Three-loop Debye mass and effective coupling in thermal QCD

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Background

QCD at finite temperature is afflicted with infrared divergences that can be cured, in part, in the framework of a dimensionally reduced effective theory, Electrostatic QCD. Within this theory we compute the screening mass of the chromo-electric fields and the effective coupling to three-loop order. The screening mass enters the QCD pressure at $\mathcal{O}(g^7)$ whereas the effective coupling will be ultimately used to determine the spatial string tension of QCD, σ_s.

EQCD

Thermal QCD develops three different scales:

1. The hard scale $\propto 2\pi T$: Comes from the non-zero Matsubara modes of bosonic fields and from all the modes of the fermionic fields.
2. The soft scale $\propto gT$: Comes from the chromo-electric screening, generated by resumming Matsubara zero modes.
3. The ultra-soft scale $\propto g^2T$: Comes from the chromo-magnetic screening and is a pure non-perturbative effect.

At high enough Temperature, $g(T) \ll 1$, a scale separation is performed by isolating the soft scales into effective Lagrangians and by performing a perturbative matching to the original theory.
Separation of the hard scale generates the EQCD Lagrangian truncated to dim-4 operators:

$$\mathcal{L}_{\rm EQCD} = -\frac{1}{4}F^a_{ij}F^a_{ij} + {\rm Tr}[D_i, A_0]^2 + m_{\rm E}^2 {\rm Tr}[A_0^2] + \lambda_{\rm E}^{(1)}({\rm Tr}[A_0^2])^2 + \lambda_{\rm E}^{(2)}{\rm Tr}[A_0^4] + \cdots \,,$$
$$D_i = \partial_i - i g_{\rm E} A_i \,, \quad i,j \in \{1,2,3\} \,.$$

$\mathcal{L}_{\rm EQCD}$ is super-renormalizable with only one mass counter-term arising a 2-loops:

$$\delta m_{\rm E}^2 = (N_c^2+1)\frac{\mu_3^{-4\epsilon}}{4\epsilon}(-g_{\rm E}^2\lambda_{\rm E} C_A + \lambda_{\rm E}^2) = -\frac{10 C_A^3}{3\epsilon}\frac{T^2}{(4\pi)^4} g(\bar\mu)^6 \mu_3^{-4\epsilon}\mu^{-2\epsilon} + \mathcal{O}(g^8) \,.$$

Matching computation

Matching: Compute various quantities in both QCD and EQCD and require that they match up to $\mathcal{O}(g^6)$.

- Matching of $m_{\rm E}^2$: Compute the pole of the static propagator of A_0.

$$\text{QCD}: \; {\rm p}^2 + \Pi_{00}(0,{\rm p}^2)\big|_{{\rm p}^2=-m_{\rm el}^2} = 0$$
$$\text{EQCD}: \; {\rm p}^2 + m_{\rm E}^2 + \Pi(0,{\rm p}^2)\big|_{{\rm p}^2=-m_{\rm el}^2} = 0$$

- Matching of $g_{\rm E}^2$: The background field gauge imposes explicit gauge invariance on the background fields, reducing the computation to:

$$g_{\rm E}^2 = \frac{1}{1+\Pi_T'(0,{\rm p}^2)} g^2 T \,.$$

- QCD: Thus, the matching requires the gluonic polarization tensor (self-energy):

$$\Pi^{\rm QCD}_{\mu\nu}({\rm p}^2) = \sum_{n=1}^{\infty}\Pi_{\mu\nu,n}(0)(g^2)^n + {\rm p}^2\sum_{n=1}^{\infty}\Pi'_{\mu\nu,n}(0)(g^2)^n + \ldots \,.$$

- EQCD: Scaleless integrals vanish in dim. reg.:

$$\Pi_{\rm EQCD} = 0 \,.$$

Higher order operators: Part I

...however, one higher order operator [1] does contribute to the self-energy, since it generates a propagator-like counter-term and not a vanishing vacuum integral.

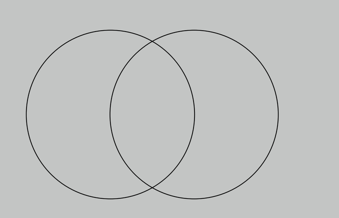
$$\mathcal{L}_{\rm dim6}\big|_{A_0^2} = -\frac{17N_c}{60}\zeta(3) \times \frac{g^2}{(4\pi)^4T^2}(\partial_i\partial_i)^2 A_0^a A_0^a \,,$$
$$m_{\rm el}^2 = m_{\rm E}^2\left(1 - \frac{17N_c}{60}\zeta(3)\frac{g^2 m_{\rm E}^2}{(4\pi)^4 T^2}\right) + \mathcal{O}(g^8) \,.$$

Automation

- Feynman graph generation with QGraf (≈ 500 diagrams).
- Lorentz contraction, color algebra, Taylor expansion with FORM $\Rightarrow 10^7$ sum-integrals.
- Integration By Parts [2] reduction to a set of $\mathcal{O}(10)$ master sum-integrals [3].
- Change of basis in order to avoid divergent pre-factors in ϵ.

3-loop sum-integrals

Remaining non-trivial topologies:



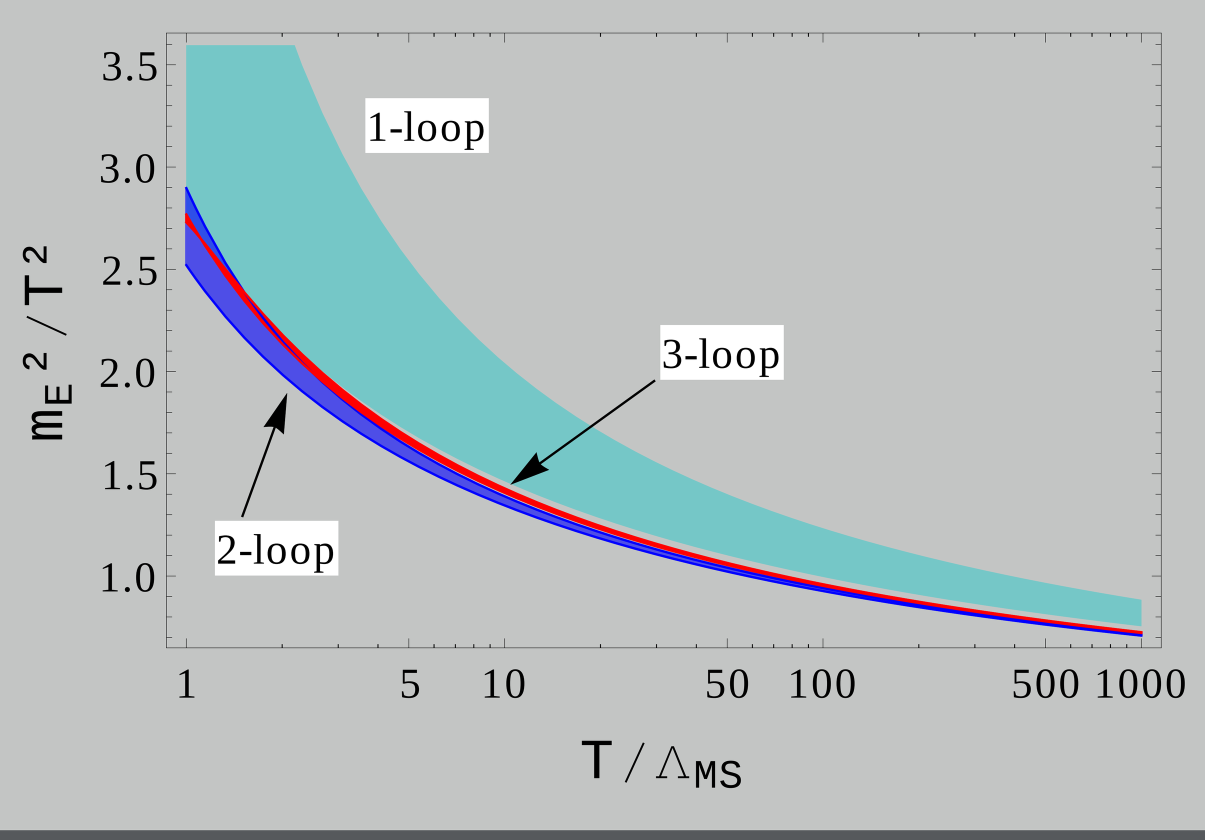
- The calculation [4] is based on the pioneering work of Arnold and Zhai [5].
- We extended the method to a large class of V-type sum-integrals.
- Highly IR divergent pieces are solved by further IBP reduction.
- Tensor structure resolved by a mapping to higher dimensional sum-integrals [6,7].
- n-tensor in d-dim $= \sum$ scalar in $d{+}2n$-dim.

Results: $m_{\rm E}$

$$\frac{m_{\rm E,ren}^2}{(4\pi T)^2} = \frac{g^2(\bar\mu)C_{\rm A}}{(4\pi)^2\;3}\left\{1 + \frac{g^2(\bar\mu)C_{\rm A}}{(4\pi)^2\;3}(22L+5) + \frac{g^4(\bar\mu)}{(4\pi)^4}\left(\frac{C_{\rm A}}{3}\right)^2\left(484L^2 + 244L - 180L_3 + \frac{1091}{2} - \frac{207\zeta(3)}{20}\right) + \mathcal{O}(g^6)\right\}$$

$$L = \ln\frac{\bar\mu e^{\gamma_{\rm E}}}{4\pi T}$$
$$L_3 = \ln\frac{\mu_3 e^{Z_1}}{4\pi T}$$
$$Z_1 = \frac{\zeta'(-1)}{\zeta(-1)}$$

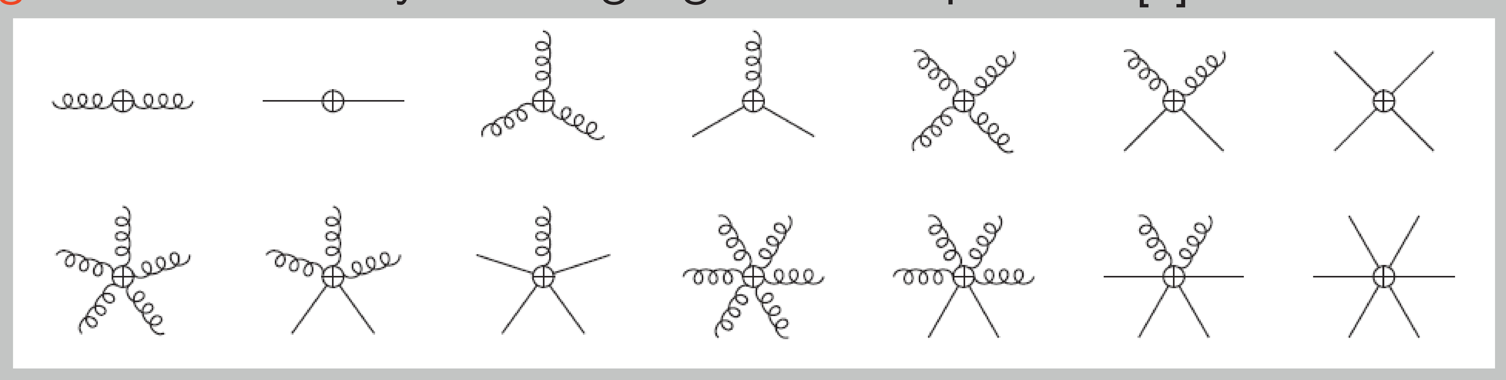
- Good convergences.
- Low dependence on arbitrary scales μ, μ_3.
- Enters the QCD pressure at $\mathcal{O}(g^7)$.



Higher order operators: Part II

$$\frac{g_{\rm E}^2}{T} = \cdots + \frac{g^8 C_A^3}{(4\pi)^6}\left[\frac{-61\zeta(3)}{5\epsilon} + \frac{10648}{27}L^3 + \frac{1408}{3}L^2 + \left(\frac{14584}{27} - \frac{4394\zeta(3)}{45}\right)L + 155.4\right]$$

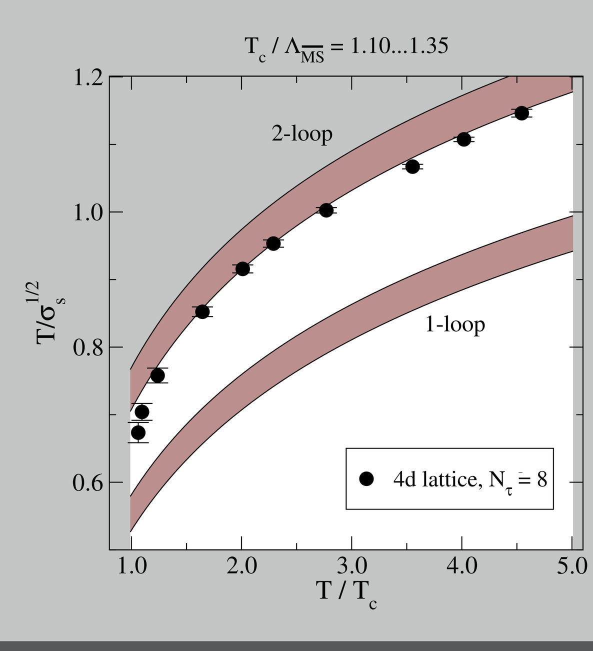
- Divergence removable by including higher order operators [1].

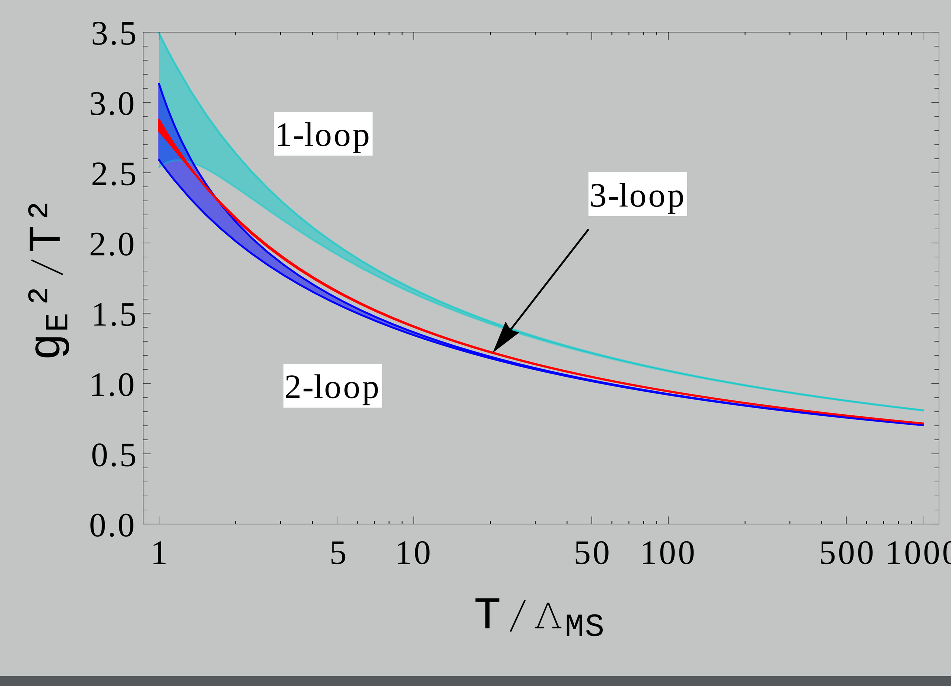


- $\mathcal{L}_{\rm EQCD}$ becomes non-renormalizable. $\Rightarrow$ coupling renormalization starting at $\mathcal{O}(g^8)$.

Preliminary results: $g_{\rm E}$

"Drop" divergence $\Rightarrow$ Good convergence. Little dependence on μ.
Goal: Compute the spatial string tension, σ_s, through matching of $g_{\rm E}$ to $g_{\rm M}$ and compare it with lattice results as in [8].





Bibliography

[1] S. Chapman, Phys. Rev. D 50 (1994) 5308.
[2] S. Laporta, Int. J. Mod. Phys. A 15 (2000) 5087.
[3] J. Möller and Y. Schröder, JHEP 1208 (2012) 025
[4] P. B. Arnold and C. -X. Zhai, Phys. Rev. D 50 (1994) 7603
[5] I. Ghişoiu and Y. Schröder, JHEP 1209 (2012) 016
[6] O. V. Tarasov, Phys.Rev. D54 (1996) 6479-6490.
[7] I. Ghişoiu and Y. Schröder, JHEP 1211 (2012) 010.
[8] M. Laine and Y. Schröder, JHEP 0503 (2005) 067.

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