Triality selection rules of Octonion in QCD

and

Astrophysics

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Abstract

Élie Cartan studied coupling of 4 dimensional spinors A, B, C, D and 4 dimensional vectors E, E' using Clifford algebra including symmetry of quaternion and octonion. In this model there appear a triality symmetry, and one can construct sectors of E and E' which cannot be detected by fermions in a definite sector.

I) Teikyo group applied this theory of É. Cartan to the analysis of the response of the bifurcation to perturbations applied in the electronic circuit of Chua and Muthuswamy. To the voltage of the memristic circuit $\dot{y}(t) \simeq \dot{i}_L(t)$, we added an external sinusoidal oscillation $\gamma \sin \omega t$, when the $\dot{x}(t) \simeq \dot{v}_C(t)$ is given by $y(t)/C$.

We observed the frequency of the single bifurcation mode divided by 8 is close to that of 7 bifurcation mode divided by 7. The data suggests the importance of octonion in the response.

We compared the frequency of the driving oscillation f_s and the frequency of the response f_d in the window and assigned

$W = f_s/f_d$ to each window. The É. Cartan's spinor was successful in reproducing the Devil's staircase route to chaos in the Muthuswamy-Chua system.

II) We applied the model to the decay of π^0, η, η' to $\gamma\gamma$. The Pauli spinor is treated as a quaternion and the Dirac spinor is treated as an octonion. In the π^0 decay, the two final vector fields belong to the same group (EE or $E'E'$), and we call the diagram rescattering diagram. In the decay of η, and η', there are twisted diagram processes, which allow production of vector particles in a different group (EE' or $E'E$). Qualitative differences of the η, η' decay and π^0 decay can be explained by the É. Cartan's spinor.

III) Assuming the triality selection rules of octonions, dark matter is interpreted as matter emitting photons in a different triality sector than that of electromagnetic probes in our world. The octonion makes the running coupling constant of electroweak interaction and gravitation at the Planck scale different.

I. Introduction

- In 2007, a possibility of a sector of unparticle, which is scale invariant and very weakly interacting with the rest of the standard model was discussed[Georgi,2007]. The unparticle field that interacts with particles of the standard model will give mass to these particles, which is scale invariant. As the universe expands, the unparticle will produce Higgs particles, and as the universe expands, since the density of the Higgs is assumed to be invariant, the total mass of the universe will increase. It violates energy conservation rule. A possible solution is to assume that there are invisible universes which are contracting.

- É. Cartan extended the SU(2) spinor $\phi = {}^t(A, B)$ with elements ξ_{**} which has an even number of indices and spinor $\psi = {}^t(C, D)$ with elements ξ_* which has an odd number of indices[E.Cartan, 1966],

$$
\begin{aligned}
A &= \xi_{14}\sigma_x + \xi_{24}\sigma_y + \xi_{34}\sigma_z + \xi_0\mathbf{I} \\
B &= \xi_{23}\sigma_x + \xi_{31}\sigma_y + \xi_{12}\sigma_z + \xi_{1234}\mathbf{I} \\
C &= \xi_1\sigma_x + \xi_2\sigma_y + \xi_3\sigma_z + \xi_4\mathbf{I} \\
D &= \xi_{234}\sigma_x + \xi_{314}\sigma_y + \xi_{124}\sigma_z + \xi_{123}\mathbf{I}, \qquad (1)
\end{aligned}
$$

where σ_x, σ_y and σ_z are Pauli spinors, and added vector fields expressed as

$$
\begin{aligned}
E &= x_1\mathbf{i} + x_2\mathbf{j} + x_3\mathbf{k} + x_4\mathbf{I} \\
E' &= x'_1\mathbf{i} + x'_2\mathbf{j} + x'_3\mathbf{k} + x'_4\mathbf{I}. \qquad (2)
\end{aligned}
$$

The spinors A, B, C, D and the vectors E, E' transform by superspace transformations $G_{23}, G_{12}, G_{13}, G_{123}$ and G_{132}, and there is triality symmetry between the six spaces. The operation on spinors and vectors are summarized in Table 1[S.Furui, 2012].

Table 1a. Transformations of A,B,C,D,E and E' by G_{23}, G_{12}, G_{123}.

G_{23}	G_{12}	G_{123}
$A \to (C_1, C_2, C_3, C_4)$	$A \to (x'_1, x'_2, x'_3, x_4)$	$A \to (x'_1, x'_2, x'_3, x'_4)$
$B \to (D_1, D_2, D_3, D_4)$	$B \to (x_1, x_2, x_3, x'_4)$	$B \to (x_1, x_2, x_3, x_4)$
$C \to (A_1, A_2, A_3, A_4)$	$C \to (C_1, C_2, C_3, D_4)$	$C \to (A_1, A_2, A_3, B_4)$
$D \to (B_1, B_2, B_3, B_4)$	$D \to (D_1, D_2, D_3, C_4)$	$D \to (B_1, B_2, B_3, A_4)$
$E \to (x_1, x_2, x_3, x'_4)$	$E \to (B_1, B_2, B_3, A_4)$	$E \to (D_1, D_2, D_3, C_4)$
$E' \to (x'_1, x'_2, x'_3, x_4)$	$E' \to (A_1, A_2, A_3, B_4)$	$E' \to (C_1, C_2, C_3, D_4)$

Table 1b. Transformations of A,B,C,D,E and E' by G_{13}, G_{132}.

G_{13}	G_{132}	
$A \to (A_1, A_2, A_3, B_4)$	$A \to (C_1, C_2, C_3, D_4)$	$A = (A_1, A_2, A_3, A_4)$
$B \to (B_1, B_2, B_3, A_4)$	$B \to (D_1, D_2, D_3, C_4)$	$B = (B_1, B_2, B_3, B_4)$
$C \to (x'_1, x'_2, x'_3, x_4)$	$C \to (x'_1, x'_2, x_3,' x_4)$	$C = (C_1, C_2, C_3, C_4)$
$D \to (x_1, x_2, x_3, x'_4)$	$D \to (x_1, x_2, x_3, x'_4)$	$D = (D_1, D_2, D_3, D_4)$
$E \to (D_1, D_2, D_3, D_4)$	$E \to (B_1, B_2, B_3, B_4)$	$E = (x_1, x_2, x_3, x_4)$
$E' \to (C_1, C_2, C_3, C_4)$	$E' \to (A_1, A_2, A_3, A_4)$	$E' = (x'_1, x'_2, x'_3, x'_4)$

II. Octonion in the memristor circuit

- In 2010, chaotic oscillation was observed in an electronic system containing inductor, capacitor and non-linear memristors [B.Muthuswamy and L.O. Chua, 2010]. The three-element circuit with the voltage across the capacitor $x(t) = v_C(t)$, the current through the inductor $y(t) = i_L(t)$ and the internal state of the memristor $z(t)$ satisfy a coupled equation[J-M.Ginoux, Ch.Letellier and L.O. Chua, 2010].

- We added an oscillation $\gamma \sin \omega t$ to the memristor. The coupled differential equation was studied, with $\gamma = 0.2, \alpha = 0.1$ fixed and ω changed.

$$\dot{x} = \frac{y}{C}, \quad \dot{y} = \frac{-1}{L}[x + \beta(z^2 - 1)y] + \gamma \sin \omega t, \quad \dot{z} = -y - \alpha z + y\, z.$$

- We observed bifurcation diagrams and the frequency of the bifurcation. The response frequency of the oscillation ω as well as ω/n were fitted as a function of n, in the case of $C = 1.2$ and $C = 1.0$

- The frequency of the single oscillation divided by 8 ($\omega_1/8$) is close to the frequency of the 7 oscillations divided by 7 ($\omega_7/7$). The data suggest 8 fold degeneracy in the single mode.

- The 4component spinors $(A, B), (C, D)$ and input vectors x and x' belong to a definite triality sector, and since the x and x' belong to a different triality sector, the spinor (A, B) produced by an operation of G_{12} on (E', E) has diffferent triality sectors, and (A, B) cannot be a Dirac fermion.

Fig.1 The bifurcation diagram of the driven memristor.

$C = 1.2, L = 3.3, 0.24 \leq \omega \leq 0.36$(left) and $0.36 \leq \omega \leq 0.46$(right).

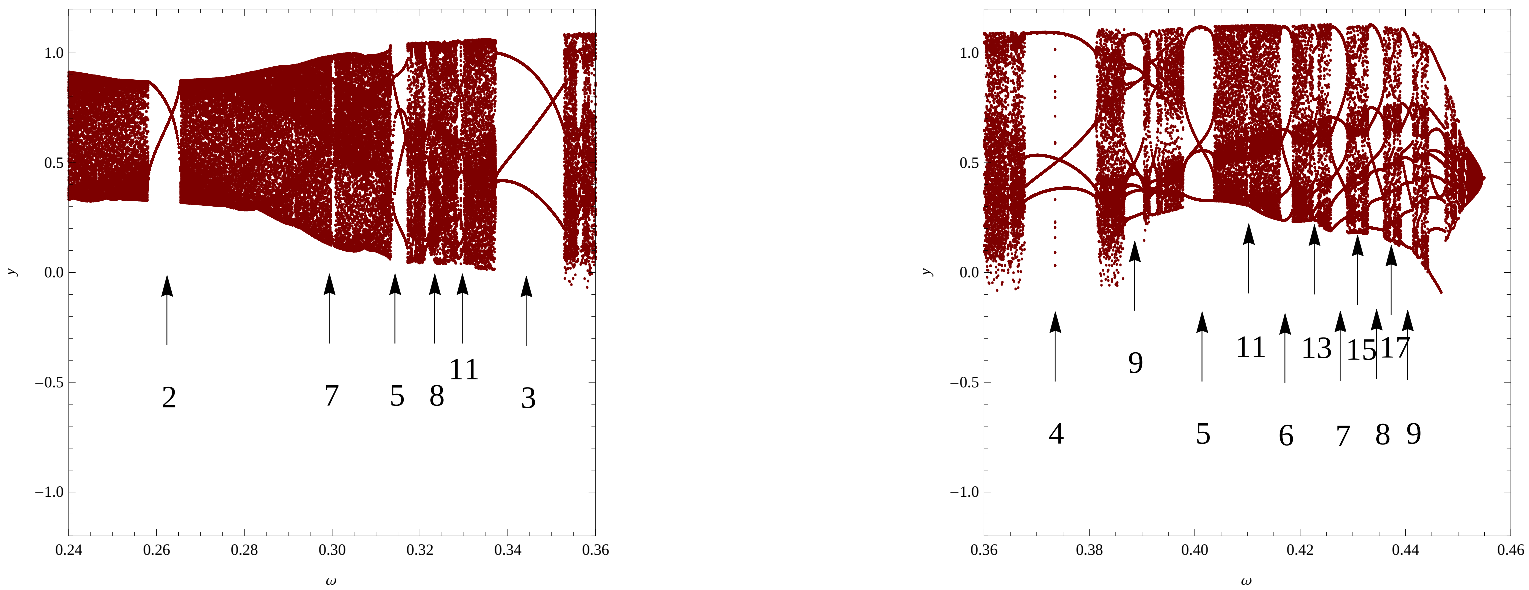
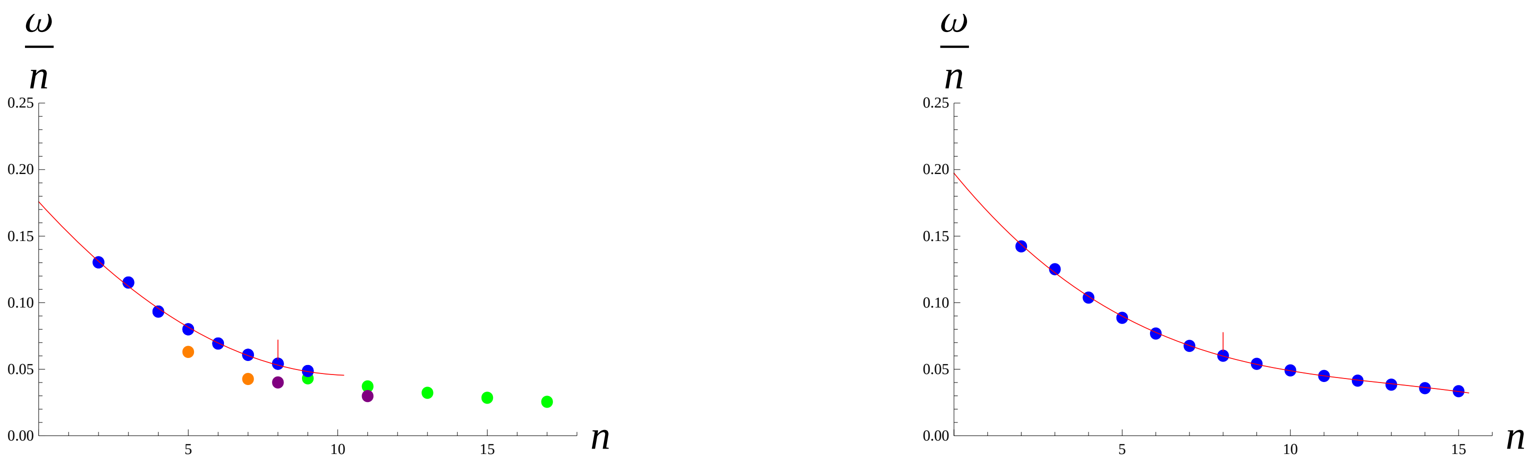
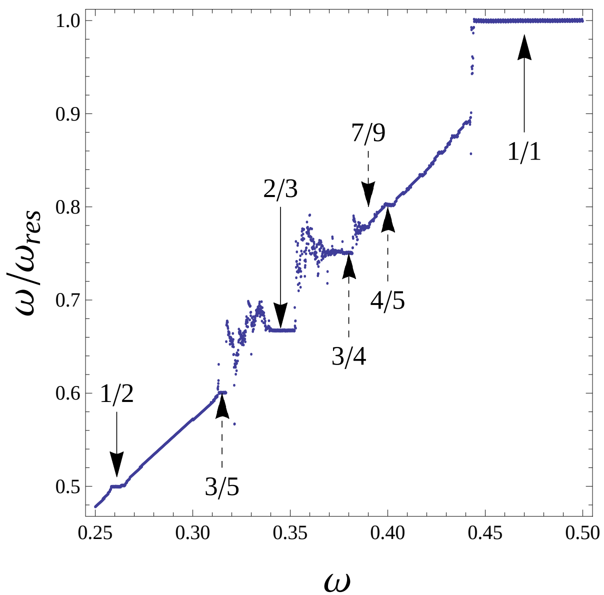


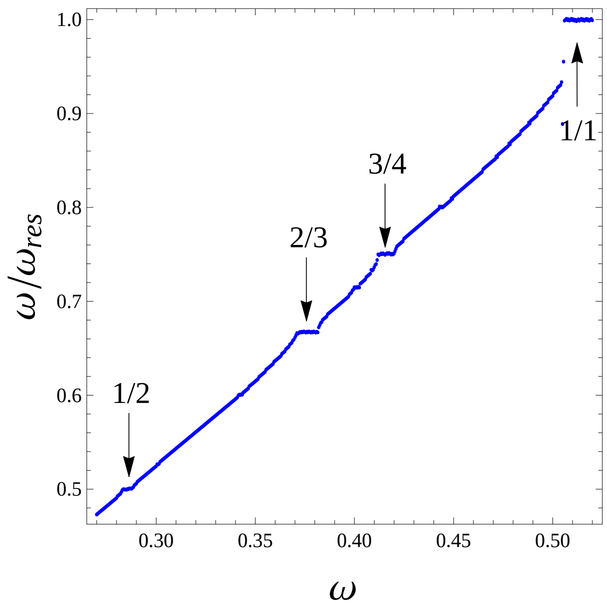
Fig2 The fit of ω/n as a function of the torus loop n.
$L = 3.3, \alpha = 0.2, \beta = 0.5,\ C = 1.2$(left) and $C = 1.0$(right)



- The Devil's staircase structures in the ratio of the frequency of the driving oscillation and that of the response oscillation were observed[S. Dos Santos 1998].

Fig.3 The Devil's staircase of memristic circuit.
$C = 1.2$ (left) and $C = 1$ (right).





- The systems of $C = 1.2$ is unstable, when $\omega < \omega_{res}$, but the system of $C = 1$ is stable. Near $\omega \geq \omega_{res}$, there is no qualitative difference.

- The perturbation $\dot{y} \simeq \frac{\gamma}{L} \sin \omega t$ on $\dot{x} = y/C$ is less serious in the case of $C = 1$ as compared to the case of $C = 1.2$.

- The Hölder exponent of the cycle limit $\nu(x)$, where x is given by the ratio of the frequency of the perturbation f_s and that of response f_d defined as $x = \frac{f_s}{f_d}$, checks the stability of the system[M. Planat and P. Koch, 1993],

$$||\nu(x) - \nu(y)|| = ||x - y||^{\alpha}.$$

We found $x = \frac{2}{3}, y = \frac{3}{4}$ of $C = 1.2$ yields $\alpha = 0.695 < 1$, and $x = \frac{2}{3}, y = \frac{1}{2}$ of $C = 1$ yields $\alpha = 1.211 > 1$. It indicates the system of $C = 1.2$ is unstable or chaotic.

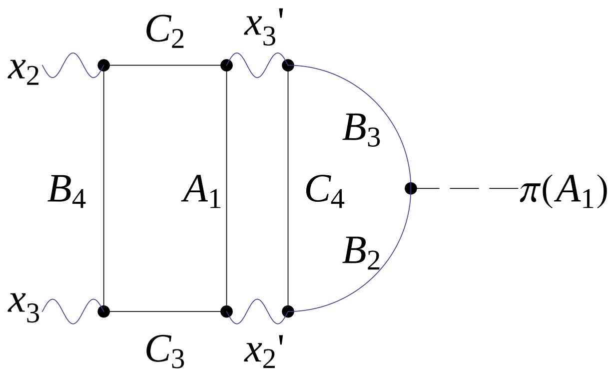
III. Quaternion in π^0, η, η' decays into $\gamma\gamma$

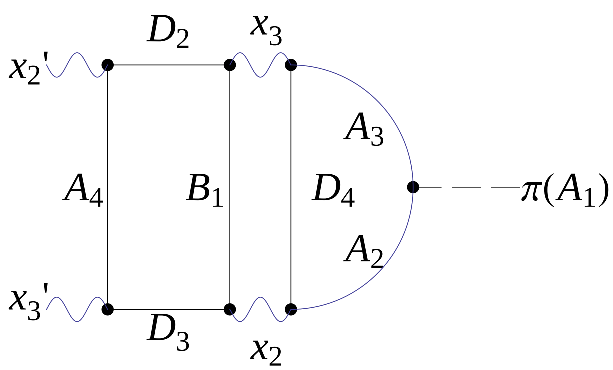
- In the universe, there are fermions which are transformed by quaternions and expressed by two-component spinors, and which are transformed by octonions and expressed by four component spinors. Three kinds of fermions (e, μ and τ) that interact electromagnetically and neutrinos which interact weakly are called leptons; and protons, neutrons etc. which interact strongly are called hadrons.

- In Quantum Chromo Dynamics(QCD), complex numbers and quaternions are used. A quaternion operates on a two-component spinor, i.e. Pauli spinor. The Dirac spinor is a four component spinor, but the octonion operates on four component spinors, which have the triality symmetry.

- Phenomenologically, the decay of a pion into two gamma rays is well described by a divergence of the axial current, since a pion can be regarded as a Nambu-Goldstone boson. The Adler-Bardeen's theorem says that higher order effects in the triangular diagram can be incorporated in the renormalization, represented by a rescattering diagram.

Fig.4 The half circle diagrams of axial anomaly with rescattering. $\pi(A_1)x_3'x_2'$ type and $\pi(A_1)x_3x_2$ type.





- The theoretical decay width of π^0 and that of η into two γ are

$$\Gamma(\pi^0 \to \gamma\gamma) = \frac{\alpha^2}{32\pi^3}\frac{{m_\pi}^3}{{f_\pi}^2} = 7.7\text{eV}$$

and

$$\Gamma(\eta \to \gamma\gamma) = \frac{\alpha^2}{32\pi^3}\frac{1}{3}\frac{{m_\eta}^3}{{f_\eta}^2} = 0.13\text{keV}.$$

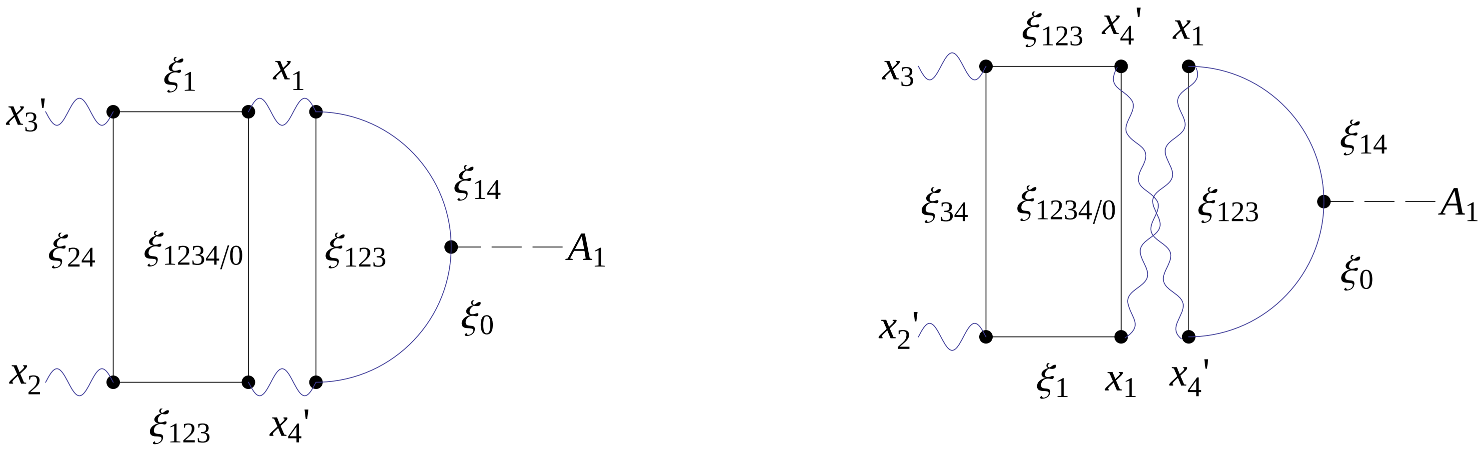
- The theoretical decay width of π^0 is consistent with experiment, but that of η is 4 times smaller [Exp. 0.510$\pm$0.026keV].

- Two γ in π^0 decay belong to the same sector of vector particles of Cartan E or E'.

- $\eta(\eta')$ can decay into two γ, in different sectors E and E', which could enhance the decay width.

- The contribution of Fig.5 enhances the isovector Goldstone boson (π^0) decay by a factor 4, and when the effective quark-gluon coupling α is reduced by a factor 2, both pion and eta decay width become consistent.

Fig.5 The half circle diagrams of $\eta(\eta')$ decay with rescattering. Twisted $\eta(\eta') \to x_2 x'_3$ type and $\eta(\eta') \to x_3 x'_2$ type.



IV. The Dark Matter and the triality symmetry

- In the É. Cartan's spinor theory based on Clifford algebra, a lepton has superpartners, but they cannot be detected by our electromagnetic probes. Leptons (A, B, C, D) and vector field (E, E') are transformed by three transformations G_{ij} and two transformations G_{ijk}. Electromagnetic probes will not detect electromagnetic waves (E, E') transformed by G_{12}, G_{13}, G_{123} and G_{132}, and 4/6 of the electromagnetic waves will appear as dark energy.

- The transformation G_{23} interchanges two components of the four component spinors, or it transforms matter to anti-matter. Then 5/6 of the matter in the universe appears different from actuality and rather appears as dark matter. The WMAP space craft confirms that almost 5 times more dark matter (24%) than normal matter (4.6%) are observed.

- Photons emitted from matter made of quarks that belong to a triality sector different from that of electromagnetic probes, will not be detected, and the matter will be assigned as a dark matter.

- We considered three massless neutrinos in different triality sectors interacting with each other and proposed a theory with a heavy and two degenerate light neutrinos. ν_e, ν_μ and ν_τ have their lepton partners. We expect e, μ and τ are sensitive to flavors, but blind to the triality of neutrinos, quarks and gluons, and that they are sensitive to the triality of electromagnetic waves. If electromagnetic waves from different triality sectors cannot be detected by electromagnetic probes in our universe, we can understand the presence of dark matter.

- If the electromagnetic interaction of leptons obeys the selection rule of triality and detects light from hadrons in the same triality sector, one may be able to interpret influence from the world transformed by 5 operators as that of the dark matter.

V. Discussion and Conclusion

- Clifford Algebra Cl_3^+ which has bases $\boldsymbol{R}\oplus\boldsymbol{R}^3\oplus\wedge^2\boldsymbol{R}^3\oplus\wedge^3\boldsymbol{R}^3$ can produce $Spin(3)$ which is expressed by quaternion $\boldsymbol{H}=\{1,\boldsymbol{i},\boldsymbol{j},\boldsymbol{k}\}$, and an extension $\boldsymbol{H}\oplus\boldsymbol{H}\boldsymbol{l}$ makes an octonion $\boldsymbol{O}$.

  É.Cartan showed that $Spin(8)$ constructed from $\boldsymbol{R}^8$, $Cl_8^+\boldsymbol{f}_+$ and $Cl_8^+\boldsymbol{f}_-$ where $\boldsymbol{f}_\pm=\frac{1}{8}(1+\boldsymbol{w})\frac{1}{2}(1\pm\boldsymbol{e}_{12\ldots8})$ has the triality symmetry. Here, $\boldsymbol{w}=\boldsymbol{v}\boldsymbol{e}_{12\ldots7}^{-1}\in\wedge^4\boldsymbol{R}^{0,7}$, $\boldsymbol{e}_{12\ldots7}^{-1}=\boldsymbol{e}_{12\ldots7}$ and

  $$\boldsymbol{v}=\boldsymbol{e}_{124}+\boldsymbol{e}_{235}+\boldsymbol{e}_{346}+\boldsymbol{e}_{457}+\boldsymbol{e}_{561}+\boldsymbol{e}_{672}+\boldsymbol{e}_{713}\in\wedge^3\boldsymbol{R}^8.$$

- One can assign 3 physical states like (e,μ,τ) leptons, (ν_1,ν_2,ν_3) neutrinos, (u,d,s) quarks etc. on definite triality states

- Fermions in our universe are transformed by G_{12}, G_{13}, G_{123} and G_{132} to vectors, but the vectors produced by these transformations cannot be detected by electromagnetic probes in our universe.

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