CHIRAL SYMMETRY RESTORATION: PATTERNS AND PARTNERS

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OUTLINE:

- ★ **Chiral partners from effective theory:** (χ_S, χ_P)
- ★ **ChPT (model independent) results**
- ★ **Direct lattice analysis. Screening masses and** S/P **degeneration**
- ★ **Unitarizing: thermal** $f_0(500)$ **saturation for** χ_S

AGN, J.Ruiz Elvira, R.Torres, Phys.Rev. D88 (2013) 076007

SEWM LAUSANNE 14-18 JULY 2014

Basic Theory Ingredients

Chiral Perturbation Theory (ChPT):

★ **Based on Chiral Symmetry Breaking** $SU_L(N_f) \times SU_R(N_f) \to SU_V(N_f)$
$\Rightarrow$ **model independent low-energy predictions for** $N_f = 2, 3$
$\Rightarrow$ **Chiral expansion formally organized in** F_π^{-2} **powers.**

★ **Systematic and consistent Meson Gas description for** T **below** T_c
$\Rightarrow$ **Predicts (extrapolated)** $\langle \bar{q}q \rangle_T$ **melts**
$\Rightarrow$ **Reliable near** $T_{FO} \sim$ **100 MeV**
$\Rightarrow$ **Also useful near** $m_q \to 0^+$ **(chiral limit scaling e.g. in lattice)**

Unitarity (UChPT):

Improves the ChPT analytical description of scattering
$\Rightarrow$ **essential for generating resonances** $(\rho, \sigma, \dots)$
$\Rightarrow$ **accurate description of collisions for thermal width, transport,** $\dots$

Meson gas: Recent Progress within ChPT+UChPT

- ★ **Hot and dense light resonances:** ρ **broadening, chiral restoration in** $\sigma/f_0(500)$ **channel, threshold enhanc.**

  D.Cabrera, A.Dobado, D.Fernández-Fraile, AGN, F.J.Llanes-Estrada, J.R.Peláez,E.T.Herruzo: PLB 02, PRC 02, PLB 05, PRD 07, EPJC 09

- ★ **Transport coefficients:** Γ^{-1} **chiral power counting,** σ_{el}, η/s, ζ/s **OK pheno and theoretically**

  D.Fernández-Fraile,AGN: PRD 06, NPA 07, EPJA 07,EPJC 09, PRL 09

- ★ **Chemical nonequilibrium for interacting pions:** T_{CFO}, T_{FO} **reduced, BEC accesible via** M_π **dropping**

  D.Fernández-Fraile,AGN: PRD 09

- ★ **Isospin breaking, EM effects:** $\chi_S^{con,dis}$ **scaling,** $\Sigma_{\pi^\pm} - \Sigma_{\pi^0}$. $\langle \bar{q}q\bar{q}q \rangle$, χ_S/χ_P

  AGN, J.R.Elvira, J.R.Peláez, R.Torres: PRD 11, 13, 14

Scalar-Pseudoscalar Degeneration

★ **Old problem of $O(4)$ chiral partners (σ, π^a) addresed in LSM through $M_\sigma(T)\downarrow \langle\sigma\rangle(T)\downarrow$ with explicit σ field (not physical!)**

T.Hatsuda, T.Kunihiro PRL 85

★ **Vector-Axial degeneration @ chiral restoration well established with physical (ρ, a_1) states**

R.Rapp, J.Wambach ANP 00

★ **Crossover Ch.Sym.Rest $N_f = 2$ in lattice @ $T_c \sim$ 150-160 MeV consistent with $O(4)$ pattern**

Y.Aoki et al JHEP 09, S.Ejiri et al PRD 09 , A.Bazavov et al PRD 12

$\Rightarrow$ **S/P degeneration expected from χ_S maximum onwards**

Look at correlators: S/P Susceptibilities

$$\chi_P(T)\delta^{ab} = \int_0^\beta \int d^3\vec{x}\, \langle \mathcal{T}\left(\bar{q}\gamma_5\tau^a q\right)(x)\left(\bar{q}\gamma_5\tau^b q\right)(0)\rangle$$

$$\chi_S(T) = -\frac{\partial}{\partial m}\langle\bar{q}q\rangle_T = \int_0^\beta d\tau \int d^3\vec{x}\left[\langle\mathcal{T}(\bar{q}q)(x)(\bar{q}q)(0)\rangle_T - \langle\bar{q}q\rangle_T^2\right]$$

Expected to be saturated by π and σ-like poles:

$$\chi_P = 4B_0^2F_\pi^2 G_\pi(p^2=0) \sim 4B_0^2\frac{F_\pi^2}{M_\pi^2} = -\frac{\langle\bar{q}q\rangle}{m_q}$$

from PCAC+GOR $(T=0)$
or LO ChPT

$$B_0 = M_\pi^2/2m_q$$

$$\chi_S = 4B_0^2F_\pi^2 G_\sigma(p^2=0) \sim \frac{4B_0^2F_\pi^2}{M_\sigma^2}$$

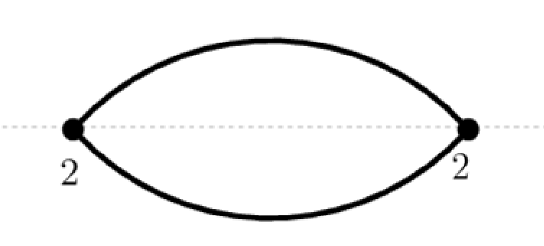
from $\mathcal{L}_{SB} = 2B_0F_\pi s(x)\sigma(x)$

But no need to deal with a particle-like σ state.
$\Rightarrow$ suitable for ChPT (model independent) and UChPT

ChPT calculation at *T≠0* to NLO

SCALAR CORRELATOR:

$\langle(\bar{q}q)^2\rangle_T$ **factorization breaking term** $\mathcal{O}(F^0)$



$$\chi_S^{ChPT}(T) = B_0^2\left[8\left(l_3^r(\mu)+h_1^r(\mu)\right) - 12\nu_\pi + 6 g_2(M_\pi,T)\right] + \mathcal{O}\left(F_\pi^{-2}\right)$$

$$\begin{aligned}
\nu_i &= \frac{1}{32\pi^2}\left(1+\log\frac{M_i^2}{\mu^2}\right)\\
g_1(M,T) &= \frac{1}{2\pi^2}\int_0^\infty dp\,\frac{p^2}{E_p}\frac{1}{e^{\beta E_p}-1} \to \frac{T^2}{12} \quad \text{for} \quad T\gg M\\
g_2(M,T) &= -\frac{dg_1(M,T)}{dM^2} = \frac{1}{4\pi^2}\int_0^\infty dp\,\frac{1}{E_p}\frac{1}{e^{\beta E_p}-1} \to \frac{T}{8\pi M} \quad \text{for} \quad T\gg M
\end{aligned}$$

ChPT calculation at $T\neq 0$ to NLO

Coupling external pseudoscalar sources, the Euclidean correlator:

$$K_P(p) = a - 4B_0^2F^2\frac{Z_\pi(T)}{p^2 - M_\pi^2(T)} - \frac{c(T)}{p^2 - M_\pi^2(T)} + \mathcal{O}(F_\pi^{-2})$$

$T=0$ **LEC**

Not just proportional to G_π^{NLO}!. Actually the residue:

$$4B_0^2F^2Z_\pi(T) + c(T) = \frac{\langle\bar{q}q\rangle_T}{\langle\bar{q}q\rangle_0}\frac{F_\pi^2(0)M_\pi^2(0)}{m_q^2}M_\pi^2(T) + \mathcal{O}(F_\pi^{-2})$$

$$K_P(0) = \chi_P^{ChPT}(T) = -\frac{\langle\bar{q}q\rangle_T}{m_q} + \mathcal{O}(F^{-2})$$

$$= 4B_0^2\left[\frac{F^2}{M^2} + \frac{1}{32\pi^2}(4\bar{h}_1 - \bar{l}_3) - \frac{3}{2M^2}g_1(M,T)\right] + \mathcal{O}(F_\pi^{-2})$$

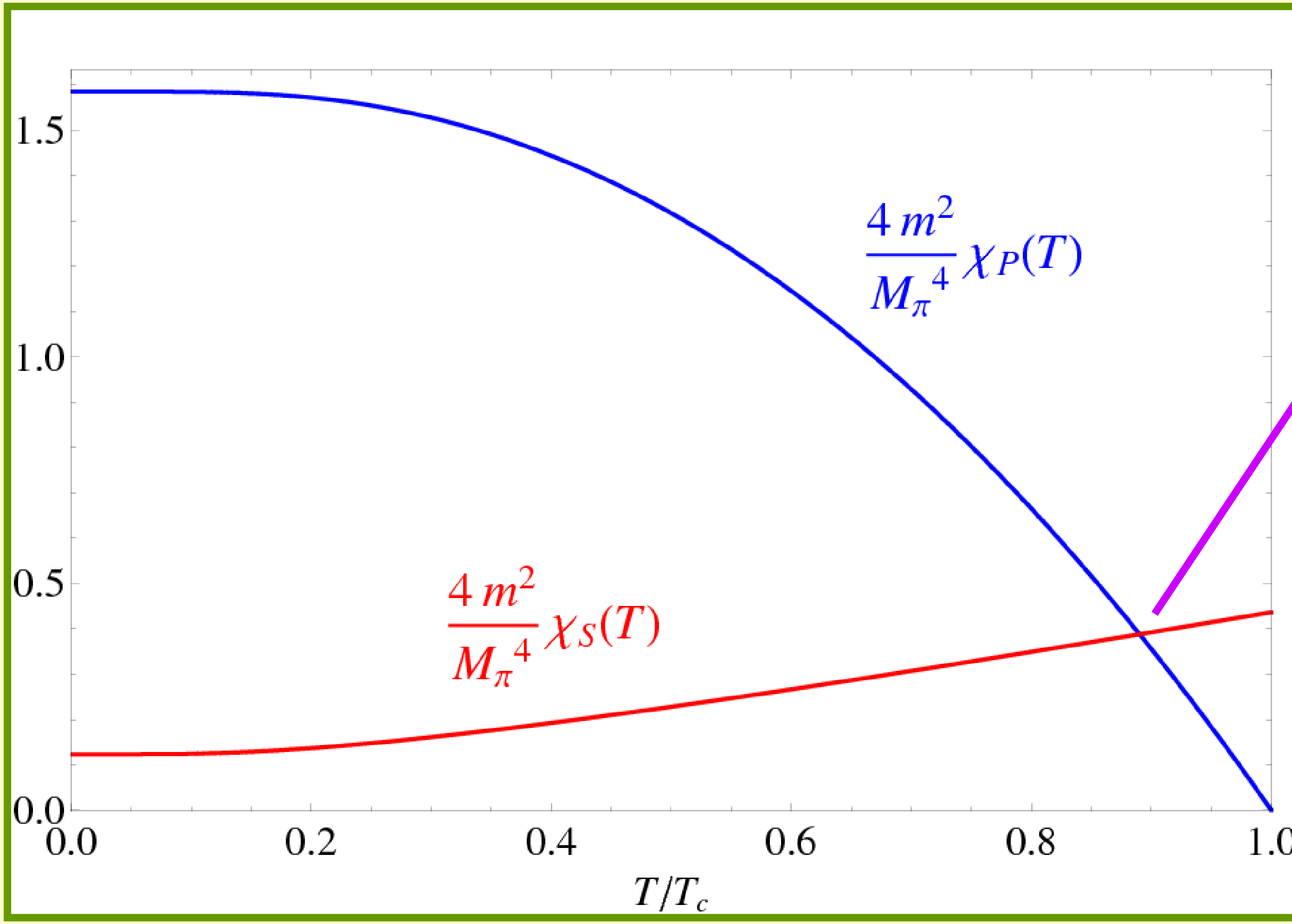
ChPT calculation at $T \neq 0$ to NLO

$$\chi_P^{ChPT}(T) = -\frac{\langle \bar{q} q \rangle_T}{m_q}$$

- ★ **Model independent. Finite and scale-independent**
- ★ $\frac{\langle \bar{q} q \rangle_T}{\langle \bar{q} q \rangle_0} = \frac{\chi_P(T)}{\chi_P(0)}$ **also LEC and m_q independent**
- ★ χ_P **drops as the condensate (not as softer $\sim M_\pi^{-2}(T)$)**
- ★ $\chi_P(\rho) \sim \langle \bar{q} q \rangle(\rho)$ **also in nuclear matter** G.Chanfray, M.Ericson EPJA 03
- ★ **Formal (bare and chiral symmetric) QCD Ward Identity.**

D.J.Broadhurst NPB 75
M.Bochicchio et al NPB 85
P.Boucaud et al PRD 10

Results: ChPT



$\langle \bar{q}q \rangle (T_c) = 0$

$$T_d \sim 0.9 T_c$$

$\chi_S(T_d) = \chi_P(T_d)$

$$T_d \simeq T_c - \frac{3M_\pi}{4\pi}$$

$$(M_\pi \ll T)$$

$T_d \to T_c$ **in chiral limit**

(exact chiral sym.rest.)

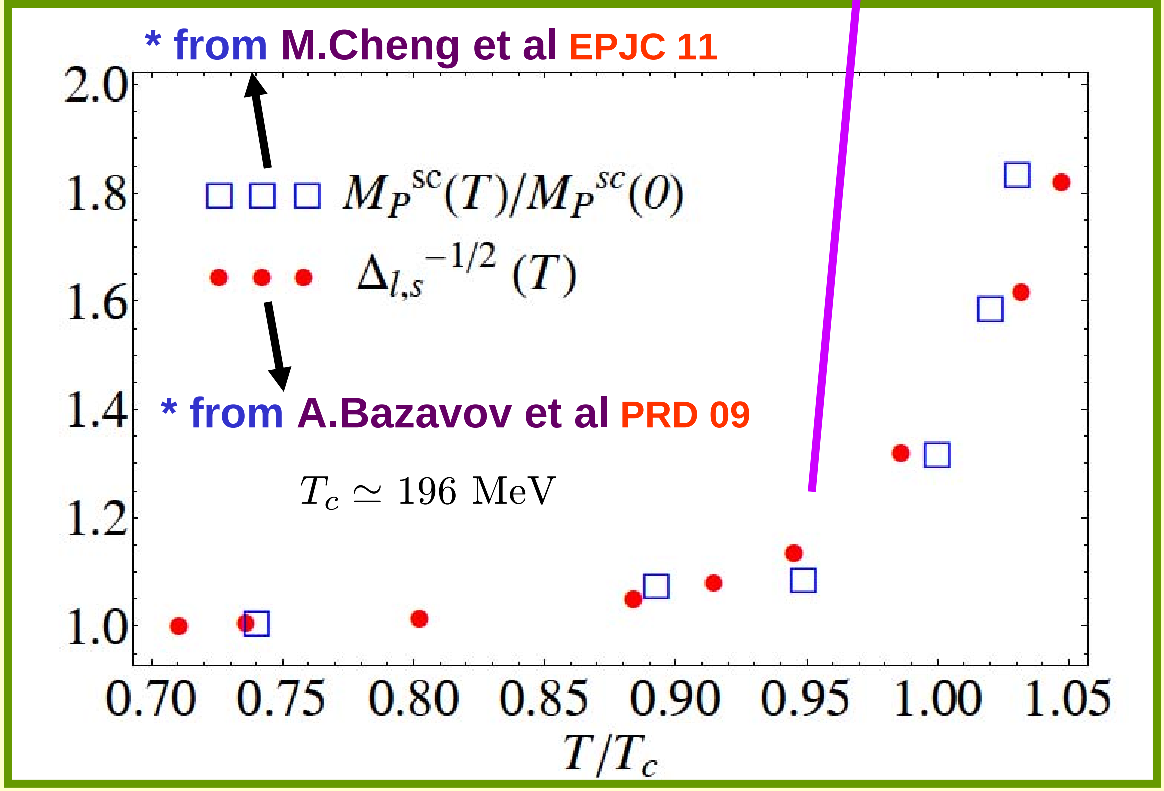
Direct Analysis of Lattice Data

1) $\dfrac{\langle\bar{q}q\rangle_T}{\langle\bar{q}q\rangle_0} = \dfrac{\chi_P(T)}{\chi_P(0)}$ **from K_P (screening) masses:**

$$\frac{M_P(T)}{M_P(0)} \sim \left[\frac{\chi_P(0)}{\chi_P(T)}\right]^{1/2} \sim \left[\frac{\langle\bar{q}q\rangle_0}{\langle\bar{q}q\rangle_T}\right]^{1/2}$$

Explains sudden increase near T_c

$$\Delta_{l,s} = \frac{\langle\bar{q}q\rangle_T - (2m_q/m_s)\langle\bar{s}s\rangle_T}{\langle\bar{q}q\rangle_0 - (2m_q/m_s)\langle\bar{s}s\rangle_0}$$



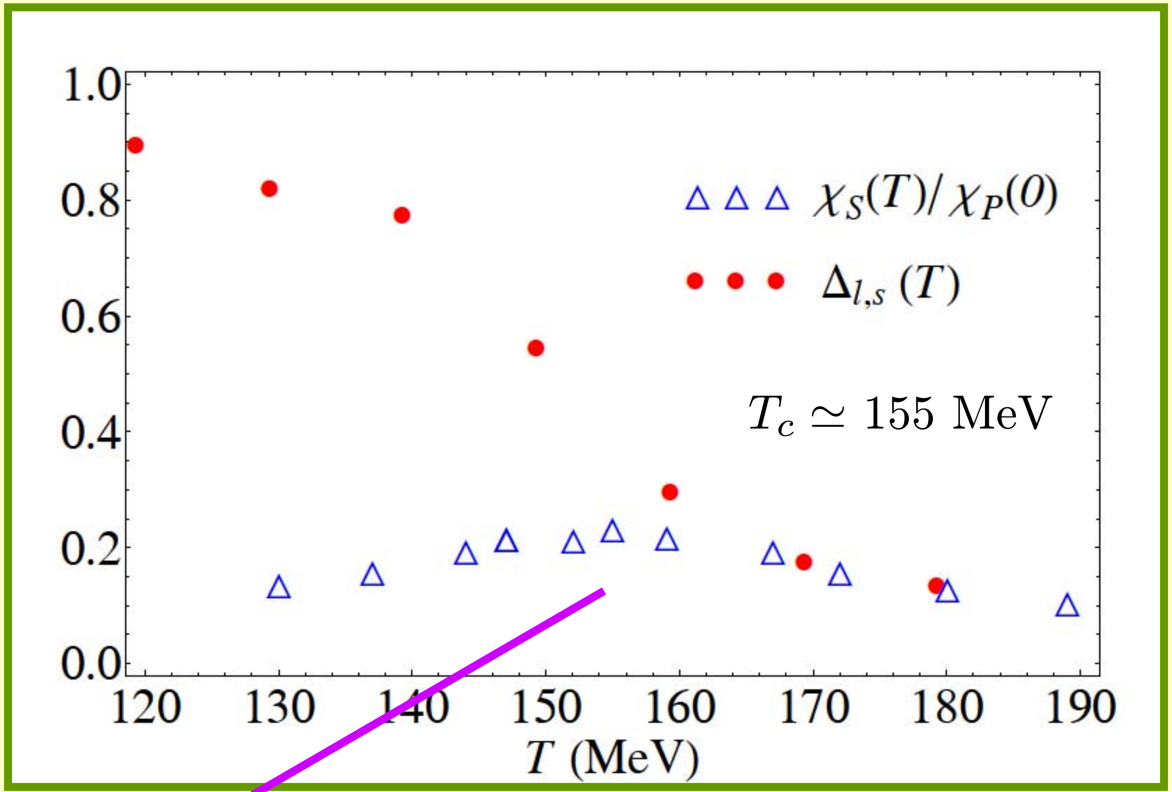
Uncertainties:

- $\langle\bar{s}s\rangle$ **subtr $\sim$ 15% @ T_c**
- M_P^{sc}/M_P^{pole} **smoothness**
- **Lattice effects**

*** same lattice conditions for masses and condensate**

Direct Analysis of Lattice Data

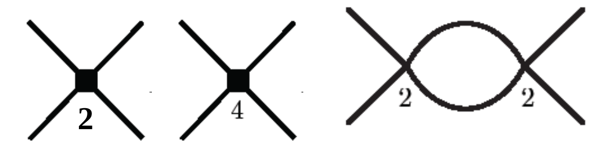
2) χ_S/χ_P degeneration:



Data from Y.Aoki et al JHEP 09

Degeneration from the χ_S maximum onwards

Unitarizing ChPT: scattering vs susceptibility



ChPT Partial waves $\boxed{t^{IJ} = t_2^{IJ} + t_4^{IJ} + \ldots}$

+

Unitarity $\rightarrow$ **Im** $t(s) = \sigma(s)|t(s)|^2 \;\; (s \geq 4M^2) \Rightarrow$ $\boxed{\textbf{Im } t^{-1} = -\sigma}$

$$\sigma(s) = \sqrt{1 - \frac{4M^2}{s}} = \frac{p_{CM}}{\sqrt{s}}$$ **two-particle phase space**

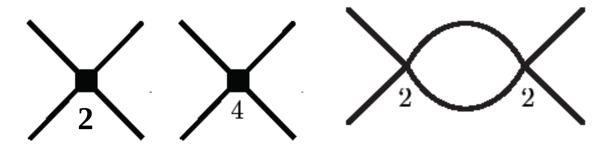
$$\boxed{t^U(s) = \frac{[t_2(s)]^2}{t_2(s) - t_4(s)}}$$

Resonances dynamically generated as poles in 2nd RS, no assumptions about their nature or couplings. Formally justified by dispersion relations.

Successful for scattering data up to 1 GeV & low-lying resonance multiplets.

Dobado, Peláez, Oset, Oller, AGN.

Unitarizing ChPT: scattering



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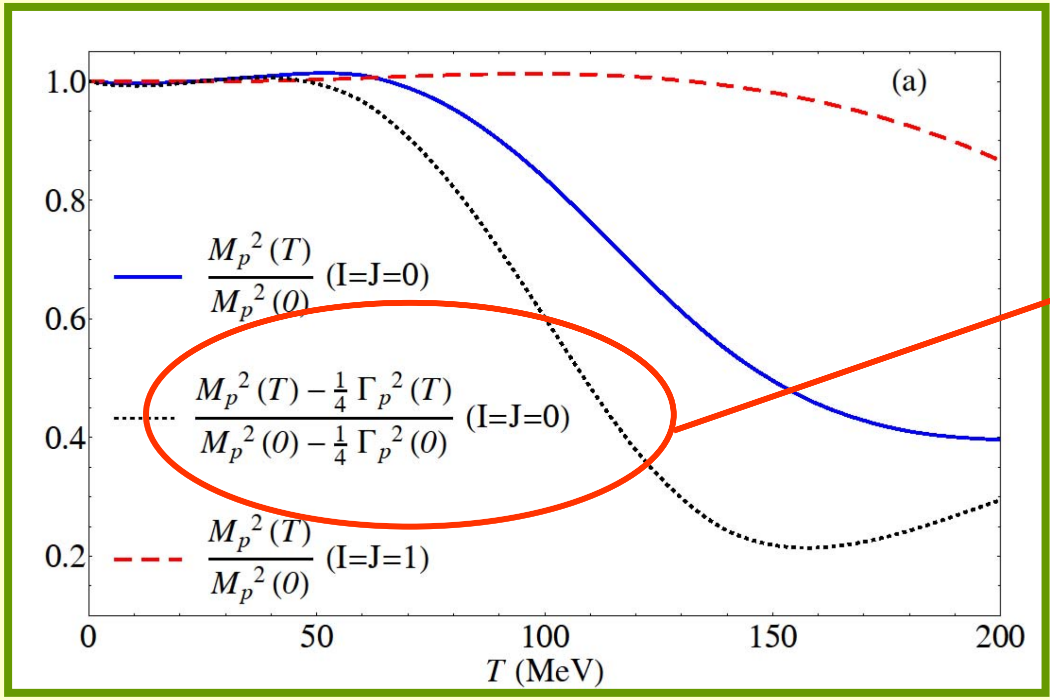
FINITE TEMPERATURE:

$$t_4(s) \rightarrow t_4(s;T)$$

$$\sigma \rightarrow \sigma\left[1 + 2n_B(\sqrt{s}/2)\right] \equiv \sigma_T$$

Thermal phase Space.
Bose net enhancement $(1+n)^2 - n^2$

A.Dobado, D.Fernández-Fraile, AGN, F.J.Llanes-Estrada, J.R.Peláez, E.Tomás-Herruzo, '02 '05 '07



$$M_S^2(T) = M_p^2(T) - \Gamma_p^2(T)/4$$

scalar pole mass

Chiral restoring behaviour !

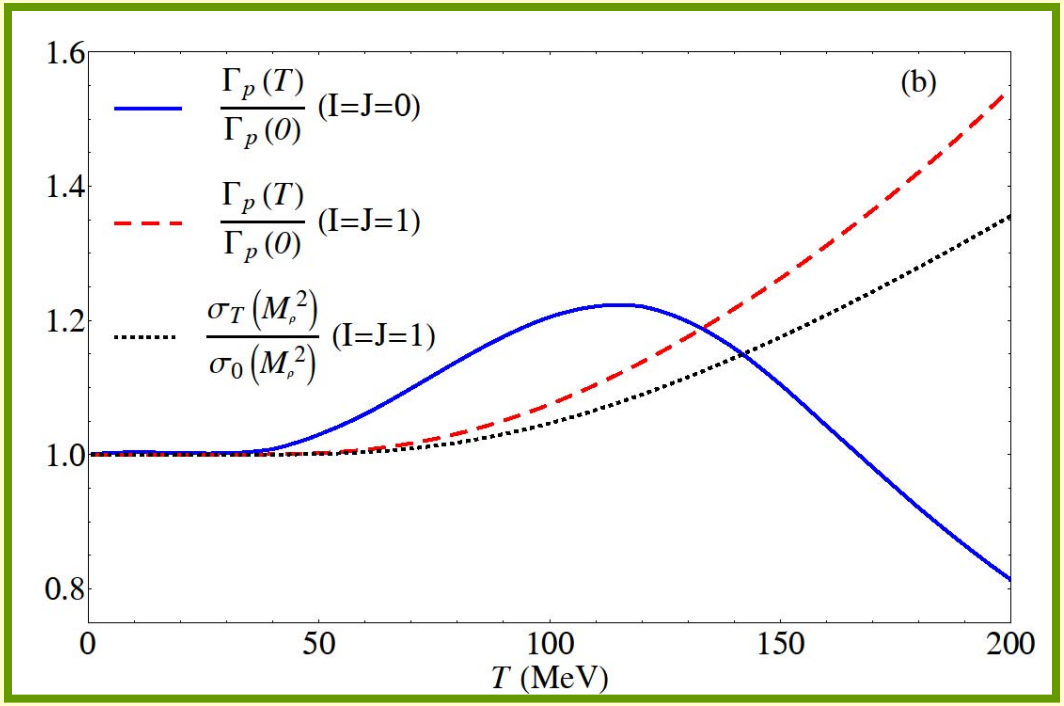
$$I = J = 0: \; f_0(500)$$

$$I = J = 1: \; \rho(770)$$

Pole position:

$$s_p(T) = [M_p(T) - i\Gamma_p(T)/2]^2$$

(2nd Riemann sheet)



Unitarized Scalar Susceptibility

★ **Saturate the scalar correlator with the $f_0(500)$ thermal state:**

(assuming $p=0$ pole not very diff. from s_p)

$$\boxed{\chi_S^U(T) = \frac{\chi_S^{ChPT}(0) M_S^2(0)}{M_S^2(T)}}$$

Normalization to match $T=0$ ChPT.
Compensates pole diff.

★ **Unitarized condensate from χ^U requires additional scaling assumptions (holding in ChPT):**

$$\delta f(T) = f(T) - f(0)$$

$$\delta \langle \bar{q} q \rangle^U (T, M) = B_0 T^2 g(T/M)$$

$x_0 \ll 1$ matching point

$$\delta \chi_S^U = B_0^2 h(T/M)$$

$$\boxed{\begin{aligned} g(x) &= g(x_0) + \int_{x_0}^{x} \frac{h(y)}{y^3} dy \quad (x > x_0) \\ g(x) &= g_{ChPT}(x) \quad (x \le x_0) \end{aligned}}$$

Results: ChPT & UChPT

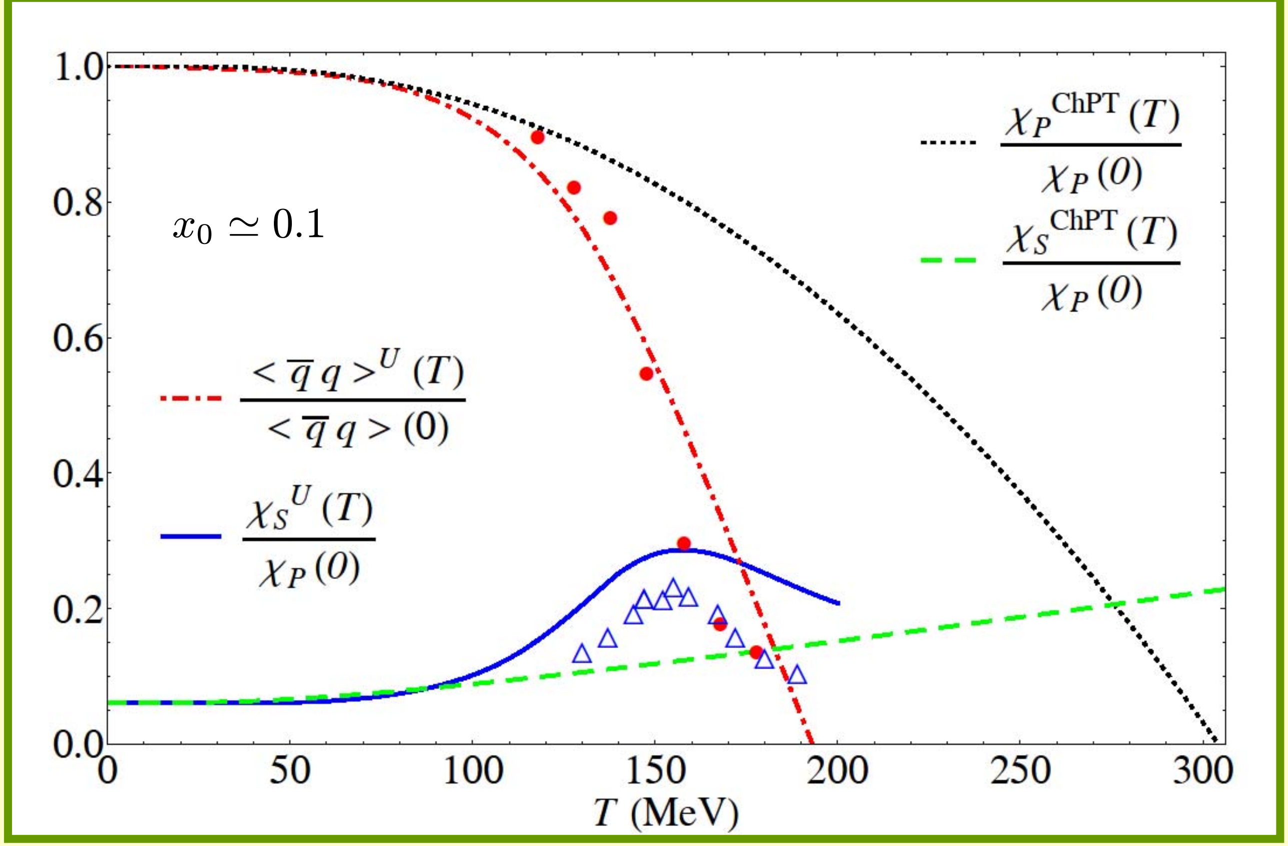
Data from
Y.Aoki et al JHEP 09

Not a fit for Unit.

LEC fixed to $T=0$ comp. with PDG:

$M_p^{00} = 441$ **MeV**
$\Gamma_p^{00} = 466$ **MeV**

$M_p^{11} = 756$ **MeV**
$\Gamma_p^{11} = 151$ **MeV**



- ★ **Improving of critical behaviour** $\rightarrow$ χ_S^U **peak at** $T_c =$ **157 MeV**
  $T_c \downarrow$ **and more abrupt** χ_S^U **near chiral limit**
- ★ **low-**T χ_S^U **and** $\langle \bar{q}q \rangle^U$ **OK with ChPT**
- ★ **S/P intersection near** χ_S^U **peak**

Conclusions

★ **Chiral partner degeneration at T_c within $O(4)$ pattern holds for $(\chi_S, \chi_P) \Rightarrow$ eff.theory & lattice analysis.**

★ $\chi_P(T) = -\langle \bar{q}q \rangle (T)/m_q$ **proved to NLO ChPT (model ind). Holds also for lattice data $\Rightarrow$ explains sudden growth of pseudoscalar screening mass.**

★ **Saturation by thermal $f_0(500)/\sigma$ state (dyn.gen. via unitariz.) is a very relevant effect to get χ_S and S/P degeneration in accordance with lattice data**

★ **In progress: $SU(3)$, $a_0(980)$...**

Backup Slides

Chiral lagrangians and power counting

$$\mathcal{L}[U, s, v, a, p, \theta] = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_4^{WZW} + \ldots$$

systematic expansion in $D_\mu U$ **and external fields** $s \sim m_q \sim M^2, v, a, p, \theta$

$U = \exp[i \sum_a \pi_a t^a / F]$ $\qquad D_\mu U \; = \; \partial_\mu U - i(v_\mu + a_\mu)U + iU(v_\mu - a_\mu)$

A given diagram $\mathcal{O}(p^D)$ $\quad E, |\vec{p}|, T, eA_\mu, \ldots$

$\Rightarrow$ **more precisely,** $p/\Lambda_\chi, T/T_c, \ldots$ **with** $\Lambda_\chi \sim$ **1 GeV.**

$$\boxed{D = 2 + \sum_n N_n (n-2) + 2L}$$

Number of loops

Number of vertices from $\mathcal{L}_n$

Chiral Lagrangians: leading order non-linear sigma model

$$\boxed{\mathcal{L}_2 = \frac{F^2}{4} \mathrm{tr}\left[(D_\mu U)^\dagger D^\mu U + 2B_0 \mathcal{M}\left(U + U^\dagger\right)\right]}$$

$$U = \exp[i\Phi/F]$$

$$\mathbf{SU(2)}: \Phi = \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix}, \quad \mathbf{SU(3)}: \Phi = \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & \frac{-2}{\sqrt{3}}\eta \end{pmatrix}$$

$\Rightarrow \mathcal{M} = \mathrm{diag}(m_u, m_d, m_s)$ **explicit ch.sym.Breaking perturbations**

$$M_\pi^2 = 2B_0(m_u + m_d)\left[1 + \mathcal{O}(m_q)\right] \; ; \; F_\pi = F\left[1 + \mathcal{O}(m_q)\right] \; ; \; \langle \bar{q}q \rangle = -2F^2 B_0\left[1 + \mathcal{O}(m_q)\right]$$

$\Rightarrow \mathcal{L}$ **chiral invariant** $U \to RUL^\dagger$ **in the chiral limit** $\mathcal{M} = 0$

$\Rightarrow$ **isospin invariant** $L = R = V$ **for** $\mathcal{M} = m\mathbf{1}$

- **Systematic construction of the effective lagrangian order by order in (covariant) derivatives and masses.**

- **All possible independent terms compatible with the symmetries:**

$$\begin{aligned}\mathcal{L}_4 &= l_1\left(\mathrm{tr}\left[(D_\mu U)^\dagger D^\mu U\right]\right)^2 \\ &+ l_2 \mathrm{tr}\left[(D_\mu U)^\dagger D_\nu U\right]\mathrm{tr}\left[(D^\mu U)^\dagger D^\nu U\right] + \ldots\end{aligned}$$

$+\mathcal{L}_{WZW}$ also of $\mathcal{O}(p^4)$ accounting for anomalous processes

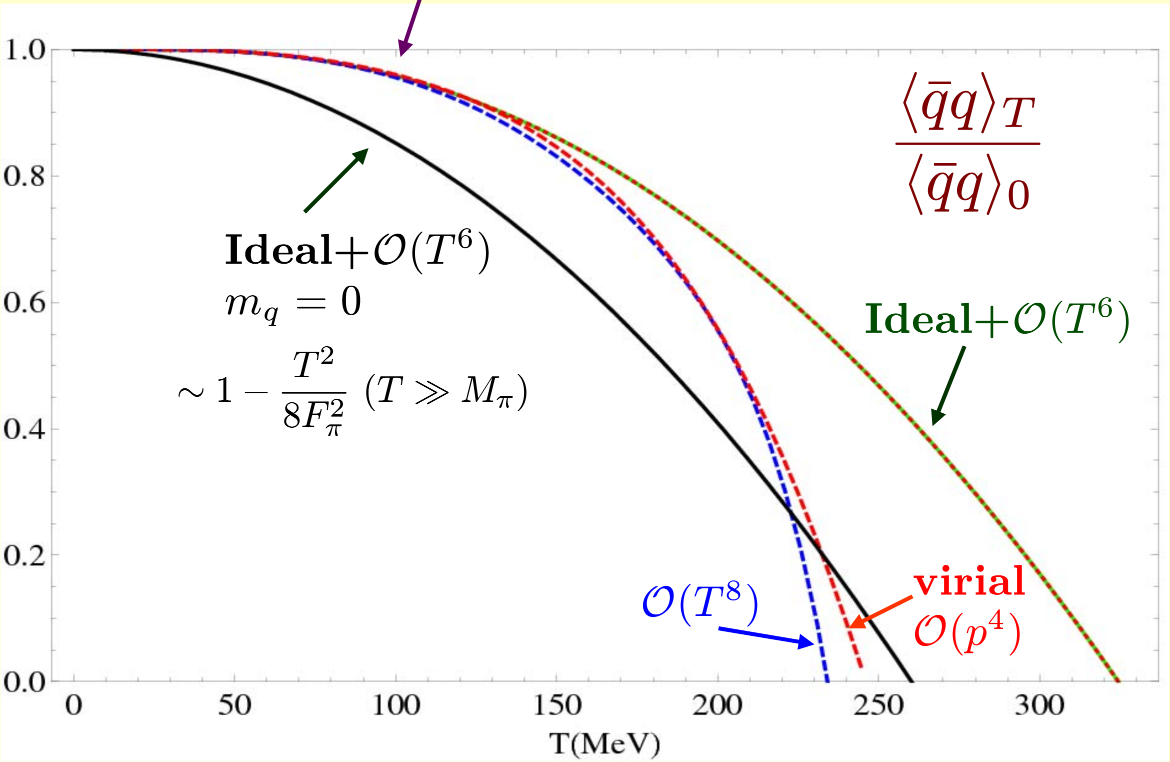
- **Low-Energy Constants (LEC) L_i absorb one-loop UV divergences**

- **Their finite part to be fixed by data or estimated theoretically with QCD models. Encode the underlying particle and field dynamics of heavier states.**

Quark Condensate: Pure pion gas in ChPT & Virial

$$\sim 1 - \frac{3T^2}{2(2\pi)^{3/2}}\sqrt{\frac{M_\pi}{T}}e^{-M_\pi/T} \quad (T \ll M_\pi)$$

$$\langle \bar{q}q\rangle_T = \langle \bar{q}q\rangle_0 + \frac{\partial P}{\partial m_q}$$



$\Rightarrow$ **Chiral limit only qualitative.**
For the relevant region, T, M_π of same chiral order

$\Rightarrow$ **Improving the perturbative expansion reduces (extrapolated) T_c**

Pole vs Screening masses

$$\chi_P = K_P(p=0) \sim (M_P^{pole})^{-2}$$

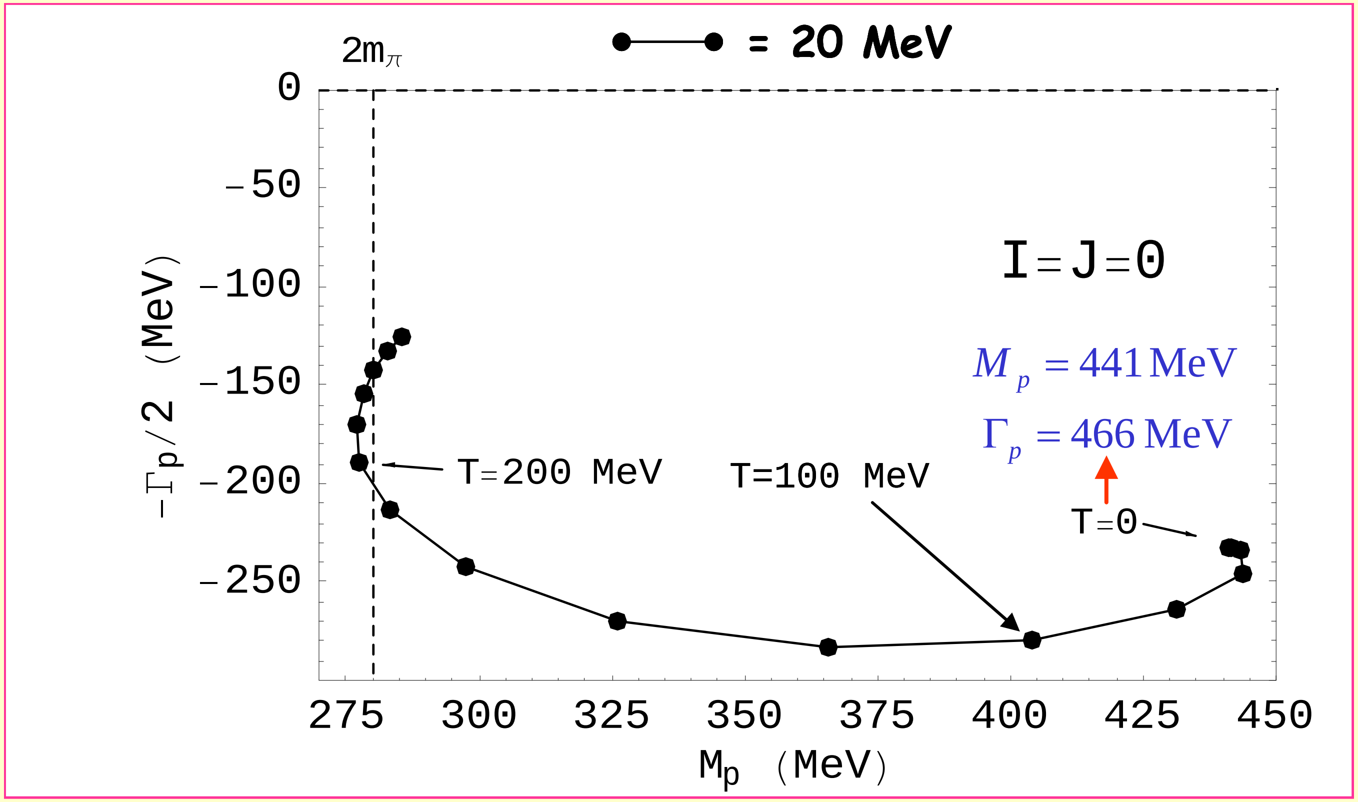
General (lattice-like) parametrization:

$$K_P^{-1}(\omega,\vec{p}) = -\omega^2 + A^2(T)|\vec{p}|^2 + M_P^{pole}(T)^2$$

$A(T) = M_P^{pole}(T)/M_P^{sc}(T)$ **assumed smooth**

($A=1$ **in one-loop ChPT, although may change near** T_c**)**

f_0(500) thermal pole:



Pole position $s_p(T) = [M_p(T) - i\Gamma_p(T)/2]^2$ **(2nd Riemann sheet)**

ρ (770) thermal pole

$s_{pole} = (M_p - i\Gamma_p / 2)^2$ **(2nd Riemann sheet)**