Lattice QCD: from Hard Thermal Loops to the Hadron Gas

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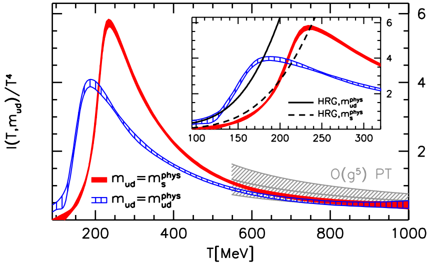
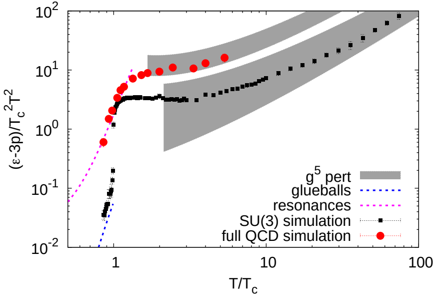
Where does the perturbative regime start?

Two data:
Physical quark masses;
up and down quark at the strange mass



Comparison:
physical point QCD
vs quarkless SU(3) theory



Quark masses play no role above $\approx 2T_c$.
The g^5 order perturbative results are consistent with data.

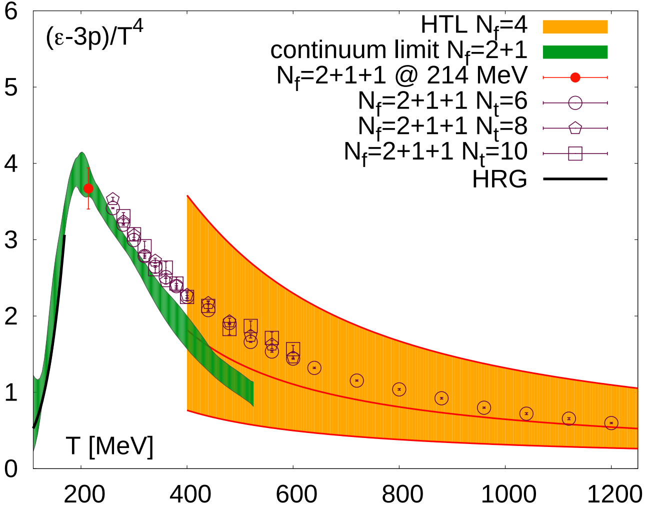
Simulation volume

If we keep the aspect ratio constant (LT) the lattice size will be very small at high temperature: $LT_c = LT \cdot T_c/T$

We keep the physical volume constant: $LT_c \gtrsim 2$.

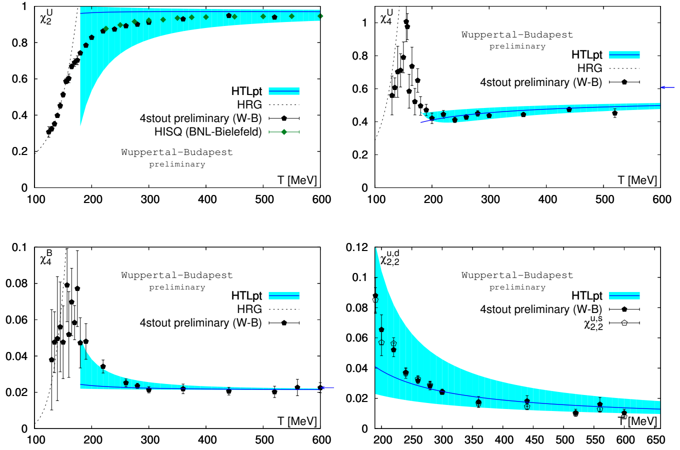
T [MeV]	$N_t = 6$	$N_t = 8$	$N_t = 10$	$N_t = 12$	$N_t = 16$
200	$24^3 \times 6$	$32^3 \times 8$	$40^3 \times 10$	$48^3 \times 12$	$64^3 \times 16$
220	$24^3 \times 6$	$32^3 \times 8$	$40^3 \times 10$	$48^3 \times 12$	$64^3 \times 16$
240	$24^3 \times 6$	$32^3 \times 8$	$40^3 \times 10$	$48^3 \times 12$	$64^3 \times 16$
260	$24^3 \times 6$	$32^3 \times 8$	$40^3 \times 10$	$48^3 \times 12$	$64^3 \times 16$
280	$24^3 \times 6$	$32^3 \times 8$	$40^3 \times 10$	$48^3 \times 12$	$64^3 \times 16$
300	$24^3 \times 6$	$32^3 \times 8$	$40^3 \times 10$	$48^3 \times 12$	$64^3 \times 16$
360	$48^3 \times 6$	$64^3 \times 8$	$64^3 \times 10$	$64^3 \times 12$	$80^3 \times 16$
440	$48^3 \times 6$	$64^3 \times 8$	$64^3 \times 10$	$64^3 \times 12$	$96^3 \times 16$
520	$48^3 \times 6$	$64^3 \times 8$	$80^3 \times 10$	$80^3 \times 12$	$112^3 \times 16$
560	$48^3 \times 6$	$64^3 \times 8$	$80^3 \times 10$	$96^3 \times 12$	
600	$48^3 \times 6$	$64^3 \times 8$	$80^3 \times 10$	$96^3 \times 12$	

Trace anomaly with and without charm



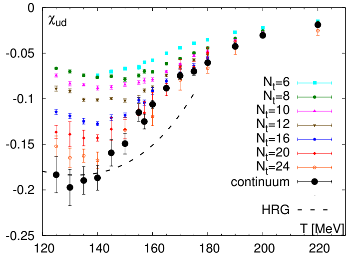
The data are running very close to the $2+1$ flavor result up to $\approx$ 350 MeV. HTLpt result is approached smoothly.

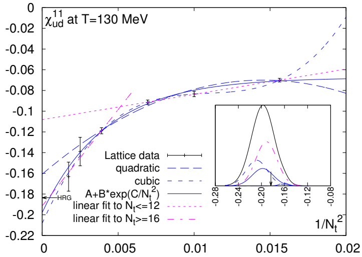
Quark number susceptibilities: lattice vs HTL



HTL results: [Haque et al 1309.3968,1402.6907]

This correlator is driven by pions in the confined phase and is extremely sensitive to taste breaking. Below T_c: agreement with HRG



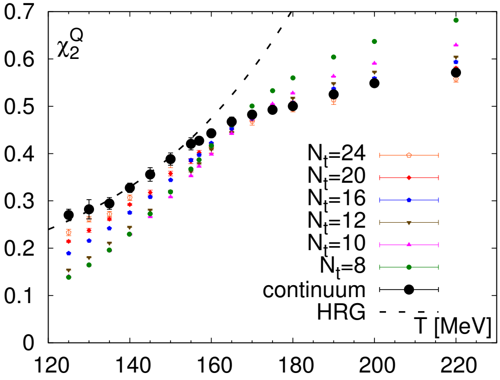


Systematic errors are calculated from the spread of various fit models:

Histogram method *[BMW Science* ***322*** *1224]*

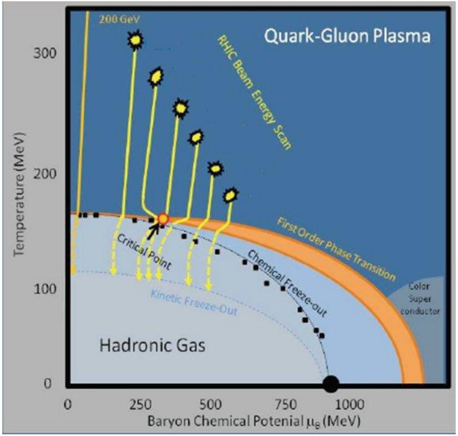
Weight: using the Akaike Information Criterion (AIC)

We update the continuum limit of the charge susceptibility:
agreement with HRG shown to high precision



For earlier calculations see [Wuppertal-Budapest: 1112.4416, HotQCD: 1203.0784]

Beam energy scan and freeze-out curve



Chiral crossover region from lattice:
$T_c = 147 \ldots 157$

[BW hep-lat/0611014,hep-lat/0609068,0903.4155,1005.3508]

Curvature: $\kappa = 0.0066(2)(4)$ [WB: 1102.1356]

At RHIC a broad energy range $\sqrt{s_{\mathrm{NN}}} = 7.7 \ldots 200$ has been scanned with heavy ion collisions.
Last inelastic scattering:
chemical freeze-out.
For each energy the chemical freeze-out is described as a grand canonical ensemble with one temperature and chemical potential.
Traditional method:
Hadron Resonance Gas (HRG)-based statistical fit of pion, kaon, proton, etc yields.
Fit result at $\sqrt{s_{\mathrm{NN}}} = 130\mathrm{GeV}$
$\mu_B = 38(12)\ \mathrm{MeV}$ and
$T_{ch} = 165(5)\ \mathrm{MeV}$.

[Andronic et al nucl-th/0511071]

The idea

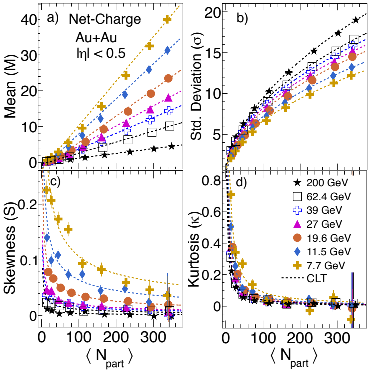
Let's not look at yields but things that exist on a lattice: conserved charges. Lattice calculates the grand canonical ensemble for a given charge (baryon number, electric charge or strangeness) and this is matched to the event-by-event statistics from the experiment

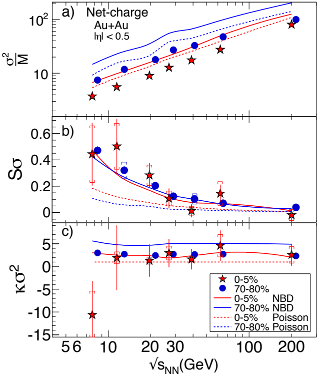
Net proton: number of protons - number of antiprotons
Net electric charge: number of positive - negative particles

Mean: $\langle N_X \rangle = -T \frac{\partial \log Z}{\partial \mu_X}$ Variance: $\langle \delta N_X^2 \rangle = -T^2 \frac{\partial^2 \log Z}{\partial {\mu_X}^2}$
On the lattice we have access to normalized quark number susceptibilities:

$$\chi_2^X = \frac{1}{VT^3} \frac{\partial^2 \log Z}{(\partial \mu_X / T)^2}$$

At RHIC **STAR** has measured the mean, variance, skewness and kurtosis of the event-by-event **net charge** distribution at various energies and centralities.





[STAR: 1402.1558]

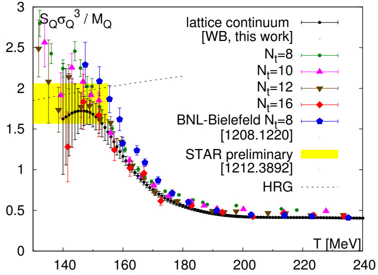
Thermometer from the skewness

A possible thermometer [BNL-Bielefeld 1208.1220] T_{ch} is found through

$$S\sigma^3/M\big|_{\text{experiment}}(\text{beam energy}) = S\sigma^3/M\big|_{\text{lattice}}(T_{ch})$$

Comparing Wuppertal-Budapest lattice results with STAR data:

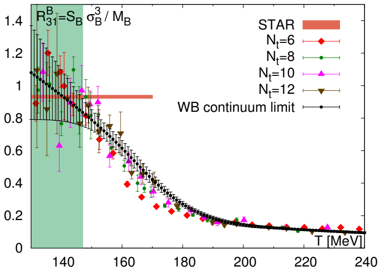
Net electric charge:



[Wuppertal-Budapest 1304.5161], [STAR 1402.1558].

Conclusion $T_{ch} \leq 157\text{MeV}$

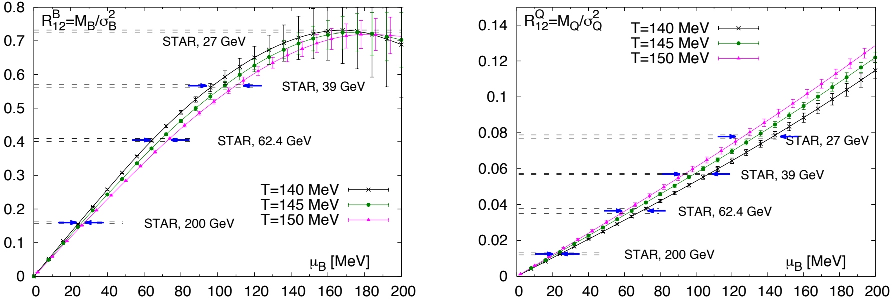
Net baryon number:



[Wuppertal-Budapest 1403.4576], [STAR 1309.5681] (protons).

Conclusion $T_{ch} \leq 151\text{MeV}$

Baryometers from proton and charge fluctuations



The second order fluctuations are accurate both on the lattice and in experiment.

Notice that we use here about 10 MeV lower temperatures than before, which is motivated by the skewness data.

Are these chemical potentials consistent between baryon and charge?

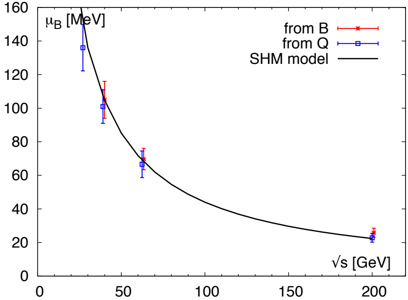
[Wuppertal-Budapest 1403.4576]

Baryometers from proton and charge fluctuations

$\sqrt{s}[GeV]$	μ_B^f [MeV] (from B)	μ_B^f [MeV] (from Q)
200	25.8±2.7	22.8±2.6
62.4	69.7±6.4	66.6±7.9
39	105±11	101±10
27	-	136±13.8

The chemical potentials are now consistent and they also agree with the statistical model's parametrization

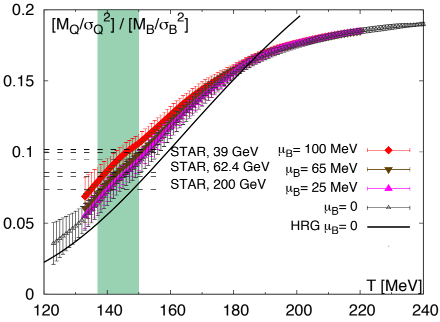
[Andronic 0812.1186]



[Wuppertal-Budapest 1403.4576]

Our thermometer, the skewness, is difficult to work with both on lattice and in experiment.

If we **assume** that the consistency between proton and charge freeze-out holds exactly (same temperature and chemical potential), then a ratio of ratios can be defined.



Our conclusion:

$$T_f = 144 \pm 6 MeV$$

Summary

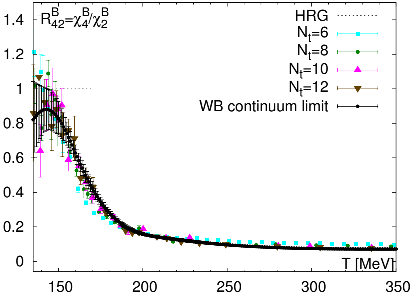
we calculated the free energy and its derivatives with respect to chemical potentials.
We calculated selected fluctuations on very fine staggered lattices: $N_t = 6, 8, 10, 12, 16, 20$ and 24.

- Chemical freeze-out temperature estimated from proton and charge fluctuations: $T_{ch} \lesssim 151$ MeV.
- Freezez-out chemical potentials are consistent between charge and proton fluctuations as well as the statistical models.
- Charge kurtosis can pinpoint the freeze-out temperatures
- At low temperatures HRG mostly describe lattice data up to at least 140 MeV, except for the charge kurtosis.
- At high temperature all studied observables match the NNLO HTLpt results

Backup slides

At LHC energies $\mu_B \approx 0 \Rightarrow$ mean, skewness $\equiv 0$.
What could be a good thermometer?

Baryon kurtosis:



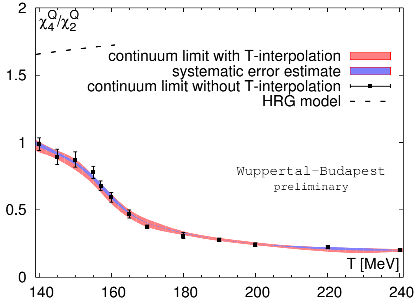
[Wuppertal-Budapest 1304.5161]

Lattice: 2stout $N_t = 6, 8, 10, 12$

STAR at 200 GeV [1309.5681]:

$\chi_4^B/\chi_2^B = 0.897(29)(20)$

Electric charge kurtosis:



[Wuppertal-Budapest preliminary]

Lattice: 4stout $N_t = 6, 8, 10, 12, 16, 20, 24$

T_f can be pinpointed as soon as LHC data are available

Caveats

- *Effects due to volume variation because of finite centrality bin width*
  Experimentally corrected by centrality-bin-width correction method
- *Finite reconstruction efficiency*
  Experimentally corrected based on binomial distribution
  [A. Bzdak, V. Koch, PRC (2012)]
- *Spallation protons*
  Experimentally removed with proper cuts in p_T
- *Canonical vs Gran Canonical ensemble*
  Experimental cuts in the kinematics and acceptance
  [V. Koch, S. Jeon, PRL (2000)]
- *Proton multiplicity distributions vs baryon number fluctuations*
  Numerically very similar once protons are properly treated
  [M. Asakawa and M. Kitazawa], [PRC (2012), M. Nahrgang et al., 1402.1238]
- *Final-state interactions in the hadronic phase* [J.Steinheimer et al., PRL (2013)]
  Consistency between different charges = fundamental test