Plasmons in anisotropic quark-gluon plasma

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Strong Electroweak Matter

14 -18 July 2014 Lausanne, Switzerland

Outline

- Motivation
- General dispersion equation
- Isotropic system
- Weakly anisotropic system
- Finite prolateness or oblateness
- Extremely prolate system
- Extremely oblate system
- Conclusions

Motivation

- Spectrum of collective excitations is an important characteristics of any many body system.
- Anisotropic plasma is qualitatively different than the isotropic one.
- QGP from relativistic heavy-ion collisions is anisotropic.
- Existing analyses of collective excitations are not complete.

Momentum distribution

The anisotropic momentum distribution is obtained from an isotropic one by rescaling it in one direction

$$f_\xi(\mathbf{p}) = C_\xi \, f_{\text{iso}}\left(\sqrt{\mathbf{p}^2 + \xi(\mathbf{p}\cdot\mathbf{n})^2}\right)$$

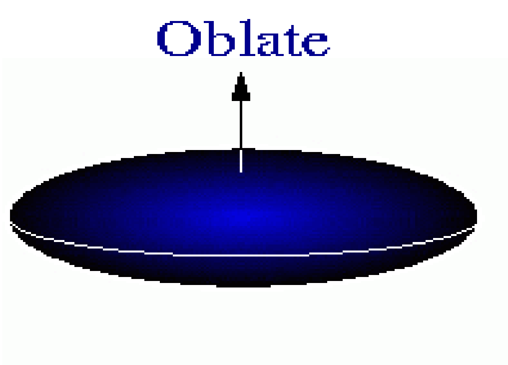
$$\xi \in (-1, \infty)$$

$$f_\sigma(\mathbf{p}) = C_\sigma \, f_{\text{iso}}\left(\sqrt{(\sigma+1)\mathbf{p}^2 - \sigma(\mathbf{p}\cdot\mathbf{n})^2}\right)$$

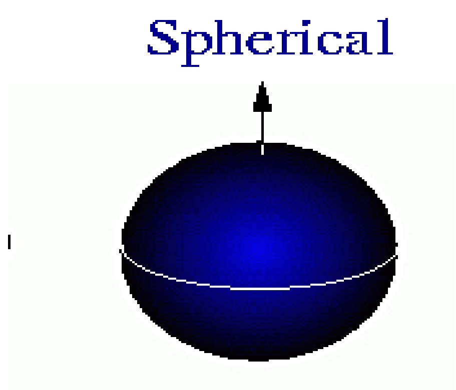
$$\sigma \in (-1, \infty)$$

P.Romatschke and M. Strickland, Phys. Rev. D **68**, 036004 (2003)

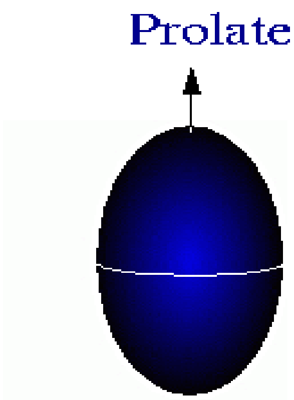
$$\xi > 0; \quad \sigma < 0$$



$$\xi = \sigma = 0$$



$$\xi < 0; \quad \sigma > 0$$



Momentum distribution

There is a freedom in choosing the normalization constants

$$f_\xi(\mathbf{p}) = C_\xi \, f_{\mathrm{iso}}\left(\sqrt{\mathbf{p}^2 + \xi(\mathbf{p}\cdot\mathbf{n})^2}\right) \qquad f_\sigma(\mathbf{p}) = C_\sigma \, f_{\mathrm{iso}}\left(\sqrt{(\sigma+1)\mathbf{p}^2 - \sigma(\mathbf{p}\cdot\mathbf{n})^2}\right)$$

1) $$\int \frac{d^3p}{(2\pi)^3} f_\xi(\mathbf{p}) = \int \frac{d^3p}{(2\pi)^3} f_{\mathrm{iso}}(|\mathbf{p}|) = \int \frac{d^3p}{(2\pi)^3} f_\sigma(\mathbf{p})$$

$$C_\xi = \sqrt{1+\xi} \qquad C_\sigma = \sigma + 1$$

2) $$m^2 \equiv \int \frac{d^3p}{(2\pi)^3} \frac{f_\xi(\mathbf{p})}{|\mathbf{p}|} = \int \frac{d^3p}{(2\pi)^3} \frac{f_{\mathrm{iso}}(|\mathbf{p}|)}{|\mathbf{p}|} = \int \frac{d^3p}{(2\pi)^3} \frac{f_\sigma(\mathbf{p})}{|\mathbf{p}|}$$

$$C_\xi = \frac{\sqrt{\xi}}{\mathrm{Arctanh}\sqrt{\xi}} \qquad C_\sigma = \frac{\sqrt{\sigma(\sigma+1)}}{\mathrm{Arctan}\sqrt{\frac{\sigma}{\sigma+1}}}$$

General dispersion equation

Gluon polarization tensor can be written down, as

$$\Pi^{ij}(\omega,\mathbf{k}) = \frac{g^2}{2}\int \frac{d^3p}{(2\pi)^3}\frac{f(\mathbf{p})}{|\mathbf{p}|}\left[\delta^{ij} + \frac{k^i v^j + v^i k^j}{\omega - \mathbf{k}\cdot\mathbf{v}} + \frac{(\mathbf{k}^2-\omega^2)v^i v^j}{(\omega-\mathbf{k}\cdot\mathbf{v})^2}\right]$$

The dielectric tensor is related to the retarded gluon polarization tensor

$$\varepsilon^{ij}(\omega,\mathbf{k}) = \delta^{ij} - \frac{1}{\omega^2}\Pi^{ij}(\omega,\mathbf{k})$$

Definition of the matrix sigma

$$\Sigma^{ij}(\omega,\mathbf{k}) \equiv (\omega^2-\mathbf{k}^2)\delta^{ij} + k^i k^j - \Pi^{ij}(\omega,\mathbf{k})$$

inverse gluon propagator in temporal axial gauge

The dispersion equation

$$\det\left[\Sigma(\omega,\mathbf{k})\right] = 0$$

$\omega(\mathbf{k})$ - collective mode in a plasma system

How to invert matrix Σ?

Method to inverse the matrix Σ

Inversion of the matrix Σ which depends on **k** and **n**

$$\Sigma = \alpha A + \beta B + \gamma C + \delta D$$

basis of matrices

$$A^{ij} = \delta^{ij} - \frac{k^i k^j}{\mathbf{k}^2}, \quad B^{ij} = \frac{k^i k^j}{\mathbf{k}^2}$$

$$C^{ij} = \frac{n_T^i n_T^j}{\mathbf{n}_T^2}, \quad D^{ij} = n_T^i k^j + k^i n_T^j$$

$$n_T^i = \left(\delta^{ij} - \frac{k^i k^j}{\mathbf{k}^2} \right) n^j$$

$$\Sigma^{-1} = \bar{\alpha} A + \bar{\beta} B + \bar{\gamma} C + \bar{\delta} D$$

$$\Sigma \Sigma^{-1} = \mathbf{1} \quad \Rightarrow \quad \bar{\alpha}, \bar{\beta}, \bar{\gamma}, \bar{\delta}$$

The coefficients α, β, γ, δ are determined by the following contractions:

$$k^i \Sigma^{ij} k^j = k^2 \beta, \qquad n_T^i \Sigma^{ij} k^j = n_T^2 k^2 \delta,$$

$$n_T^i \Sigma^{ij} n_T^j = n_T^2 (\alpha + \gamma), \qquad \mathrm{Tr}\Sigma = 2\alpha + \beta + \gamma,$$

P. Romatschke and M. Srickland, Phys. Rev. D **68**, 036004 (2003)

Collective mode in isotropic QGP

In isotropic plasma the matrix Σ is decomposed as: $\Sigma^{ij} = \alpha_{\rm iso} A^{ij} + \beta_{\rm iso} B^{ij}$

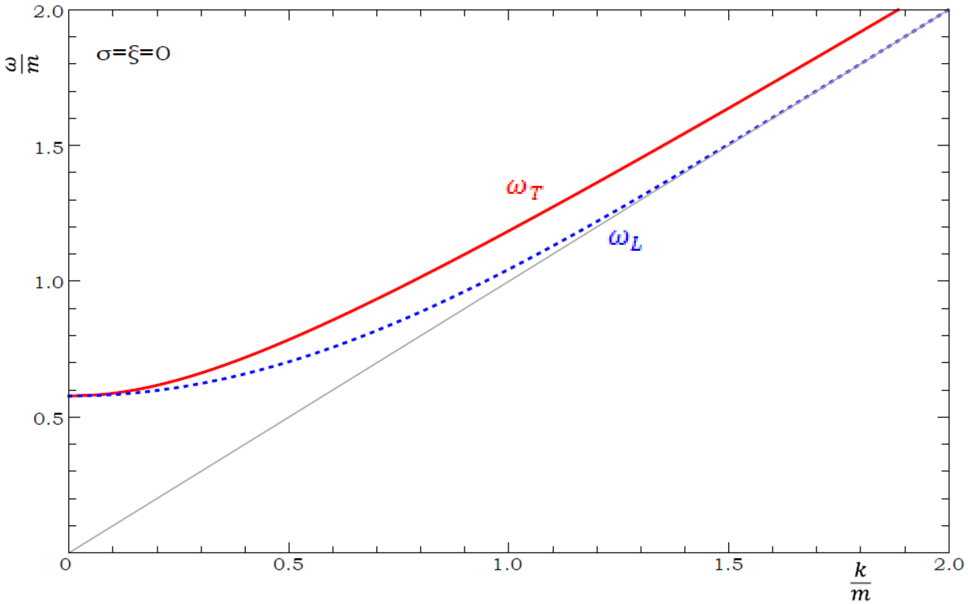
$$\alpha_{iso}(\omega,\mathbf{k}) = \omega^2 - k^2 - \frac{m^2\omega^2}{2k^2}\left[1-\left(\frac{\omega}{2k}-\frac{k}{2\omega}\right)\ln\left(\frac{\omega+k}{\omega-k}\right)\right]$$

and

$$\beta_{iso}(\omega,\mathbf{k}) = \omega^2 + \frac{m^2\omega^2}{k^2}\left[1-\frac{\omega}{2k}\ln\left(\frac{\omega+k}{\omega-k}\right)\right]$$

Only four real solutions
(two positive & two negative)

$$m^2 = g^2\int\frac{d^3p}{(2\pi)^3}\frac{f_\xi(\mathbf{p})}{|\mathbf{p}|}$$ Debye mass



Weakly anisotropic system

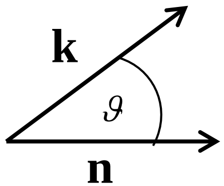
Weakly anisotropic distribution $|\xi| << 1$

$$f_{\xi}(\mathbf{p}) \approx \left(1+\frac{\xi}{3}\right) f_{\mathrm{iso}}(p) + \frac{\xi}{2}\frac{d\, f_{\mathrm{iso}}(p)}{d\, p}\, p(\mathbf{v}\cdot\mathbf{n})^2$$

Coefficients α, β, γ, δ are found in analytic form

$$\alpha(\omega,\mathbf{k}) = \left(1+\frac{\xi}{3}\right)\alpha_{iso}(\omega,\mathbf{k}) + \xi\frac{m^2}{8}\left\{\frac{8}{3}\cos^2\vartheta + \frac{2}{3}\left(5-19\cos^2\vartheta\right)\frac{\omega^2}{k^2} - 2\left(1-5\cos^2\vartheta\right)\frac{\omega^4}{k^4}\right.$$
$$\left. + \left[1-3\cos^2\vartheta - \left(2-8\cos^2\vartheta\right)\frac{\omega^2}{k^2} + \left(1-5\cos^2\vartheta\right)\frac{\omega^4}{k^4}\right]\frac{\omega}{k}\ln\left(\frac{\omega+k}{\omega-k}\right)\right\},$$

$$\beta(\omega,\mathbf{k}) = \ldots,$$
$$\gamma(\omega,\mathbf{k}) = \ldots,$$
$$\delta(\omega,\mathbf{k}) = \ldots$$



Weakly anisotropic system

Dispersion equations:

1) $\alpha(\omega,\mathbf{k})-\omega^2+k^2=0$

2) $\delta^2(\omega,\mathbf{k})k^2n_T^2-\left(\beta(\omega,\mathbf{k})-\omega^2\right)\left(\alpha(\omega,\mathbf{k})+\gamma(\omega,\mathbf{k})-\omega^2+k^2\right)=0$

In the limit of weak anisotropy, we have three dispersion equations because $\delta^2=O(\xi^2)$

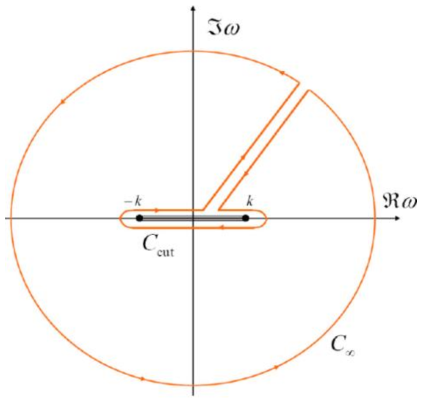
1) $\alpha(\omega,\mathbf{k})-\omega^2+k^2=0$

2) $\beta(\omega,\mathbf{k})-\omega^2=0$

3) $\alpha(\omega,\mathbf{k})+\gamma(\omega,\mathbf{k})-\omega^2+k^2=0$

Nyquist analysis

Nyquist analysis allows one to find the number of solution of the equation



$$f(\omega)=0$$

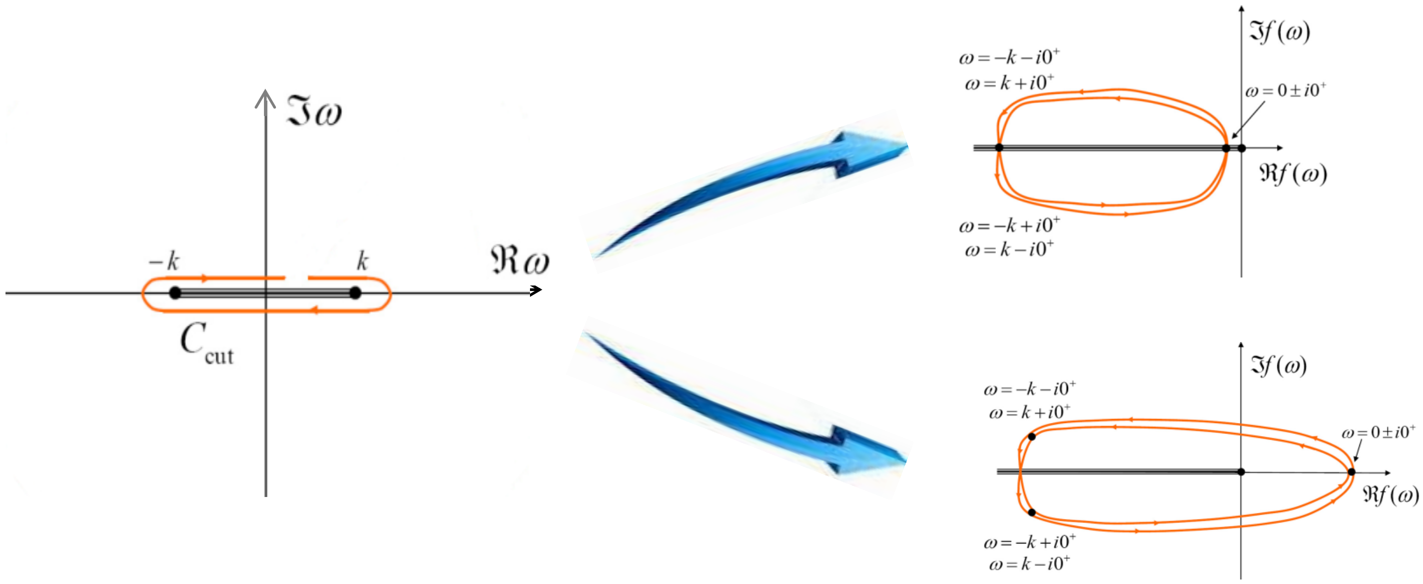
$$\oint_C \frac{d\omega}{2\pi i}\frac{f'(\omega)}{f(\omega)} = n_Z - n_P$$

$$\oint_C \frac{d\omega}{2\pi i}\frac{f'(\omega)}{f(\omega)} = \oint_{C_\infty} \frac{d\omega}{2\pi i}\frac{f'(\omega)}{f(\omega)} + \oint_{C_{cut}} \frac{d\omega}{2\pi i}\frac{f'(\omega)}{f(\omega)}$$

- C_∞ $$\oint_{C_\infty} \frac{d\omega}{2\pi i}\frac{f'(\omega)}{f(\omega)} = \lim_{|\omega|->\infty} \omega \frac{f'(\omega)}{f(\omega)} \equiv n_\infty$$

- C_{cut} $$\oint_{C_{cut}} \frac{d\omega}{2\pi i}\frac{f'(\omega)}{f(\omega)} = \frac{1}{2\pi i}\oint_{C_{cut}} \frac{d}{d\omega}\ln f(\omega) = \frac{1}{2\pi i}(\ln f(\omega_e) - \ln f(\omega_s)) \equiv n_W$$

Nyquist analysis



$n_W = 0$

$n_W = 2$

The number of zeros of the function $f(\omega)$: $n_Z = n_P + n_\infty + n_W$

n_P – number of poles $f(\omega)$

n_∞ – contribution from C_∞

n_W – contribution from C_{cut}

Equation $\alpha(\omega,\mathbf{k})-\omega^2+k^2=0$

$k^2-\xi\frac{m^2}{3}\cos^2\vartheta\geq 0$ 2 solutions

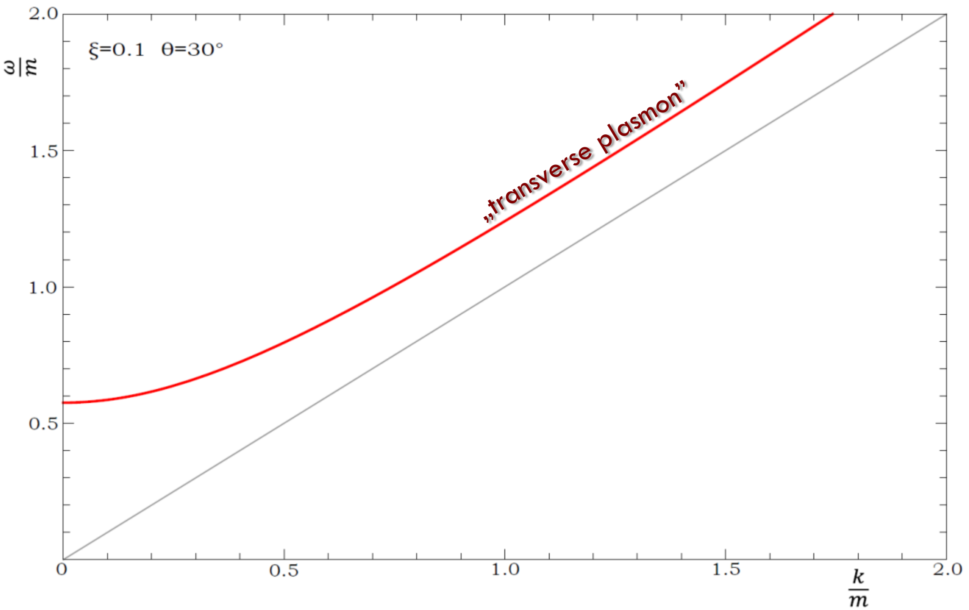
$k^2-\xi\frac{m^2}{3}\cos^2\vartheta<0$ 4 solutions

Equation $\alpha(\omega,\mathbf{k}) - \omega^2 + k^2 = 0$

$k^2 - \xi \frac{m^2}{3} \cos^2 \vartheta \geq 0$ **2 solutions**

$k^2 - \xi \frac{m^2}{3} \cos^2 \vartheta < 0$ **4 solutions**

$$\omega^2(\mathbf{k}) \underset{k<<m}{\approx} \frac{m^2}{3}\left(1 - \frac{\xi}{15}\right) + \frac{6}{5}\left[1 + \frac{\xi}{14}\left(\frac{4}{15} + \cos^2\vartheta\right)\right] k^2$$



Equation $\alpha(\omega,\mathbf{k})-\omega^2+k^2=0$

$k^2-\xi\frac{m^2}{3}\cos^2\vartheta\geq 0$ **2 solutions**

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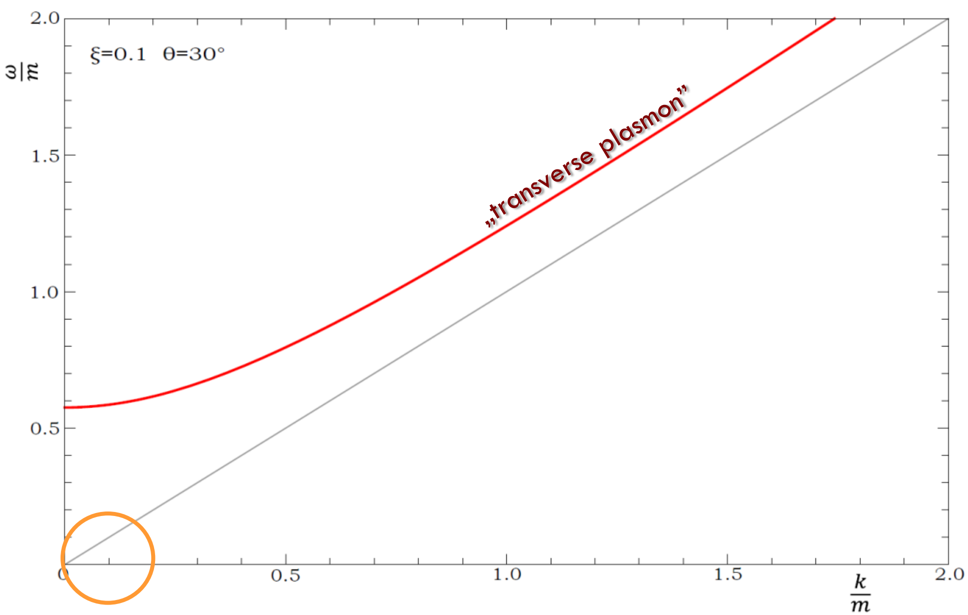
$$\omega(\mathbf{k})=\pm i\gamma(\mathbf{k})$$

$$\gamma(\mathbf{k})\underset{k>>\gamma}{\approx}\frac{1}{2}\left(\sqrt{\frac{\lambda^2}{k^2}+4\left(k_A^2-k^2\right)}-\frac{\lambda}{k}\right)$$



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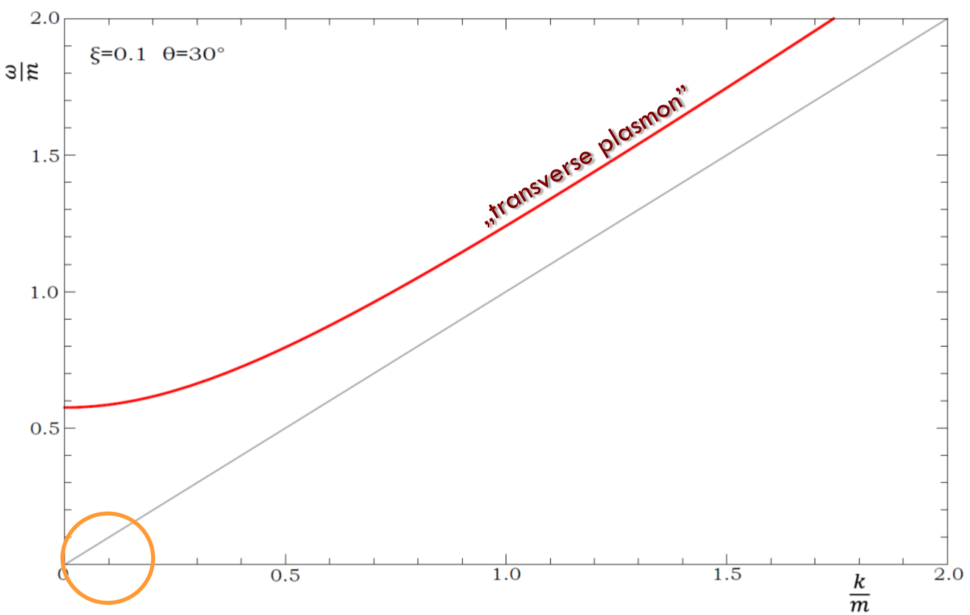
$$\gamma(\mathbf{k})\underset{k>>\gamma}{\approx}\frac{1}{2}\left(\sqrt{\frac{\lambda^2}{k^2}+4\left(k_A^2-k^2\right)}-\frac{\lambda}{k}\right)$$

where

$$\lambda\equiv\frac{\pi}{4}\left[1-\frac{\xi}{2}\left(\frac{1}{3}-3\cos^2\vartheta\right)\right]m^2$$

$$k_A\equiv\sqrt{\frac{\xi}{3}}\left|\cos\vartheta\right|$$



Equation $\alpha(\omega,\mathbf{k}) - \omega^2 + k^2 = 0$

$k^2 - \xi \frac{m^2}{3} \cos^2 \vartheta \geq 0$ **2 solutions**

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$$\omega^2(\mathbf{k}) \underset{k<<m}{\approx} \frac{m^2}{3}\left(1 - \frac{\xi}{15}\right) + \frac{6}{5}\left[1 + \frac{\xi}{14}\left(\frac{4}{15} + \cos^2 \vartheta\right)\right] k^2$$

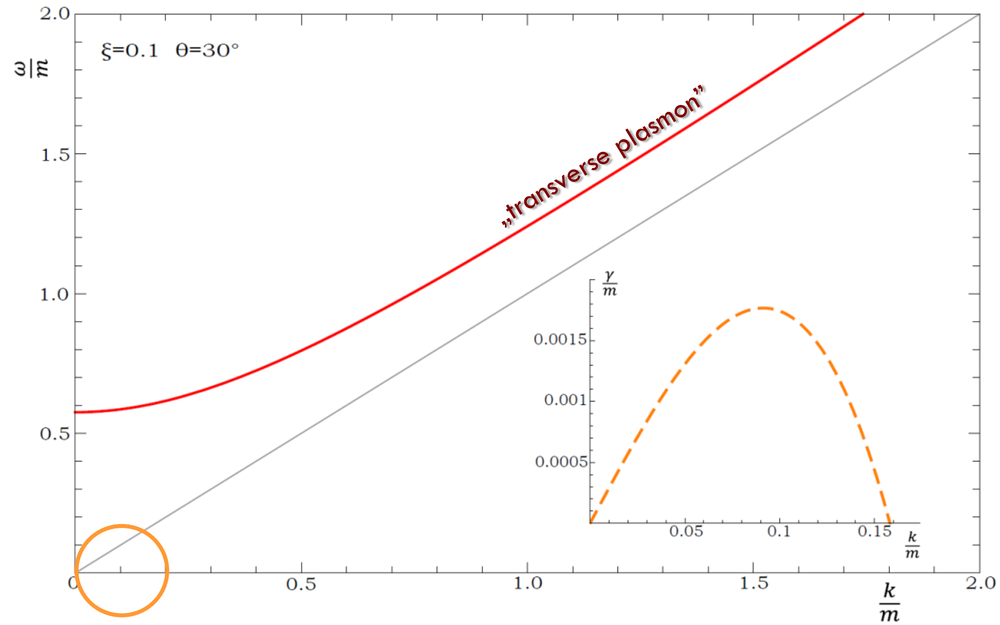
$$\omega(\mathbf{k}) = \pm i\gamma(\mathbf{k})$$

$$\gamma(\mathbf{k}) \underset{k>>\gamma}{\approx} \frac{1}{2}\left(\sqrt{\frac{\lambda^2}{k^2} + 4\left(k_A^2 - k^2\right)} - \frac{\lambda}{k}\right)$$

where

$$\lambda \equiv \frac{\pi}{4}\left[1 - \frac{\xi}{2}\left(\frac{1}{3} - 3\cos^2 \vartheta\right)\right] m^2$$

$$k_A \equiv \sqrt{\frac{\xi}{3}} |\cos \vartheta|$$



Equation $\beta(\omega, \mathbf{k}) - \omega^2 = 0$

There are always two solutions

$$\omega^2(\mathbf{k}) \underset{k<<m}{\approx} \frac{m^2}{3}\left(1 + \frac{\xi}{5}\left(-\frac{1}{3} + \cos^2 \vartheta\right)\right) + \frac{3}{5}\left[1 + \frac{4\xi}{35}\left(1 - 3\cos^2 \vartheta\right)\right]k^2$$

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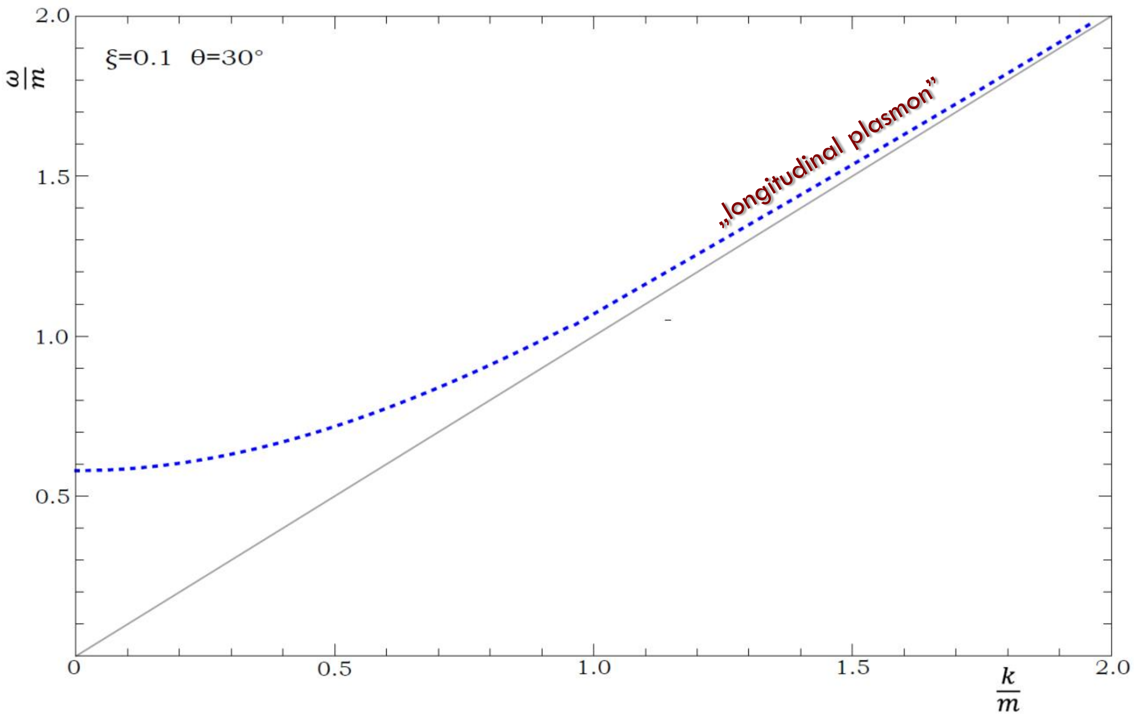

Equation $\alpha(\omega,\mathbf{k})+\gamma(\omega,\mathbf{k})-\omega^2+k^2=0$

$k^2+\xi\frac{m^2}{3}\left(1-2\cos^2\vartheta\right)\geq 0$ **2 solutions**

$k^2+\xi\frac{m^2}{3}\left(1-2\cos^2\vartheta\right)< 0$ **4 solutions**

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$k^2+\xi\frac{m^2}{3}\left(1-2\cos^2\vartheta\right)\geq 0$ **2 solutions**

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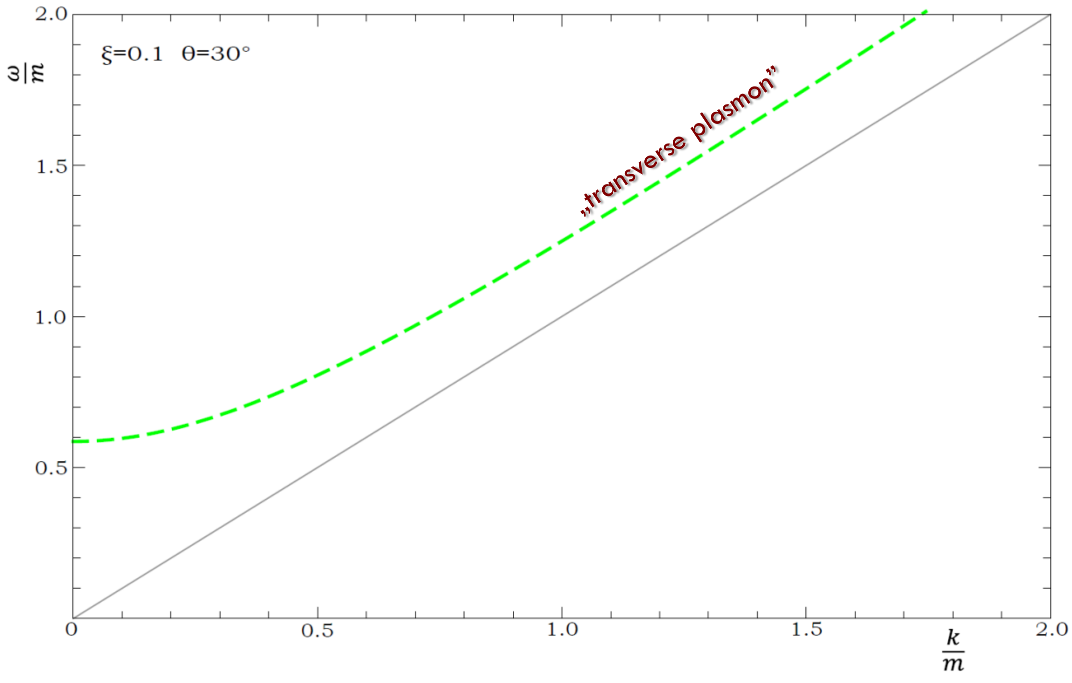
$$\omega^2(\mathbf{k})\underset{k<<m}{\approx}\frac{m^2}{3}\left(1+\frac{\xi}{5}\left(\frac{2}{3}-\cos^2\vartheta\right)\right)+\frac{6}{5}\left[1+\frac{\xi}{5}\left(\frac{23}{42}-\cos^2\vartheta\right)\right]k^2$$



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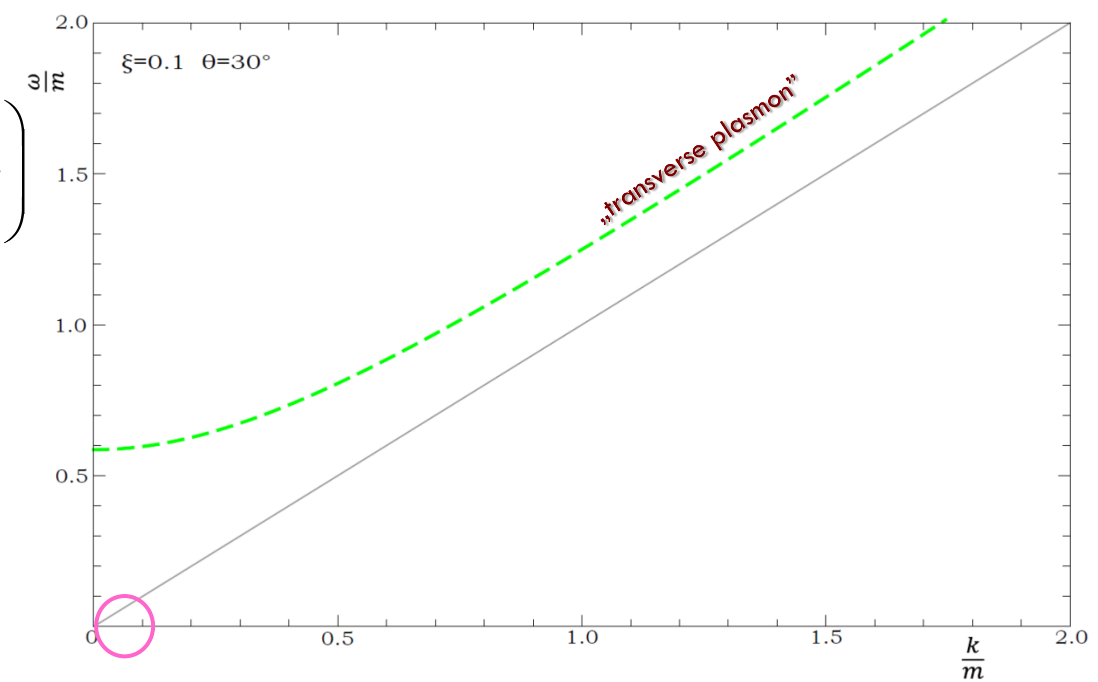
$$\omega(\mathbf{k})=\pm i\gamma(\mathbf{k})$$

$$\gamma(\mathbf{k})\underset{k>>\gamma}{\approx}\frac{1}{2}\left(\sqrt{\frac{\lambda^2}{k^2}+4\left(k_A^2-k^2\right)}-\frac{\lambda}{k}\right)$$



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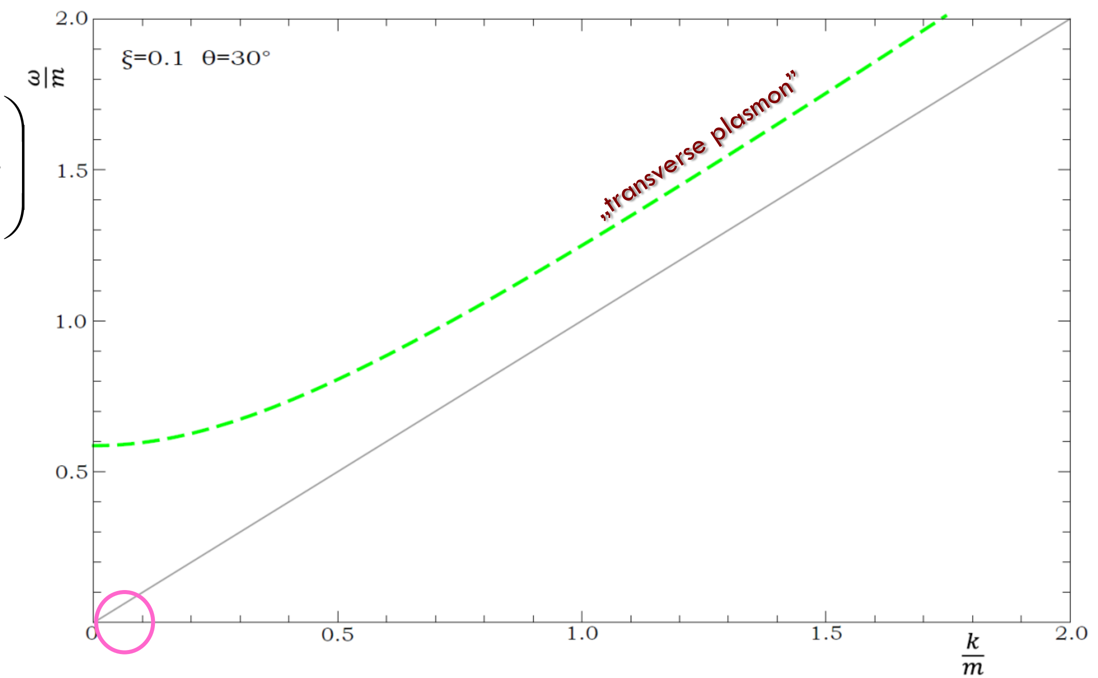
where

$$\lambda=\frac{\pi}{4}\left[1-\frac{\xi}{2}\left(\frac{7}{3}-5\cos^2\vartheta\right)\right]m^2$$

$$k_A\equiv m\Re\sqrt{\frac{\xi}{3}\left(2\cos^2\vartheta-1\right)}$$



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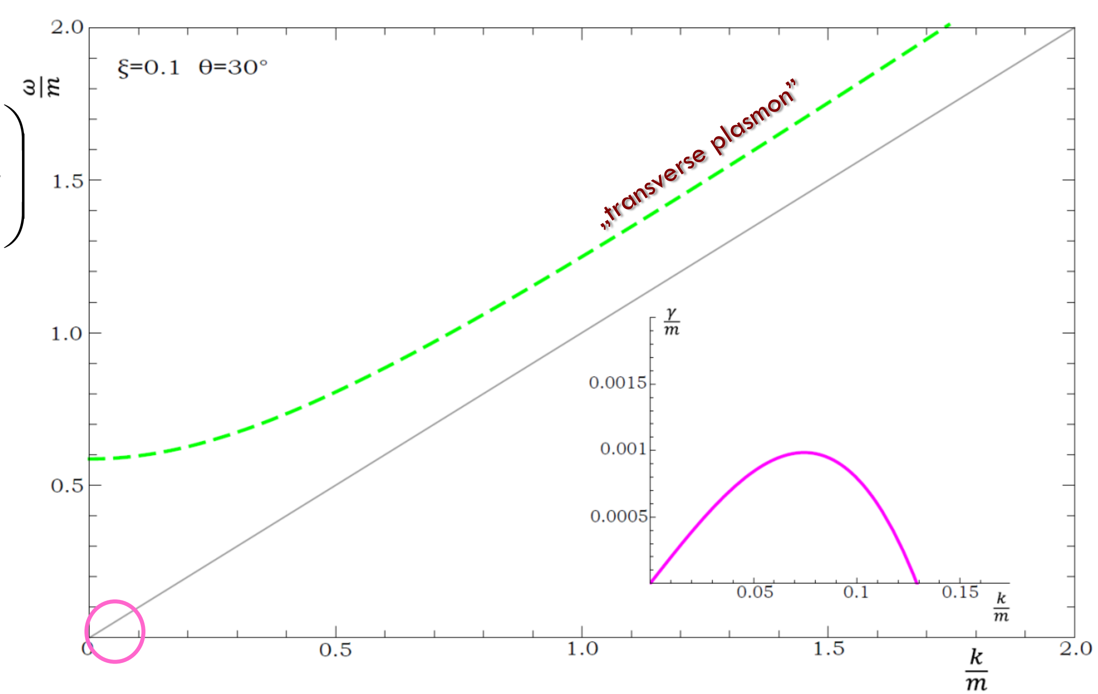
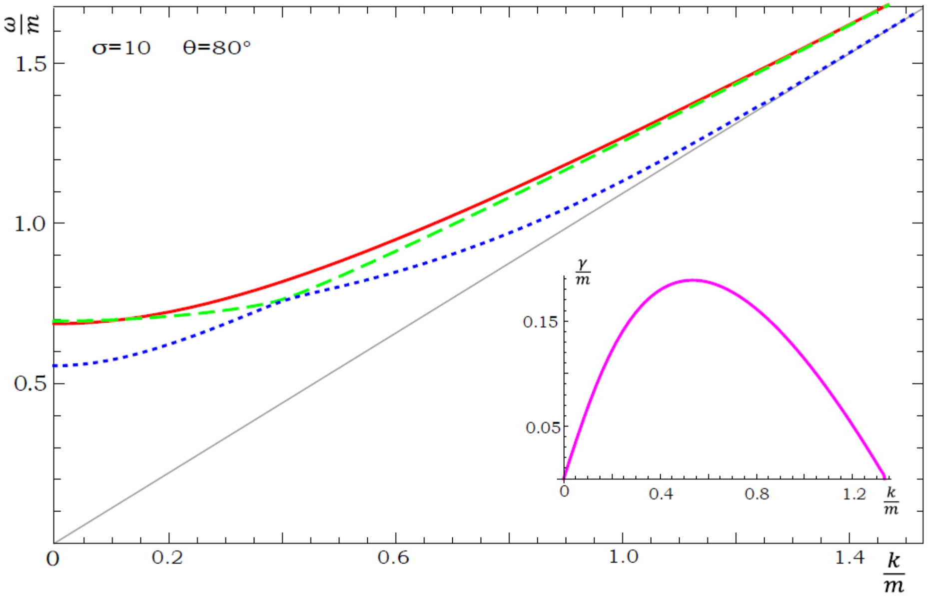
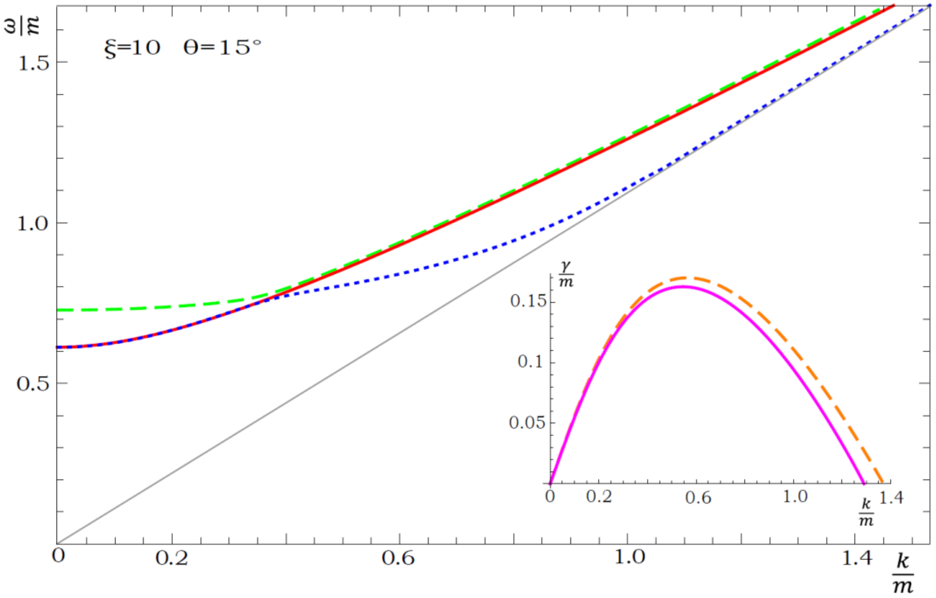


Finite prolateness or oblateness

$\sigma = 10$ and $\theta = 80^0$

$\xi = 10$ and $\theta = 15^0$

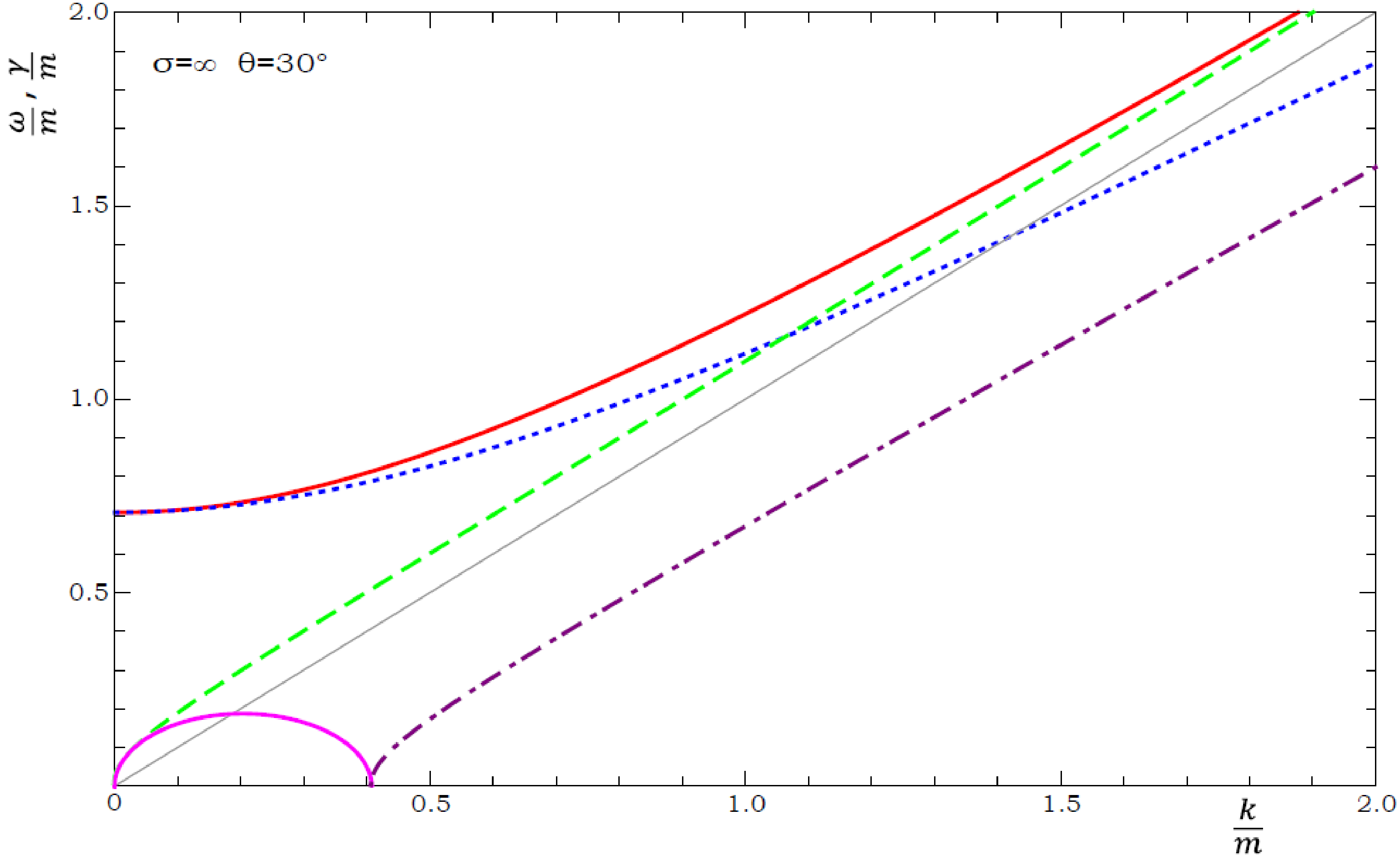




Extremly prolate QGP $\sigma \to \infty$

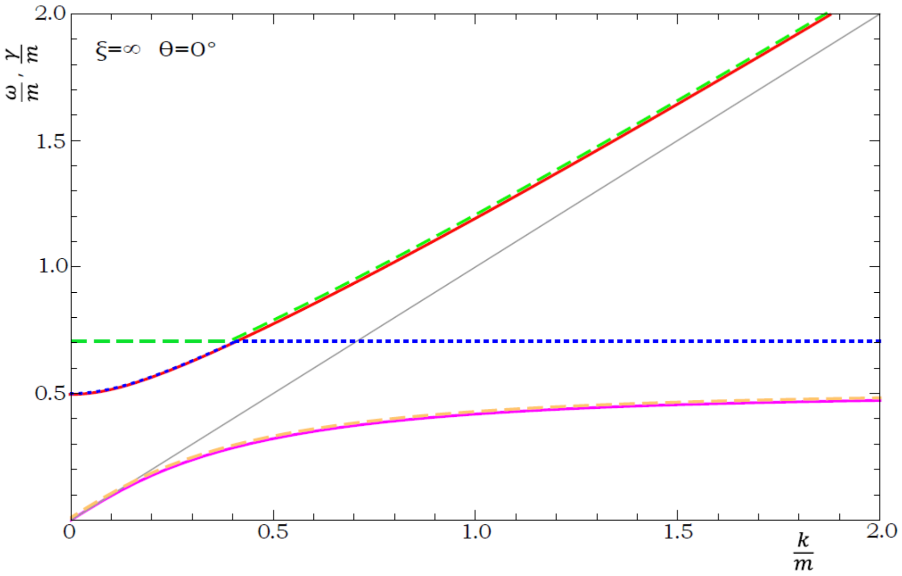
Extremly prolate distribution: $f(\mathbf{p}) \sim \delta(p_T)$

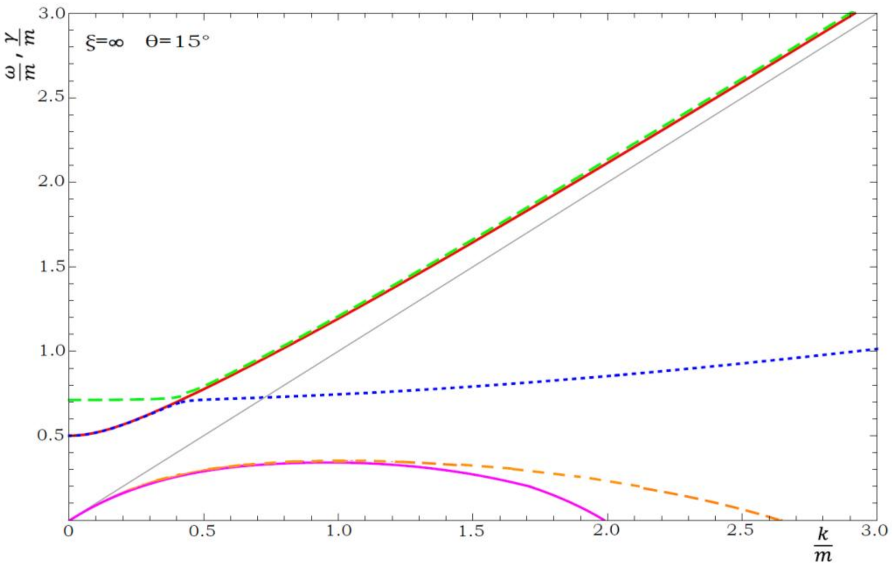
8 solutions



Extremly oblate QGP $\xi \to \infty$

Extremly oblate distribution: $f(\mathbf{p}) \sim \delta(p_L)$ 8 or 10 solutions

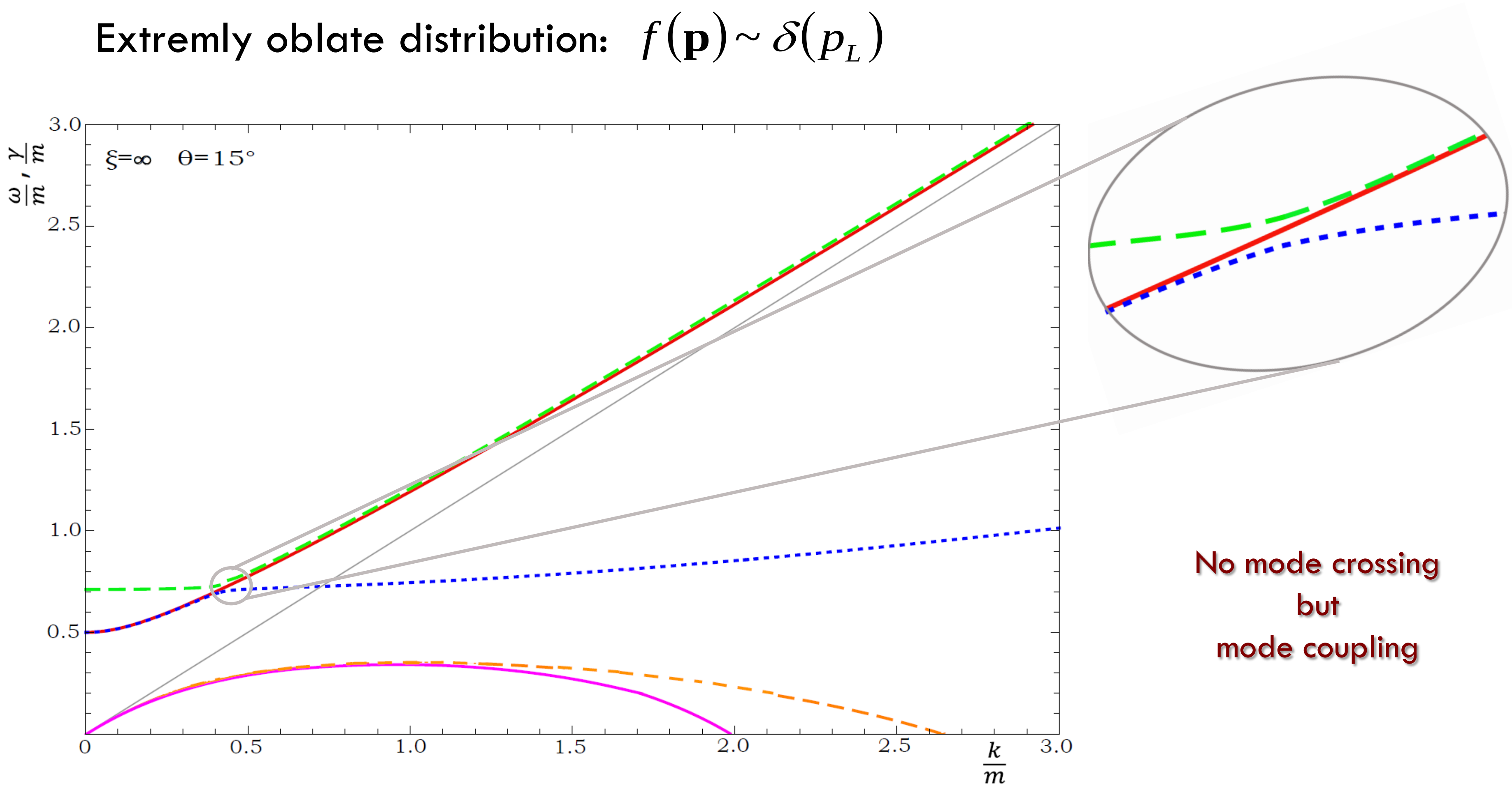




Mode crossing ?

Extremly oblate QGP $\xi \to \infty$

Extremly oblate distribution: $f(\mathbf{p}) \sim \delta(p_L)$



No mode crossing
but
mode coupling

Number of solutions

The number of modes for each system

Momentum distribution	Number of real modes	Number of imaginary modes	Total number of modes	Maximal number of modes
extremely prolate	$6 + 2\Theta(k - k_{\rm p})$	$2\Theta(k_{\rm p} - k)$	8	8
weakly prolate	6	$2\Theta(k_{\rm C} - k)$	$6 + 2\Theta(k_{\rm C} - k)$	8
isotropic	6	0	6	6
weakly oblate	6	$2\Theta(k_{\rm A} - k) + 2\Theta(k_{\rm C} - k)$	$6 + 2\Theta(k_{\rm A} - k) + 2\Theta(k_{\rm C} - k)$	10
extremely oblate	6	$2\Theta(k_{\rm oA} - k) + 2\Theta(k_{\rm oG} - k)$	$6 + 2\Theta(k_{\rm oA} - k) + 2\Theta(k_{\rm oG} - k)$	10

Conclusions

- Systematical analysis of the complete mode spectrum is performed.
- The number of modes is found in every case.
- Analytical and numerical solutions are found.
- <u>Complete</u> spectrum of modes is needed to compute various plasma characteristics e.g. the energy loss in anisotropic QGP.