Properties of strong interactions in strong magnetic fields

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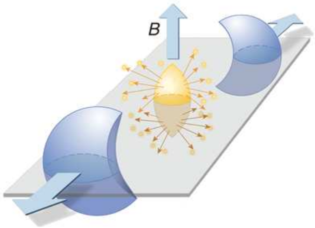
Outline

Why QCD in external magnetic field?

External magnetic fields can be relevant for the phenomenology of

- primordial universe and cosmological EWSM, $B \sim 10^{16}$Tesla
  Vachaspati, Grasso & Rubinstein
- neutron star and magnetars, $B \sim 10^{10}$Tesla
  Duncan & Thompson
- non central heavy-ion collision, $B \sim 10^{15}$ Tesla
  Skokov & Illarionov & Toneev, Tuchin

$10^{15}\, e\, \mathrm{Tesla} \sim 0.06 \mathrm{GeV}^2 \sim 3.3 m_\pi^2$
Possible modifications of the strong interactions dynamics.



The aim of our study

- A determination of the magnetic susceptibility of the QCD medium at finte T.
  Phys. Rev. Lett. **111**, 182001 (2013) [arXiv:1307.8063 [hep-lat]],
  Phys. Rev. D **89**, 054506 (2014) [arXiv:1310.8656 [hep-lat]]
- An analysis of effect of the magnetic field on the heavy-quark potential (by now just for $T = 0$).
  Phys. Rev. D **89**, 114502 (2014) [arXiv:1403.6094 [hep-lat]]

Need for a first principles non-perturbative study of QCD in background e.m. fields.

Lattice QCD (LQCD) is an ideal tool to study such questions, at least in the limit of vanishing baryon density where no algorithmic problems are present (*i.e.* no magnetars!)

The magnetic properties of the QCD medium

We are interested in the magnetic properties of QCD at finite temperature.

The free energy can be expanded in B as

$$F(B,T) = F(B=0,T) + F_1(T)B - \frac{1}{2}\chi(T)B^2 + \mathcal{O}(B^3)$$

$F_1 \equiv 0$ if there is no ferromagnetism
$\chi > 0$ for paramagnetic media and $\chi < 0$ for diamagnetic media.

Our aims are:

- study $\chi(T)$
- check for which B values the linear approximation $F \sim F_0 - \frac{1}{2}\chi B^2$ is reliable

The standard way and a no-go

The determination of magnetic susceptibilities is a standard problem in statistical physics. An estimator for χ is obtained by using the relation

$$\chi(T) = -\left.\frac{\partial^2 F(B,T)}{\partial B^2}\right|_{B=0}$$

and it is enough to compute the mean value of some well defined lattice observable at $B = 0$.

In LQCD this is not possible: to reduce finite size effects simulations are performed on compact manifold without boundary and as a consequence the possible values of the *homogeneous* magnetic field are quantized.

$\frac{\partial}{\partial B}$ on the lattice is not well defined!

The finite difference method

We are interested in studying the B dependence of F, *i.e.*

$$\Delta F(B,T) \equiv F(B,T) - F(0,T) \qquad a^2 qB = \frac{2\pi b}{L_x L_y} \quad b \in \mathbb{Z}$$

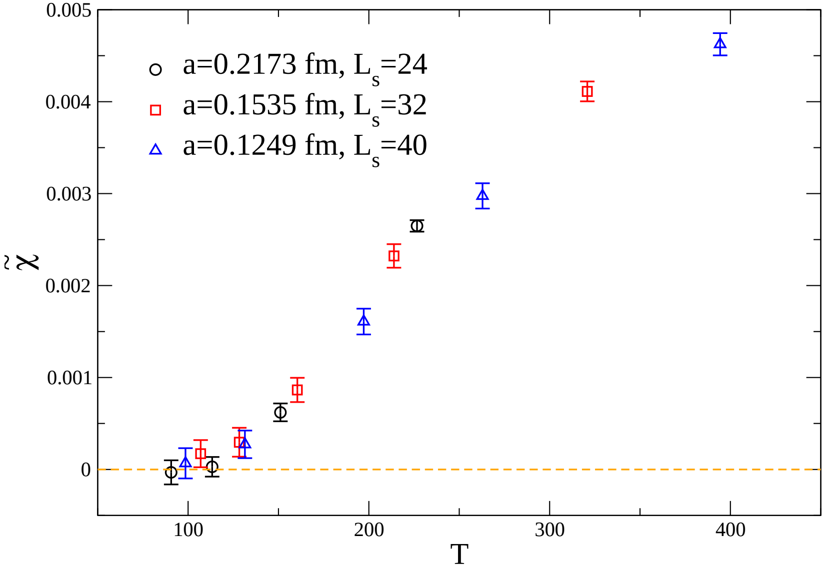
$M(b,T) \equiv \frac{\partial F(B,T)}{\partial b}$ *is not* the magnetization, but we can evaluate it at non quantized values of B (*i.e.* $b \in \mathbb{R}$) in order to get

$$\Delta F(B,T) = \int_0^b M(\tilde{b},T)\mathrm{d}\tilde{b}$$

Our procedure is thus the following:

1. compute the "magnetization" M for different temperatures and for non quantized B values
2. integrate M to get $\Delta F(B,T)$ for the quantized B values
3. compute the renormalized magnetic free energy $(\Delta F)_R(B,T) \equiv \Delta F(B,T) - \Delta F(B,T=0)$

The result for $\tilde{\chi}$



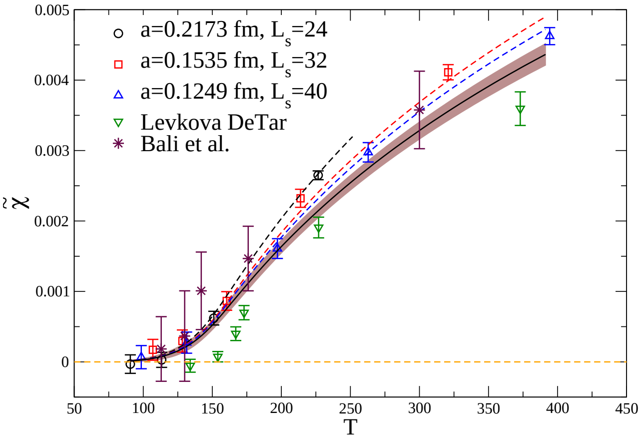
Comments & physical interpretation

- The system is paramagnetic in the explored regime
- The QCD medium is linear up to $eB \approx 0.2\mathrm{GeV}^2$
- UV effects are small
- Expectation for the low-T region: Hadron Resonance Model
  $\tilde{\chi} \approx A \exp(-M/T)$
- Expectation for the high-T region: pQCD, $\tilde{\chi} = A' \log(T/M')$
  Elmfors *et al.*, Phys. Rev. Lett. **71**, 480 (1993)
- Data are well described by a function

$$\tilde{\chi}(T) = \begin{cases} A \exp(-M/T) & T \leq T_0 \\ A' \log(T/M') & T > T_0 \end{cases}$$

with $\mathcal{C}^1$ matching at T_0. The fit gives $M \approx 900\,\mathrm{MeV}$ (lightest hadrons with magnetic moment) and $T_0 \approx 160\,\mathrm{MeV} \approx T_c$.

Continuum limit and comparison with other methods



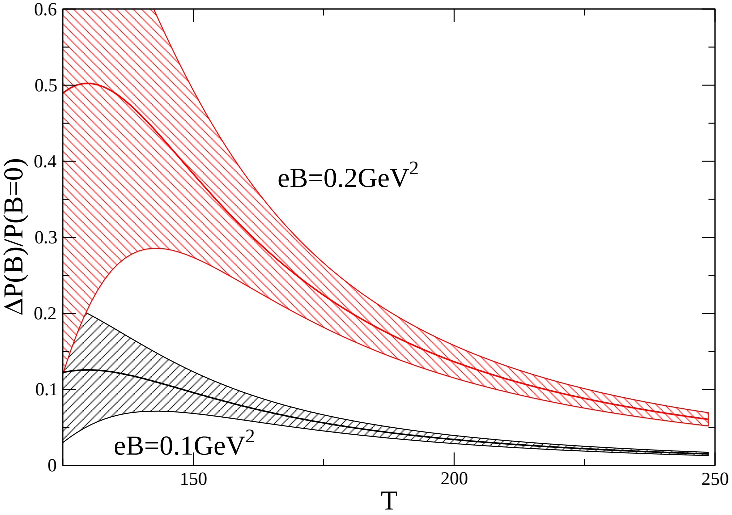
G. S. Bali et al. JHEP **1304**, 130 (2013) [arXiv:1303.1328 [hep-lat]]

L. Levkova, C. DeTar Phys. Rev. Lett. **112**, 012002 (2014) [arXiv:1309.1142 [hep-lat]]

G. S. Bali et al. [arXiv:1406.0269 [hep-lat]].

Magnetic contribution to the pressure

$$\Delta P(B,T) = \frac{1}{2}\chi(T)B^2$$



The heavy quark potential

The zero temperature (spin averaged) heavy quark potential can be determined in LQCD by using

$$V(R) = -\lim_{R_t\to\infty} \frac{1}{R_t} \log\langle W(R, R_t)\rangle$$

where $W(R, R_t)$ is the Wilson loop of spatial size R and temporal size R_t.
When $B = 0$ the potential is central and is usually parametrized by

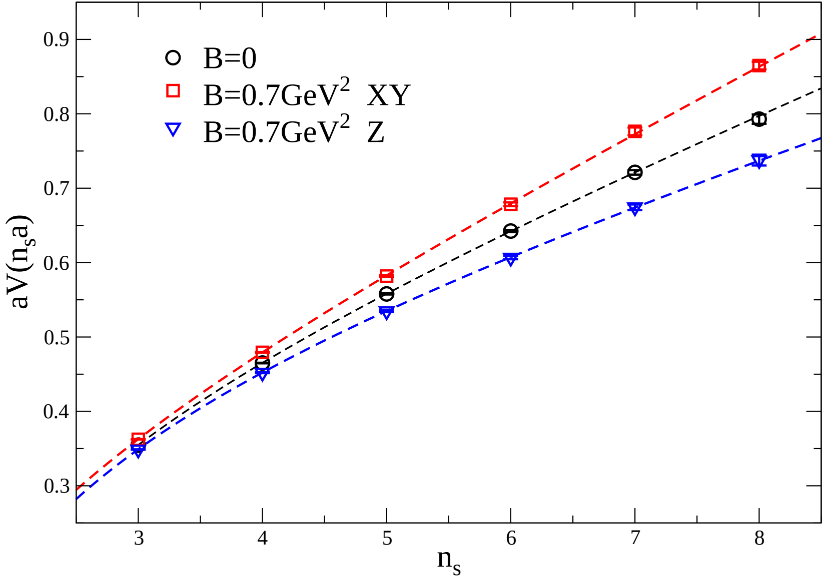
$$V(r) = c + \sigma r + \frac{\alpha}{r}$$

When $B \neq 0$, the symmetry reduces from $O(4)$ to $O(2) \times O(2)$ and the potential is no more central:

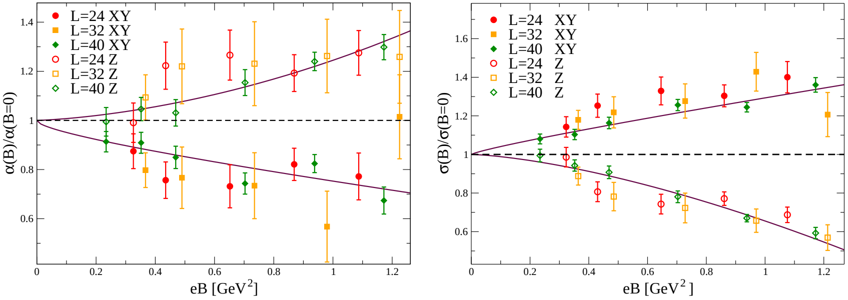
$$V_d(r) = c_d + \sigma_d r + \frac{\alpha_d}{r}$$

If $\vec{B} = B\hat{z}$, then $d = XY, Z$.

An example



$\alpha(B)$ and $\sigma(B)$



Data can be nicely parametrized by the functional form $1 + A(|e|B)^C$, with A, C depending both on the observable and on the direction.

The string tension increases in the directions orthogonal to the magnetic field $\vec{B}$ and decreases in the direction of $\vec{B}$. The Coulomb term displays an opposite behaviour.

Conclusions (I)

- The confined phase is weakly paramagnetic in the explored regime, the deconfined phase is strongly paramagnetic.
- The QCD medium is linear up to $B \approx 0.2\,\mathrm{GeV}^2$
- The magnetic contribution to the pressure for $B = 0.1 \div 0.2\,\mathrm{GeV}^2$ can be of order of $10 \div 50\%$ and can play an important role in heavy-ion collision phenomenology.
- For $B > 0.2\,\mathrm{GeV}^2$ nonlinear susceptibilities can play a dominant role (both at zero and finite temperatures). Their study is on the way.

Conclusions (II)

- The heavy-quark potential is significantly affected by an external magnetic field.
- Consequences on heavy-quarkonia masses and decay rates?
  Previous studies considered only the effect of the hyperfine interactions between $\vec{B}$ and the quark spins.
  Study on the way in collaboration with A. Rucci.
- Complete angular dependence of the potential?
- Can $\sigma_Z(B)$ vanishes for high enough B?
- What about light mesons?
- Do all these results change at finite temperature?

Backup slides with some more details

The magnetic field on the lattice

For an homogeneous magnetic field $B\hat{z}$ we have

$$\oint A_\mu \mathrm{d}x_\mu = \mathcal{A}B \qquad \oint A_\mu \mathrm{d}x_\mu = -(\ell_x\ell_y - \mathcal{A})B$$

This does not affect the motion of a particle of charge q if we impose

$$\exp(iqB\mathcal{A}) = \exp(iqB(\mathcal{A} - \ell_x\ell_y)) \quad \Rightarrow \quad \boxed{qB = \frac{2\pi b}{\ell_x\ell_y} \quad b \in \mathbb{Z}}$$

(the ℓ_μ's are the lengths in physical units)

In LQCD gauge fields enter through parallel transports. A simple choice of the lattice discretization for the e.m. field is

$$u_y(n) = e^{ia^2 qBn_x} \quad u_x(L_x - 1) = e^{-ia^2 qBL_x n_y} \quad \text{otherwise} \quad u_j(n) = 1$$

Extracting the quadratic term

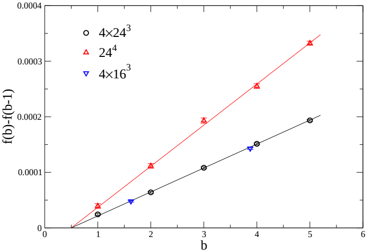
We need to estimate f_2 defined by $\Delta F(B,T) \approx \frac{1}{2} f_2(T) b^2$ ($B \propto b$). In order to minimize the error propagation in the integration we fit

$$\Delta F(B_b, T) - \Delta F(B_{b-1}, T) = \int_{b-1}^{b} M(\tilde{b}, T) \mathrm{d}\tilde{b}$$

with the function

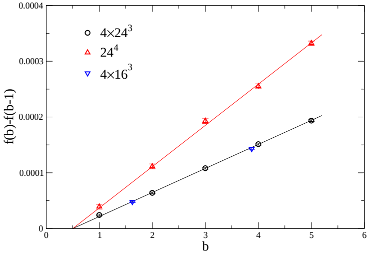
$$\frac{1}{2} f_2(T) \left[b^2 - (b-1)^2 \right] = \frac{1}{2} f_2(T)(2b-1)$$

Results for $4 \times 16^3, 4 \times 24^3$ and 24^4 lattices with physical masses and $a \approx 0.22\,\mathrm{fm}$ ($T \approx 225\mathrm{MeV}$).



Check for systematics

- dependence on the volume



- dependence on the spline interpolation and/or the number of points

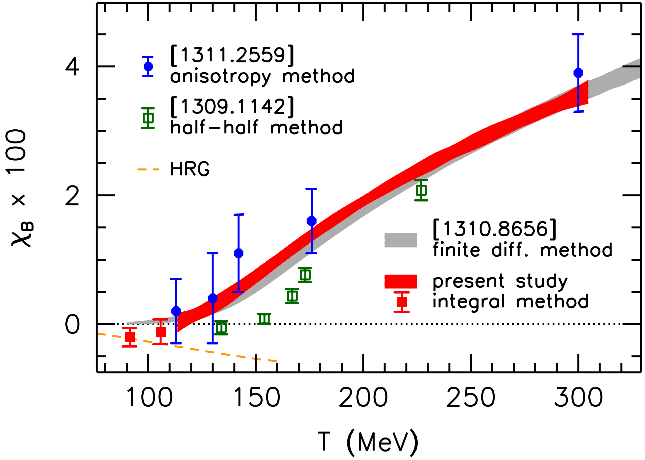
s	16 points	32 points
1	0.001192(32)	0.001187(25)
2	0.001188(35)	0.001186(25)
3	0.001184(35)	0.001188(25)
4	0.001183(34)	0.001188(27)

- dependence on the B field extension out of integers

one string	0.00211(5)
two strings	0.00208(4)

Systematics are always less than statistical errors

Other comparison



from G. S. Bali et al. [arXiv:1406.0269 [hep-lat]].

Effect of the non-dynamical e.m. field on χ

In an usual linear medium we have (in SI units)

$$\mathbf{H} = \frac{1}{\mu_0}\mathbf{B} - \mathbf{M} \qquad \mathbf{M} = \chi \mathbf{H} \qquad \mathbf{M} = \frac{\tilde{\chi}}{\mu_0}\mathbf{B} \qquad \chi = \frac{\tilde{\chi}}{1-\tilde{\chi}}$$

In our simulations the external field is a background field, so we have to subtract the energy of the magnetic field in vacuum from the free energy:

$$\Delta f_R = -\int \mathbf{M} \cdot \mathrm{d}\mathbf{B} = -\frac{\tilde{\chi}}{\mu_0}\int \mathbf{B} \cdot \mathrm{d}\mathbf{B} = -\frac{\tilde{\chi}}{2\mu_0}\mathbf{B}^2$$

B is the total field felt by the medium, but in our simulations the medium has no backreaction, so **B** is just the external field. Once $\tilde{\chi}$ is determined, we can extract the real world behaviour by using

$$\Delta f_R = -\frac{\mu_0 \chi (1+\chi)}{2}\mathbf{H}^2$$

Stability of the potential anisotropy against smearing

SM = number of smearing steps