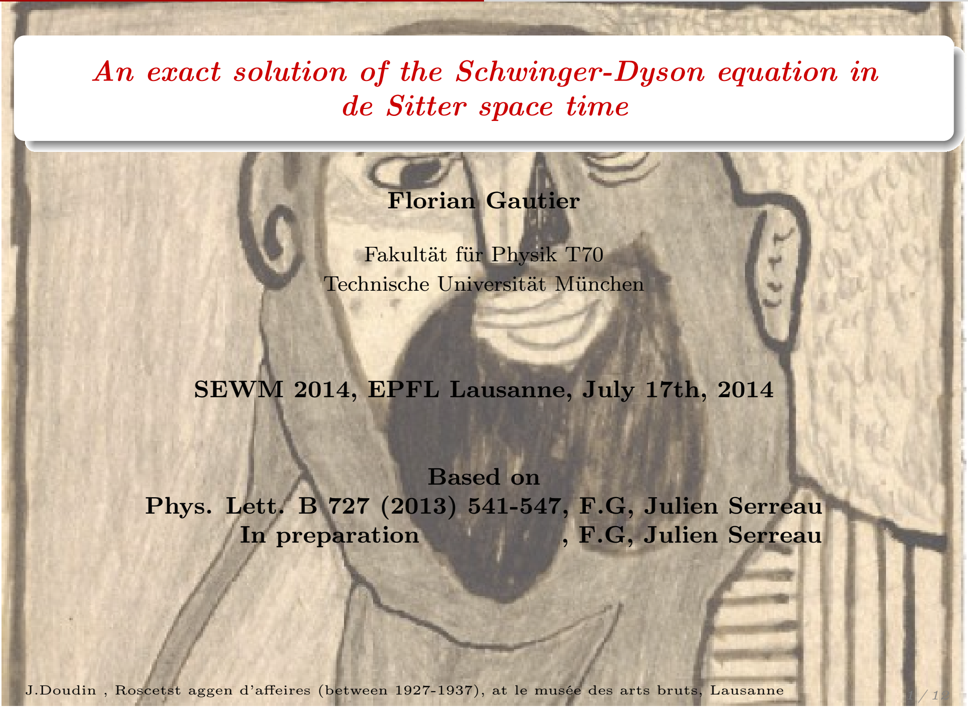

An exact solution of the Schwinger-Dyson equation in de Sitter space time

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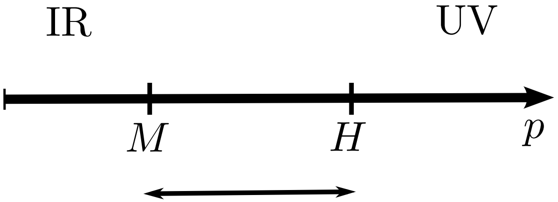
SEWM 2014, EPFL Lausanne, July 17th, 2014

Based on
Phys. Lett. B 727 (2013) 541-547, F.G, Julien Serreau
In preparation , F.G, Julien Serreau

J.Doudin , Roscetst aggen d'affeires (between 1927-1937), at le musée des arts bruts, Lausanne

Why Quantum Field Theory in de Sitter space ?

- First step towards Quantum Gravity
- Maximally symmetric space-time
- Perfect test lab for periods of accelerated expansion of the Universe



Specific effects

Outline

1. *Momentum representation of correlators*

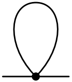
2. *Resummation of non-local logarithms*

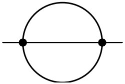
3. *Solution and discussion*

4. *conclusions and prospects*

Radiative corrections

- Scalar field φ with quartic self-interaction $\mathcal{S}_{\rm int} = -\int \mathrm{d}^4x\sqrt{-g}\ \frac{\lambda}{4!}\varphi^4$
- Infrared divergent radiative corrections

 $\sim \dfrac{\lambda H^4}{m^2}$

 $\sim \dfrac{\lambda^2 H^6}{m^2}\ln p \sim t$

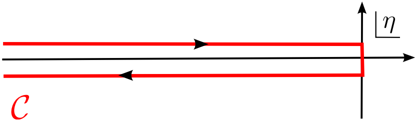
- Line element in conformal time $-\infty < \eta < 0$ and comoving spatial coordinates $\mathbf{X}$

$$ds^2 = \eta^{-2}(-\mathrm{d}\eta^2 + \mathrm{d}\mathbf{X}.\mathrm{d}\mathbf{X})$$

$\rightarrow$ Intrinsically out of equilibrium.

calls for resummation

$1/N$ techniques, Non-Equilibrium Renormalisation Group, Stochastic approaches, 2-PI techniques, etc...



- With the Schwinger-Dyson equation $G^{-1}(x,x') = G_0^{-1}(x,x') - \Sigma(x,x')$

Momentum representation of correlators

R.Parentani, J.Serreau ('12)

- Searching for de Sitter invariant solutions. In principle
$$G(x,x') = G(z)$$
- But Inverting $G^{-1}(z) = G_0^{-1}(z) - \Sigma(z)$ is "hard".
- In Minkowsky: momentum representation $\rightarrow$ algebraic equation "easy"

Not (fully) possible in de Sitter space-time

- Exploit partly de Sitter space symmetries
- $\rightarrow$ Scaling property: $G(\eta,\eta',K) \equiv \frac{1}{K}\hat{G}(-K\eta,-K\eta') = \frac{1}{K}\hat{G}(p,p')$

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Physical momentum representation of correlators

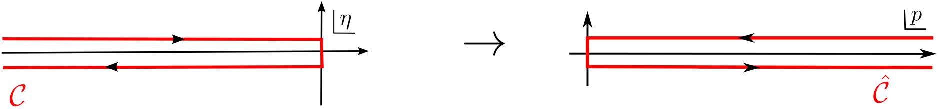
Momentum representation - 2

R.Parentani, J.Serreau ('12)

- Scaling property consistent with the Schwinger-Dyson equation, i.e,

$$\hat{G}^{-1}(p,p') = \hat{G}_0^{-1}(p,p') - \hat{\Sigma}(p,p')$$

- trade contour:



- Momentum flow like SD-equations

$$\left[\partial_p^2 + 1 - \frac{\nu^2 - \frac{1}{4}}{p^2}\right]\hat{F}(p,p') = \int_{p'}^{+\infty} \mathrm{d}s\ \hat{\Sigma}_F(p,s)\hat{\rho}(s,p') - \int_{p}^{+\infty} \mathrm{d}s\ \hat{\Sigma}_\rho(p,s)\hat{F}(s,p') \qquad \text{with } \nu = \sqrt{d^2/4 - M^2}$$

$$\left[\partial_p^2 + 1 - \frac{\nu^2 - \frac{1}{4}}{p^2}\right]\hat{\rho}(p,p') = -\int_{p}^{p'} \mathrm{d}s\ \hat{\Sigma}_\rho(p,s)\hat{\rho}(s,p')$$

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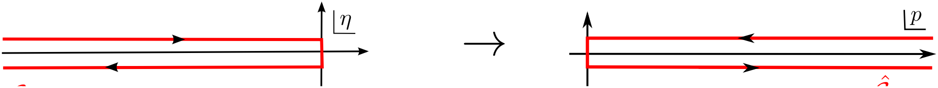

$(0+1)$ effective description

- Momentum flow like SD-equations

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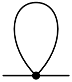
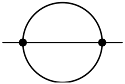
The resummation

- Remember two type of divergences

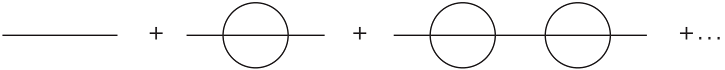
 $\sim \dfrac{\lambda H^4}{m^2}$ $\longrightarrow$ Mass term M

 $\sim \dfrac{\lambda^2 H^6}{m^2} \ln p \sim t$ $\longrightarrow$ This talk

- Take for granted the mass generation mechanism

  [A.A.Starobinsky, J.Yokoyama ('96)], [B.Garbrecht, G.Rigopoulos ('11)], [Serreau ('11)], [D.Boyanosky ('12)],...

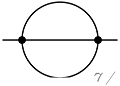
- We wish to calculate



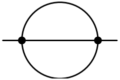
- Schwinger-Dyson equation

$$\hat{G}^{-1} = \hat{G}_M^{-1} - \hat{\Sigma}_{nl}[\hat{G}_M]$$

with

$\hat{\Sigma}_{nl} =$ 

Infrared self-energy

$\hat{\Sigma}_{nl} =$  for super-horizon modes $p, p' \lesssim H$

- de Sitter "plane waves": $\hat{G}_M(p,p') = \frac{\pi}{4}\sqrt{pp'}H_\nu(p)H^\star_\nu(p')$
- Infrared enhancement of correlators $\rightarrow$ Infrared contributions dominate

$$\begin{aligned}\hat{F}^{\mathrm{IR}}_M(p,p') &= \sqrt{pp'}F_\nu e^{-\nu(x+x')},\\ \hat{\rho}^{\mathrm{IR}}_M(p,p') &= -\sqrt{pp'}\mathcal{P}_\nu(x-x'),\end{aligned}$$

with $x = \ln\left(\frac{p}{H}\right)$ and $\mathcal{P}_\nu(x) = \sinh(\nu x)/\nu$

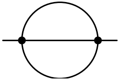
- The self energies are

$$\begin{aligned}\hat{\Sigma}^{\mathrm{IR}}_F(p,p') &= -(pp')^{-3/2}\, F_\nu \sigma_\rho\, s(x)s(x')\\ \hat{\Sigma}^{\mathrm{IR}}_\rho(p,p') &= (pp')^{-3/2}\, \sigma_\rho\, \sigma(x-x')\end{aligned}$$

with $s(x) = e^{-(\nu-2\varepsilon)x}$, $\sigma(x) = \mathcal{P}_\nu(x)e^{-2\varepsilon|x|}$, $\nu \equiv \frac{d}{2} - \varepsilon$, i.e., $\varepsilon \simeq \frac{M^2}{d}$ and

$$\sigma_\rho \sim \frac{\lambda^2}{M^4}$$

Infrared self-energy

$\hat{\Sigma}_{nl} =$ 

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Factorisation property

- The self energies are

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$$\sigma_\rho \sim \frac{\lambda^2}{M^4}$$

One dimensional dynamics

- One dimensional reduction: $\hat{\rho}(p,p') = \sqrt{pp'}\,\rho(x-x')$

- Statistical part of the formal solution:

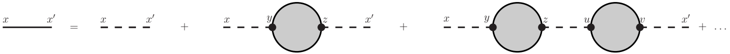
$$\hat{F}^{\mathrm{IR}}(p,p') = \sqrt{pp'}\,\tilde{F}_\nu \left\{ f_R(x) f_R(x') + \sigma_\rho\, f_\sigma(x) f_\sigma(x') \right\}$$

with $\quad f_R(x) = \rho'(x) - \nu\rho(x)$ and $\quad f_\sigma(x) = \int_x^0 dy\ \rho(x-y)\, s(y)$

- Non trivial part of the spectral function:

$$\rho''(x) - \nu^2 \rho(x) = \sigma_\rho \int_0^x dy\ \sigma(x-y)\,\rho(y)$$

This is the equation resuming all



Full solution

- Exact solution
- Non-trivial spectral part:

$$\rho(x) = \frac{1}{2\tilde{\nu}}\left\{(\tilde{\nu}+\varepsilon)\mathcal{P}_{\bar{\nu}_+}(x) + (\tilde{\nu}-\varepsilon)\mathcal{P}_{\bar{\nu}_-}(x) + \mathrm{sign}(x)\left[\mathcal{P}'_{\bar{\nu}_+}(x) - \mathcal{P}'_{\bar{\nu}_-}(x)\right]\right\}e^{-\varepsilon|x|}$$

- statistical parts:

$$f_R(x) = \left\{(\nu_- + \bar{\nu}_+)A_{\tilde{\nu}}(\bar{\nu}_+)e^{-\bar{\nu}_+ x} + (\nu_- - \bar{\nu}_+)A_{\tilde{\nu}}(-\bar{\nu}_+)e^{\bar{\nu}_+ x}\right\}e^{\varepsilon x} + (\tilde{\nu} \to -\tilde{\nu})$$

and

$$f_\sigma(x) = \left\{\frac{A_{\tilde{\nu}}(\bar{\nu}_+)}{\nu_- - \bar{\nu}_+}e^{-\bar{\nu}_+ x} + \frac{A_{\tilde{\nu}}(-\bar{\nu}_+)}{\nu_- + \bar{\nu}_+}e^{\bar{\nu}_+ x}\right\}e^{\varepsilon x} + (\tilde{\nu} \to -\tilde{\nu}),$$

with

$$A_{\tilde{\nu}}(z) = (z + \tilde{\nu} + \varepsilon)/4\tilde{\nu}z, \qquad \tilde{\nu} = \sqrt{\nu^2 + \tfrac{\sigma_\rho}{4\varepsilon^2}}, \qquad \varepsilon \simeq \tfrac{M^2}{d}$$

and

$$\bar{\nu}_\pm^2 = \nu^2 \pm 2\varepsilon\tilde{\nu} + \varepsilon^2$$

Moderate infrared

- Simpler form for $|x| \gtrsim 1$ and at first order in infrared enhancement $1/M^2$
- Superposition of two massive solutions:

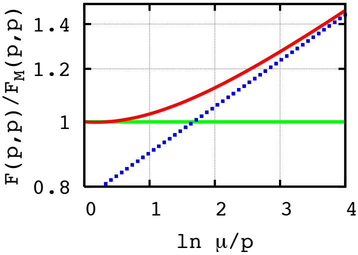
$$\rho(x) = \left\{ c_+ \mathcal{P}_{\bar{\nu}_+}(x) + c_- \mathcal{P}_{\bar{\nu}_-}(x) \right\} e^{-\varepsilon |x|}$$

with $c_\pm = (\tilde{\nu} \pm \nu)/2\tilde{\nu}$

- Similarly for the statistical part

$$\hat{F}^{\mathrm{IR}}(p,p') = \sqrt{pp'} F_\nu \left\{ c_+ e^{-\bar{\nu}_+(x+x')} + c_- e^{-\bar{\nu}_-(x+x')} \right\} e^{\varepsilon(x+x')}$$

- Interpolate between two free massive solutions



- Agreement on "Dynamical mass" : $\langle \varphi^2(x) \rangle = \int \frac{d^d p}{(2\pi)^d} \frac{\hat{F}(p,p)}{p} \propto \frac{1}{m_{dyn}^2}$

[[A.A.Starobinsky, J.Yokohama ('94)], [M.Beneke, P.Moch ('12)], [B.Garbrecht, G.Rigopulous, Y.Zhu ('13)]]

Conclusion

Conclusions

- Exact solution of the Schwinger-Dyson equation.
- Rich and infrared finite structure of the propagator.
- Dangerous logarithms are resummed in modified power laws (c.f. anomalous dimensions)

Prospects

- Non-perturbative methods: large-N [in preparation], 2-PI, ...
- Extend the method to quasi de Sitter space-time: Quantum corrections to inflation.

  [[Weinberg ('05)], [M.van der Meulen, J.Smit ('07)], [Sloth ('07)], [M.Herranen, T.Markkanen, A.Tranberg ('13)]]