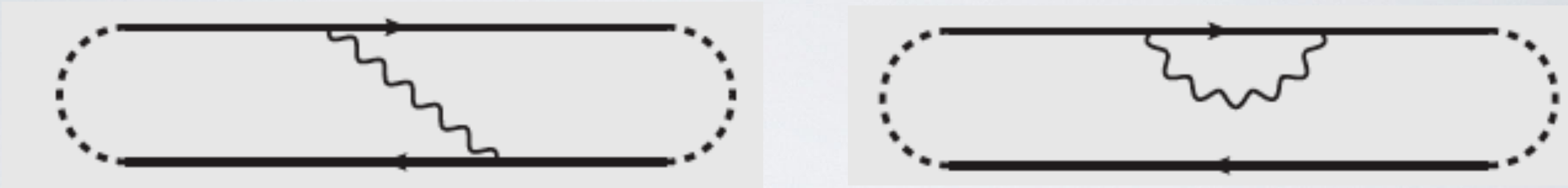
$$\tau_{\rm br}(\omega) \sim \sqrt{\frac{2\omega}{\hat{q}}}$$

Medium of finite extent $\tau_{\rm br} \lesssim L \Rightarrow \omega \lesssim \omega_c$ $\quad \omega_c \sim \hat{q}L^2$

Typical branching kT and angle

$$k_{\rm br}^2 = \hat{q}\tau_{\rm br} \qquad \theta_{\rm br} \sim k_{\rm br}/\omega \sim \left(\hat{q}/\omega^3\right)^{1/4}$$

Radiative corrections to momentum broadening



Double logarithmic correction

$$\langle k_\perp^2 \rangle_{\rm typ} \simeq \hat{q} L \left(1 + \frac{\bar{\alpha}}{2} \ln^2 \frac{L}{\tau_0} \right)$$

(Mueller, Liou, Wu, 2013)

Can be interpreted as a renormalization of $\hat{q}$

(Y. Mehtar-Tani)

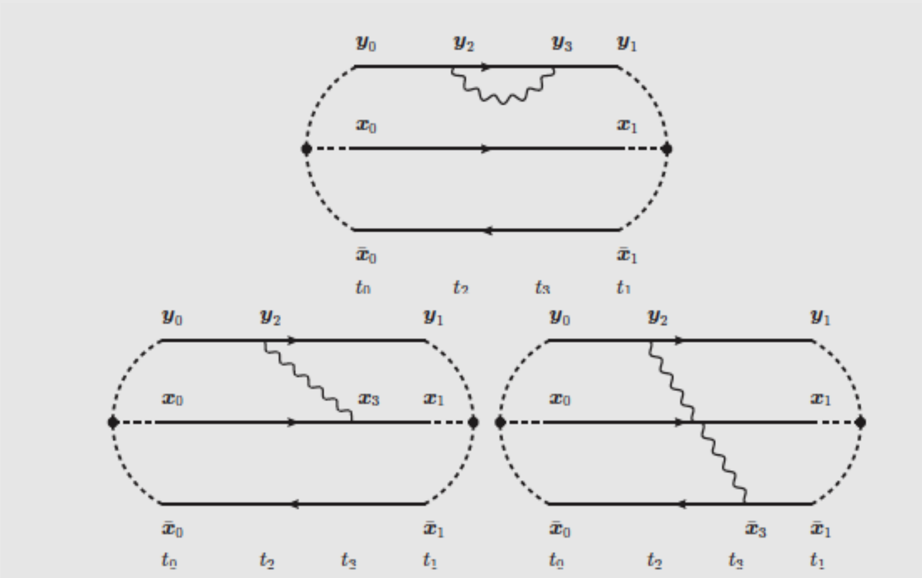
Universality of the dominant radiative correction

$$\Delta\hat{q}(\tau_{\max}, p^2) \equiv \frac{\alpha_s N_c}{\pi} \int_{\tau_0}^{\tau_{\max}} \frac{\mathrm{d}\tau}{\tau} \int_{\hat{q}\tau}^{p^2} \frac{\mathrm{d}q^2}{q^2}\, \hat{q}(q^2)$$

Same correction for momentum broadening

$$\langle k_\perp^2 \rangle_{\mathrm{typ}} \simeq \hat{q}L \left(1 + \frac{\bar{\alpha}}{2}\ln^2\frac{L}{\tau_0}\right)$$

and energy loss



$$\langle \omega \rangle \sim \hat{q}L^2 \left(1 + \frac{\bar{\alpha}}{2}\ln^2\frac{L}{\tau_0}\right)$$

From talk by Yacine Mehtar-Tani at the h3QCD workshop, ECT*, Trento, June 2013

Renormalization of the quenching parameter

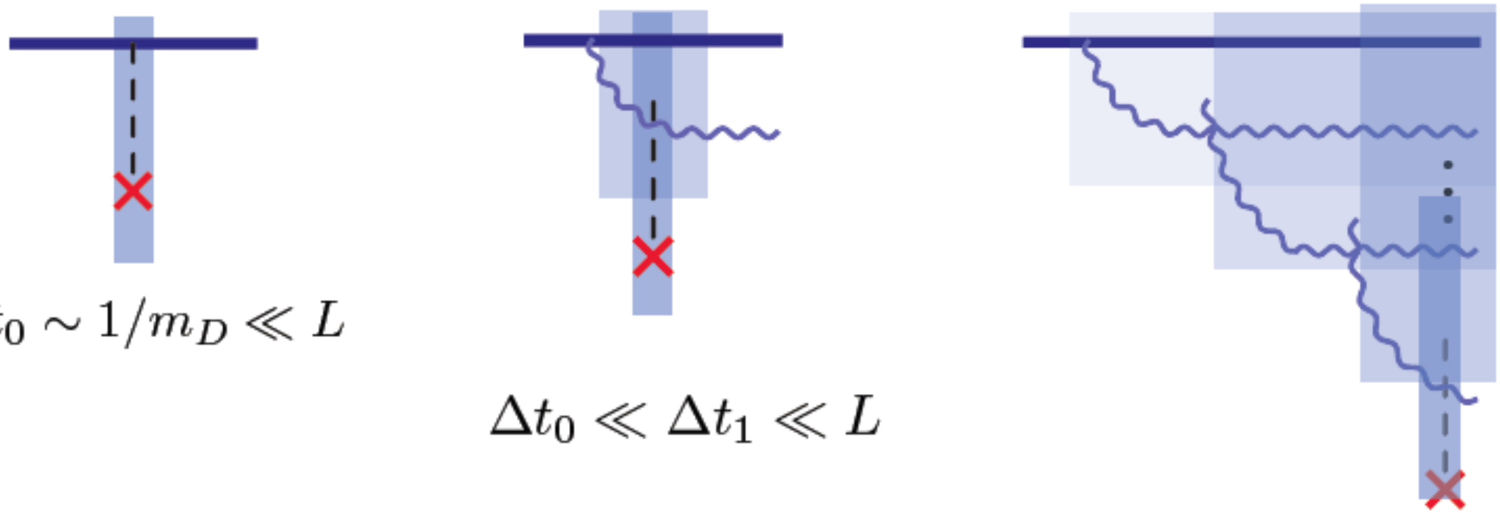
➡ The DL's are resummed assuming strong ordering in formation time (or energy) and transverse mom. of overlapping successive gluon emissions (coherent branchings!)

$$\frac{\partial\,\hat{q}(t,\boldsymbol{k}^2)}{\partial \log(t/t_0)} = \int_{\hat{q}t}^{\boldsymbol{k}^2} \bar{\alpha}_s(\boldsymbol{q}^2)\,\frac{d\boldsymbol{q}^2}{\boldsymbol{q}^2}\,\hat{q}(t,\boldsymbol{q}^2)$$

$\hat{q}t_0 \equiv Q_0^2$

$\hat{q}t \equiv \boldsymbol{k}^2$

$\hat{q}_0$ $\hat{q}_1$ $\hat{q}_n$



$\Delta t_0 \sim 1/m_D \ll L$

$\Delta t_0 \ll \Delta t_1 \ll L$

$\Delta t_0 \ll \Delta t_1 \ll \ldots \Delta t_n \ll L$

$$\hat{q}(\boldsymbol{k}) \sim \hat{q}_0 \left(\frac{\boldsymbol{k}^2}{m_D^2}\right)^{\sqrt{\frac{\alpha_s N c}{\pi}}}$$

with $\boldsymbol{k}^2 \sim \hat{q}_0 L$

Radiative Energy Loss

As a consequence, the DL's not only affects the pt-broadening but also the radiative energy loss expectation:

$$\Delta E \equiv \int d\omega\, \omega\, dN/d\omega$$

Typically the transport coefficient runs up to the scale $\boldsymbol{k}^2 \sim \hat{q}_0 L$

$$\Delta E \simeq \alpha_s \hat{q}_0 L^2 \;\rightarrow\; \Delta E \simeq \alpha_s \hat{q}_0 L^2 \left[1 + \frac{\alpha_s C_A}{2\pi} \log^2\left(\hat{q}_0 L/m_D^2\right)\right]$$

When the logs become large (asymptotic behavior)

$$\rightarrow\; \Delta E \simeq \frac{\alpha_s \hat{q}_0 L^2}{4\sqrt{\pi}\,\bar{\alpha}_s^{3/4} \log^{3/2}(\hat{q}_0 L/m_D^2)} \left(\frac{\hat{q}_0 L}{m_D^2}\right)^{\sqrt{\frac{4\alpha_s C_A}{\pi}}}$$

Path length dependence of mean energy loss

Weak coupling

$$\Delta E \sim L^2$$

Strong coupling

$$\Delta E \sim L^3$$

Weak coupling + Radiative corrections

$$\Delta E \sim L^{2+\gamma} \qquad 0 < \gamma \equiv \sqrt{4\alpha_s N_c/\pi} < 1$$

BDMPSZ spectrum

$$\omega \frac{\mathrm{d}N}{\mathrm{d}\omega} \simeq \frac{\alpha_s N_c}{\pi} \sqrt{\frac{\omega_c}{\omega}} \equiv \bar{\alpha} \sqrt{\frac{\omega_c}{\omega}} = \bar{\alpha} \frac{L}{\tau_{\mathrm{br}}(\omega)}$$

Hard emissions

- rare events, with probability $\sim \mathcal{O}(\alpha_s)$
- dominate energy loss: $E_{\mathrm{hard}} \sim \alpha_s \omega_c$
- small angle, not important for di-jet asymmetry

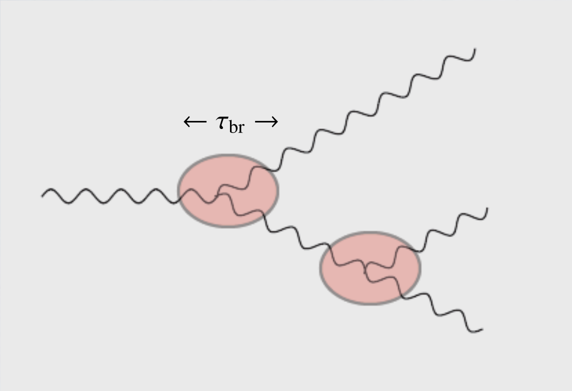
Soft emissions

- frequent, with probability $\sim \mathcal{O}(1)$
- weaker energy loss: $E_{\mathrm{soft}} \sim \alpha_s^2 \omega_c$
- but arbitrary large angles: control di-jet asymmetry

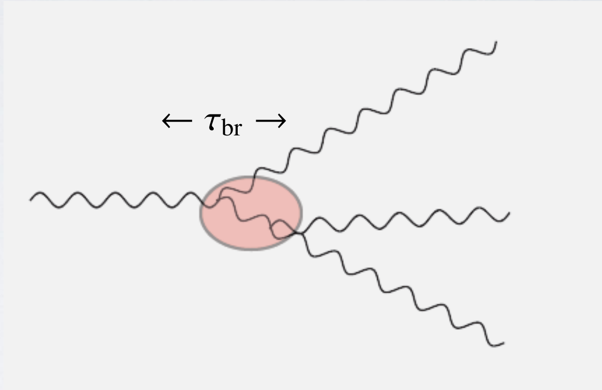
large angles emissions are dominated by soft multiple branchings

Multiple emissions

In medium, interference effects are subleading, and independent emissions are enhanced by a factor $L/\tau_{\rm br}$



$$\sim \left(\alpha_s \frac{L}{\tau_{\rm br}}\right)^2$$



$$\sim \alpha_s^2 \frac{L}{\tau_{\rm br}}$$

When $\bar{\alpha} L/\tau_{br} \sim 1$ all powers of $\bar{\alpha} L/\tau_{br} \sim 1$ need to be resummed.

Since independent emissions dominate, the leading order resummation is equivalent to a probabilistic cascade, with nearly local branchings

JPB, F. Dominguez, E. Iancu, Y. Mehtar-Tani, arXiv: 1209.4585

Inclusive Gluon Distribution

Density of gluons with momentum k inside a parton with momentum p:

$$x\frac{dN}{dx\, d^2\boldsymbol{k}} \equiv D(x,\boldsymbol{k},t) \qquad\qquad x=\omega/E$$

Leading order equation

$$\frac{\partial}{\partial t}D(x,\boldsymbol{k}) = \frac{1}{t_*}\int dz\, \mathcal{K}(z)\left[\frac{1}{z^2}\sqrt{\frac{z}{x}}D\left(\frac{x}{z},\frac{\boldsymbol{k}}{z}\right) - \frac{z}{\sqrt{x}}D\left(x,\boldsymbol{k}\right)\right] + \int\frac{d^2\boldsymbol{q}}{(2\pi)^2}\mathcal{C}(\boldsymbol{q})D(x,\boldsymbol{k}-\boldsymbol{q})$$

$$\frac{1}{t_*} \equiv \frac{\bar{\alpha}}{\tau_{\rm br}(E)} = \bar{\alpha}\sqrt{\frac{\hat{q}}{E}} \qquad\qquad \bar{\alpha}\equiv\frac{\alpha_s N_c}{\pi}$$

Energy flow

Integrating over transverse momentum yields equation for energy flow

$$\frac{\partial D(x,\tau)}{\partial \tau} = \int \mathrm{d}z\, \mathcal{K}(z) \left[\sqrt{\frac{z}{x}} D\left(\frac{x}{z},\tau\right) - \frac{z}{\sqrt{x}} D(x,\tau)\right]$$

$$\mathcal{K}(z) = \frac{\bar{\alpha}}{2} \frac{f(z)}{[z(1-z)]^{3/2}}, \qquad f(z) = [1 - z(1-z)]^{5/2}$$

(See also R. Baier, A. H. Mueller, D. Schiff, D.T. Son (2001) S. Jeon, G. D. Moore(2003))

Formally analogous to DGLAP. But very different kernel... and physics.

⟶ **A QCD cascade of a new type**

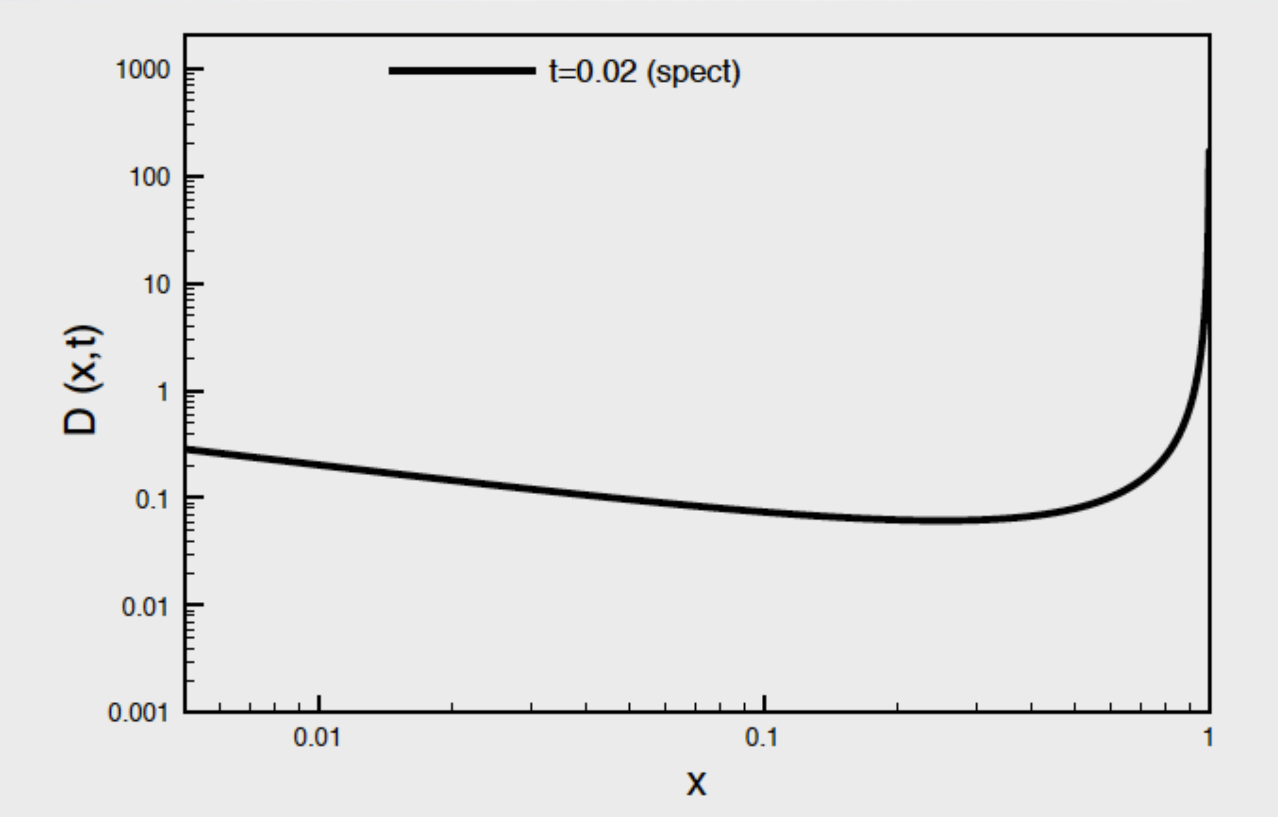
Exhibits wave turbulence

Short times

$$\frac{\partial D(x,\tau)}{\partial \tau} = \int \mathrm{d}z\, \mathcal{K}(z) \left[\sqrt{\frac{z}{x}} D\left(\frac{x}{z},\tau\right) - \frac{z}{\sqrt{x}} D(x,\tau) \right]$$

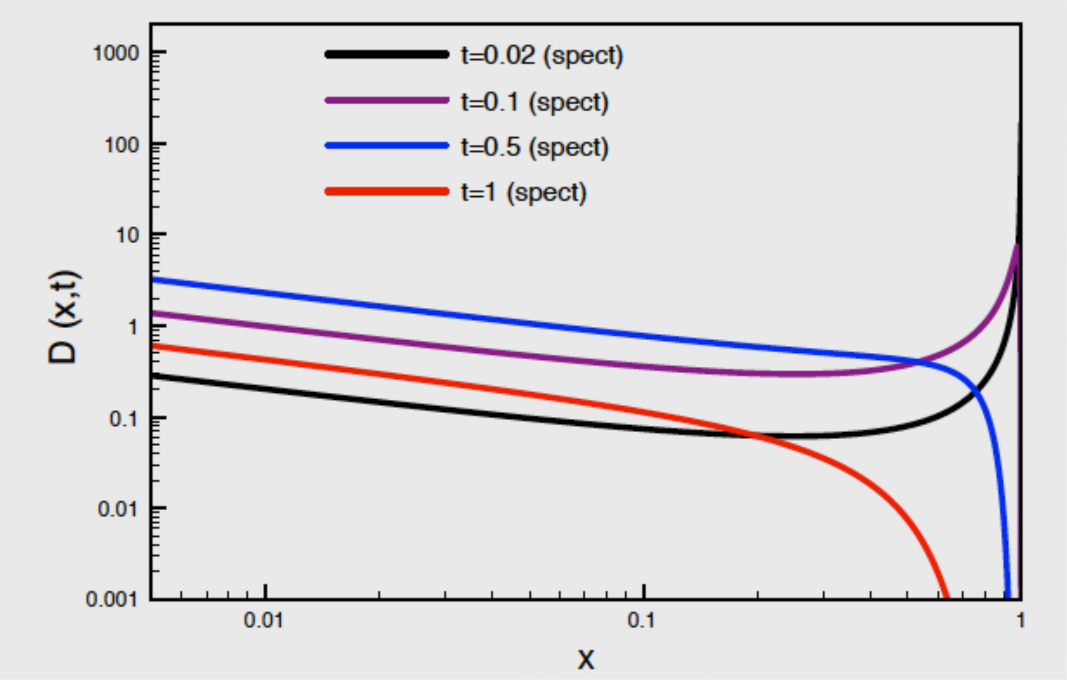
At short time, single emission by the leading particle $(D_0(\tau = 0, x) = \delta(x-1))$

D is the BDMSZ spectrum



How do multiple branchings affect this spectrum ?

One finds (exact result) $$D(x,t) \simeq \frac{t}{\sqrt{x}}\, e^{-\pi t^2} \qquad \text{for } x \ll 1$$



Fine (local) cancellations between gain and loss terms

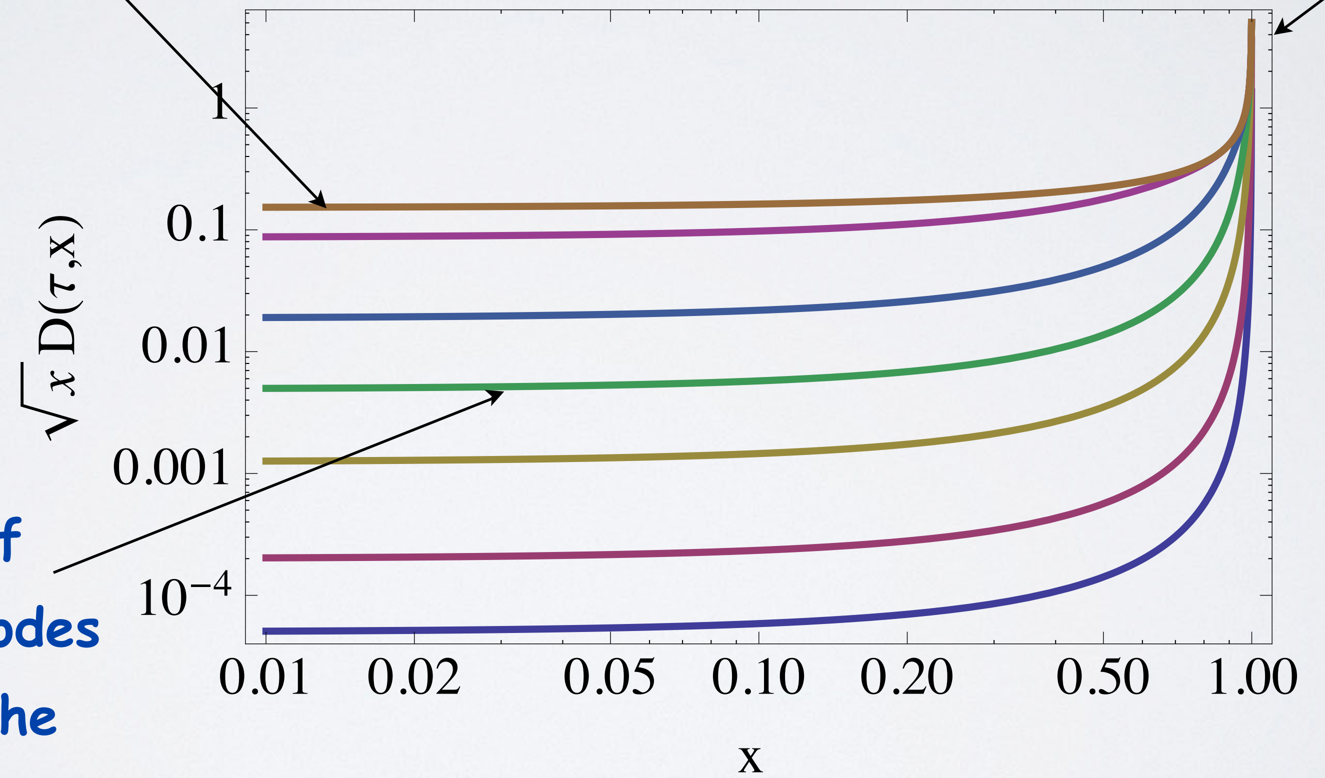
BDMPS spectrum emerges as a fixed point, scaling spectrum (energy conservation and spectrum in $1/\sqrt{x}$)

Characteristic features of wave turbulence

Digresssion: source problem

At this (fixed) point ALL the energy flows through the whole system

Energy is injected at x=1, at a constant rate



The population of the various x-modes grows, keeping the shape of the spectrum at small x

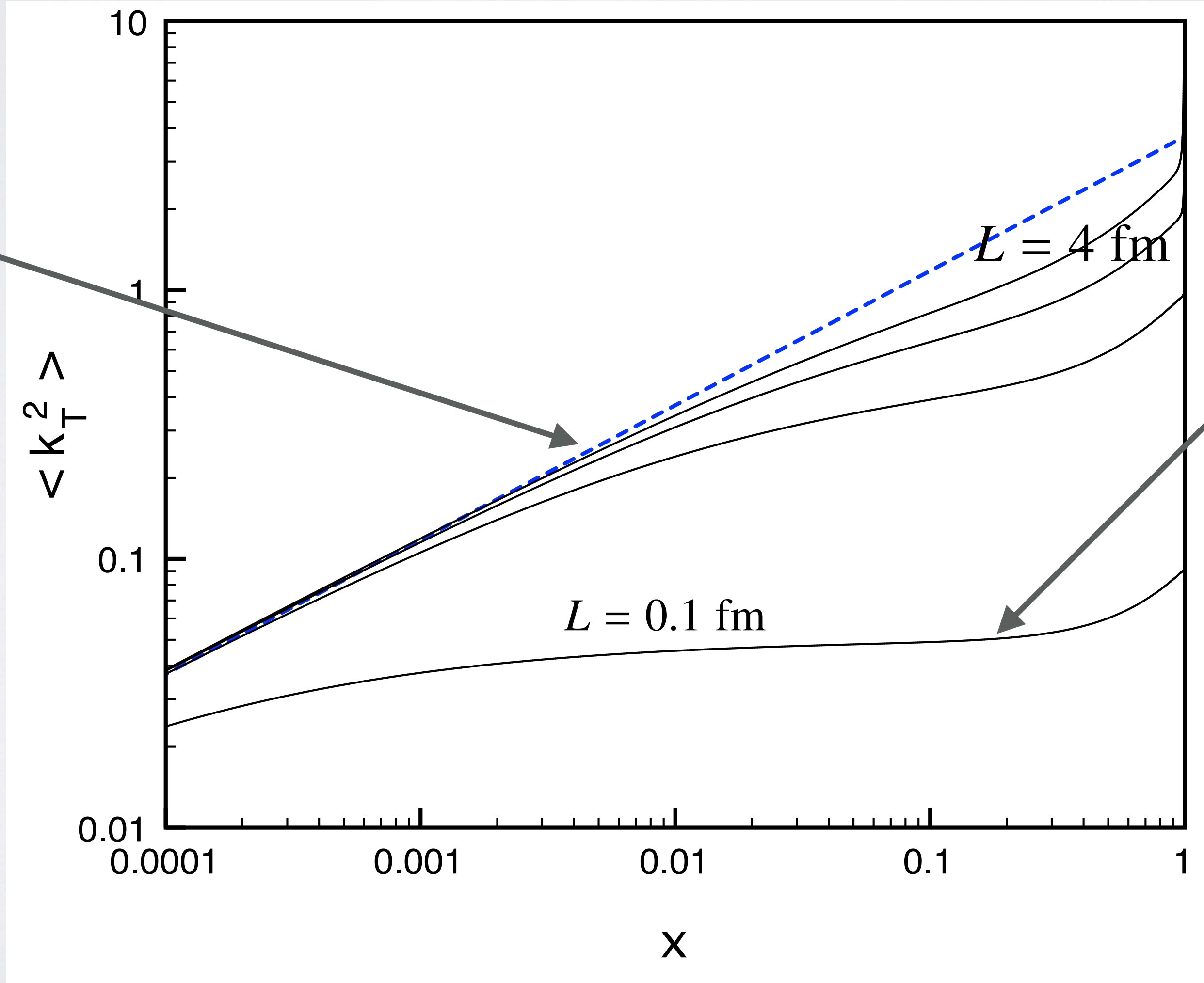
The angular structure of the in-medium cascade

JPB, Y. Mehtar-Tani, M. Torres
A. Kurkela, U. Wiedemann
JPB, L. Fister, Y. Mehtar-Tani

Angular structure

$$\langle k_\perp^2 \rangle_{t,x} = \frac{\int_{\boldsymbol{k}} \boldsymbol{k}^2\, D(x,\boldsymbol{k},t)}{\int_{\boldsymbol{k}} D(x,\boldsymbol{k},t)}$$

$$\langle k_\perp^2 \rangle \approx \frac{k_{\rm br}^2(x)}{4\bar{\alpha}}$$



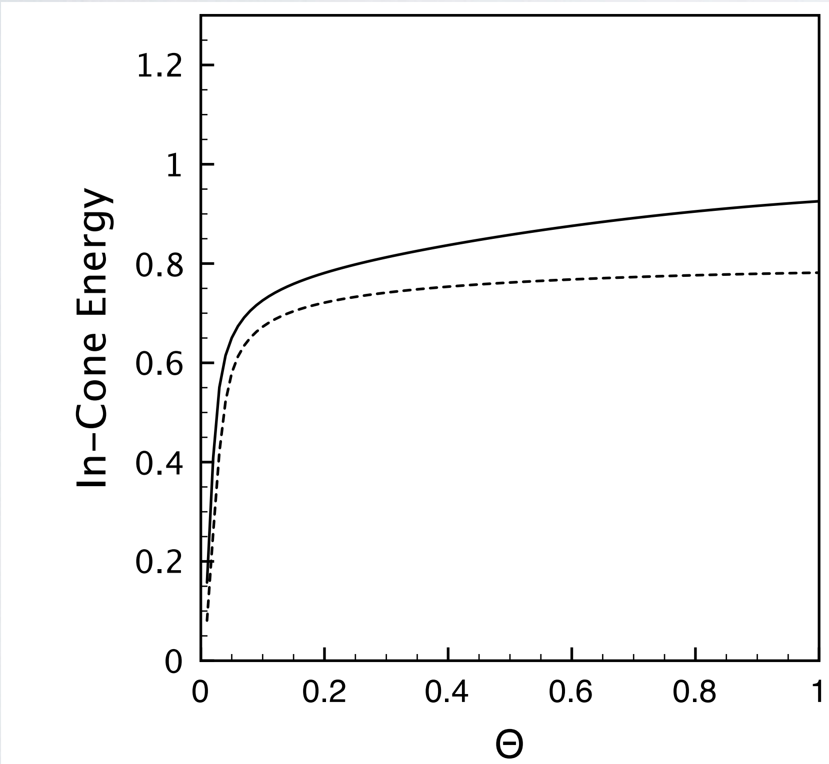
$$\frac{1+x^2}{2}\,\hat{q}L$$

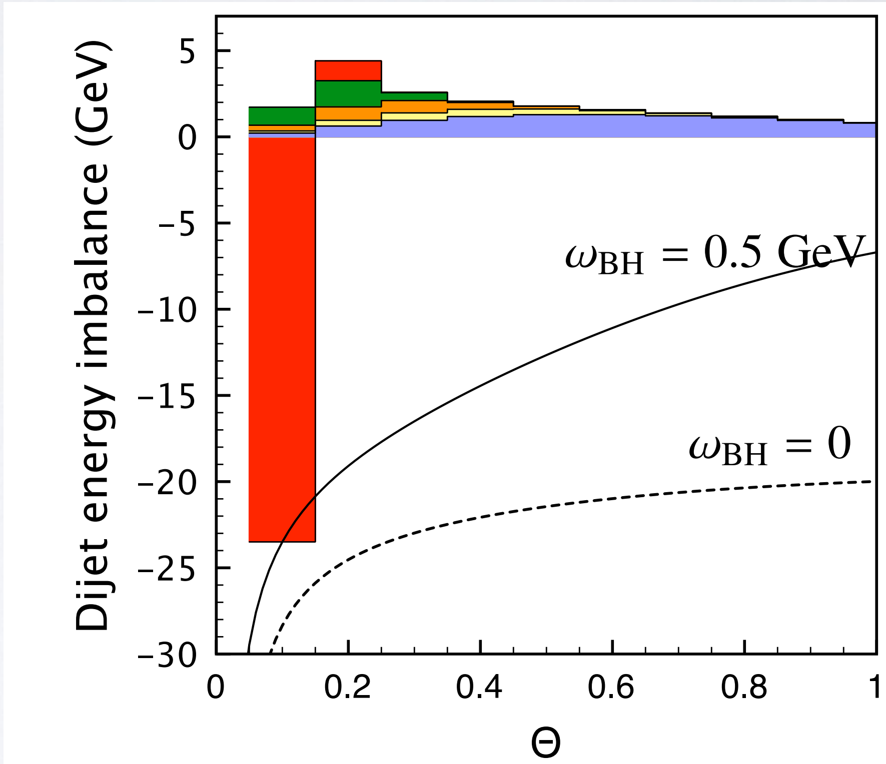
Angular structure

Broadening of the leading particle is small

$$\Delta\Theta = \frac{\sqrt{\hat{q}L}}{E}$$

red: 8-100 GeV
green: 4-8 GeV
yellow; 1-2 GeV
blue: 0-1 GeV





Summary

There are large radiative corrections to the jet quenching parameter

In a medium of large size, the successive branchings can be treated as independent, giving rise to a cascade that is very different from the vacuum cascade (no angular ordering, turbulent flow)

This cascade provides a simple and efficient mechanism for the transfer of jet energy towards very large angles. The mechanism is intrinsic, not related to a specific coupling between the jet and the medium.

The angular structure is qualitatively compatible with the data

This turbulent cascade may play a role in the latest stages of the thermalization of the quark-gluon plasma produced in ultra-relativistic heavy ion collisions