Correlations as a Signature of the QCD Critical Point in Heavy Ion Collisions

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1 Motivation

The neighborhood of the QCD chiral critical point is characterized by the arising of intense fluctuations of the chiral field which could, in principle, generate pronounced experimental signatures of its presence.

However, experimental uncertainties which are inherent to heavy ion collisions, as well as the modest size and duration of the formed plasma in these collisions, might severely attenuate these signatures, demanding a careful search for robust and reliable signals of the critical point neighborhood.

Here, we use Monte Carlo techniques to study the viability of second-order correlations of the pions as signatures of the chiral critical point in a realistic scenario, similar to the ones which are found in RHIC.

2 Effective Theory [1]

We follow Ref. [1] and use a three-dimensional effective theory for the chiral field with an effective action

$$\Omega = \int d^3x \left[\frac{1}{2}(\nabla\sigma)^2 + \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{3}\lambda_3\sigma^3 + \frac{1}{4}\lambda_4\sigma^4 + \dots\right] \quad (1)$$

and consider, as a first approximation, only the zero momentum mode of the σ field, $\sigma_0 = \frac{1}{V}\int d^3x\ \sigma(x)$.

We also consider a coupling of the form

$$\mathcal{L}_{int} = G\ \sigma_0\pi\pi \quad , \quad (2)$$

where the pions are, for now, treated as scalar bosons.

2.1 Leading order contribution [1, 2]

Let us approximate our problem as a system in thermal equilibrium and suppose its energy is a function $E[\{n_{\vec{p}}\}]$ of the set of occupation numbers $\{n_{\vec{p}}\}$ of the momentum modes.

This way, we can calculate contributions to $\langle\Delta n_{\vec{p}}\ \Delta n_{\vec{k}}\rangle$ and use this quantity to calculate any given second-order moment of the pions.

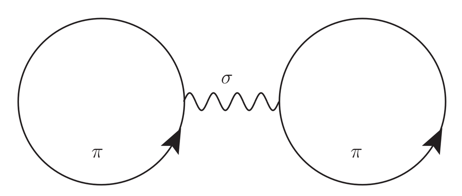


FIGURE 1: Leading order contribution for $\langle\Delta n_{\vec{p}}\ \Delta n_{\vec{k}}\rangle$.

The diagram in Figure 1 gives the leading order chiral critical contribution to $\langle\Delta n_{\vec{p}}\ \Delta n_{\vec{k}}\rangle$ with $\vec{p} \neq \vec{k}$. Ref. [2] calculates the corresponding contribution to be

$$\delta\langle\Delta n_{\vec{p}}\ \Delta n_{\vec{k}}\rangle_2^{(a)} = \frac{(G\xi)^2}{V}\frac{f_{\vec{p}}(f_{\vec{p}}+1)}{\omega_{\vec{p}}}\frac{f_{\vec{k}}(f_{\vec{k}}+1)}{\omega_{\vec{k}}} \quad . \quad (3)$$

Notice that it has quadratic dependence on the correlation length ξ, so that correlations are, as expected, strongly enhanced near the critical point.

3 The effective Boltzmann factor [3]

Since we want to seek for effects of criticality in fluctuations, we must introduce moments such as the one in Eq. (3) in our Monte Carlo simulation. However, in order to do so, we need some kind of probability distribution which reproduces them.

Hence, we introduce a probability distribution for $\{n_{\vec{p}}\}$ with an effective Boltzmann factor $e^{-\beta\delta E[\{n_{\vec{p}}\}]}$, along with the regular, non-interacting one:

$$\mathcal{P}[\{n_{\vec{p}}\}] \propto e^{-\beta(E_0[\{n_{\vec{p}}\}]-\mu N+\delta E[\{n_{\vec{p}}\}])} \quad , \quad (4)$$

and choose δE so as to reproduce the correlations among different modes.

Eq. (4) yields, to first order in $\delta E/T$ and for $\vec{p}_i \neq \vec{p}_j$,

$$\delta\langle\Delta n_{\vec{p}_1}\dots\Delta n_{\vec{p}_N}\rangle = -\beta\ \langle\Delta n_{\vec{p}_1}\dots\Delta n_{\vec{p}_N}\ \delta E\rangle_0 \quad , \quad (5)$$

which can be used to find a suitable effective Boltzmann factor that introduces the desired moments. The correction δE should be chosen to be as small as possible and to keep $\mathcal{P}[\{n_{\vec{p}}\}]$ stable enough, while not changing the correct features of E_0 (e.g. the rest mass of the particles).

3.1 Monte Carlo algorithm

We now turn to the generation of event samples that reproduce the probability distribution $\mathcal{P}$. In order to do so, we rewrite Eq. (4):

$$\mathcal{P}[\{n_{\vec{p}}\}] \propto \left(\prod_{\vec{k}}\mathcal{P}_0^{(\vec{p})}(n_{\vec{k}})\right)\cdot e^{-\beta\,\delta E[\{n_{\vec{p}}\}]} \quad . \quad (6)$$

Our algorithm consists in separately drawing each occupation number according to $\mathcal{P}_0^{(\vec{p})}(n_{\vec{k}})$ to generate non-interacting events and then filtering these events as a whole according to $\bar{\mathcal{P}} \propto e^{-\beta\,\delta E[\{n_{\vec{p}}\}]}$ (rejection sampling method).

Simplified Algorithm

1. For each event i:

1.1 For each mode $\vec{p}$ such that $|\vec{p}| < p_{max}$:

i. Draw an integer $n_{\vec{p}}$ according to $\mathcal{P}_0^{(\vec{p})}(n_{\vec{p}})$ (exponential)

1.2 Uniformly draw $y \in [0,1]$

1.3 If $y < \bar{\mathcal{P}}[\{n_{\vec{p}}^i\}]$ record $\{n_{\vec{p}}^i\}$ as the i-th event otherwise go back to (1.1)

2. Repeat (1) until N_{evt} events are generated

3.2 Particular case

In our particular case of interest, we can use Eq. (5), along with $\langle\Delta n_{\vec{p}}\Delta n_{\vec{k}}\rangle_0 = \delta_{\vec{p}\vec{k}}\ f_{\vec{p}}(f_{\vec{p}}+1)$, to find an effective energy correction which reproduces the moment in Eq. (3):

$$\delta E_2^{(a)} = -\frac{(G\xi)^2}{2V}\sum_{\vec{k}_1\neq\vec{k}_2}\frac{\Delta^0 n_{\vec{k}_1}}{\omega_{\vec{k}_1}}\frac{\Delta^0 n_{\vec{k}_2}}{\omega_{\vec{k}_2}}, \quad (7)$$

which will be used in our simulations to generate correlation among different modes.

4 Heavy ion collisions

We wish to test whether the critical non-monotonic behavior on second-order moments of the pions, which is expected near the chiral critical point E, is sufficiently strong to be experimentally detected.

We choose, restricted by available experimental data, freeze-out conditions corresponding to STAR-RHIC $Au + Au@\sqrt{s_{NN}} = 64.2$ MeV ($(T,\mu_B)_{freezeout} = (99,63)$ MeV) [4] and pretend the critical point is near the freeze-out parameters.

Experimental limitations should also be considered:

- Background contribution ⇒ spurious fluctuations;
- Fluctuations of the freeze-out parameters!

4.1 Temperature fluctuations

- How does the freezeout temperature fluctuate?
- Model: Gaussian fluctuations.
- Uncertainty in $T_{freezeout}$ of $\sim 10\%$:

$$\sigma_T \lesssim 10\%\ T \quad . \quad (8)$$

- Conservative choice: the maximum value.

4.2 Geometrical Fluctuations

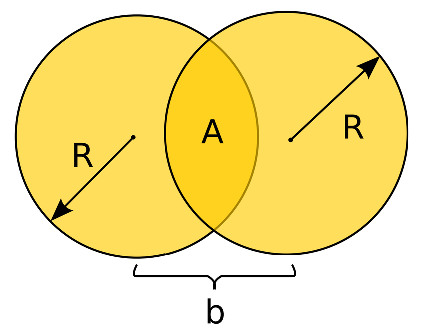
- Impact parameter distribution ⇒ $P(b) \propto b$ (normalized).
- Model: $V_{freezeout}(b,R) = C \cdot A(b,R)$, with A being the intersection area between the cross section of each nuclei:

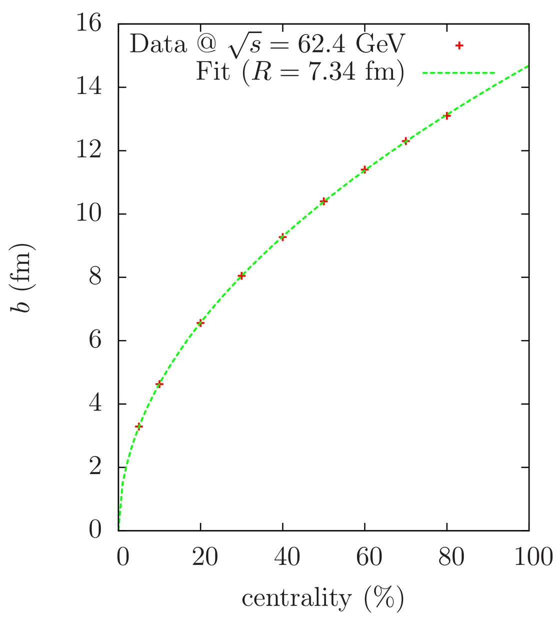
$$A(b,R) = 2R^2\cos^{-1}\left(\frac{b}{2R}\right) - b\sqrt{R^2 - \frac{b^2}{4}} \quad . \quad (9)$$

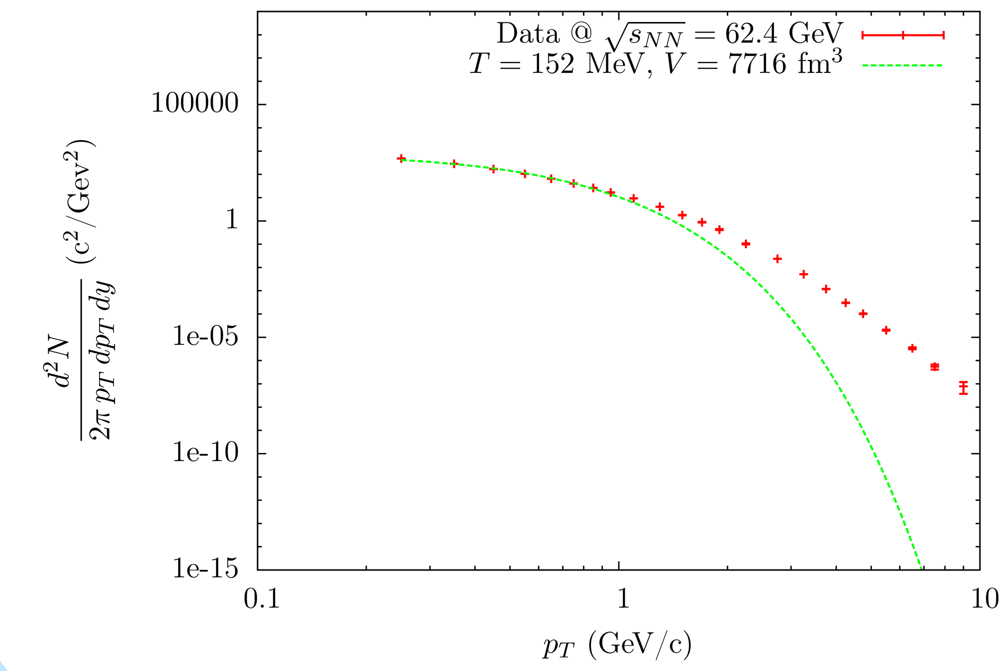
- The radius R of the nucleus can be found by considering the impact parameter distribution of the $Y\%$ most central collisions:

$$Y\% = \int_0^{b_{max}}\mathcal{P}(b)\,db = \left(\frac{b_{max}}{2R}\right)^2 \quad . \quad (10)$$

- C can be found by fixing the volume for a given centrality class.
- Geometrical fluctuations affect higher moments.

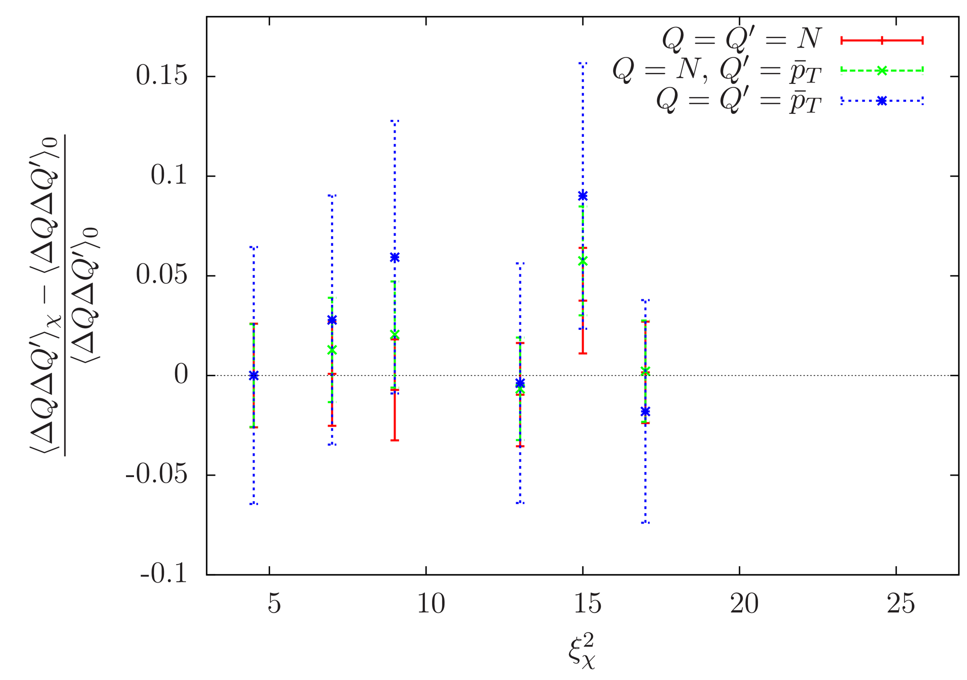






5 Preliminary Results [3]

- Results for moments of the multiplicity N and the average transverse momentum $\bar{p}_T$;
- simulations with $10,000$ events for charged pions;
- Defining $\xi_\chi^2 := (G\,\xi_\sigma)^2$.
- $\xi_\chi^2 \approx 4.5 - 13.0$, with 4.5 being the reference value.



- Spurious contributions and statistical fluctuations have hidden any significant critical signatures.

6 Perspectives

The present work can be improved in a number of ways. For instance, it would be interesting to include:

- different/higher-order moments, expected to exhibit stronger dependence on ξ (straightforward);
- moments of the protons, also expected to display more relevant signatures;
- radial flow and resonance decay as sources of background;
- more realistic sets of freeze-out conditions.

We are also working on improving our algorithm so as to obtain a larger number of events for a shorter amount of computing time.

Acknowledgments

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