Gravitational waves from phase transitions in the early universe

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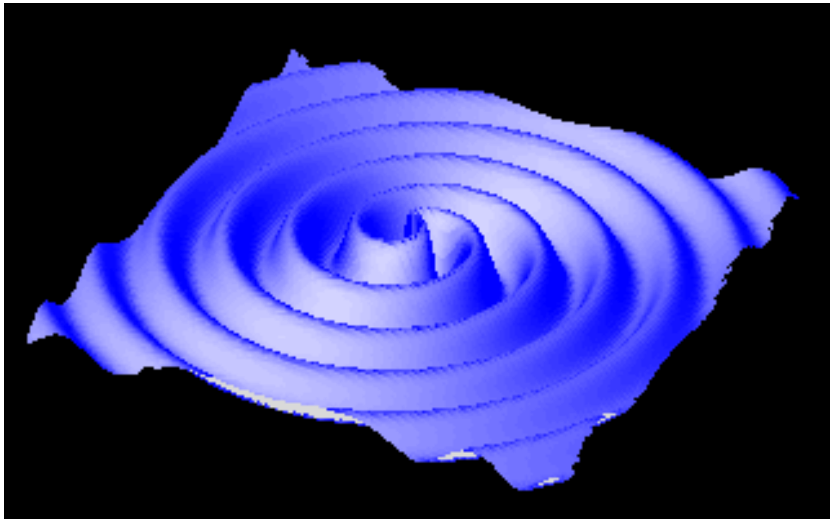
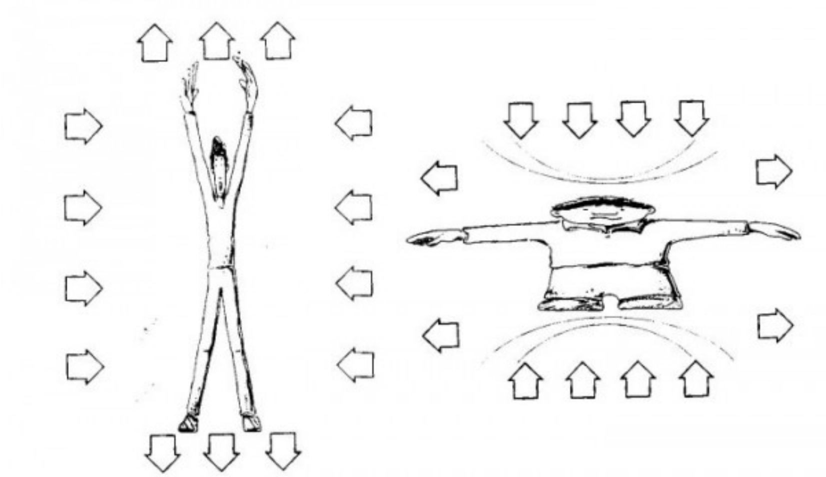
Gravitational waves

- Predicted by Einstein, 1919
- Generated by accelerating, asymmetric mass-energy (quadrupole moment)
- Astrophysical sources: binary compact objects (white dwarves, neutron stars, black holes); supernovae
- Cosmological sources: early universe

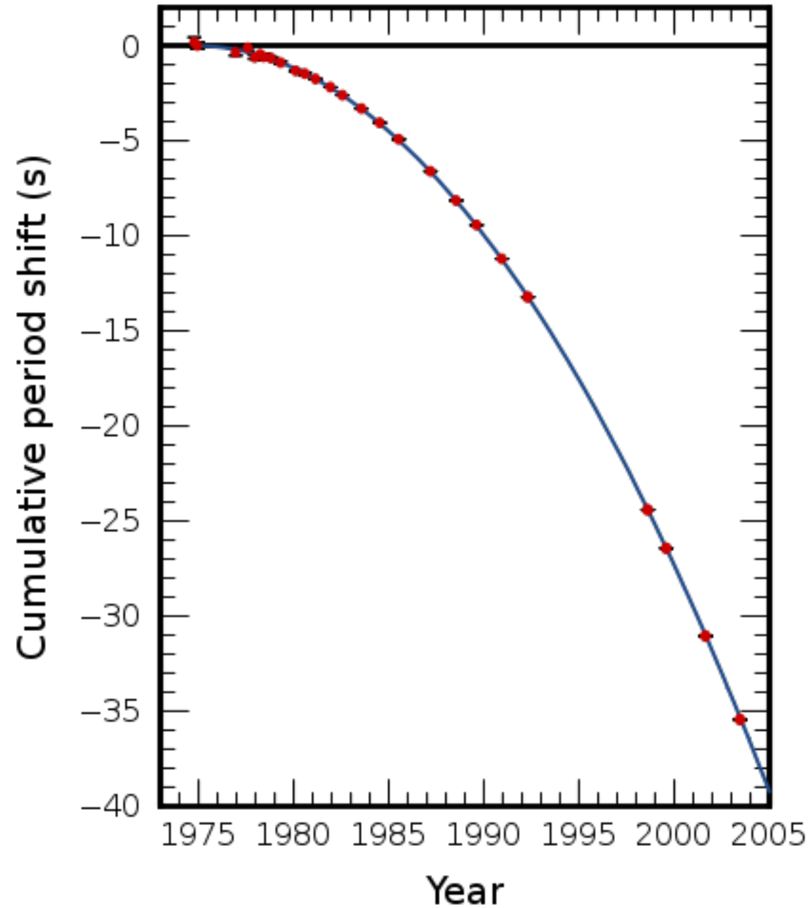




Evidence for gravitational waves

- Binary pulsar B1913+16 (Hulse, Taylor 1975)
- Orbital period decreases with emission of GWs
- Measured rate of change agrees with GR prediction to 0.2%

$$\begin{aligned} \dot{P}_b &= 2.4184(9) \times 10^{-12} \\ \dot{P}_{b,GR} &= 2.40242(2) \times 10^{-12} \end{aligned}$$



See Weisberg, Taylor (2004)

Gravitational waves in the early universe

- Sources
  - Inflation
  - Preheating after inflation
  - Topological defects
  - First-order phase transitions
- Early Universe is transparent to GWs
  - Unique probe of physics at high energies

Gravitational waves from the early universe

- Events at time *t* generate waves with minimum frequency $f \approx 1/t$
- Redshifted to a frequency now: $f_0 = (a(t)/a(t_0))f$
- Minimum frequencies:

Event	Time/s	Temp/GeV	f_0/Hz
QCD transition	10^{-3}	0.1	10^{-8}
EW transition	10^{-11}	100	10^{-5}
End of inflation	$\geq 10^{-36}$	$\leq 10^{16}$	$\leq 10^{8}$

Detecting gravitational waves

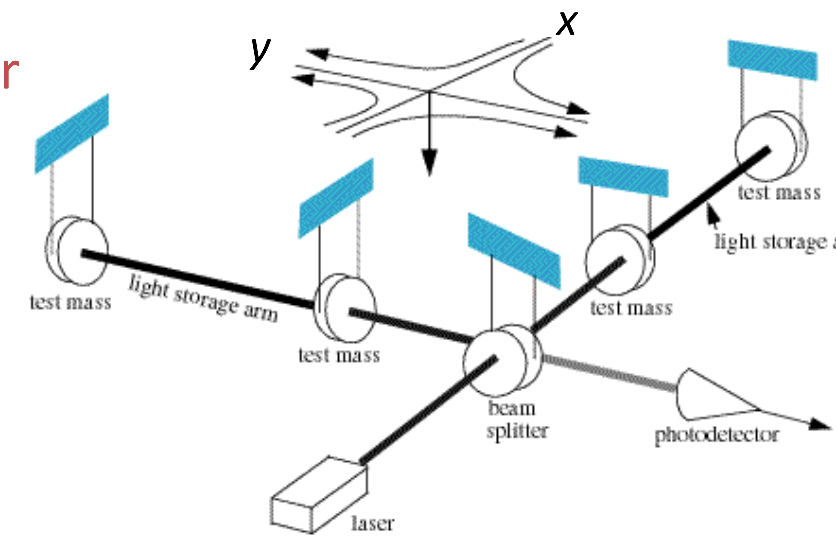
- Compare distances between test masses in two directions with laser interferometer
- Strain ≈ Metric perturbation

$$\frac{\Delta l}{l} = \frac{1}{2}(h_{xx} - h_{yy})$$

- Sensitivity: $\Delta l \approx 10^{-18}$ m @ 10^{2} Hz
-

Name	Location	Arm length
aLIGO	USA (2)	4km
GEO	Germany	600m
VIRGO	Italy	3km
KAGRA*	Japan	3km

- Future: Einstein Telescope

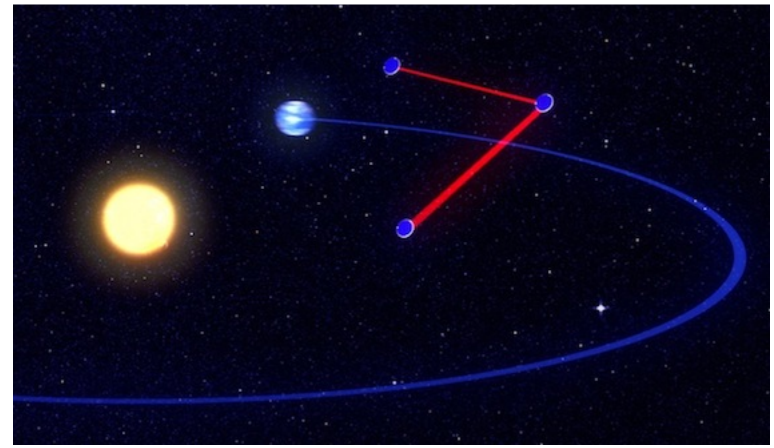


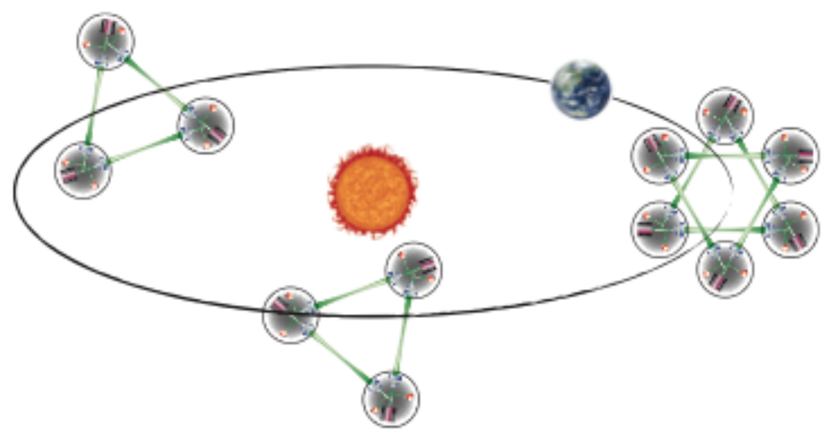
LIGO

LIGO

Space-based gravitational wave detectors

- Approved:
  - eLISA (ESA, 2034)
- Proposed:
  - DECIGO (Japan, ?)
  - Big Bang Observer (USA, ?)
- eLISA sensitivity
  - Peak: 10^{-3} - 10^{-2} Hz
  - $l \approx 10^{9}$ m
  - $\Delta l \approx 10^{-12}$ m
  - $h \approx 10^{-21}$





Measures of gravitational waves

- Unit vectors along interferometer arms: l_i m_i
- Fourier transform of strain

$$\tilde{h}(f) = \frac{1}{2}\int_{-\infty}^{\infty} dt\, e^{-i2\pi f t} h_{ij}(l_i l_j - m_i m_j)$$

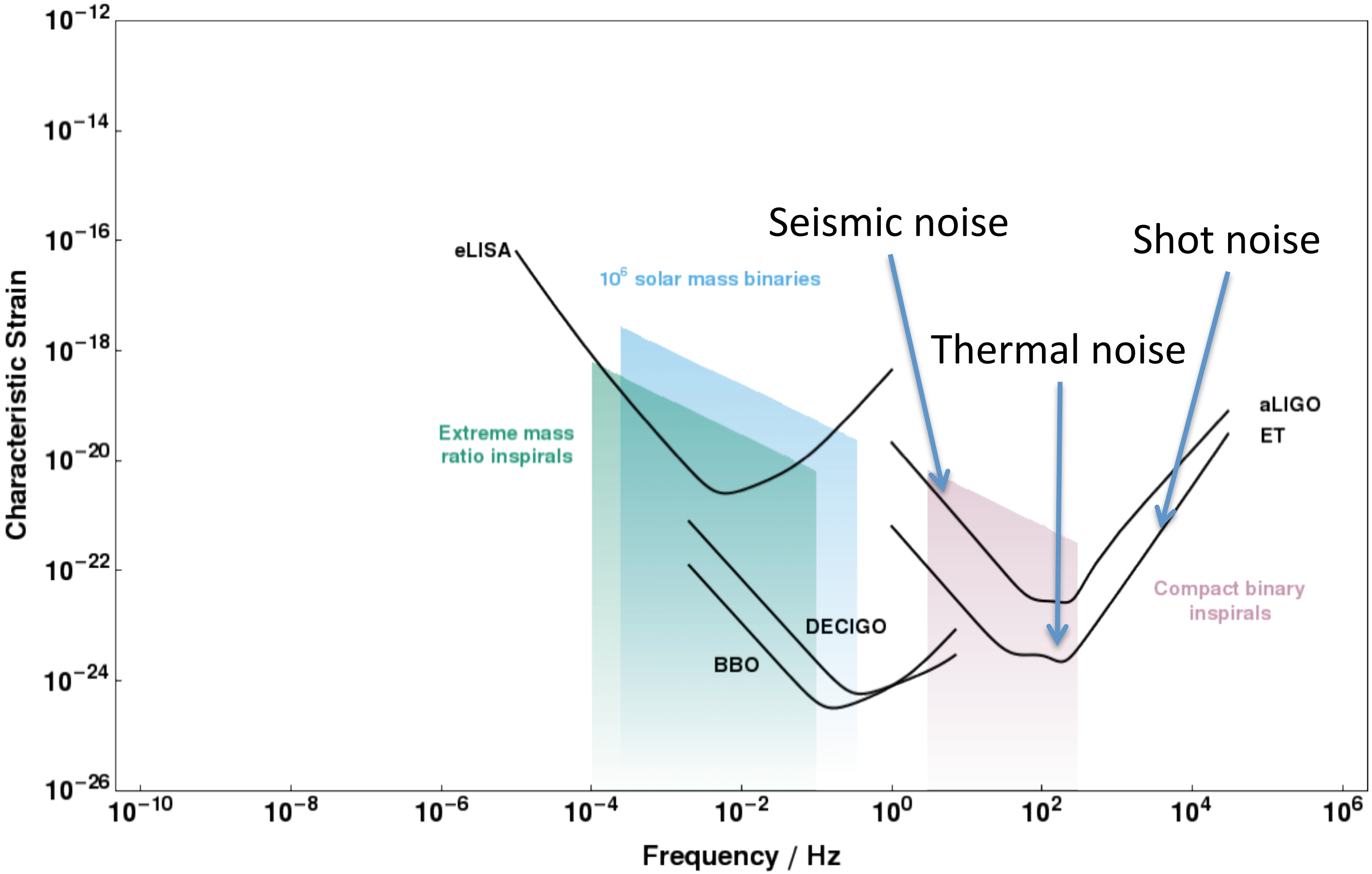
- One-sided power spectrum $S_h(f)$ ($f > 0$)

$$\langle \tilde{h}(f)\tilde{h}^*(f')\rangle = \frac{1}{2} S_h(f)\delta(f - f')$$

  - Characteristic strain (dimensionless) $h_c(f) = \sqrt{f S_h(f)}$
  - Root power spectral density ($\sqrt{\mathrm{Hz}^{-1}}$) $h(f) = \sqrt{S_h(f)}$
  - Energy density per logarithmic frequency interval:

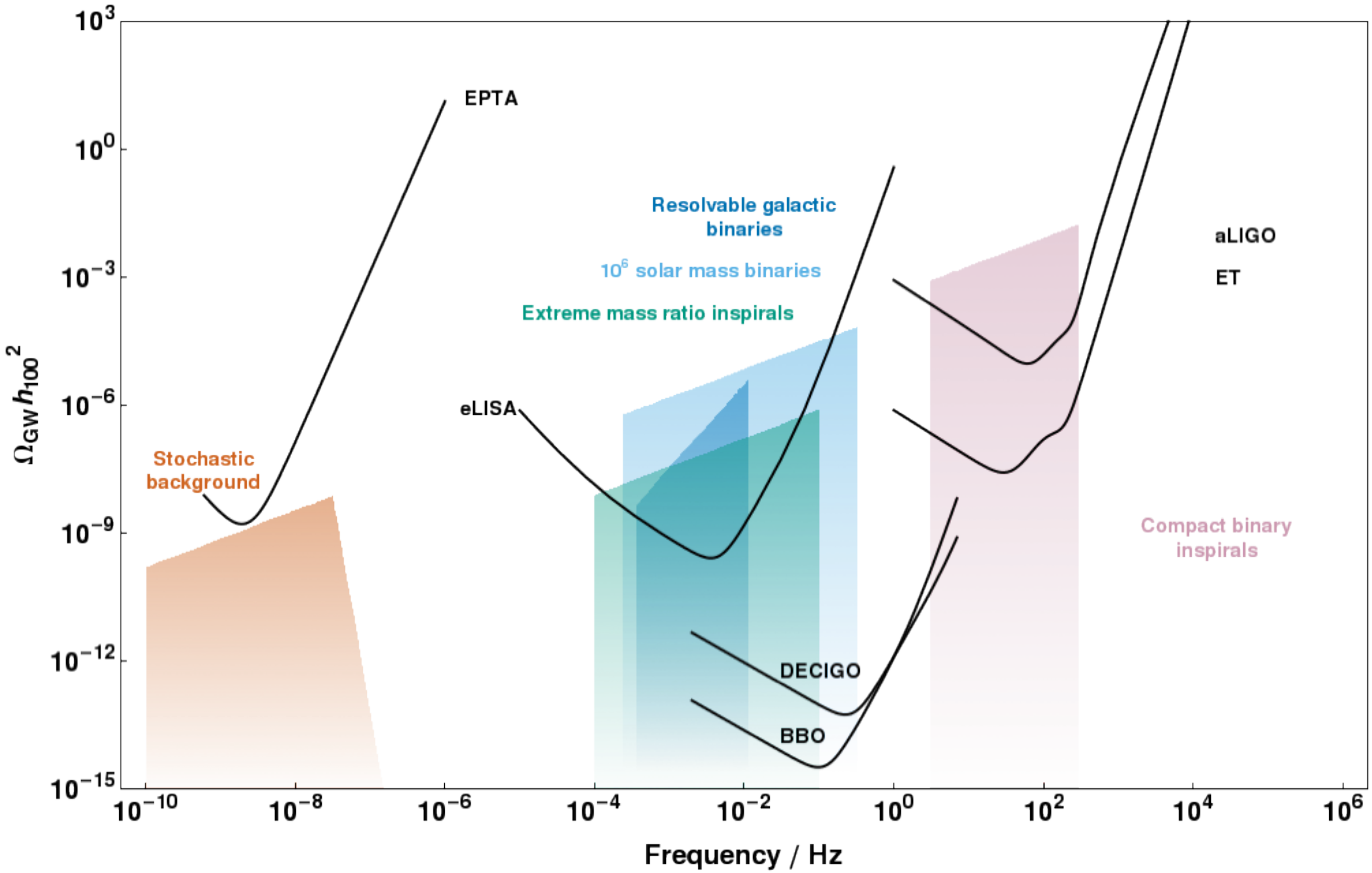
$$\frac{d\rho_{\mathrm{GW}}(f)}{d\ln f} = \frac{\pi}{G} f^5 S_h(f)$$

Sensitivity curves, astrophysical sources



http://www.ast.cam.ac.uk/~rhc26/sources/

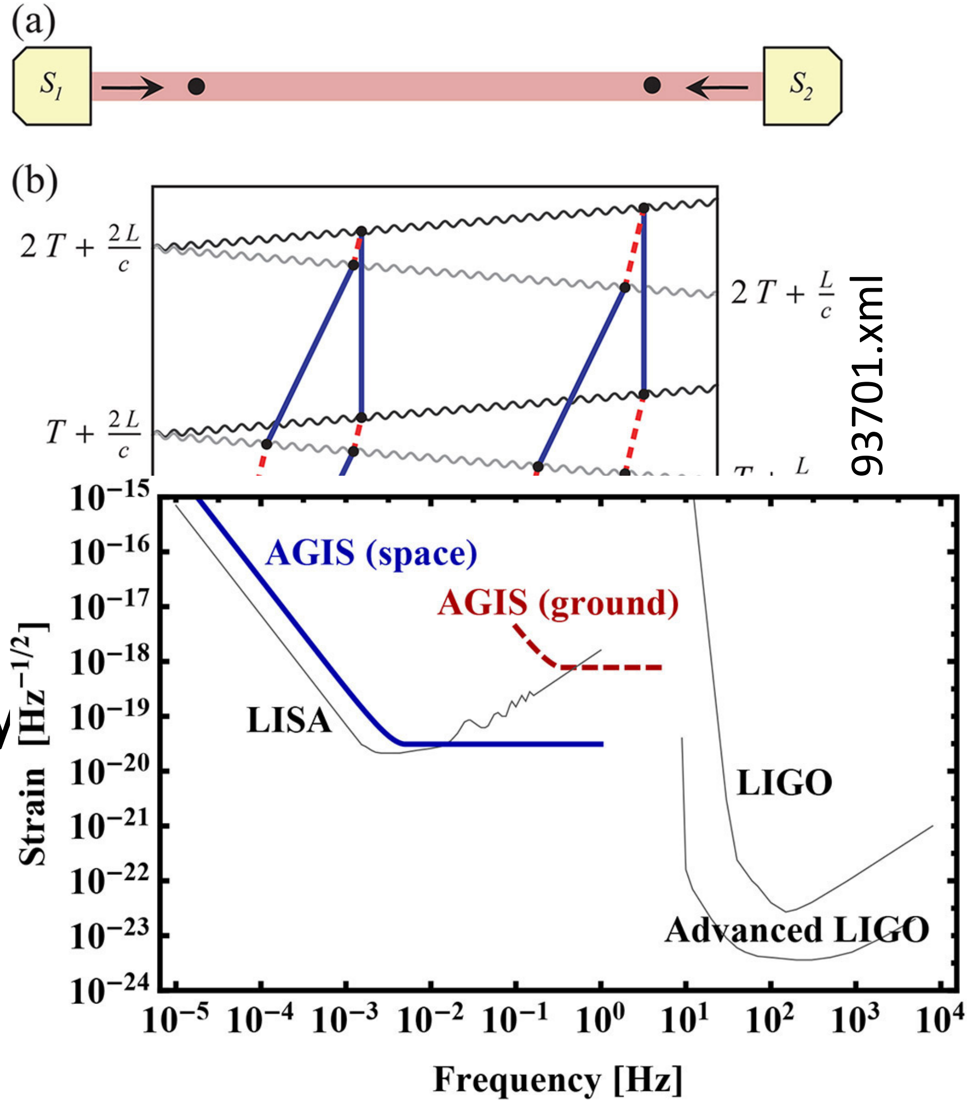
GW energy density sources and sensitivity



http://www.ast.cam.ac.uk/~rhc26/sources/

Future: atom interferometers?

- AGIS (Stanford)
- Test masses are clouds of ultracold atoms
- Interference between wavepackets split by laser pulses
- Phase differences between two clouds are produced by same laser
- Reduces laser frequency noise



100 MHz GW detector

- Gravitational wave rotates plane of polarisation of E field in toroidal resonant cavity
- Two detectors in correlation increase sensitivity
- Prototype:

  $h_{noise} \sim 10^{-10}$
- Other designs:
  - Tabletop interferometer ($h \sim 10^{-17}$)
  - Magnetic conversion

Cruise, Ingley (2006)

Akutsu et al (2008)
Cruise (2012)

Gravitational waves from the early universe

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- Topological defects and inflation: waves on all scales

Gravitational waves from inflation

- Action (Einstein + inflaton + matter):

$$S = \int d^4x \sqrt{-g} \left(\frac{R}{16\pi G} - \frac{1}{2}\partial\phi^2 - V(\phi) + \mathcal{L}_m \right)$$

- Perturb metric around inflating background (~ de Sitter):

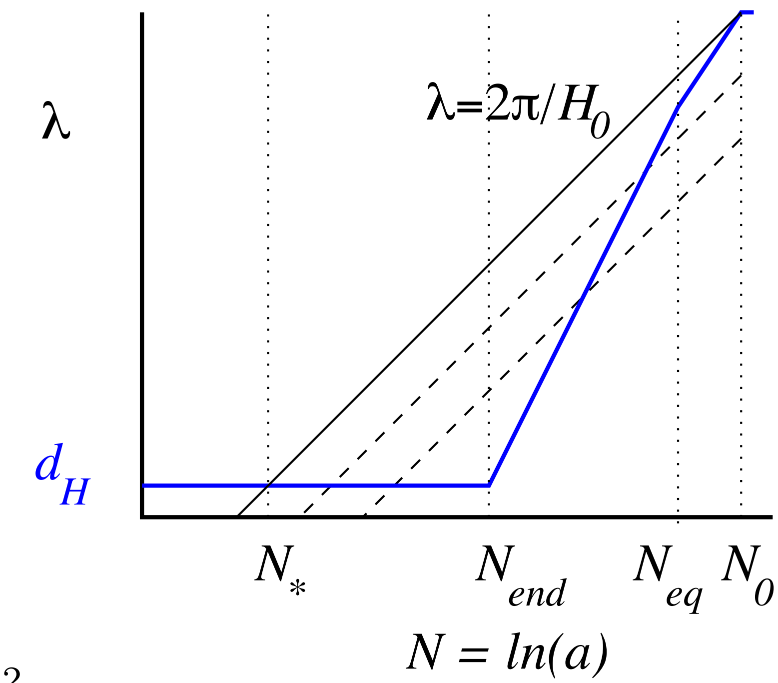
$$ds^2 = a^2(\tau)\left[-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j\right]$$
$$a(\tau) = -1/H_\mathrm{i}\tau$$

- Quantise metric perturbations
- Energy density power spectrum:

$$\frac{d\rho_\mathrm{GW}(k)}{d\ln k} = \frac{1}{32\pi G}\frac{k^3}{2\pi^2}\left\langle \left|\dot{h}_{ij}(t,k)\right|^2 \right\rangle$$

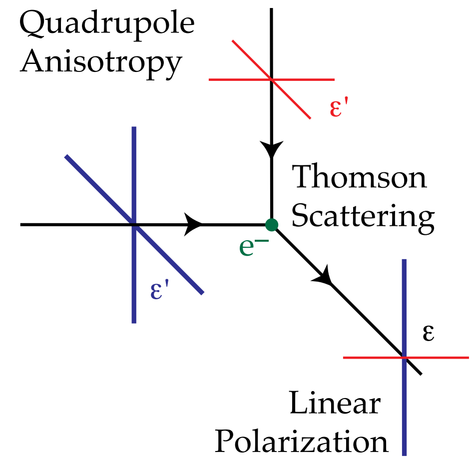
- Scale-invariant spectrum of GWs

$$\Omega_\mathrm{GW}(k) = \frac{1}{\rho_\mathrm{c}}\frac{d\rho_\mathrm{GW}}{d\ln k} = 16\pi G \left(\frac{H_k}{2\pi}\right)^2$$

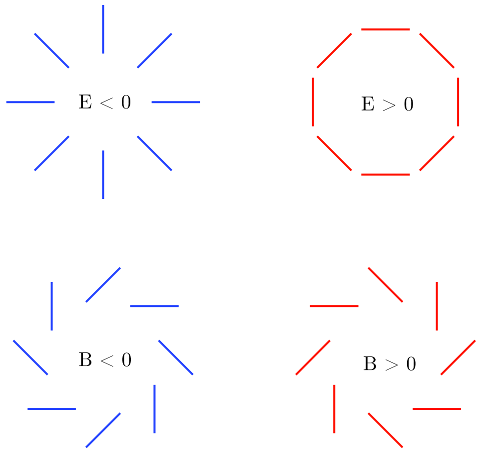


GWs and CMB polarisation

- Density quadrupole in last scattering surface polarises the Cosmic Microwave Background radiation
- Density (scalar) perturbations produce only even-parity polarisation: “**E-mode**”
- GWs (tensor perturbations) produce odd-parity polarisation: “**B-mode**”
- Gravitational lensing by intervening matter converts E-mode into B-mode



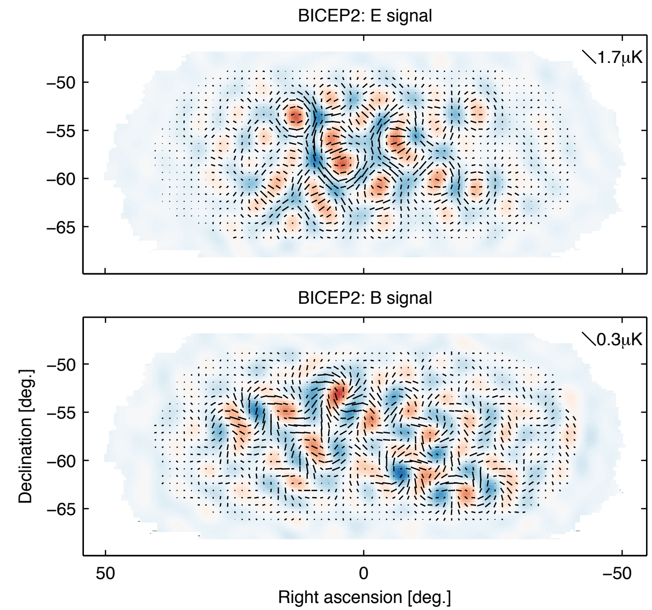
Hu & White (1997)



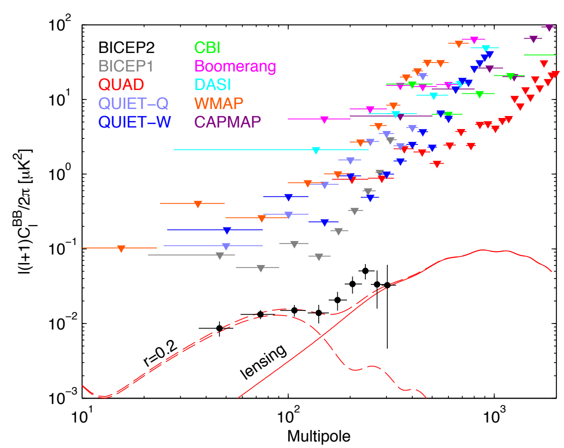
Krauss, Dodelson, Meyer (2010)

BICEP2

- Detection of B-mode polarisation
- Initial claim (March ‘14):
  - *“available [foreground] models … considerably smaller than the observed signal”*
  - Primordial GWs detected
  - $E_{inf} \approx 2 \times 10^{16}$ GeV
- Now: (June ‘14):
  - *“[foreground] models are not sufficiently constrained by external public data to exclude the possibility of dust emission bright enough to explain the entire excess signal”*
  - Need better understanding of polarised dust emission (Planck)

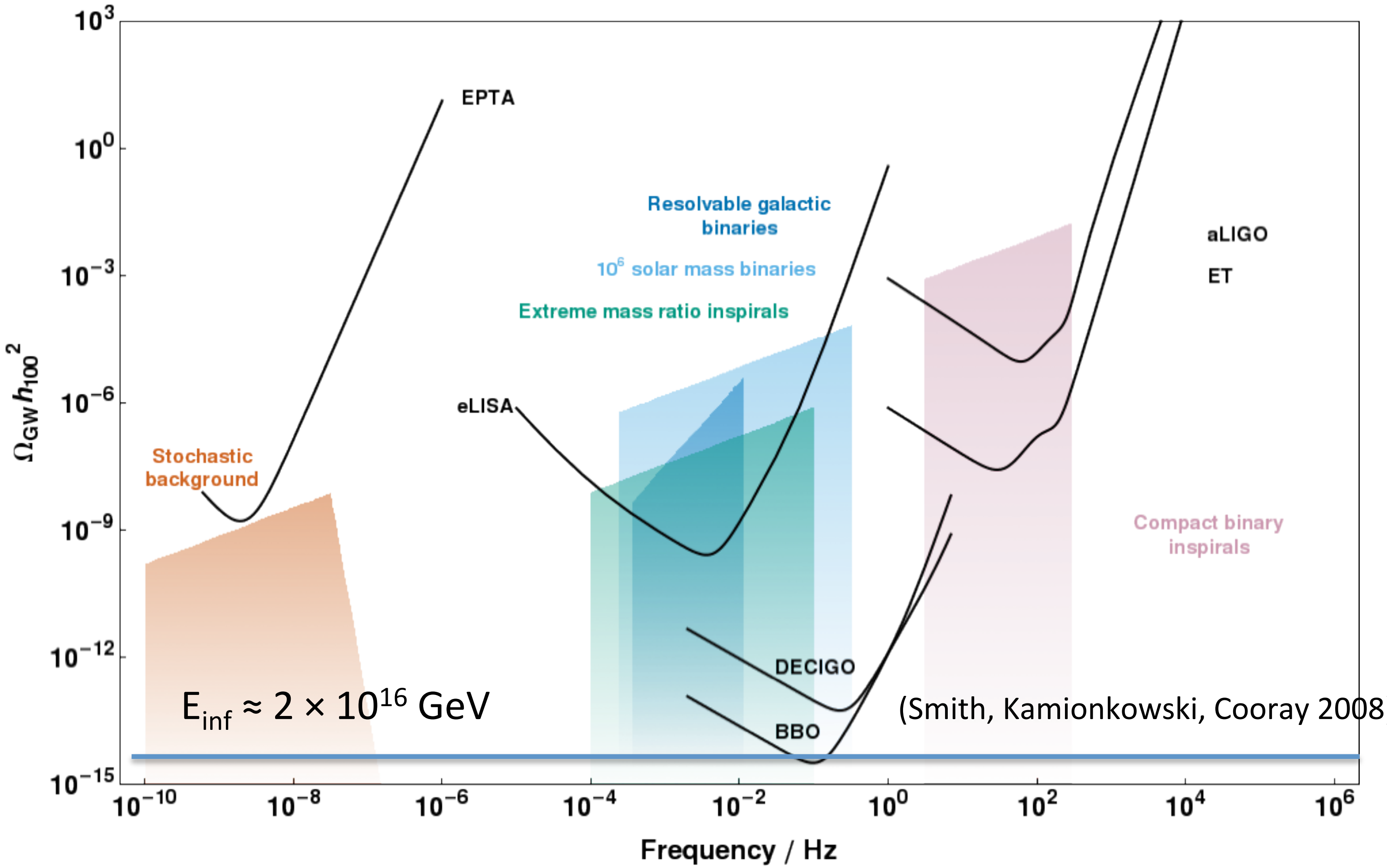


Ade et al. (2014)



Krauss, Dodelson, Meyer (2010)

Direct detection of primordial GWs



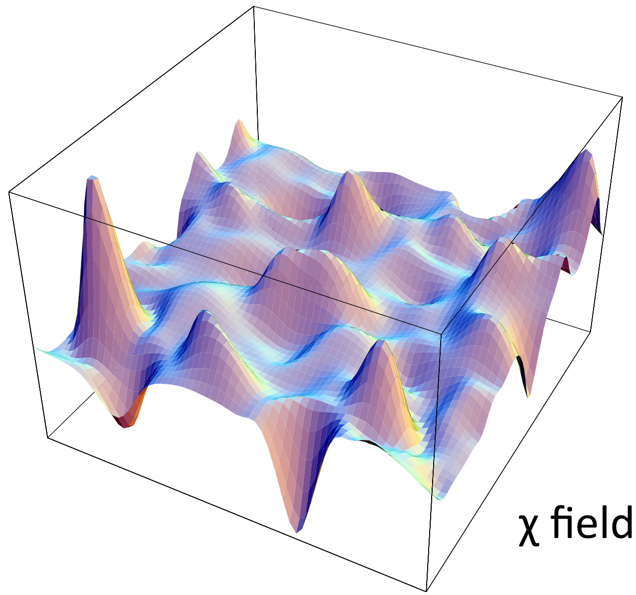
http://www.ast.cam.ac.uk/~rhc26/sources/

Gravitational waves from preheating

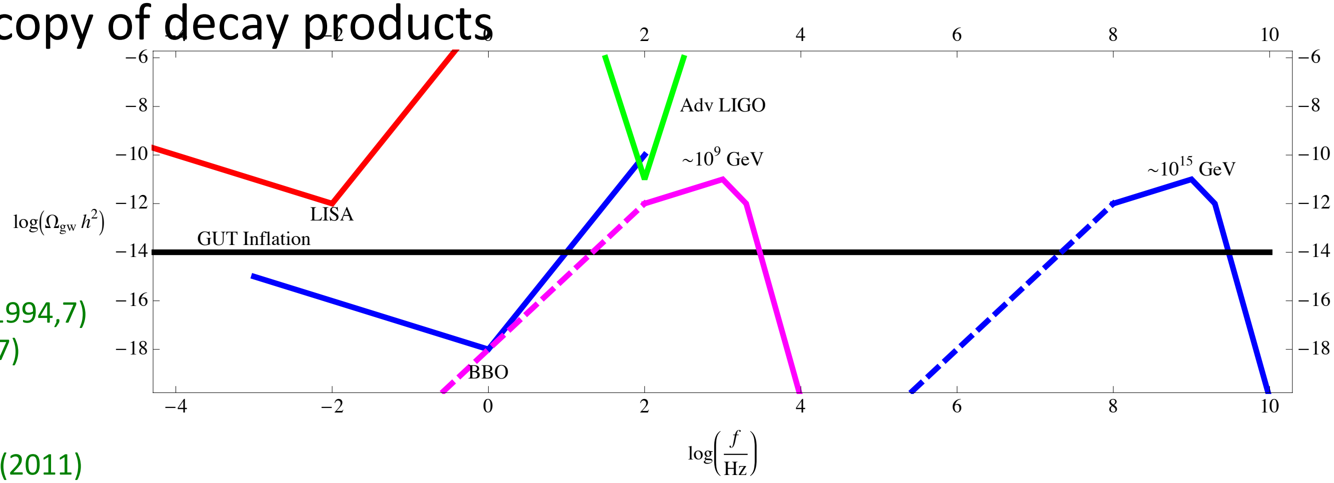
- Transfer of energy from homogeneous mode of inflaton to higher *k* modes of all fields

$$V(\phi, \chi) = \frac{1}{4}\lambda\phi^4 + \frac{1}{2}g^2\phi^2\chi^2$$

- Inhomogeneous field configurations
- Shear stresses ➔ GWs
- GW spectroscopy of decay products of inflaton?



Dufaux et al (2007)



Easther, Giblin, Lim (2007)

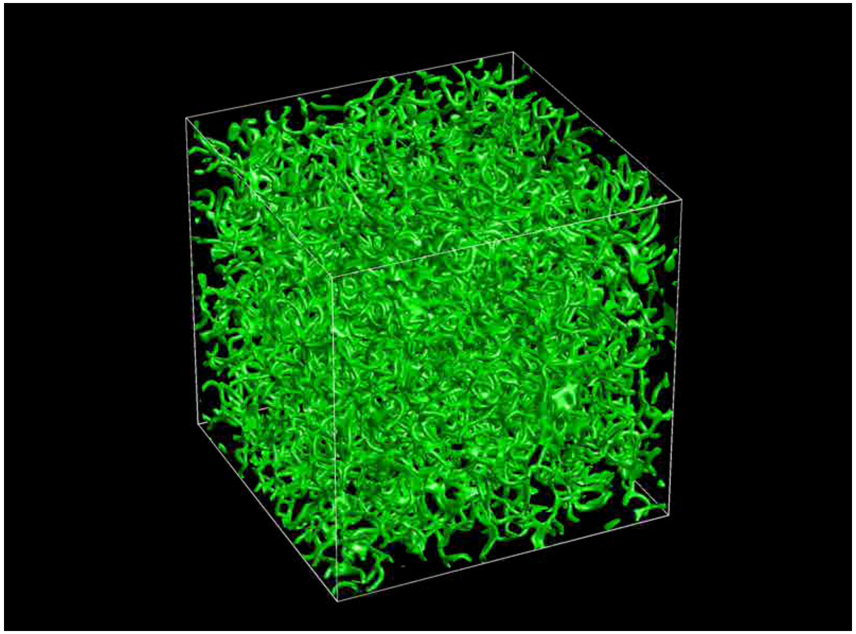
Kofman, Linde, Starobinsky (1994,7)
Garcia-Bellido, Figueroa (2007)
Easther, Giblin, Lim (2007)
Dufaux et al (2007,8)
Enqvist, Figueroa, Meriniemi (2011)
Figueroa (2014)

Gravitational waves from defects

- Further beyond the Standard ModelNew symmetries
  - New phase transitions
  - New exotic objects: topological defects
- Scale-invariant GW spectrum
- GW density at least (symmetry-breaking scale *v*)

$$\Omega_{\rm GW}(f) \sim \left(Gv^2\right)^2$$

Krauss (1992)
Jones-Smith, Krauss, Mathur (2008)
Fenu et al (2009)
Figueroa, Hindmarsh, Urrestilla (2013)



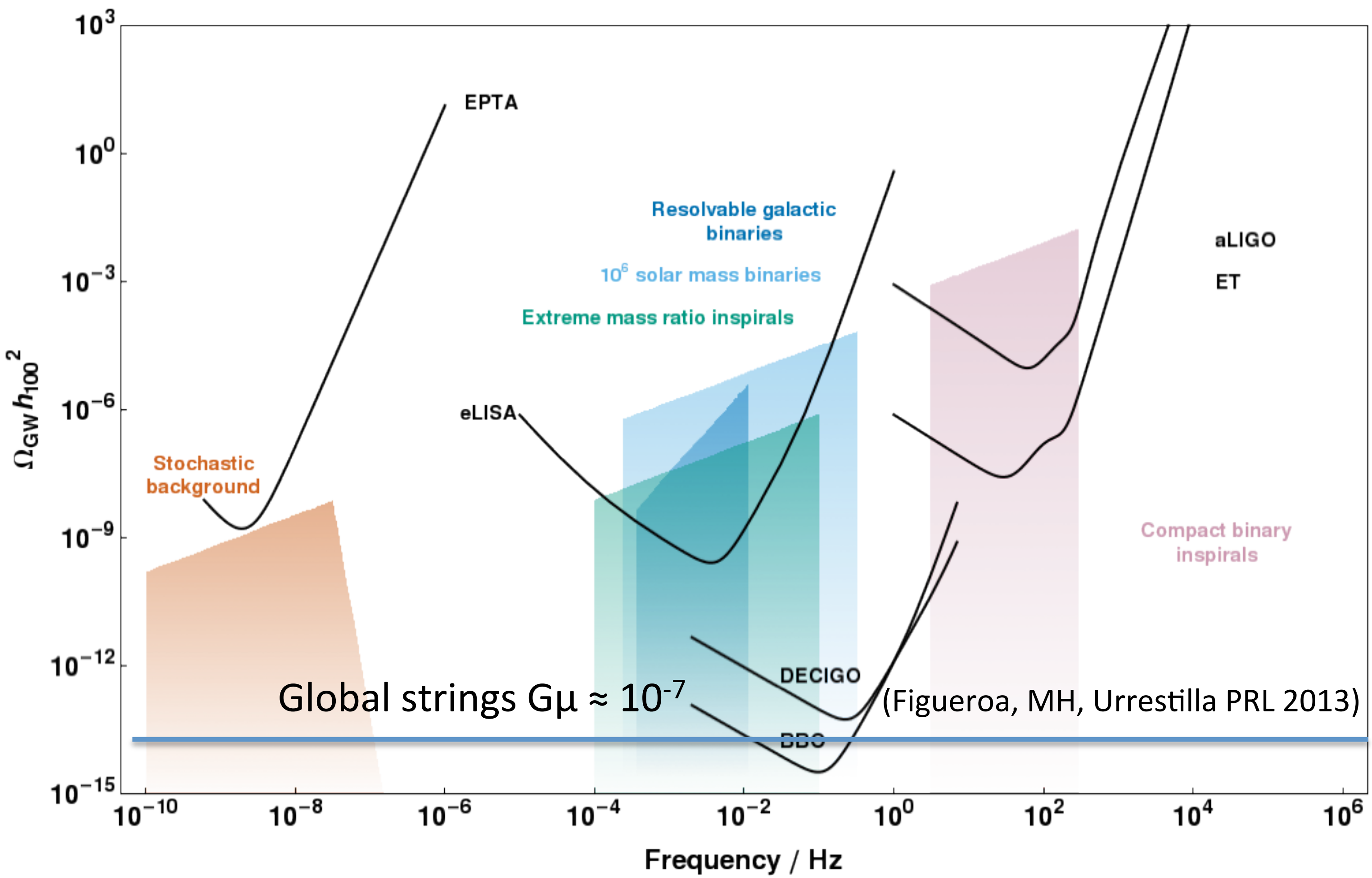
Abelian Higgs cosmic strings

- Cosmic string GW signal enhanced if particle production suppressed

$$\Omega_{\rm GW}(f) \sim \left(Gv^2\right)$$

Vilenkin (1982) …
Binetruy et al (2012)

GWs from defects



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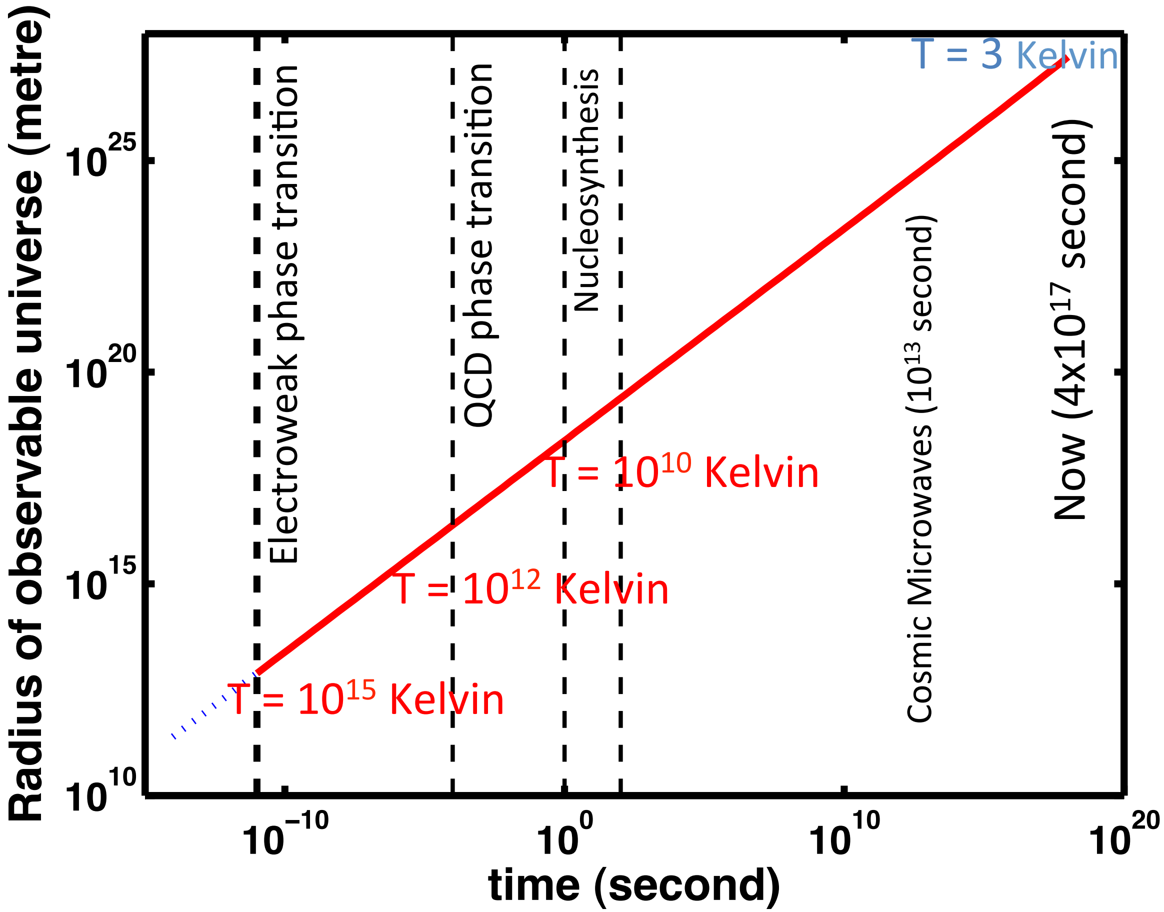
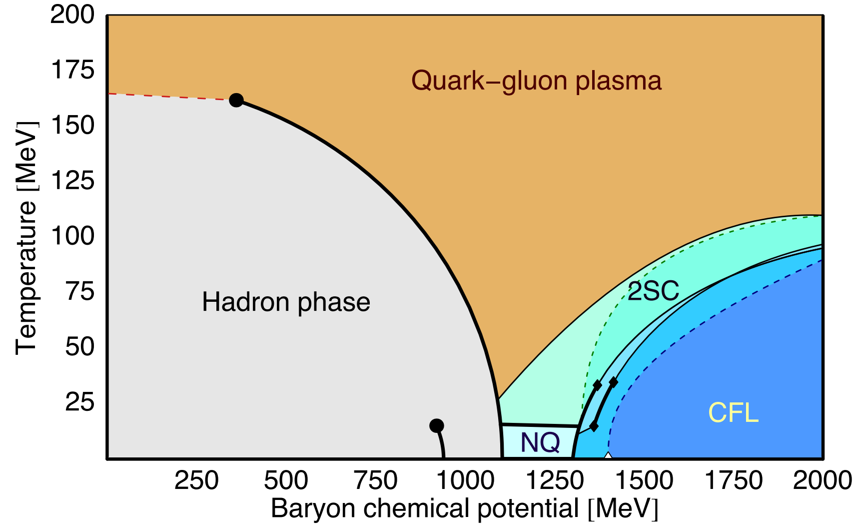
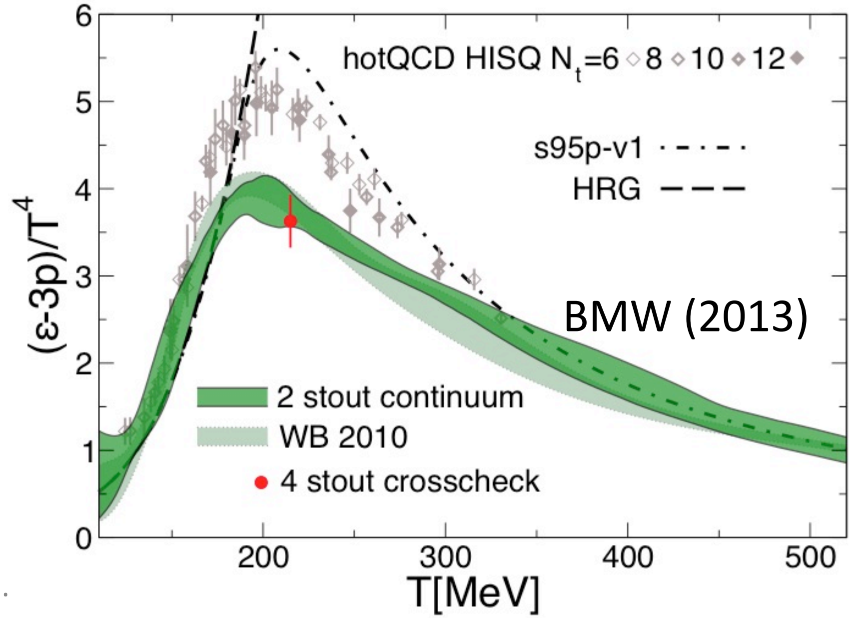
Time, size, temperature



QCD phase transition

- QCD: rich phase diagram
- Universe: $n_B/n_\gamma \approx 6.1 \times 10^{-10}$
- Behaviour at low chemical potential well-established
- Transition from QGP to hadronic phase is a cross-over
- Peak in trace anomaly at $T_c \approx 200$ MeV (10^{12} K)

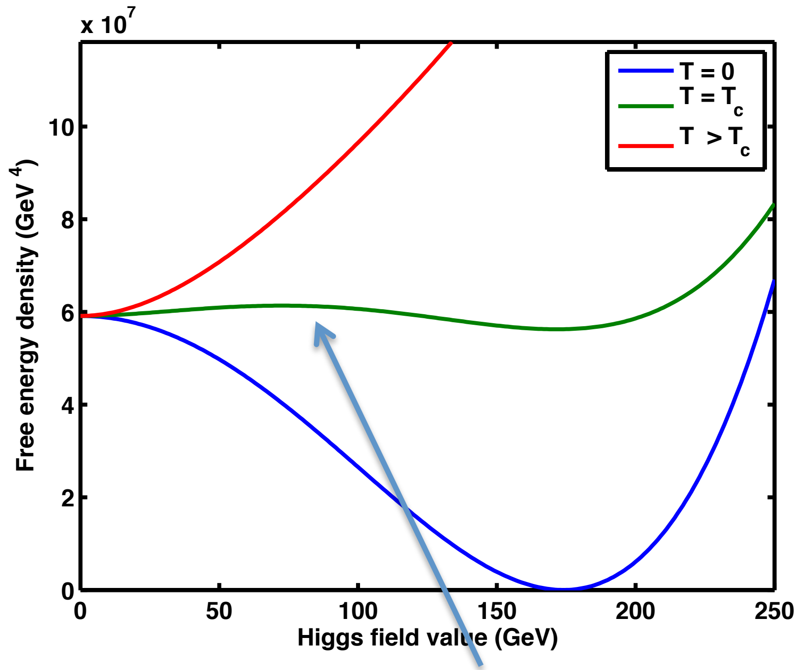




Borsanyi et al 2013 (BMW)

Electroweak phase transition

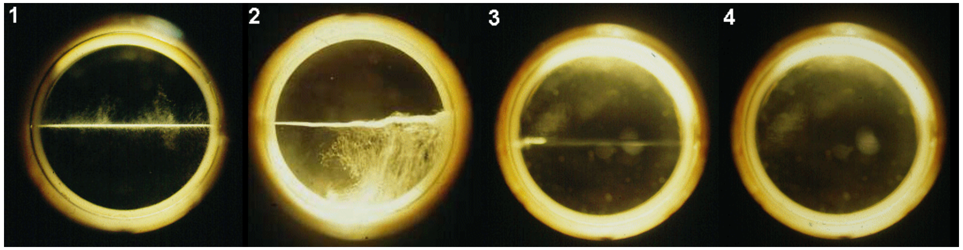
- Free energy density of plasma depends on
  - Temperature T
  - Particle masses $m_i(\phi)$
- High T: reduce free energy by forcing Higgs ϕ to zero
- Phase transition in weakly coupled gauge theories: Kirzhnits 1972
- Electroweak phase transition: $T_c \approx v_{EW} \approx 100$ GeV (10^{15} K)
- High T (>> $m_i(\phi)$): $V(\phi, T) \simeq \frac{1}{2}\alpha(T^2 - T_0^2)\phi^2 - \frac{1}{3}\gamma T\phi^3 + \frac{1}{4}\lambda\phi^4$

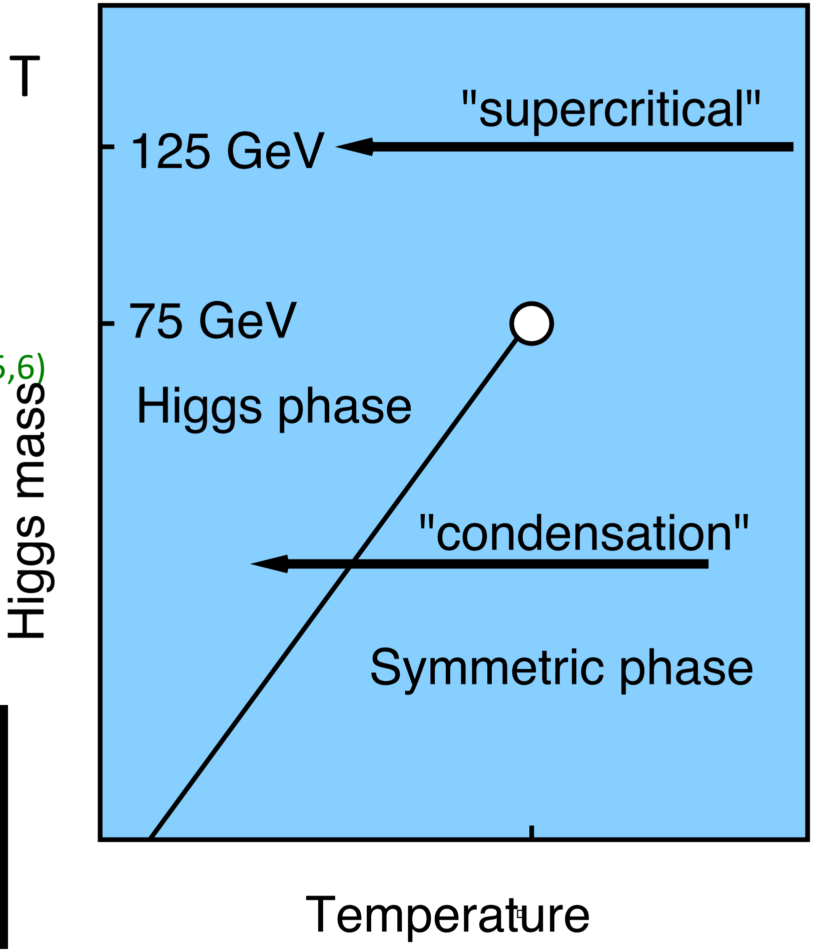


Potential barrier from cubic term in perturbative high-T expansion. First order transition?

Standard Model electroweak phases

- SM is not weakly coupled at high T
- Non-perturbative techniques:
  - Dimensional reduction + effective field theory + 3D lattice
    Kajantie, Laine, Rummukainen, Shaposhnikov (1995,6)
  - 4D lattice
    Czikor, Fodor, Heitger (1998)
- SM transition at $m_h \approx 125$ GeV
  like a supercritical fluid





1^{st} order phase transitions Beyond the Standard Model

- MSSM with light stop
  - Light stop contributes to cubic term in $V(\phi,T)$.
  - Phase transition is first order enough for
    $$m_{\tilde{t}} \lesssim 120 \text{ GeV}$$
  - Increased gluon fusion rate can be compensated by invisible Higgs decays to neutralino, if
    $$m_{\tilde{\chi}} \lesssim 60 \text{ GeV}$$

Carena, Quiros, Wagner (1996)
Delepine et al (1996)
Cline, Kainulainen (1996)
Carena, Nardini, Quiros, Wagner (2013)
Laine, Nardini, Rummukainen (2013)

- SM + dim 6

$$V_{\text{eff}}(H) = -\frac{\mu^2}{2}H^2 + \frac{\lambda}{4}H^4 + \frac{1}{8M^2}H^6$$

  - Tree level first-order phase transition for $\lambda < 0$

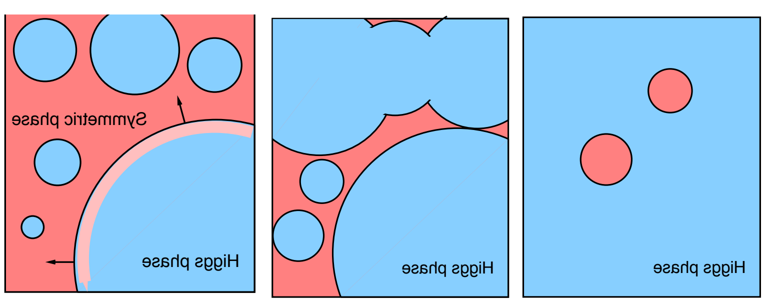
Zhang (1993)
Grojean, Servan, Wells (2004)
Ham, Oh (2004)
Bodeker et al (2004)

- SM + singlet
  - NMSSM
  - nMSSM

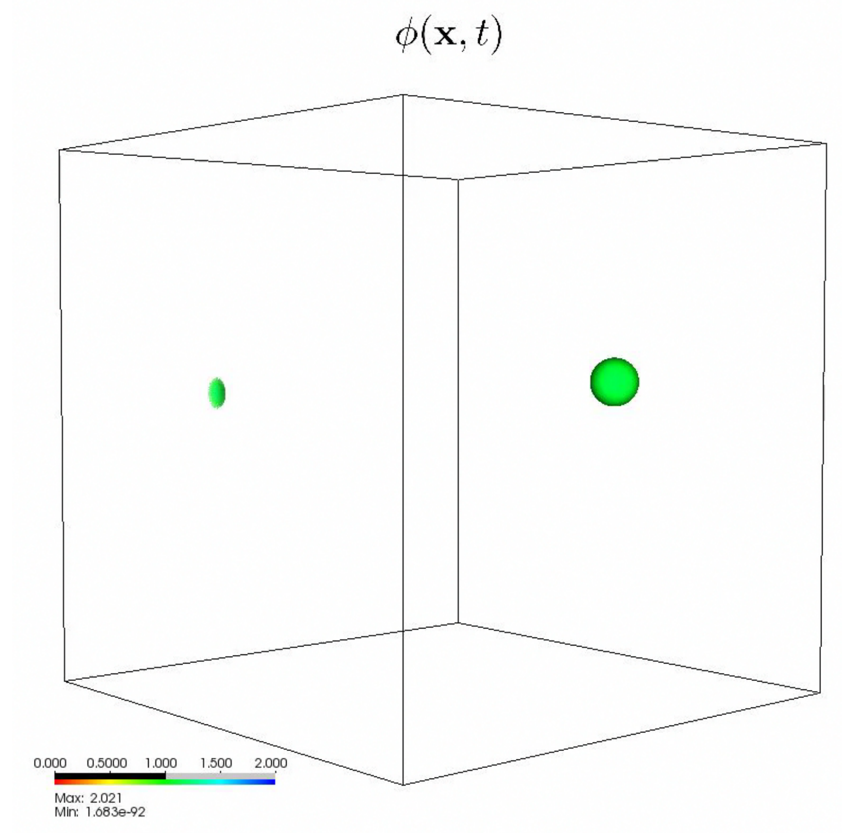
Pietroni (1993)
Davis, Froggatt, Moorhouse (1996)
Huber, Schmidt (1999,2001)
Panagiotakopoulos, Tamvakis (1999)
Menon, Morrissey, Wagner (2004)

Little bangs in the Big Bang

- 1st order transition proceeds by nucleation of bubbles of Higgs phase
- Expanding bubbles generate pressure waves in hot fluid
- Detectable gravitational waves?



Steinhardt (1982); Gylulassy et al (1984); Witten (1984); Enqvist et al (1992);

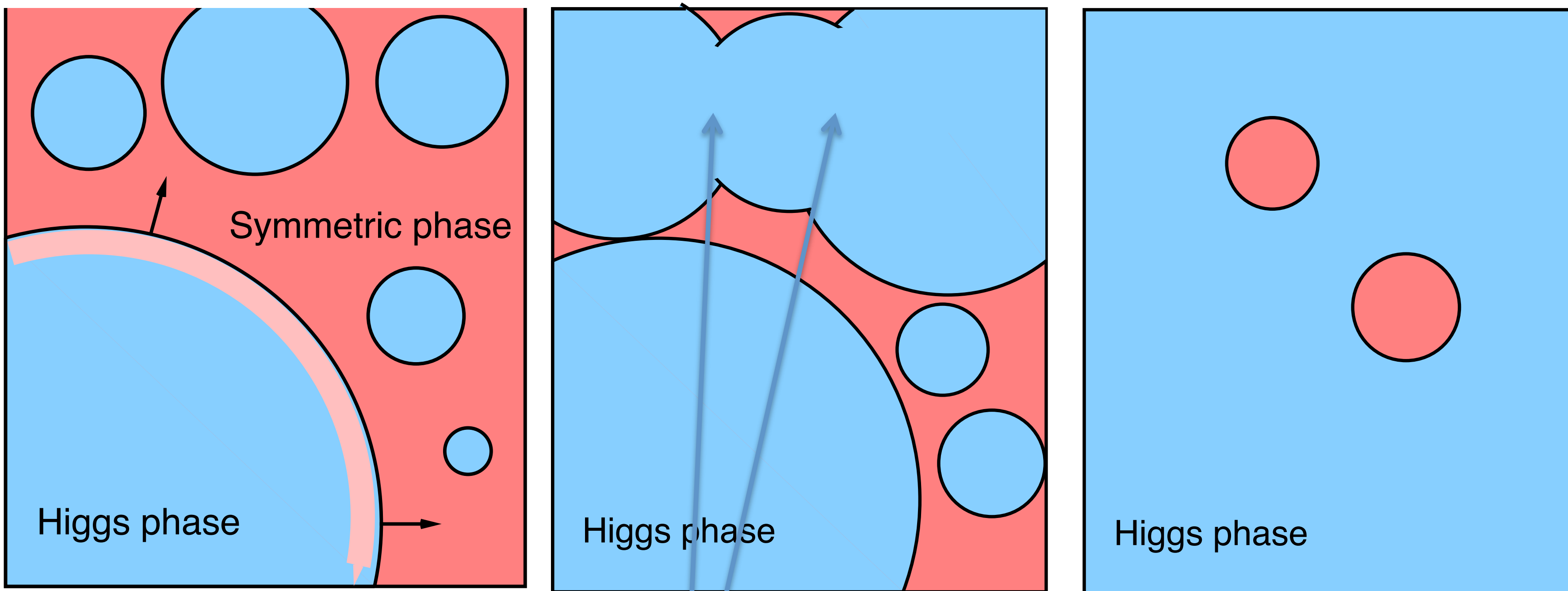


David Weir, Helsinki University

Scalar field

Hindmarsh, Huber, Rummukainen, Weir (2013)
Scalar only: Child, Giblin (2012)

GWs from first order transitions: envelope approximation



Spherical matter distributions do not radiate

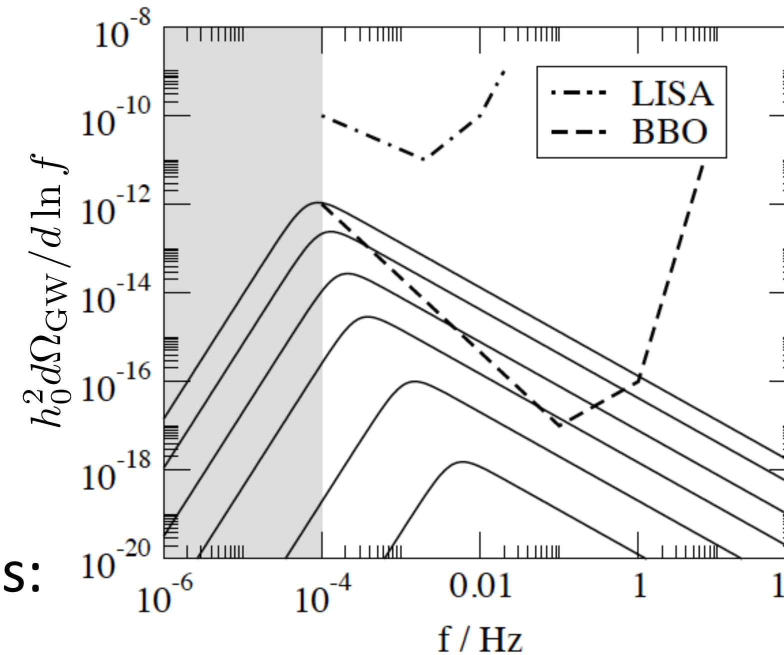
Assume: Gravitational waves generated by colliding bubbles
(Kosowski, Turner, Watkins 1992)

GWs from the EW transition: envelope approximation

- Parametrise transition:
  - α = (Latent heat)/(Total energy)
  - v_w = Bubble wall speed
  - d_h = horizon distance ($2t$)
  - R_* = mean bubble separation
  - κ = conversion efficiency of latent heat to fluid kinetic energy
- Fraction of energy density in GWs:

$$\Omega_{\rm GW} = \frac{\rho_{\rm GW}}{\rho_{\rm Tot}}$$

$$\Omega_{\rm GW} \sim v_w \left(\frac{R_*}{d_{\rm h}}\right)^2 \kappa^2 \alpha^2$$



Huber, Konstandin 2008

GW power spectrum:
α = 0.2 … 0.03
R_b / d_h = 0.1 … 0.003

Direct numerical simulation of an early universe phase transition

- Ingredients:

Ignatius et al (1994), Kurki-Suonio, Laine (1996)

  - Higgs field $$-\ddot{\phi} + \nabla^2\phi - \frac{\partial V}{\partial \phi} = \eta W(\dot{\phi} + V^i \partial_i \phi)$$
    - η coupling to fluid (models energy transfer)
  - Relativistic fluid

$$\dot{E} + \partial_i(EV^i) + P[\dot{W} + \partial_i(WV^i)] - \frac{\partial V}{\partial \phi} W(\dot{\phi} + V^i \partial_i \phi) = \eta W^2 (\dot{\phi} + V^i \partial_i \phi)^2.$$

$$\dot{Z}_i + \partial_j(Z_i V^j) + \partial_i P + \frac{\partial V}{\partial \phi} \partial_i \phi = -\eta W(\dot{\phi} + V^j \partial_j \phi) \partial_i \phi.$$

    - E energy density, Z_i momentum density, V_i velocity, W γ-factor
  - Discretisation

Wilson & Matthews (2003)
Different approach: Giblin, Mertens (2013)

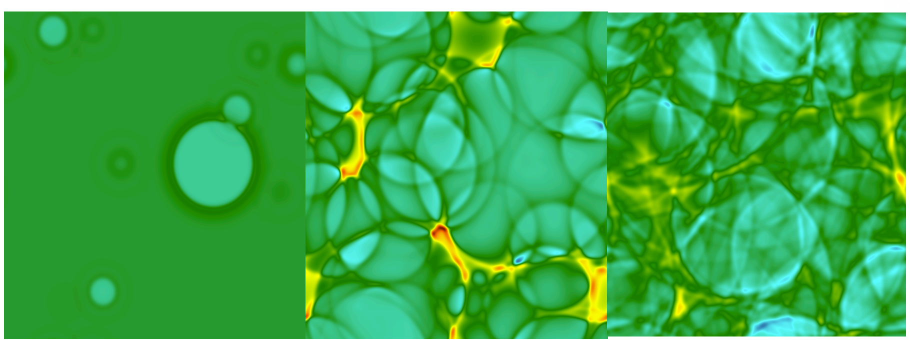
  - Metric perturbation

$$\ddot{h}_{ij} - \nabla^2 h_{ij} = 16\pi G T_{ij}^{\mathrm{TT}}$$

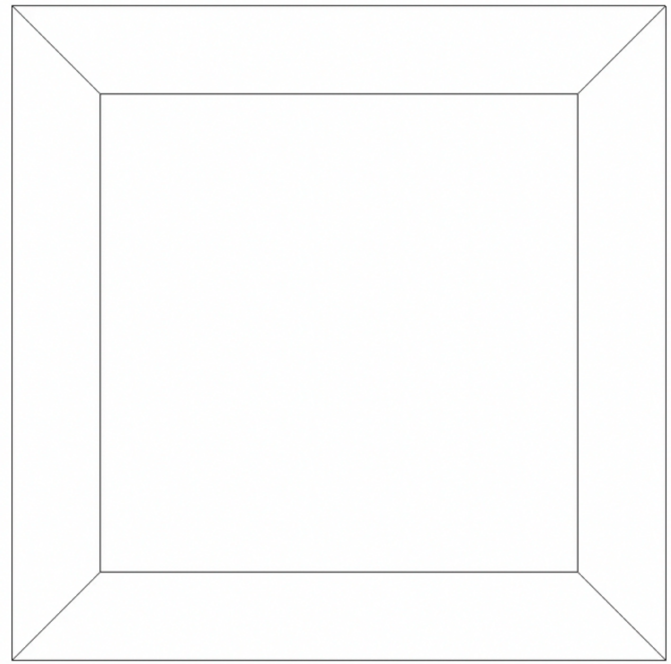
Garcia-Bellido, Figueroa, Sastre (2008)

Hydrodynamics of phase transitions

- Latent heat of transition goes into fluid compression waves – **sound** Hogan (1986)
- New: sound waves source gravitational waves long after transition is complete



Gas density as transition proceeds



Fluid kinetic energy density

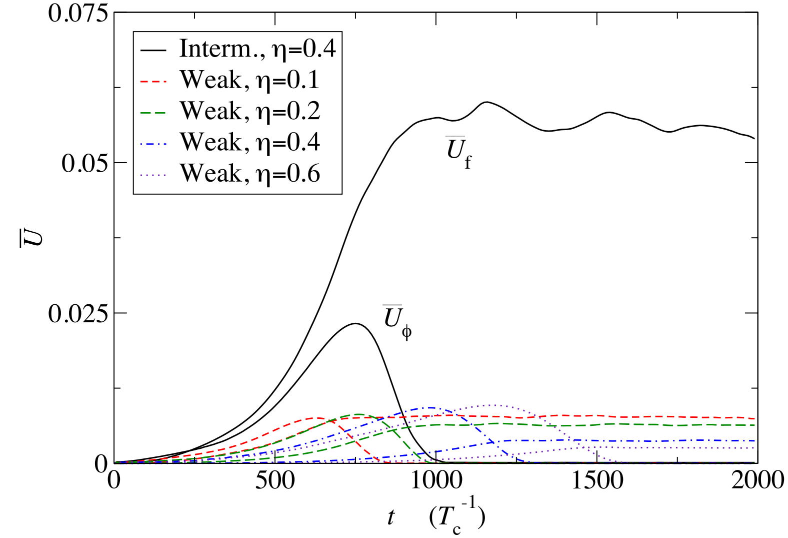
Hindmarsh, Huber, Rummukainen, Weir (2013)
See also Giblin, Mertens (2013)

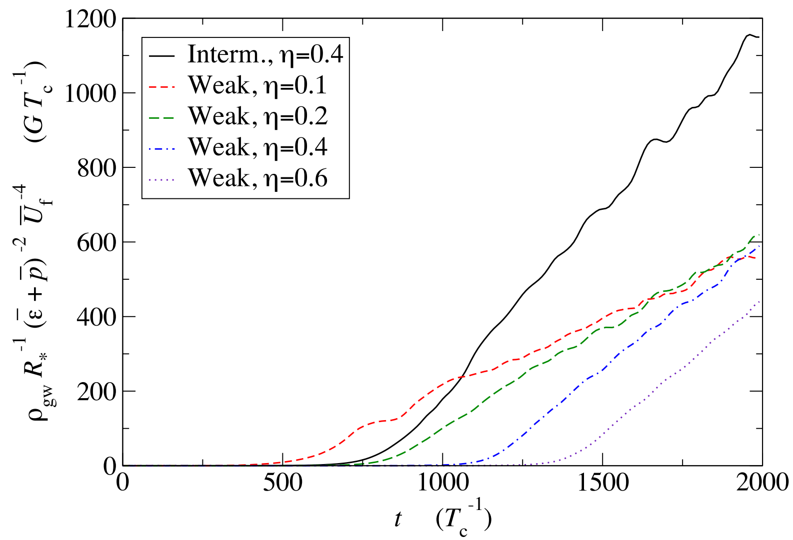
Sources of gravitational waves

- Look at
  - rms fluid velocity $\bar{U}_{\mathrm{f}}$
  - equivalent field quantity

$$\bar{U}_\phi = \sqrt{\langle(\nabla\phi)^2\rangle}$$

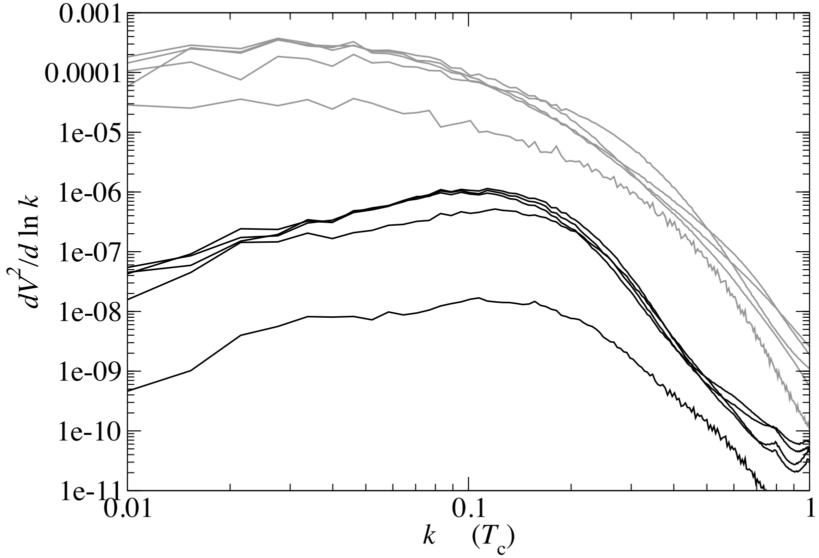
- Gravitational wave energy density rises linearly after transition
- Source: fluid

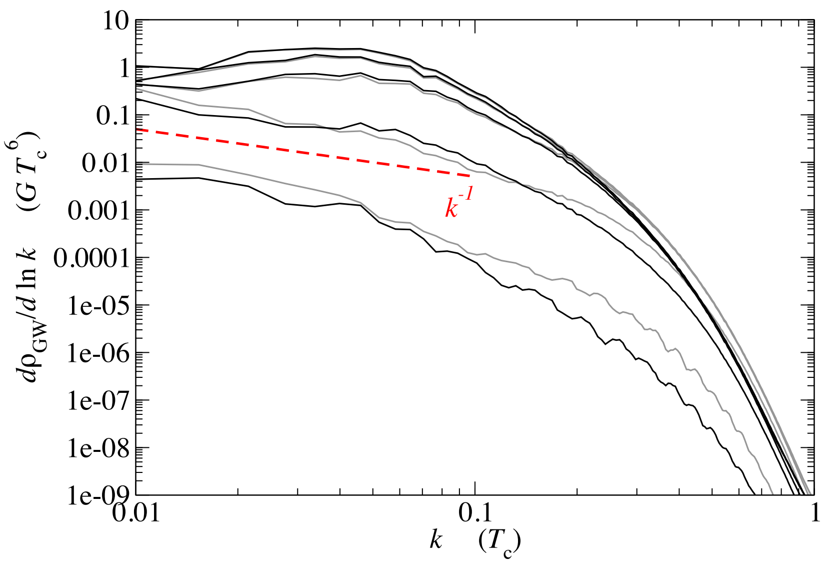




Evolution of power spectra

- Velocity power spectrum:
  - Peaked at wavenumber corresponding to mean bubble separation
  - Mostly compressional (grey)
  - Small rotational component (black)
- G-wave power spectrum
  - Peaked at wavenumber corresponding to mean bubble separation



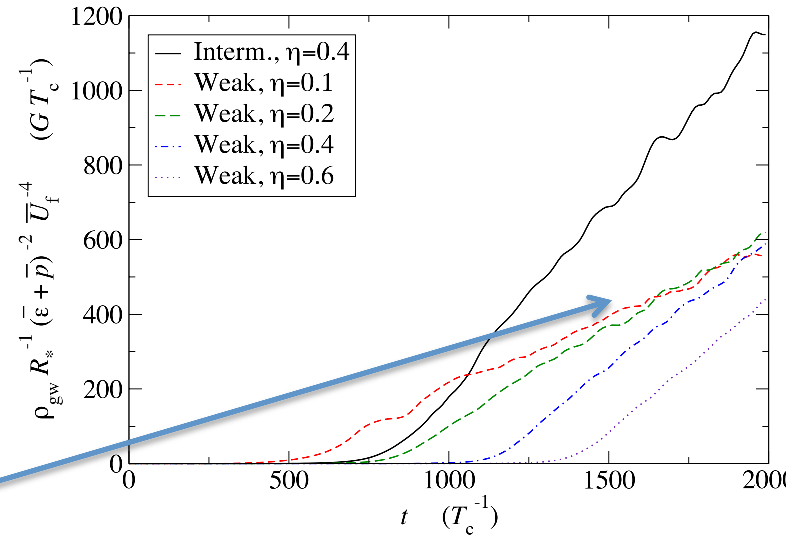


A new estimate for GWs

- Our model:

$$\frac{d}{dt}\Omega_{\rm GW} \sim \frac{1}{t}\left(\frac{R_*}{d_{\rm h}}\right)\bar{U}_{\rm f}^4$$

  - Source "on" for time *t*
  - Slopes should be the same



- Hence:

$$\Omega_{\rm GW} \sim \left(\frac{R_{\rm b}}{d_{\rm h}}\right)\bar{U}_{\rm f}^4$$

  - Compare with envelope approximation:

$$\Omega_{\rm GW} \sim v_w \left(\frac{R_{\rm b}}{d_{\rm h}}\right)^2 \kappa^2\alpha^2$$

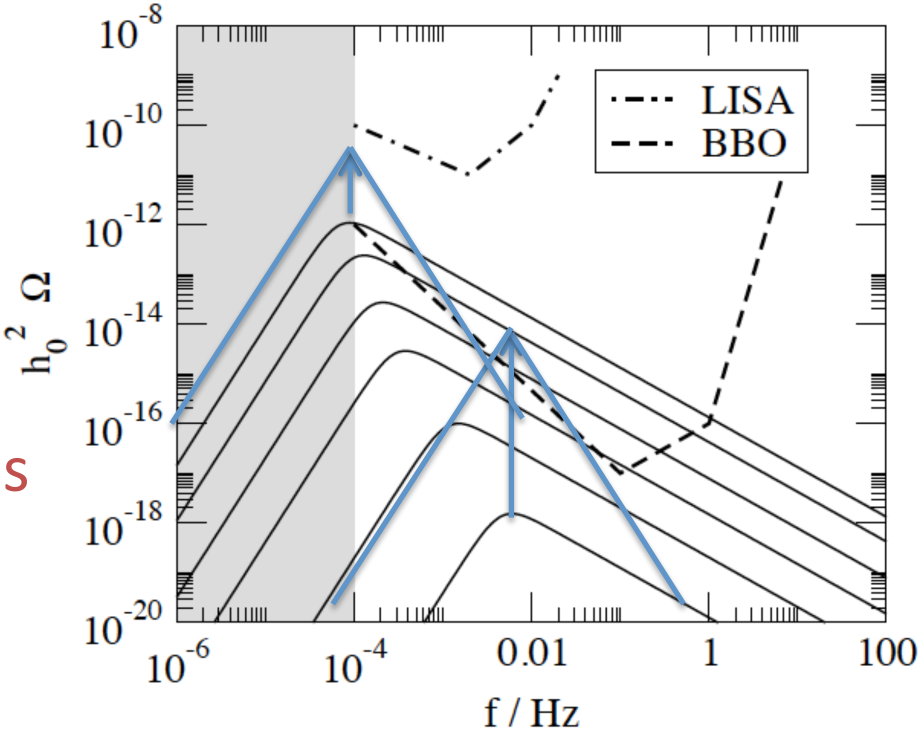
- GWs are parametrically larger by a factor

$$d_{\rm h}/R_* v_{\rm w}$$

- E.g. 10 – 3000

Implications for GW detection

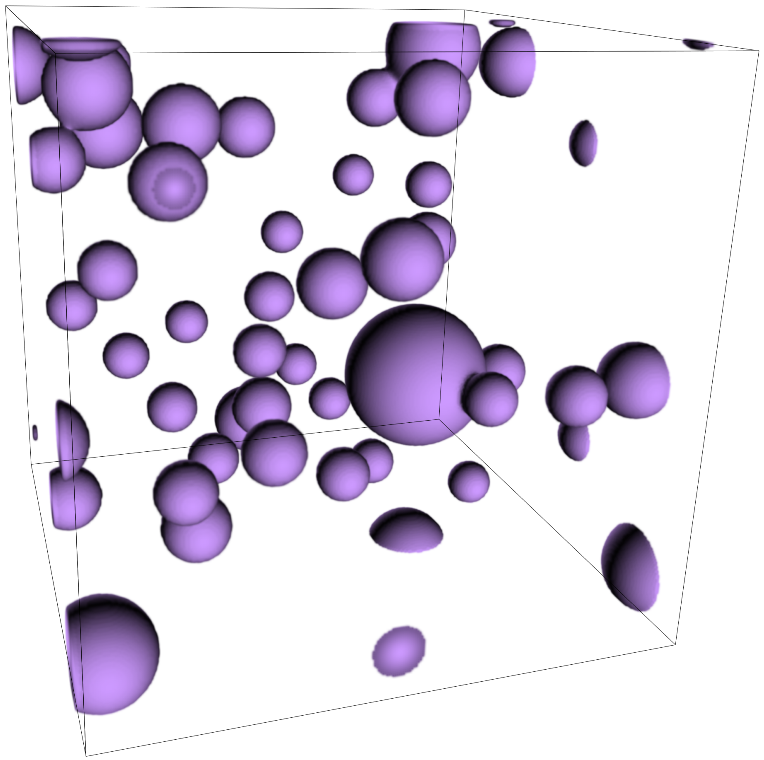
- Preliminary sketch:
  - Peak amplitude increases
  - High frequency GW power spectrum ~ f^{-3}



- More simulations in progress
- See David Weir's talk
  - Friday 16:40

Summary

- GWs unique probe of physics of early universe
- 1st order phase transitions in the early universe can produce gravitational waves
  - Source: sound waves from the nucleating droplets of the low temperature phase
  - Significantly larger than previous estimates
  - GW spectrum contains information about phase transition
- eLISA 2034, DECIGO, BBO
- Far future: physics of the end of inflation



Back-up slides

Direct detection of gravity waves in Brighton

- Here they are!

- No direct detection of gravitational waves yet