CHIRAL SYMMETRY RESTORATION: PATTERNS AND PARTNERS

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OUTLINE:

★ Chiral partners from effective theory: \((\chi_S, \chi_P)\)

★ ChPT (model independent) results

★ Direct lattice analysis. Screening masses and S/P degeneration

★ Unitarizing: thermal \(f_0(500)\) saturation for \(\chi_S\)


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Basic Theory Ingredients

Chiral Perturbation Theory (ChPT):

★ Based on Chiral Symmetry Breaking $SU_L(N_f) \times SU_R(N_f) \to SU_V(N_f)$
  ⇒ model independent low-energy predictions for $N_f = 2, 3$
  ⇒ Chiral expansion formally organized in $F^{-2}_\pi$ powers.

★ Systematic and consistent Meson Gas description for $T$ below $T_c$
  ⇒ Predicts (extrapolated) $\langle \bar{q}q \rangle_T$ melts
  ⇒ Reliable near $T_{FO} \sim 100$ MeV
  ⇒ Also useful near $m_q \to 0^+$ (chiral limit scaling e.g. in lattice)

Unitarity (UChPT):

Improves the ChPT analytical description of scattering
⇒ essential for generating resonances ($\rho$, $\sigma$, ...)
⇒ accurate description of collisions for thermal width, transport, ...
Meson gas: Recent Progress within ChPT+UChPT

★ Hot and dense light resonances: $\rho$ broadening, chiral restoration in $\sigma/f_0(500)$ channel, threshold enhanc.

D.Cabrera, A.Dobado, D.Fernández-Fraile, AGN, F.J.Llanes-Estrada, J.R.Peláez, E.T.Herruzo: PLB 02, PRC 02, PLB 05, PRD 07, EPJC 09

★ Transport coefficients: $\Gamma^{-1}$ chiral power counting, $\sigma_{el}, \eta/s, \zeta/s$ OK pheno and theoretically

D.Fernández-Fraile, AGN: PRD 06, NPA 07, EPJA 07, EPJC 09, PRL 09

★ Chemical nonequilibrium for interacting pions: $T_{CFO}, T_{FO}$ reduced, BEC accesible via $M_\pi$ dropping

D.Fernández-Fraile, AGN: PRD 09

★ Isospin breaking, EM effects: $\chi_{S,\text{dis}}^{\text{con},\text{dis}}$ scaling, $\Sigma_{\pi^\pm} - \Sigma_{\pi^0}$.

AGN, J.R.Elvira, J.R.Peláez, R.Torres: PRD 11, 13, 14
Scalar-Pseudoscalar Degeneration

★ Old problem of $O(4)$ chiral partners $(\sigma, \pi^a)$ addressed in LSM through $M_\sigma(T) \downarrow \langle \sigma \rangle(T) \downarrow$ with explicit $\sigma$ field (not physical!)

T.Hatsuda, T.Kunihiro PRL 85

★ Vector-Axial degeneration @ chiral restoration well established with physical $(\rho, a_1)$ states

R.Rapp, J.Wambach ANP 00

★ Crossover Ch.Sym.Rest $N_f = 2$ in lattice @ $T_c \sim 150$-$160$ MeV consistent with $O(4)$ pattern

Y.Aoki et al JHEP 09, S.Ejiri et al PRD 09, A.Bazavov et al PRD 12

$\Rightarrow \ S/P$ degeneration expected from $\chi_S$ maximum onwards
Look at correlators: S/P Susceptibilities

\[
\chi_P(T)\delta^{ab} = \int_0^\beta \int d^3\vec{x} \left\langle \mathcal{T} (\bar{q}\gamma_5\tau^a q)(x) (\bar{q}\gamma_5\tau^b q)(0) \right\rangle
\]

\[
\chi_S(T) = -\frac{\partial}{\partial m} \langle \bar{q}q \rangle_T = \int_0^\beta d\tau \int d^3\vec{x} \left[ \langle \mathcal{T}(\bar{q}q)(x)(\bar{q}q)(0) \rangle_T - \langle \bar{q}q \rangle_T^2 \right]
\]

Expected to be saturated by \(\pi\) and \(\sigma\)-like poles:

\[
\chi_P = 4B_0^2F_\pi^2G_\pi(p^2 = 0) \sim 4B_0^2\frac{F_\pi^2}{M_\pi^2} = -\frac{\langle \bar{q}q \rangle}{m_q} \quad \text{from PCAC+GOR (}T = 0\text{)}
\]

\[
\text{or LO ChPT} \quad B_0 = \frac{M_\pi^2}{2m_q}
\]

\[
\chi_S = 4B_0^2F_\pi^2G_\sigma(p^2 = 0) \sim \frac{4B_0^2F_\pi^2}{M_\sigma^2} \quad \text{from } \mathcal{L}_{SB} = 2B_0F_\pi s(x)\sigma(x)
\]

But no need to deal with a particle-like \(\sigma\) state.

\[\Rightarrow\text{ suitable for ChPT (model independent) and UChPT}\]
**ChPT calculation at $T \neq 0$ to NLO**

**SCALAR CORRELATOR:**

$\langle (\bar{q}q)^2 \rangle_T$ factorization breaking term $O(F^0)$

$$\chi_{S}^{ChPT}(T) = B_0^2 \left[ 8 \left( l_3^r(\mu) + h_1^r(\mu) \right) - 12 \nu_\pi + 6 g_2(M_\pi, T) \right] + O \left( F_\pi^{-2} \right)$$

$$\nu_i = \frac{1}{32\pi^2} \left( 1 + \log \frac{M_i^2}{\mu^2} \right)$$

$$g_1(M,T) = \frac{1}{2\pi^2} \int_0^\infty dp \frac{p^2}{E_p} e^{\beta E_p} - 1 \rightarrow \frac{T^2}{12} \quad \text{for} \quad T \gg M$$

$$g_2(M,T) = -\frac{d g_1(M,T)}{dM^2} = \frac{1}{4\pi^2} \int_0^\infty dp \frac{1}{E_p} e^{\beta E_p} - 1 \rightarrow \frac{T}{8\pi M} \quad \text{for} \quad T \gg M$$
Coupling external pseudoscalar sources, the Euclidean correlator:

$$K_P(p) = a - 4B_0^2F^2 \frac{Z_\pi(T)}{p^2 - M_\pi^2(T)} - \frac{c(T)}{p^2 - M_\pi^2(T)} + \mathcal{O}(F^{-2})$$

$$T = 0 \text{ LEC}$$

Not just proportional to $G_{\pi}^{NLO}$! Actually the residue:

$$4B_0^2F^2 Z_\pi(T) + c(T) = \frac{\langle \bar{q}q \rangle_T}{\langle \bar{q}q \rangle_0} \frac{F_{\pi}^2(0)M_\pi^2(0)}{m_q^2}M_\pi^2(T) + \mathcal{O}(F^{-2})$$

$$K_P(0) = \chi_P^{ChPT}(T) = -\frac{\langle \bar{q}q \rangle_T}{m_q} + \mathcal{O}(F^{-2})$$

$$= 4B_0^2 \left[ \frac{F^2}{M^2} + \frac{1}{32\pi^2}(4\bar{h}_1 - \bar{l}_3) - \frac{3}{2M^2}g_1(M, T) \right] + \mathcal{O}(F^{-2})$$
ChPT calculation at $T \neq 0$ to NLO

$$
\chi'^{ChPT}(T) = -\frac{\langle \bar{q}q \rangle_T}{m_q}
$$

★ Model independent. Finite and scale-independent

★ $\frac{\langle \bar{q}q \rangle_T}{\langle \bar{q}q \rangle_0} = \frac{\chi_P(T)}{\chi_P(0)}$ also LEC and $m_q$ independent

★ $\chi_P$ drops as the condensate (not as softer $\sim M_\pi^{-2}(T)$)

★ $\chi_P(\rho) \sim \langle \bar{q}q \rangle(\rho)$ also in nuclear matter

★ Formal (bare and chiral symmetric) QCD Ward Identity.

G.Chanfray, M.Ericson EPJA 03

D.J.Broadhurst NPB 75
M.Bochicchio et al NPB 85
P.Boucaud et al PRD 10
\( \langle \bar{q}q \rangle (T_c) = 0 \)

\( T_d \sim 0.9T_c \)

\( \chi_S(T_d) = \chi_P(T_d) \)

\( T_d \simeq T_c - \frac{3M_\pi}{4\pi} (M_\pi \ll T) \)

\( T_d \rightarrow T_c \) in chiral limit 
(exact chiral sym.rest.)
Direct Analysis of Lattice Data

1) \( \frac{\langle \bar{q}q \rangle_T}{\langle \bar{q}q \rangle_0} = \frac{\chi_P(T)}{\chi_P(0)} \) from \( K_P \) (screening) masses:

\[
\frac{M_P(T)}{M_P(0)} \sim \left[ \frac{\chi_P(0)}{\chi_P(T)} \right]^{1/2} \sim \left[ \frac{\langle \bar{q}q \rangle_0}{\langle \bar{q}q \rangle_T} \right]^{1/2}
\]

Explains sudden increase near \( T_c \)

\[
\Delta_{l,s} = \frac{\langle \bar{q}q \rangle_T - (2m_q/m_s)\langle \bar{s}s \rangle_T}{\langle \bar{q}q \rangle_0 - (2m_q/m_s)\langle \bar{s}s \rangle_0}
\]

Uncertainties:
- \( \langle \bar{s}s \rangle \) subtr \( \sim 15\% @ T_c \)
- \( M_P^{sc}/M_P^{pole} \) smoothness
- Lattice effects

* from M.Cheng et al EPJC 11

* from A.Bazavov et al PRD 09

\( T_c \approx 196 \text{ MeV} \)

* same lattice conditions for masses and condensate
Direct Analysis of Lattice Data

2) $\chi_S/\chi_P$ degeneration:

Degeneration from the $\chi_S$ maximum onwards

Data from Y.Aoki et al JHEP 09

$T_c \approx 155$ MeV
ChPT Partial waves \( t^{IJ} = t^{IJ}_2 + t^{IJ}_4 + \ldots \)

**Unitarity** → Im \( t(s) = \sigma(s)|t(s)|^2 \) \( s \geq 4M^2 \) ⇒ Im \( t^{-1} = -\sigma \)

\[
\sigma(s) = \sqrt{1 - \frac{4M^2}{s}} = \frac{p_{\text{CM}}}{\sqrt{s}} \quad \text{two-particle phase space}
\]

\[
t^U(s) = \frac{\left[t^2_2(s)\right]^2}{t^2_2(s) - t^2_4(s)}
\]

Resonances dynamically generated as poles in 2nd RS, no assumptions about their nature or couplings. Formally justified by dispersion relations.

**Successful for scattering data up to 1 GeV & low-lying resonance multiplets.**
Unitarizing ChPT: scattering

ChPT Partial waves \( t_{IJ} = t_{2J}^{IJ} + t_{4J}^{IJ} + \ldots \)

Unitarity \( \implies \text{Im } t(s) = \sigma(s)|t(s)|^2 \quad (s \geq 4M^2) \implies \text{Im } t^{-1} = -\sigma \)

\[ \sigma(s) = \sqrt{1 - \frac{4M^2}{s}} = \frac{p_{CM}}{\sqrt{s}} \quad \text{two-particle phase space} \]

\[ t^U(s) = \frac{\left[t_2(s)\right]^2}{t_2(s) - t_4(s)} \]

FINITE TEMPERATURE:

\[ t_4(s) \rightarrow t_4(s; T) \]

\[ \sigma \rightarrow \sigma\left[1 + 2n_B(\sqrt{s}/2)\right] \equiv \sigma_T \]

A. Dobado, D. Fernández-Fraile, AGN, F. J. Llanes-Estrada, J. R. Peláez, E. Tomás-Herruzo, '02 '05 '07

Thermal phase Space.

Bose net enhancement \((1 + n)^2 - n^2\)
\[ M_S^2(T) = M_p^2(T) - \frac{\Gamma_p^2(T)}{4} \]

Scalar pole mass

Chiral restoring behaviour!

\[ I = J = 0: f_0(500) \]

\[ I = J = 1: \rho(770) \]

Pole position:

\[ s_p(T) = [M_p(T) - i\Gamma_p(T)/2]^2 \]

(2nd Riemann sheet)
Unitarized Scalar Susceptibility

★ Saturate the scalar correlator with the $f_0(500)$ thermal state:
(assuming $p = 0$ pole not very diff. from $s_p$)

\[
\chi^U_S(T) = \frac{\chi_S^{ChPT}(0) M_S^2(0)}{M_S^2(T)}
\]

Normalization to match $T = 0$ ChPT. Compensates pole diff.

★ Unitarized condensate from $\chi^U$ requires additional scaling assumptions (holding in ChPT):

\[
\delta \langle \bar{q}q \rangle^U(T, M) = B_0 T^2 g(T/M)
\]

\[
\delta \chi^U_S = B_0^2 h(T/M)
\]

\[
g(x) = g(x_0) + \int_{x_0}^{x} \frac{h(y)}{y^3} dy \quad (x > x_0)
\]

\[
g(x) = g_{ChPT}(x) \quad (x \leq x_0)
\]
Results: ChPT & UChPT

Data from Y. Aoki et al JHEP 09

Not a fit for Unit.
LEC fixed to $T = 0$
comp. with PDG:

$M_{p}^{00} = 441$ MeV
$\Gamma_{p}^{00} = 466$ MeV

$M_{p}^{11} = 756$ MeV
$\Gamma_{p}^{11} = 151$ MeV

★ Improving of critical behaviour $\rightarrow \chi_{S}^{U}$ peak at $T_{c} = 157$ MeV
$T_{c} \downarrow$ and more abrupt $\chi_{S}^{U}$ near chiral limit

★ low-$T$ $\chi_{S}^{U}$ and $\langle \bar{q}q \rangle^{U}$ OK with ChPT

★ S/P intersection near $\chi_{S}^{U}$ peak
Conclusions

★ **Chiral partner degeneration at** \( T_c \) **within** \( O(4) \) **pattern**
holds for \( (\chi_S, \chi_P) \) \( \Rightarrow \) eff.theory & lattice analysis.

★ \( \chi_P(T) = -\langle \bar{q}q \rangle(T)/m_q \) proved to NLO ChPT (model ind).
Holds also for lattice data \( \Rightarrow \) explains sudden growth of pseudoscalar screening mass.

★ **Saturation by thermal** \( f_0(500)/\sigma \) **state** (dyn.gen. via unitariz.)
is a very relevant effect to get \( \chi_S \) and \( S/P \) degeneration
in accordance with lattice data.

★ **In progress**: \( SU(3), \ a_0(980) \ldots \)
Backup Slides
Chiral lagrangians and power counting

\[ \mathcal{L}[U, s, v, a, p, \theta] = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_4^{WZW} + \ldots \]

systematic expansion in \( D_\mu U \) and external fields \( s \sim m_q \sim M^2, v, a, p, \theta \)

\[ U = \exp\left[i \sum a \pi_t t^a / F\right] \quad D_\mu U = \partial_\mu U - i(v_\mu + a_\mu)U + iU(v_\mu - a_\mu) \]

A given diagram \( \mathcal{O}(p^D) \) → \( E, |\vec{p}|, T, eA_\mu, \ldots \)

⇒ more precisely, \( p/\Lambda_\chi, T/T_c, \ldots \) with \( \Lambda_\chi \sim 1 \text{ GeV} \).

\[ D = 2 + \sum_n N_n (n - 2) + 2L \]

Number of vertices from \( \mathcal{L}_n \)

Number of loops
Chiral Lagrangians: leading order non-linear sigma model

\[
\mathcal{L}_2 = \frac{F^2}{4} \text{tr} \left[ (D_\mu U)^\dagger D^\mu U + 2B_0 \mathcal{M} \left( U + U^\dagger \right) \right]
\]

\[U = \exp[i\Phi/F]\]

\[
\text{SU}(2) : \Phi = \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix}, \quad \text{SU}(3) : \Phi = \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}} \eta & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}} \eta & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}K^0 & -\frac{2}{\sqrt{3}} \eta \end{pmatrix}
\]

\[\Rightarrow \mathcal{M} = \text{diag}(m_u, m_d, m_s) \text{ explicit ch. sym. Breaking perturbations}\]

\[M_\pi^2 = 2B_0 (m_u + m_d) \left[ 1 + \mathcal{O}(m_q) \right] ; \quad F_\pi = F \left[ 1 + \mathcal{O}(m_q) \right] ; \quad \langle \bar{q}q \rangle = -2F^2 B_0 \left[ 1 + \mathcal{O}(m_q) \right]\]

\[\Rightarrow \mathcal{L} \text{ chiral invariant } U \rightarrow RUL^\dagger \text{ in the chiral limit } \mathcal{M} = 0\]

\[\Rightarrow \text{isospin invariant } L = R = V \text{ for } \mathcal{M} = m_1\]
• Systematic construction of the effective lagrangian order by order in (covariant) derivatives and masses.

• All possible independent terms compatible with the symmetries:

\[
\mathcal{L}_4 = l_1 (\text{tr} [(D_\mu U)^\dagger D^\mu U])^2 \\
+ l_2 \text{tr} [(D_\mu U)^\dagger D^\nu U] \text{tr} [(D^\mu U)^\dagger D^\nu U] + \ldots
\]

+ \mathcal{L}_{WZW} \text{ also of } \mathcal{O}(p^4) \text{ accounting for anomalous processes}

• Low-Energy Constants (LEC) \( L_i \) absorb one-loop UV divergences

• Their finite part to be fixed by data or estimated theoretically with QCD models. Encode the underlying particle and field dynamics of heavier states.
Quark Condensate: Pure pion gas in ChPT & Virial

\[ \langle \bar{q}q \rangle_T = \langle \bar{q}q \rangle_0 + \frac{\partial P}{\partial m_q} \]

\[ \sim 1 - \frac{3T^2}{2(2\pi)^{3/2}} \sqrt{\frac{M_\pi}{T}} e^{-M_\pi/T} \ (T \ll M_\pi) \]

\[ \langle \bar{q}q \rangle_T \sim \langle \bar{q}q \rangle_0 \]

\[ \text{Ideal+O}(T^6) \]
\[ m_q = 0 \]

\[ \sim 1 - \frac{T^2}{8F_\pi^2} \ (T \gg M_\pi) \]

\[ \mathcal{O}(T^8) \]
\[ \text{virial} \]
\[ \mathcal{O}(p^4) \]

⇒ Chiral limit only qualitative.
For the relevant region, \( T, M_\pi \) of same chiral order

⇒ Improving the perturbative expansion reduces (extrapolated) \( T_c \)
Pole vs Screening masses

$$\chi_P = K_P(p = 0) \sim (M_P^{pole})^{-2}$$

General (lattice-like) parametrization:

$$K_P^{-1}(\omega, \vec{p}) = -\omega^2 + A^2(T)|\vec{p}|^2 + M_P^{pole}(T)^2$$

$$A(T) = \frac{M_P^{pole}(T)}{M_P^{sc}(T)} \text{ assumed smooth}$$

$$(A = 1 \text{ in one-loop ChPT, although may change near } T_c)$$
Pole position \( s_p(T) = [M_p(T) - i\Gamma_p(T)/2]^2 \) (2nd Riemann sheet)
\[ s_{\text{pole}} = (M_p - i\Gamma_p / 2)^2 \quad \text{(2nd Riemann sheet)} \]