EFTs for non-relativistic particles in a medium: Application to quarkonium and Majorana neutrinos

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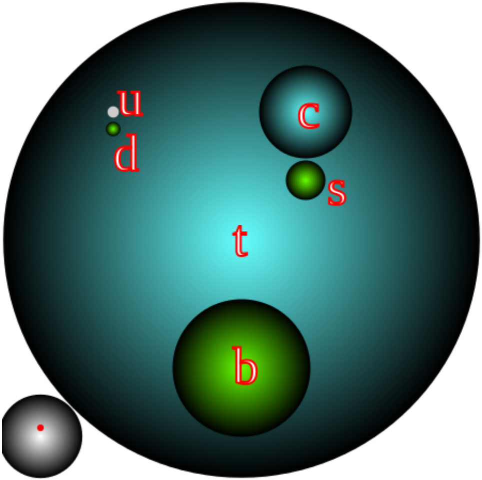
Outline

Introduction. Quarkonium example

Why are heavy particles interesting?



Why are heavy particles interesting?

In many situations we will find $M \gg T \gg m$.

- Particles of mass M are non-relativistic. There are more difficult to create but they give interesting information about the medium because they see it as "spectators". They tend to be out of equilibrium.
- Particles of mass m can be approximated as massless. They will form the bulk of the particles in the medium.

Interesting heavy particles

In heavy-ion collisions

- B and D mesons.
- Heavy quark energy loss.
- Heavy Quarkonium.

Dark matter and baryogenesis

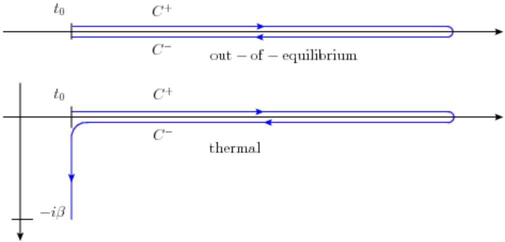
- Heavy sterile neutrinos, resonance scenario.
- Lightest supersymetric particle (still heavier than most SM particles).

HP in thermal field theory. Advantages

There are small parameter that lead to simplifications, double expansion.

- $\frac{T}{M}$
- $\rho_{HP} \ll 1$. For example in thermal equilibrium $e^{-M/T} \ll 1$.

Small densities and the doubling of degrees of freedom



Picture taken from T. Konstandin (2013)

Small densities and the doubling of degrees of freedom

For a static quark in thermal equilibrium

$$\begin{pmatrix} \frac{i}{k_0+i\epsilon} & 0 \\ 2\pi\delta(k_0) & \frac{-i}{k_0-i\epsilon} \end{pmatrix} + \mathcal{O}(e^{-T/M})$$

If I want to compute a time-ordered correlator of heavy particles fields I only need to consider heavy fields living in C^{+}.

HP in thermal field theory, complications

HP are non-relativistic, $v \ll 1$.
In a perturbative computation of the binding energy.

$$E = m_Q \alpha_s \sum_{n=0}^{\infty} \alpha_s^n A_n(v)$$

because v is small we can not know the size of $A_n(v)$, for example, it could go like $1/v$.
If we use EFT the computation is an expansion in both v and α_s.

$$E = m_Q \alpha_s v^2 \sum_{n,m} \alpha_s^n v^m B_{n,m}$$

now $B_{n,m}$ is of order 1.
In perturbation theory $v \sim \alpha_s$.

Heavy quarkonium is non-relativistic

- EFT philosophy, one energy scale at a time.
- When a heavy quark annihilates with a heavy antiquark the energy involved is of order m_Q. Hard scale.
- The momentum of a heavy quark in the reference frame in which heavy quarkonium is at rest is of order $m_Q v$, where $v \ll 1$. Soft scale. This is why we say it is a non-relativistic system.
- In a bound state the total energy is of the order of magnitude of the kinetic energy, $\frac{p^2}{m_Q} \sim m_Q v^2$. Ultrasoft scale.

EFTs for non-relativistic particles. NRQCD

NRQCD, EFT for heavy quarks inside quarkonium.
Caswell and Lepage (1986), Bodwin, Braaten and Lepage (1995)

$$\mathcal{L} = \psi^\dagger \left(iD_0 + \frac{D^2}{2M} \right) \psi + sub - leading$$

- Matching can be done always at T=0. Also useful on the lattice.
- A field ψ for the heavy quark, and analog ξ to the anti-quark.
- Infinite number of terms. Given a desire precision there is a systematic way to know how many terms are needed, power counting. But...

Potential NRQCD

- In NRQCD's operators there are two scales, the soft and ultrasoft. This does not follow the one scale at a time approach.
- As a result power counting in NRQCD is not trivial.

A solution is to integrate out also the soft scale. This is pNRQCD Soto and Pineda (1998)

$$\mathcal{L} = \int d^3r \left[S^\dagger(t,\mathbf{r},\mathbf{R})(i\partial_0 - V_s(r))S(t,\mathbf{r},\mathbf{R}) \right.$$
$$\left. + O^\dagger(t,\mathbf{r},\mathbf{R})(iD_0 - V_o(r))O(t,\mathbf{r},\mathbf{R}) \right] + sub - leading$$

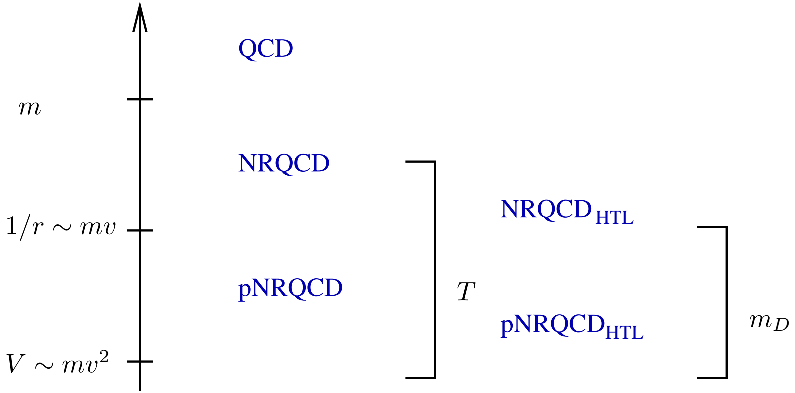
Potential NRQCD

$$\mathcal{L} = \int d^3r \Big[S^\dagger(t,\mathbf{r},\mathbf{R})(i\partial_0 - V_s(r))S(t,\mathbf{r},\mathbf{R})$$
$$+ O^\dagger(t,\mathbf{r},\mathbf{R})(iD_0 - V_o(r))O(t,\mathbf{r},\mathbf{R}) \Big] + sub - leading$$

- S field is a singlet, O field is an octet.
- Quantum mechanics plus corrections due to the interaction with ultrasoft gluons. Quarkonium for ultrasoft gluons is a color dipole.
- The potential can depend on the medium.

Now power counting is trivial.

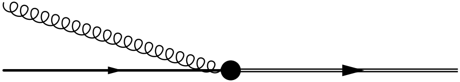
EFTs for heavy quarkonium



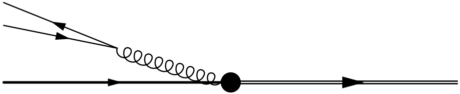
Brambilla, Ghiglieri, Petreczky and Vairo (2008), M.A.E and Soto (2008)

Perturbative computations of cross-section for quarkonia in the literature

Gluo-dissociation

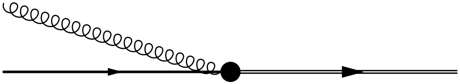


Bhanot and Peskin (1979)
Quasi-free dissociation



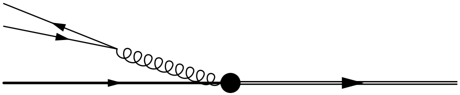
Combridge (1978), Park, Kim, Song, Lee and Wong (2007)

Gluo-dissociation



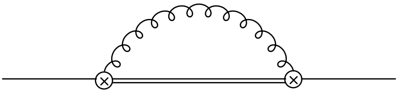
- Leading-order in α_s.
- Small phase-space.

Quasi-free dissociation



- NLO in α_s.
- Bigger phase-space.

Power counting for gluo-dissociation at $T \gg E$



- The gluon is on-shell.
- The energy difference between a heavy quarkonium state and two free heavy quarks is of order $m_Q v^2$.

This effect is found when taking into account the energy region $m_Q v^2$. It can be studied using pNRQCD

$$\delta\Gamma \propto \alpha_s r^2 T(\Delta E)^2$$

Power counting

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- Multipole expansion in the Lagrangian.
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-

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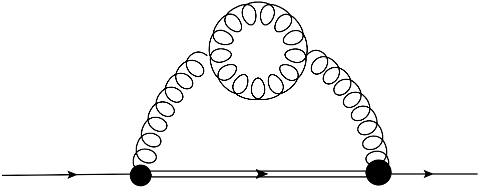
- Bose-enhancement.
-

Power counting

$$\delta\Gamma \propto \alpha_s r^2 T (\Delta E)^2$$

- Multipole expansion in the Lagrangian.
- Bose-enhancement.
- The only scale that appears in the integral, dimensional analysis.

Power counting for quasi-free dissociation, case $1/r \gg T$



- There are two possible cuts. One contributes a NLO correction to gluo-dissociation and the other contributes to quasi-free dissociation.
- In the cut contributing to quasi-free the gluon attached to the heavy quark can have any momentum. Dominated by scale T for $T \gg E$.

Power counting for quasi-free dissociation, case $1/r \gg T$

$$\delta\Gamma \propto \alpha_s^2 r^2 T^3$$

- If $E \sim T$ or $E \gg T$ then this is always a sub-leading diagram. But if $T \gg E$ different things can happen.
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- Multipole expansion in the Lagrangian.
- The only scale that appears is T.

Conclusion. What is the dominant mechanism?

It only depends on the relation between $m_D \sim \sqrt{\alpha T}$ and E.

- If $E \gg m_D$ the dominant mechanism is gluo-dissociation. HQ absorbs a gluon and dissociates.
- If $m_D \gg E$ the dominant mechanism is quasi-free dissociation. HQ scatters with a parton and dissociates.

EFT results

Gluo-dissociation:Brambilla, M.A.E, Ghiglieri and Vairo (2011)

Scattering:Brambilla, M.A.E, Ghiglieri and Vairo (2013)

Heavy Majorana neutrinos in a thermal bath

EFT for heavy Majorana neutrinos

$$\mathcal{L} = N^\dagger \left(i\partial_0 + \frac{\nabla^2}{2M} \right) N + sub - leading$$

- Interaction is always given by higher order operators, always suppressed by powers of M.
- LO thermal corrections will always come from operators whose dimension is smaller.
- We have to respect the symmetries.

LO interaction with Higgs

$$N^\dagger N \phi^\dagger \phi$$

Operator of dimension 5

$$\delta\Gamma \propto \frac{T^2}{M}$$

LO interaction with relativistic fermions

$$N^\dagger N \bar{L} L$$

Operator of dimension 6. But in thermal equilibrium $\langle L\bar{L}\rangle = 0$.
Need to include a derivative D_0.

$$\delta\Gamma \propto \frac{T^4}{M^3}$$

LO interaction with gauge bosons

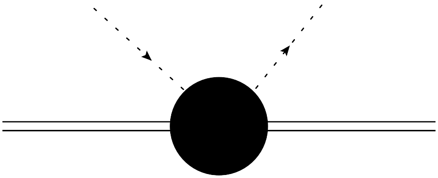
$$N^\dagger N F^{\mu\nu} F_{\mu\nu}$$

Operator of dimension 7.

$$\delta\Gamma \propto \frac{T^4}{M^3}$$

Easy way to compute thermal corrections to a process

Compute in the full theory the scattering of a heavy neutrino with a Higgs particle. Matching computation at T=0.



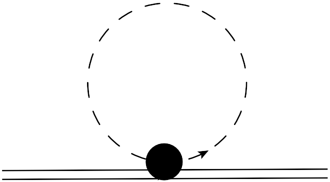
$$\delta\mathcal{L} = \frac{1}{M}\left(\Re a - \frac{3i}{8\pi}|F|^2\lambda\right) N^\dagger N \phi^\dagger \phi$$

Easy way to compute thermal corrections to a process

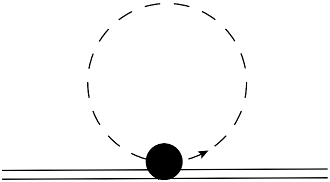
See how this operator contributes to the process you are interested

$$\delta\mathcal{L} = \frac{1}{M}\left(\Re a - \frac{3i}{8\pi}|F|^2\lambda\right) N^\dagger N \phi^\dagger \phi$$

Example, Corrections to the decay width. Tadpole diagram.



LO correction to the decay width



$$\delta\Gamma = -\frac{3|F|^2\lambda}{4\pi M}\langle\phi^\dagger\phi\rangle = -\frac{\lambda|F|^2 T^2}{8\pi M} + \mathcal{O}\left(\frac{T^4}{M^3}\right)$$

Agree with Savio, Lodone and Strumia (2011) and Laine and Schroeder (2012).

Degenerate case

Two heavy neutrinos with a degenerate mass

$$M_1 = M$$
$$M_2 = M - \Lambda$$
$$M \gg \Lambda$$

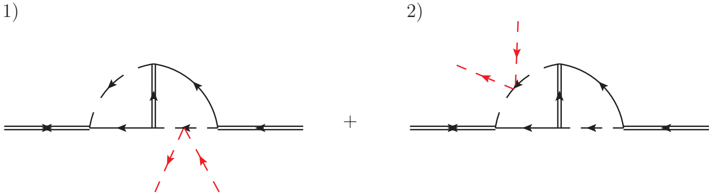
In this scenario baryogenesis is enhanced. Flanz, Pachos, Sarkar and Weiss (1996).

Direct Lepton asymmetry. Thermal corrections

$$A = \sum_i \frac{\Gamma(N_i \to leptons) - \Gamma(N_i \to antileptons)}{\Gamma(N_i \to leptons) + \Gamma(N_i \to antileptons)}$$

- Compute scattering of heavy neutrinos with Higgs.
- Using the cutting rules (at $T = 0$) keep track of which diagram contribute to decay into leptons and which to antileptons.
- Compute the same tadpole diagram in the EFT.

Direct Lepton asymmetry. Thermal corrections



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Direct Lepton asymmetry. Thermal corrections

Preliminary result

$$\delta A = \frac{\mathrm{Im}\left[(F_1 F_2^*)^2\right]}{16\pi}\frac{|F_2|^2-|F_1|^2}{|F_1|^2|F_2|^2}\left(\frac{T}{M}\right)^2 \times$$
$$\times\left[\lambda(1-2\ln 2)+(3g^2+g'^2)\frac{2-\ln 2}{24}\right]$$

Instead of three loop in thermal field theory $\rightarrow$ two loop in normal quantum field theory (but using optical theorem)+ tad-pole in thermal field theory

Conclusions

Conclusions

- Heavy particles imply several simplifications and complications that can be taken into account by using EFTs.
- Power counting is a very powerful tool.
- Heavy quarkonium in thermal equilibrium in perturbation theory is well understood in EFT. To illustrate this I showed the cross-section for the dominant decay process in a wide range of temperature.
- For heavy sterile neutrinos a lot can be known by looking at the operator that describes the interaction with the Higgs.

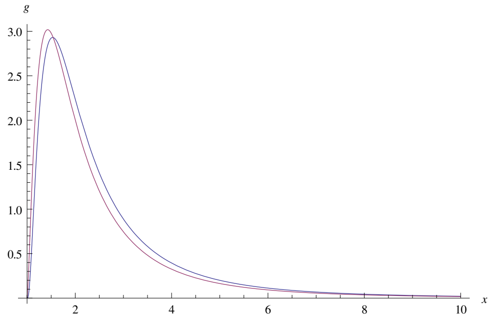
End

Back-up

Cross-section for gluo-dissociation

$$\sigma_{gd}(k) = \sigma_R g(x)$$

with $\sigma_R = \frac{32\pi C_F \alpha_{\rm s} a_0^2}{3}$ and $x = \frac{k}{|E_1|}$



Bhanot and Peskin large N_c limit.

pNRQCD. Agrees with Brezinski and Wolschin (2011).

Cross-section for quasi-free dissociation. Some notation

$$\sigma(k, m_D) = \sigma_R f(x, y)$$

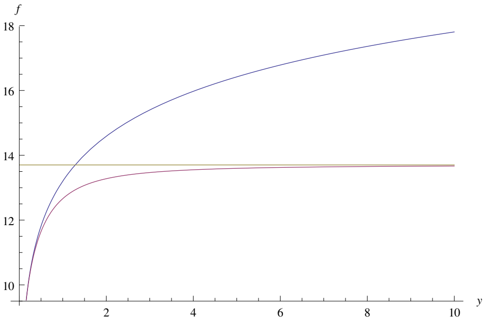
where

$$\sigma_R = 8\pi C_F \alpha_{\rm s}^2 N_F a_0^2$$
$$x = m_D a_0$$
$$y = k a_0$$

I will only show the result for the fermion part, the boson part is quantitatively and qualitatively very similar.

Cross-section for quasi-free dissociation.

$m_D a_0 = 0.001$



$\frac{1}{r} \gg T \gg m_D, T \sim \frac{1}{r} \gg m_D$ and $T \gg \frac{1}{r} \sim m_D$. Discrepancy between blue and red lines signals a failure of color dipole approximation.

Corrections in the EFT for heavy Majorana neutrinos

$$
\begin{aligned}
\mathcal{L}^{(3)}_{\text{N-SM}} = \; & b\, \bar{N}N \left(v\cdot D\phi^\dagger\right)\left(v\cdot D\phi\right) \\
& + c_1^{ff'} \left[\left(\bar{N}P_L\, iv\cdot DL_f\right)\left(\bar{L}_{f'}P_R N\right) \right. \\
& \qquad \left. + \left(\bar{N}P_R\, iv\cdot DL^c_{f'}\right)\left(\bar{L}^c_f P_L N\right)\right] \\
& + c_2^{ff'} \left[\left(\bar{N}P_L\,\gamma_\mu\gamma_\nu\, iv\cdot DL_f\right)\left(\bar{L}_{f'}\,\gamma^\nu\gamma^\mu\, P_R N\right) \right. \\
& \qquad \left. + \left(\bar{N}P_R\,\gamma_\mu\gamma_\nu\, iv\cdot DL^c_{f'}\right)\left(\bar{L}^c_f\,\gamma^\nu\gamma^\mu\, P_L N\right)\right] \\
& + c_3\, \bar{N}N\, \left(\bar{t}P_L\, v^\mu v^\nu\gamma_\mu\, iD_\nu t\right) + c_4\, \bar{N}N\, \left(\bar{Q}P_R\, v^\mu v^\nu\gamma_\mu\, iD_\nu Q\right) \\
& + c_5\, \bar{N}\,\gamma^5\gamma^\mu\, N\, \left(\bar{t}P_L\, v\cdot\gamma\, iD_\mu t\right) + c_6\, \bar{N}\,\gamma^5\gamma^\mu\, N\, \left(\bar{Q}P_R\, v\cdot\gamma\, iD_\mu Q\right) \\
& + c_7\, \bar{N}\,\gamma^5\gamma^\mu\, N\, \left(\bar{t}P_L\,\gamma_\mu\, iv\cdot Dt\right) + c_8\, \bar{N}\,\gamma^5\gamma^\mu\, N\, \left(\bar{Q}P_R\,\gamma_\mu\, iv\cdot DQ\right) \\
& - d_1\, \bar{N}N\, v^\mu v_\nu W^a_{\alpha\mu}W^{a\,\alpha\nu} - d_2\, \bar{N}N\, v^\mu v_\nu F_{\alpha\mu}F^{\alpha\nu} \\
& + d_3\, \bar{N}N\, W^a_{\mu\nu}W^{a\,\mu\nu} + d_4\, \bar{N}N\, F_{\mu\nu}F^{\mu\nu}\,.
\end{aligned}
$$

Full decay width

$$\delta\Gamma = \frac{|F|^2 M}{8\pi}\left[-\lambda\left(\frac{T}{M}\right)^2 + \frac{\lambda}{2}\frac{k^2 T^2}{M^4} - \frac{\pi^2}{80}\left(\frac{T}{M}\right)^4 (3g^2 + g'^2)\right.$$
$$\left. - \frac{7\pi^2}{60}\left(\frac{T}{M}\right)^4 |\lambda_t|^2\right]$$