Decay of the Higgs and other spectators after inflation

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inflation

dynamics unknown

slow rolling scalar(s)?

light scalar spectators exist

$H_* \approx const.$

$m \ll H_*$

$\rho_\sigma \ll \rho_{\text{inf}}$

example: the higgs

others? – the curvaton
contents:

spectator dynamics

1. during

2. after

inflation
spectators can play a dynamical role after inflation

1. because of their field perturbations
   - modulated (p)reheating  \( \Gamma_{\text{inf}} = \Gamma(\sigma) \)
   - modulated end of inflation  \( t_{\text{end}} = t(\sigma) \)
   - conversion of isocurvature into adiabatic (curvaton)

2. because of their classical evolution
   - flat directions & Affleck-Dine BG
   - moduli problems
   - relaxation to (the correct ) vacuum
DURING INFLATION

massless scalars in an expanding background

stochastic treatment (cf. Starobinsky)
Langevin (simplified):

decompose field into UV and IR parts:

\[ \Phi_{IR} \propto \int dk W(k, t) \phi_k(t) \]

\[ W(k, t) = \theta(k - x a H) \]

\[ \dot{\Phi}_{IR} = -\frac{\partial}{3H\partial \Phi} V(\Phi_{IR}) + s(x, \eta) \quad \text{for } k << a H \]

stochastic term, white noise correlators

\[ \langle SS \rangle(dN) = \left(1 + x^3\right) \frac{H^2 dN}{4\pi^2}, \quad k = x a(N) H \]

\textbf{N = \# efoldings}
inflationary fluctuations

massless field

\[ \langle \phi^2 \rangle = \frac{1}{4\pi^2} H^2 N \]

\( N = \# \) of efold

evolution of pdf: Fokker-Planck

\[ \frac{\partial P}{\partial t} = \frac{1}{3H} \frac{\partial}{\partial \phi} \left[ V' (\phi) P \right] + \frac{H^3}{8\pi^2} \frac{\partial^2}{\partial \phi^2} P \]

equilibrium pdf:

\[ P \propto \exp\left( -8\pi^2 \frac{V}{3H^4} \right) \]
example

\[ V = \frac{1}{2} m^2 \sigma^2 \]

\[ m = 0.01H \]

\[ \sigma / H \]

\[ N \]

relaxation time

\[ N_{rel} = \frac{3H_*^2}{m^2} \]

de coherence time

\[ N_{dec} = \frac{3H_*^2}{2m^2} \]
quartic potential

\[ V = \frac{1}{4} \lambda \phi^4 \]

\[ \lambda = 0.003125 \]
relaxation time

\[ N_{\text{rel}} \approx \frac{11.3}{\sqrt{\lambda}} \]

decoherence time

\[ N_{\text{dec}} \approx \frac{5.65}{\sqrt{\lambda}} \]

Example: the higgs

\[ V \approx \frac{1}{4} \lambda h^4 \]

\[ \text{RGE} \rightarrow \lambda \approx 0.01 \quad \text{at inflationary scales (?)} \]

decoherence at ~ 60 efolds

\[ h_* \approx 0.36 \lambda^{-1/4} H_* \approx 1.1 H_* \]

mean field from equilibrium dist

effective higgs mass

\[ m^2_{h_*} \approx V''(h_*) = 0.40 \lambda^{1/2} H_*^2 = 0.04 H_*^2 \]
the Higgs condensate

at equilibrium after inflation:

\[ h_* \approx 0.36 \lambda^{-1/4} H_* \approx 1.1 H_* \]

"typical value"

how does the condensate decay?

1. inflaton decays first \( \rightarrow \) thermal background (assuming thermalization)

2. Higgs decays first (in matter dominated universe)

if Higgs is to modulate inflaton decay, it can do so only while the condensate is still there
Higgs starts to move ... and becomes effectively massive at oscillations after inflation.

\[
\frac{H_{\text{osc}}}{H_*} \sim \frac{1}{4} \lambda_*^{3/4}
\]

oscillations \( t_{\text{osc}} \lesssim O(10^2) H_*^{-1} \)

decay rates depend on the value of the Higgs background field

\[ m_h, m_f, m_G \sim h \]
perturbative decays

to gauge bosons: kinematically blocked until

\[ t \sim \lambda (H_*)^{-3/8} \left( \frac{H_*}{10^2 \text{GeV}} \right)^{3/2} H_*^{-1} \]

not efficient unless \( H_* \ll 10^5 \ \text{GeV} \).
perturbative decays (cont)

to fermions: top channel kinematically blocked, others ok

\[ \Gamma(h \rightarrow bb) = \frac{3\sqrt{3}\lambda y_b^2 h_{osc}}{16\pi} \left(1 - \frac{2y_b^2}{3\lambda} \right)^{3/2} \sim 10^{-6} \lambda^{3/4} H_* \]

takes >> 10^6 Hubble times

not efficient

others (to gluons, photons) even less efficient
non-perturbative decays

resonant production of gauge bosons

W’s in the unitary gauge: (abelian approx.)

\[ \ddot{W}_\mu^\pm (z, k) + \omega_k^2 W_\mu^\pm (z, k) = 0, \quad \omega_k^2 = \frac{k^2}{a^2 \lambda h_{osc}^2} + q_W \frac{h(z)^2}{h_{osc}^2} + \Delta. \]

Higgs eq of motion

\[ \frac{\ddot{h}}{h_{osc}} + 3 \frac{H}{\sqrt{\lambda} h_{osc}} \frac{\dot{h}}{h_{osc}} + \left( \frac{\dot{h}}{h_{osc}} \right)^3 = 0 \]

= 0 for matter domination

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Table 1: Numerical values of the characteristic exponent \( \mu_k \) of \( k = 0 \) modes for a set of different values of \( H_s \).

<table>
<thead>
<tr>
<th>( H_s / \text{GeV} )</th>
<th>( \lambda )</th>
<th>( (q_W, \mu_k) )</th>
<th>( (q_Z, \mu_k) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10^4</td>
<td>0.09</td>
<td>(1.1, 0.14)</td>
<td>(1.5, 0.26)</td>
</tr>
<tr>
<td>10^6</td>
<td>0.04</td>
<td>(2.3, 0.25)</td>
<td>(3.2, 0.00)</td>
</tr>
<tr>
<td>10^8</td>
<td>0.02</td>
<td>(4.4, 0.00)</td>
<td>(6.2, 0.14)</td>
</tr>
<tr>
<td>10^{10}</td>
<td>0.005</td>
<td>(16, 0.22)</td>
<td>(24, 0.00)</td>
</tr>
</tbody>
</table>
Example: resonant Z production

$k = 0, q = 1.5$

Floquet $\sim 0.26$

no backreaction

$z = \sqrt{\lambda h^2_{osc}} (t - t_{osc})$
resonant production of fermions, Higgses: inefficient

**estimation of the end of decay time**

effective Higgs mass generated by Ws starts to affect Higgs dynamics:

\[ m_{h(W)}^2 = 2q_W \lambda \langle W^+ W^- \rangle = 2q_W \lambda \langle W^2 \rangle \]

integrate up to a cut-off scale

find when \( m_{h(W)}^2 \approx m_h^2 \)

<table>
<thead>
<tr>
<th>( H_*/\text{GeV} )</th>
<th>( \lambda )</th>
<th>( H_{osc}/H_{dec} )</th>
<th>( n_{\phi}^{\text{dec}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10^4</td>
<td>0.09</td>
<td>370</td>
<td>1000</td>
</tr>
<tr>
<td>10^6</td>
<td>0.04</td>
<td>360</td>
<td>1700</td>
</tr>
<tr>
<td>10^8</td>
<td>0.02</td>
<td>630</td>
<td>5100</td>
</tr>
<tr>
<td>10^{10}</td>
<td>0.005</td>
<td>340</td>
<td>7700</td>
</tr>
</tbody>
</table>

NB: resonant production cannot deplete the condensate completely
resonance structure

for SM, the resonance is broadish

best fit SM: $q_W = 18$, $q_Z = 29$
role of non-abelian terms?

in the Hartree approximation

(non-linear terms $\rightarrow$ vevs of linear solutions)

H = $10^{14}$ GeV

but: backreaction on Higgs mass kicks in earlier

abelian a good approximation at early stages
the opposite case:

narrow resonance (BSM)

example $q = 0.1$

need lattice simulations
Higgs and large H

how to reach SM vacuum?

\[ \lambda(h_{\text{max}}) + \frac{\beta(h_{\text{max}})}{4} = 0 \]

\[ V_{\text{SM}}^{1/4}(h) \ll (3M_{\text{P}}^2H^2)^{1/4} \approx 1.6 \times 10^{16}\text{GeV} \]

Espinosa, Giudice, Riotto
Kobakhidze, Spencer-Smith
KE, Meriniemi, Nurmi

very sensitive to RGE
fluctuations in kinetic energy $\sim H^4$

wrong vacuum unless $V^{1/4}(h_{\text{max}}) \gtrsim 10^{14}\text{GeV}$
false vacuum during inflation?

- tunneling to the top of $V$ (\(= \min \Delta F\))
- 1. stochastic epoch \(N < 20\)
- 2. classical drift \(N < 70\)
- 3. \(h \rightarrow 0\): quantum fluctuations dominate

mean field reaches equilibrium value if inflation lasts \(> O(100)\) efolds

\[\xi R^2 \rightarrow \text{RGE?} \quad 1407.3142\]
so what?

Higgs exists, inflation (probably) took place

all of this actually happened

for other spectators exact decay time may matter much
if a spectator remains around for some time after inflaton decay, it can generate the observed curvature perturbation

the curvaton

curvature perturbation generated after inflation

require

- curvaton decay products thermalize with radiation

initial curvaton isocurvature perturbation $\delta \sigma(x)$ is converted to an adiabatic perturbation

- $\zeta_{\text{inf}} \ll \zeta_{\sigma} \approx 10^{-5}$
curvaton perturbation remains $\sim$ constant (depends on potential)

curvatton dominates

perturbation grows
curvature perturbation
\[ \zeta = \frac{H_*}{3\pi\sigma_*} r_{\text{eff}} \approx 10^{-5} \]

\[ r_{\text{eff}} \approx r_{\text{dec}} = \frac{3\rho_\sigma}{3\rho_r + 4\rho_\sigma} \]

initial value \( \sigma_* \) not fixed by mean field fluctuations (?) 

simplest potential
\[ V = \frac{1}{2} m^2 \sigma^2 \]

\[ f_{NL} \approx \frac{3}{8r} \]

large non-gaussianity = subdominant curvaton

Lerner, Melville 1402.3176
the amplitude of the curvature perturbation depends on the time of decay of the curvaton

must account for the decay mechanism

OPTIONS

1. throw in a $\Gamma$

2. couple the curvaton to SM and compute
Higgs as the curvaton?

NO: \( V \approx \frac{1}{4} \lambda h^4 \)

higgs oscillations behave as radiation

relative density does not grow

but could be a field modulating either a) end of inflation or b) inflaton decay rate
Curvaton coupled to SM higgs

Only renormalisable coupling to standard model:

\[ V(\sigma, \Phi) = \frac{1}{2} m_\sigma^2 \sigma^2 + g^2 \sigma^2 \Phi^\dagger \Phi + \lambda \left( \Phi^\dagger \Phi - \frac{v^2}{2} \right)^2 \]

Free parameters: \( g, m_\sigma, \sigma^*, H^* \)

- no perturbative decay (no three-point coupling)
- but expect non-perturbative decay, just like preheating
- there is a thermal background from inflaton decay
- higgs has a thermal mass \( m^2(H) = g_T^2 T^2, \quad g_T^2 \approx 0.1 \)
resonant production of higgs particles

- curvaton is oscillating
- higgs has mass $g\sigma$
- resonant production of higgs with momentum $k$
  - depends on the dispersion relation
  - requires non-adiabacity at zero crossing

IR modes with $k < k_{cut}$

$$K_{cut}(j) = \frac{k_{cut}(j)}{a} \simeq j^{-3/8} \sqrt{g m \sigma_*}$$

$j$th zero crossing

(some differences between broad and narrow resonance)

(corrected by thermal mass)

(thermal background also induces mass for curvaton)
dispersion relation

• Higgs equation of motion: $j = \text{time} = \# \text{zero crossings}$

$$\frac{d^2 \chi_\alpha}{dx^2} + \left( \kappa^2(j) + g_T^2 a^2(j) \frac{T^2(j)}{k_{cut}^2(j)} + x^2 \right) \chi_\alpha = 0$$

• effective frequency:

$$\omega_k^2(j) = \kappa^2(j) + \frac{m_\sigma}{H_*} g_T^2 \frac{8}{14\pi} \left( \frac{T_*}{k_{cut}(j)} \right)^2 + x^2$$

\[ \kappa^2(j) \approx \left( \frac{K}{K_{cut}(j)} \right)^2 \quad x \equiv K_{cut}(j) t \]
Adiabaticity violated if...

\[ 0 \leq k^2 \leq k^2_{\text{cut}}(j) - \frac{8m_\sigma g^2_T}{14\pi H_*} T^2 > 0 \]

- RHS should be > 0
- Thermal mass of Higgs blocks resonance!
- Unblocked after **many** oscillations:

\[ j \gtrsim j_{\text{NP}|_{RD}} \equiv \frac{g^8_T}{g^4 g^2_*} \left( \frac{M_P}{\sigma_*} \right)^4 \]
need to consider

• as the curvaton is oscillating, the resonance parameter $q$ also evolves
  - unblocking: broad or narrow resonance?

• as the curvaton is oscillating, its relative energy density is increasing
  - unblocking: radiation or matter (=curvaton oscillation) dominated
it is not enough that the resonance becomes unblocked – energy must also be transferred to higgs particles

• if decay products do not thermalise:

\[
\rho_H(j) \approx 0.028 f(q) q(j)^{1/4} \left( \frac{1 + \frac{2}{e} \Delta j^{-1}}{\left( \frac{1}{3} + \frac{J_{NP} + \Delta j}{J_{EQ}} \right)^2} \right)^2 \left( \frac{\sigma_*}{M_P} \right)^6 \frac{1}{\left( 1 + \frac{\Delta j^{-1}}{(e/2+1)} \right)^{3/2}} \times (g m_{\sigma} \sigma_*)^2
\]

where

\[
f(q) \equiv 1 + \frac{2 + e}{\exp \left( g_T q^{1/4} - 1 \right)}
\]

• if decay products thermalise ( \( m_{\sigma} \ll T(j_{NP}) \))

\[
\rho_H(j_{NP} + \Delta j) \approx \rho_H(j_{NP}) \left[ 1 + \frac{1}{g_*} 0.01357 \Delta j \right]
\]
Possible timescales

- e.g. narrow resonance in matter-domination

- depends on resonance parameter $q \equiv \left(\frac{g\sigma(t)}{2m}\right)^2$

- $q$ decreases with time

- narrow resonance: $T_{NP} = \frac{m\sigma(1 + \mathcal{O}(q))}{gT}$

- narrow resonance energy transfer:
  \[
  \Delta j \sim -\frac{\log(g^2 q^{1/2}(j_{NP}))}{\pi q(j_{NP})}
  \]
but: for a range in parameters, thermal blocking persists until electroweak symmetry breaking
treatment of thermal effects: see also
Harigaya, Mukaida, JHEP 1405 (2014) 006
Mukaida, Nakayama, Takimoto, JCAP 1406 (2014) 013
Complete minimal curvaton-Higgs model

**instant reheating**

+ CW at 1 loop

+ dim 5 operators

\[ \mathcal{L}_5 \propto \frac{1}{M_P} \sigma f \Phi f \quad \Rightarrow \quad \Gamma_5 \approx \frac{m_\sigma^3}{M_P^2} \]

responsible for completing the decay

+ evaporation by thermal scattering

KE, Lerner, Takanashi
WIMP freeze-out at 10 GeV

at 1 TeV

BBN
CONCLUSIONS:

- light spectators likely to exist (Higgs, curvatons, ...)

- how do they settle in their vacua?
  thermal effects important

- dynamics depend on the inflationary Hubble scale

- High scale inflation: cosmology constrains particle physics?