

Ghosts out of equilibrium

Alina Czajka

*Institute of Physics,
Jan Kochanowski University, Kielce, Poland*

in collaboration with St. Mrówczyński

based on: A. Czajka & St. Mrówczyński, Phys. Rev. D **89**, 085035 (2014)

Outline

1. Motivation
2. Keldysh-Schwinger formalism
3. Green's functions of gluons
4. Generating functional
5. Slavnov-Taylor identities
6. Green's functions of ghosts
7. Application - polarization tensor
8. Conclusions

Motivation

- QCD computations in covariant gauges are usually much simpler than those in physical ones like the Coulomb gauge.
- Covariant gauges require ghosts to compensate unphysical degrees of freedom.



How to introduce ghosts in the Keldysh-Schwinger formalism?

What is the Green's function of free ghosts?

Keldysh–Schwinger formalism

Description of non-equilibrium many-body systems

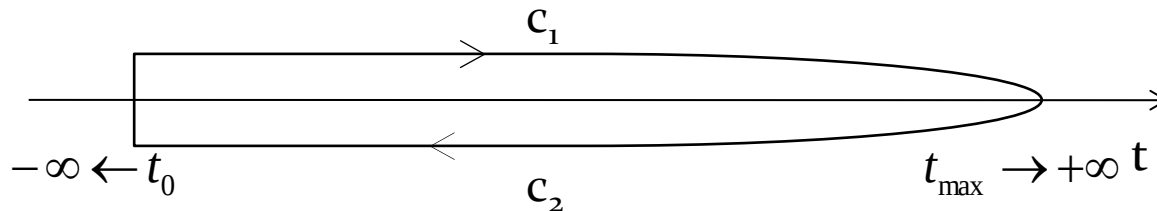
Contour Green function of gauge field

$$i\mathcal{D}_{ab}^{\mu\nu}(x, y) \stackrel{\text{def}}{=} \left\langle \tilde{T} A_a^\mu(x) A_b^\nu(y) \right\rangle$$

$$\langle \dots \rangle = \text{Tr}[\hat{\rho}(t_0) \dots]$$

$\tilde{T}$ - ordering along the contour

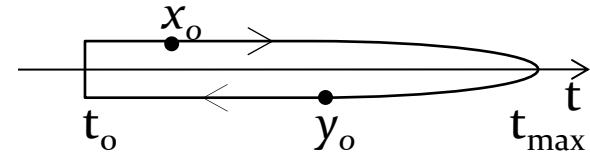
$$\tilde{T}A(x)B(y) = \Theta(x_0, y_0)A(x)B(y) \pm \Theta(y_0, x_0)B(y)A(x)$$



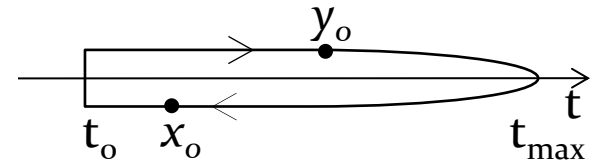
Keldysh–Schwinger formalism

Contour Green's function includes 4 Green's functions with real time arguments:

$$\left(\mathcal{D}_{ab}^{\mu\nu}\right)^{\triangleright}(x, y) = \left\langle A_a^{\mu}(x) A_b^{\nu}(y) \right\rangle$$

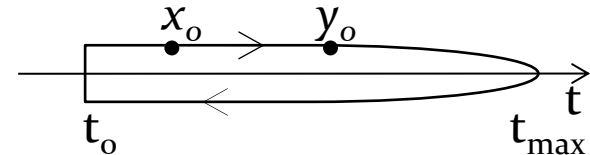


$$\left(\mathcal{D}_{ab}^{\mu\nu}\right)^{\triangleleft}(x, y) = \left\langle A_b^{\nu}(y) A_a^{\mu}(x) \right\rangle$$



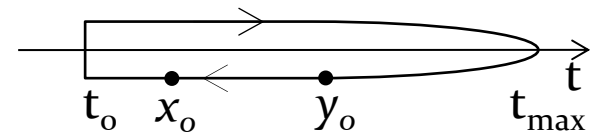
$$\left(\mathcal{D}_{ab}^{\mu\nu}\right)^c(x, y) = \left\langle T^c A_a^{\mu}(x) A_b^{\nu}(y) \right\rangle$$

Chronological time ordering



$$\left(\mathcal{D}_{ab}^{\mu\nu}\right)^a(x, y) = \left\langle T^a A_a^{\mu}(x) A_b^{\nu}(y) \right\rangle$$

Anti-chronological time ordering



Retarded, advanced & symmetric Green's functions

$$\mathcal{D}^+(x, y) = \Theta(x_0 - y_0) (\mathcal{D}^>(x, y) - \mathcal{D}^<(x, y))$$

$$\mathcal{D}^-(x, y) = \Theta(y_0 - x_0) (\mathcal{D}^<(x, y) - \mathcal{D}^>(x, y))$$

$$\mathcal{D}^{\text{sym}}(x, y) = \mathcal{D}^>(x, y) + \mathcal{D}^<(x, y)$$

Meaning of the functions

$$\mathcal{D}^{<, >}(x, y)$$

- phase-space density
- mass-shell constraint
- real particles

$$\mathcal{D}^{\pm}(x, y)$$

- retarded & advanced propagator
- no mass-shell constraint
- virtual particles

Meaning of the functions

The Green's functions $\mathcal{D}(x, y)$ are gauge dependent

Physical results obtained from Green's functions must be gauge independent

For example

The poles of $\mathcal{D}(x, y)$ - dispersion relations – are gauge independent

Green's functions of free gluon field

$$D(x, y) = D(x - y)$$

Feynman gauge

$$D^>(p) = \frac{i\pi}{E_p} g_{\mu\nu} \delta^{ab} \left[\delta(E_p - p_0) [n_g(\mathbf{p}) + 1] + \delta(E_p + p_0) n_g(-\mathbf{p}) \right]$$

$$D^<(p) = \frac{i\pi}{E_p} g_{\mu\nu} \delta^{ab} \left[\delta(E_p - p_0) n_g(\mathbf{p}) + \delta(E_p + p_0) [n_g(-\mathbf{p}) + 1] \right]$$

$$D^c(p) = -g_{\mu\nu} \delta^{ab} \left[\frac{1}{p^2 + i0^+} - \frac{i\pi}{E_p} \left(\delta(E_p - p_0) n_g(\mathbf{p}) + \delta(E_p + p_0) n_g(-\mathbf{p}) \right) \right]$$

$$D^a(p) = g_{\mu\nu} \delta^{ab} \left[\frac{1}{p^2 - i0^+} + \frac{i\pi}{E_p} \left(\delta(E_p - p_0) n_g(\mathbf{p}) + \delta(E_p + p_0) n_g(-\mathbf{p}) \right) \right]$$

$n_g(\mathbf{p})$ - gluon distribution function

Green's functions of free ghosts

$$\begin{array}{l} \Delta^>(p) \\ \Delta^<(p) \\ \Delta^c(p) \\ \Delta^a(p) \end{array} = ?$$

How to get Green's function of free ghosts?

Ghost sector should be determined by the gauge symmetry of the theory!

$$A_\mu^a \rightarrow (A_\mu^a)^U = A_\mu^a + f^{abc} \omega^b A_\mu^c - \frac{1}{g} \partial_\mu \omega^a$$

gauge symmetry of the theory



Slavnov-Taylor identities

Generating functional

$$W_0[J, \chi, \chi^*] = N_0 \int_{BC} \mathcal{D}A \mathcal{D}c \mathcal{D}c^* e^{i \int_C d^4x \mathcal{L}_{\text{eff}}(x)}$$

boundary conditions:

the fields are fixed in $t = -\infty \pm i0^+$

all fields are on the contour

Lagrangian:

$$\begin{aligned} \mathcal{L}_{\text{eff}}(x) = & -\frac{1}{4} F_a^{\mu\nu} F_{\mu\nu}^a + \bar{\psi}(i\gamma_\mu D^\mu - m)\psi - \frac{1}{2\alpha} (\partial^\mu A_\mu^a)^2 \\ & - c_a^* (\partial^\mu \partial_\mu \delta_{ab} - g \partial^\mu f^{abc} A_\mu^c) c_b + J_\mu^a A_\mu^a + \chi_a^* c_a + \chi_a c_a^* \end{aligned}$$

$$\begin{aligned} W[J, \chi, \chi^*] = & N \int DA' Dc' Dc^{*'} DA'' Dc'' Dc^{*''} \\ & \times \rho[A', c', c^{*'} | A'', c'', c^{*''}] W_0[J, \chi, \chi^*] \end{aligned}$$

density matrix

Generating functional

$$W[J, \chi, \chi^*] = N \int DA' Dc' Dc^{*'} DA'' Dc'' Dc^{*''} \\ \times \rho[A', c', c^{*'} | A'', c'', c^{*''}] W_0[J, \chi, \chi^*]$$

The full Green's function can be generated through

$$i\mathcal{D}_{\mu\nu}^{ab}(x, y) = (-i)^2 \frac{\delta^2}{\delta J_{\mu}^a(x) \delta J_{\nu}^b(y)} W[J, \chi, \chi^*] \Big|_{J=\chi=\chi^*=0}$$

density matrix $\rho[A', c', c^{*'} | A'', c'', c^{*''}]$ is not specified



the explicit form of the functional and the Green's function cannot be found

The functional provides various relations among Green's functions.

General Slavnov-Taylor identity

$$W[J, \chi, \chi^*] = N \int_{BC} \mathcal{D}A \Delta[A] e^{i \int_C d^4x \mathcal{L}(x)}$$

analog of the Fadeev-Popov determinant

$$\Delta[A] \equiv \int_{BC} \mathcal{D}c \mathcal{D}c^* e^{-i \int_C d^4x \left(-c_a^* (\partial^\mu \partial_\mu \delta_{ab} - g \partial^\mu f^{abc} A_\mu^c) c_b + \chi_a^* c_a + \chi_a c_a^* \right)}$$

The invariance of $W[J, \chi, \chi^*]$ under the transformations

$$A_\mu^a \rightarrow (A_\mu^a)^U = A_\mu^a + f^{abc} \omega^b A_\mu^c - \frac{1}{g} \partial_\mu \omega^a$$

leads to

$$\left\{ i \partial_{(y)}^\mu \frac{\delta}{\delta J_d^\mu(y)} - \int_C d^4x J_a^\mu(x) \left(\partial_\mu^{(x)} \delta^{ab} + i g f^{abc} \frac{\delta}{\delta J_c^\mu(x)} \right) M_{bd}^{-1} \left[\frac{1}{i} \frac{\delta}{\delta J} \middle| x, y \right] \right\} W[J, \chi, \chi^*] = 0$$

Slavnov-Taylor identity for gluon Green's function

$$\frac{\delta}{\delta J_e^\nu(z)} \left\{ i\partial_{(y)}^\mu \frac{\delta}{\delta J_d^\mu(y)} - \int_C d^4x J_a^\mu(x) \left(\partial_\mu^{(x)} \delta^{ab} + igf^{abc} \frac{\delta}{\delta J_c^\mu(x)} \right) M_{bd}^{-1} \left[\frac{1}{i} \frac{\delta}{\delta J} \Big|_{x,y} \right] \right\} W[J, \chi, \chi^*] = 0$$

$$J = \chi = \chi^* = 0$$

$$-p^\mu \mathcal{D}_{\mu\nu}^{ab}(p) = p_\nu \Delta_{ab}(-p)$$

free ghosts Green's function

The longitudinal component of the gluon Green's function is free.

Ghost functions

$$-p^\mu D_{\mu\nu}^{ab}(p) = p_\nu \Delta_{ab}(-p)$$

$$\Delta^>(p) = -\frac{i\pi}{E_p} \delta^{ab} \left[\delta(E_p - p_0) [n_g(\mathbf{p}) + 1] + \delta(E_p + p_0) n_g(-\mathbf{p}) \right]$$

$$\Delta^<(p) = -\frac{i\pi}{E_p} \delta^{ab} \left[\delta(E_p - p_0) n_g(\mathbf{p}) + \delta(E_p + p_0) [n_g(-\mathbf{p}) + 1] \right]$$

$$\Delta^c(p) = \delta^{ab} \left[\frac{1}{p^2 + i0^+} - \frac{i\pi}{E_p} \left(\delta(E_p - p_0) n_g(\mathbf{p}) + \delta(E_p + p_0) n_g(-\mathbf{p}) \right) \right]$$

$$\Delta^a(p) = -\delta^{ab} \left[\frac{1}{p^2 - i0^+} + \frac{i\pi}{E_p} \left(\delta(E_p - p_0) n_g(\mathbf{p}) + \delta(E_p + p_0) n_g(-\mathbf{p}) \right) \right]$$

$n_g(\mathbf{p})$ - gluon distribution function

Application - polarization tensor

The poles of $\mathcal{D}(x, y)$ give dispersion relations of quasiparticles

Dyson – Schwinger equation

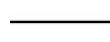
$$\mathcal{D} = D - D\Pi\mathcal{D}$$

$$\mathcal{D}^{-1} = D^{-1} + \Pi$$

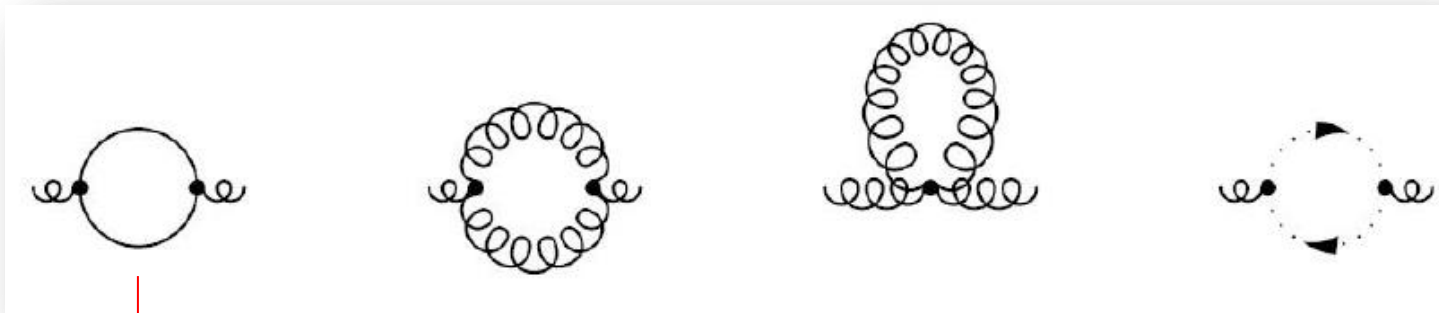
To get dispersion relations one needs the polarization tensor

Contributions to polarization tensor

 gluon

 fermion

 ghost

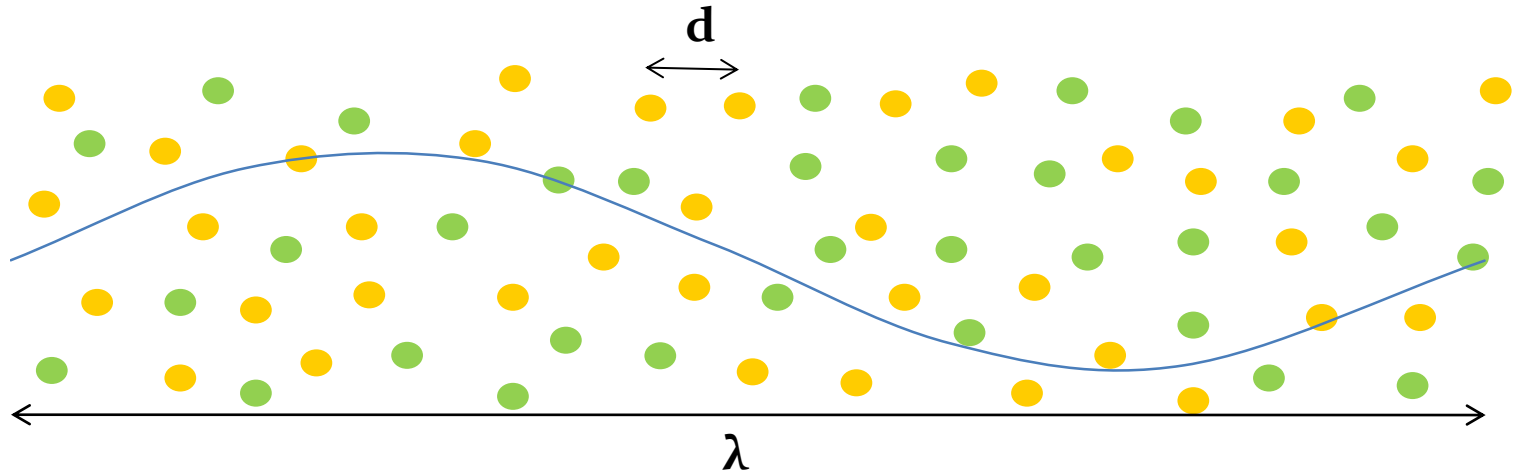


$$\Pi_{ab}^{\mu\nu}(x, y) = -ig^2 N_c \delta_{ab} \text{Tr}[\gamma^\mu S(x, y) \gamma^\nu S(y, x)]$$

quark-loop contribution to contour polarization tensor

$S(x, y)$ - fermion contour Green's function

Hard Loop Approximation



**Wavelength of a quasi-particle is much bigger
than inter-particle distance in the plasma:**

$$\lambda \gg d$$

$$k^\mu \ll p^\mu$$

Polarization tensor

$$\Pi_{ab}^{\mu\nu}(k) = g^2 \delta_{ab} \int \frac{d^3 p}{(2\pi)^3} \frac{f(\mathbf{p})}{E_p} \frac{k^2 p^\mu p^\nu - [p^\mu k^\nu + k^\mu p^\nu - g^{\mu\nu} (k \cdot p)](k \cdot p)}{(k \cdot p + i0^+)^2}$$

distribution function

$$f(\mathbf{p}) \equiv 2N_c n_g(\mathbf{p}) + n_q(\mathbf{p}) + \bar{n}_q(\mathbf{p})$$

(vacuum effect is subtracted)

➤ **symmetric**

$$\Pi^{\mu\nu}(k) = \Pi^{\nu\mu}(k)$$

➤ **transversal**

$$k_\mu \Pi^{\mu\nu}(k) = 0$$

**Gauge
independence!**

Ghosts work properly!

Conclusions

- The generating functional of QCD in the Keldysh-Schwinger formalism was constructed.
- The general Slavnov-Taylor identity was derived.
- The ghost Green's function was expressed through the gluon one.
- The computed polarization tensor in the hard loop approximation is automatically transverse.
- **QCD calculations in Keldysh-Schwinger formalism are possible in the Feynman gauge.**