Leptogenesis in crossing and runaway regime

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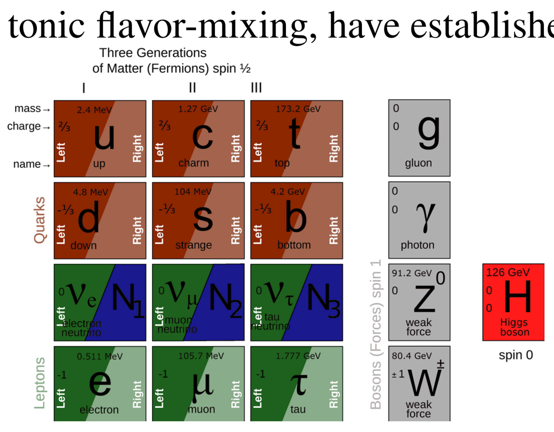
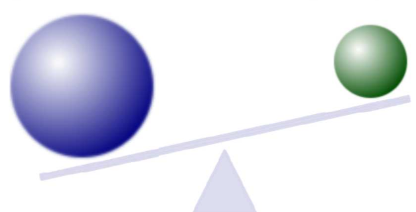
Leptogenesis

Neutrino oscillations, i.e. the experimental evidence for leptonic flavor-mixing, have established the existence of small



but nonzero neutrino masses. Through a realization of the see-saw mechanism these can find a satisfying theoretical explanation which entails further interesting phenomenological consequences. In particular CP-violating phases in the leptonic mixing open the possibility to explain the baryon asymmetry of the universe through the leptogenesis scenario. Analogous to the complex phase in the Cabibbo-Kobayashi-Maskawa matrix, CP-violating phases in the leptonic mixing may result from phases in vacuum expectation values of the Higgs fields or from complex Yukawa couplings. The heavy Majorana neutrinos can therefore induce a non-vanishing $B-L$ asymmetry through their lepton number violating decays in the early universe. This asymmetry



survives the subsequent cooling of the universe and therefore represents the seed for the structures we observe today and in which anti-matter is largely absent.

CP-properties and basis invariants

To produce a net (baryon-)charge asymmetry a model with CP-violation is needed. Consider the toy-Lagrangian $\mathcal{L} = \mathcal{L}_{kin} + \mathcal{L}_{int}$ with

$$\mathcal{L}_{kin} \equiv \frac{1}{2}\partial^\mu\psi_i\partial_\mu\psi_i + \partial^\mu\bar{b}\partial_\mu b$$

$$\mathcal{L}_{int} \equiv -\frac{1}{2}\psi_i M^2_{ij}\psi_j - m^2\bar{b}b - \frac{\lambda}{2!2!}(\bar{b}b)^2 - \frac{h_i}{2!}\psi_i bb - \frac{h^*_i}{2!}\psi_i\bar{b}\bar{b}\,.$$

In general the model contains complex phases which lead to a CP-asymmetry between b and its CP-conjugate field $\bar{b} \equiv b^*$. More precisely, the Lagrangian is CP-violating if $CP^{-1}\mathcal{L}CP$ is not equivalent to $\mathcal{L}$ if it is expressed in terms of CP-transformed fields. Here CP represents the 'generalized' CP-transform which includes a transformation that leaves the kinetic Lagrangian $\mathcal{L}_{kin}$ invariant:

$$(CP)b(x_0,\mathbf{x})(CP)^{-1} = \beta\bar{b}(x_0,-\mathbf{x})\,,$$
$$(CP)\psi_i(x_0,\mathbf{x})(CP)^{-1} = U_{ij}\psi_j(x_0,-\mathbf{x})\,.$$

If this equivalence cannot be established by choice of the parameters, the theory is CP-violating. When exactly this is the case can conveniently be captured by a 'CP-odd basis invariant' (similar to a Jarlskog-invariant):

$$J \equiv \mathrm{Im\,Tr}(HM^3H^TM) = 2\,\mathrm{Im}\,H_{12}\mathrm{Re}\,H_{12}M_1M_2(M_2^2 - M_1^2)\,.$$

If J vanishes the theory is CP-conserving and all CP-violating observables should vanish in a consistent approximation. This statement holds true after renormalization (MS-bar and OS) and $J = 0$ is RG-invariant.

Asymmetry: 1PI vs. 2PI

The processes which take place during the epoch of leptogenesis can be classified as source terms $\mathcal{S}_0$, which account for the generation of the asymmetry, and washout terms $\mathcal{W}_0$, which tend to deplete the asymmetry. In a 1PI computation and with Boltzmann-approximation the rate equation for the asymmetry in the comoving volume Y_b can be written in the form

$$dY_b/dz = \epsilon\,\mathcal{S}(z) - \mathcal{W}(z)Y_b\,.$$

The generated asymmetry is proportional to the CP-violating parameter ϵ:

$$\epsilon_i \equiv \frac{\Gamma_i - \bar{\Gamma}_i}{\Gamma_i + \bar{\Gamma}_i} \approx -\frac{1}{8\pi H_{ii}}\frac{\mathrm{Im}\,H_{12}\mathrm{Re}\,H_{12}M_1M_2(M_2^2 - M_1^2)}{(M_j^2 - M_i^2 - \frac{1}{\pi}\ln(M_j^2/M_i^2))^2 + (M_j\Gamma_j - M_i\Gamma_i)^2}\,.$$

It is proportional to J in accordance with the requirement that CP-violating observables vanish for $J = 0$. An elegant approach to the calculation of the lepton asymmetry within 2PI is based on the use of the divergence of the 'baryon' current $\partial_\mu J^\mu$. The part which vanishes in a CP-symmetric configuration may be interpreted as washout and the remainder as source term:

$$\partial_\mu J^\mu \supset S(x) \equiv 2\int_{t_0}^{x^0} dz^0 \int d^3z\left[\mathrm{Im}\,\Sigma_\rho(x,z)\mathrm{Re}D_F(z,x) - \mathrm{Im}\,\Sigma_F(x,z)\mathrm{Re}D_\rho(z,x)\right].$$

It depends on the propagators D of the complex field b and their self-energies Σ. The latter have to be determined self-consistently as functional derivatives of the 2PI effective action with respect to the propagators and depend also on the propagators of the mixing fields ψ_i. A CP-violating parameter cannot be factored out since it relies on a quasi-particle approximation.

Exact solution of the Kadanoff-Baym equations

In order to address the out-of-equilibrium phenomenon leptogenesis, consistent kinetic equation are needed. If one wants to go beyond the traditional quasi-particle picture, neQFT represents an adequate starting point. In particular the divergence of ψ_i from equilibrium is crucial for the generation of the asymmetry. Its evolution is governed by *Kadanoff-Baym equations* for the statistical propagator and spectral function. For mixing scalar fields they read

$$[\Box_x + M^2_{ik}]G^{kj}_F(x,y) = \int_{t_0}^{y^0} d^4z\,\Pi^{ik}_F(x,z)G^{kj}_\rho(z,y) - \int_{t_0}^{x^0} d^4z\,\Pi^{ik}_\rho(x,z)G^{kj}_F(z,y)\,,$$

$$[\Box_x + M^2_{ik}]G^{kj}_\rho(x,y) = \int_{x^0}^{y^0} d^4z\,\Pi^{ik}_\rho(x,z)G^{kj}_\rho(z,y)\,.$$

They are written in terms of *resummed propagators* of ψ_i, G, and their self-energy Π. If not one wants to solve them numerically or to risk loosing effects in unquantified approximations, one can only try to find exact solutions. Fortunately such a solution exists if one neglects washout. It may be expressed in terms of deviations from the equilibrium solution as

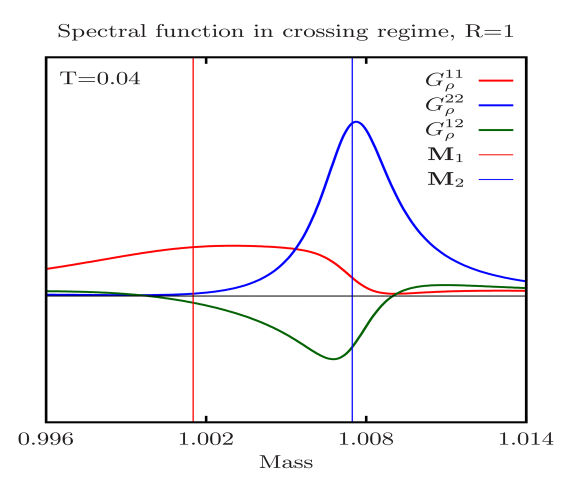
$$\Delta G^{ij}_\rho(x^0,y^0,\mathbf{q}) = 0\,,$$
$$\Delta G^{ij}_F(x^0,y^0,\mathbf{q}) = -G^{ik}_R(x^0,\mathbf{q})\Delta^{kl}_F(\mathbf{q})G^{lj}_A(-y^0,\mathbf{q})\,.$$

Physical interpretation: The system of mixing real fields coupled to a thermal bath of the complex field begins its evolution at $t_0 = -\infty$ in a thermal state (equilibrium solution). At $t = 0$ an external source *instantly* brings it out of equilibrium. After that it slowly thermalises producing some asymmetry. Because the thermal bath remains in equilibrium this asymmetry would eventually be completely erased by the washout processes. However, since we neglect the latter here, the asymmetry asymptotically reaches a constant value. With this solution the value of the asymmetry at any time may be written as

$$q_S(t) = \int\frac{d^3q}{(2\pi)^3}\Delta_F(\mathbf{q})\,\mathrm{tr}\,\eta(\mathbf{q})\,.$$

Effective masses, widths and level-crossing

The description of the asymmetry generation in terms of Boltzmann-like equations is based on the quasiparticle picture. The positions and widths of the peaks determine the effective masses $\mathbf{M}$ and decay widths $\mathbf{\Gamma}$ of the quasiparticles. Thermal corrections to the masses are $\propto H_{ii}T^2$. In a

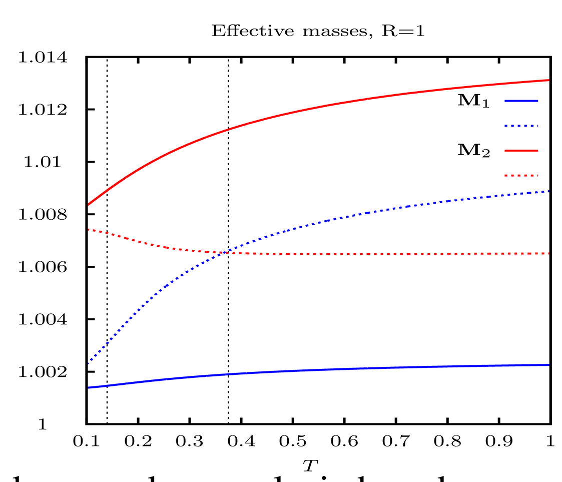


thermal resonant leptogenesis scenario (SM+3RHN) these corrections are nevertheless always comparable to the size of the mass splitting and can therefore affect the resonant enhancement, except if the model is fine-tuned. For the 2PI solution in terms of resummed propagators, masses and widths can be defined by

$$\det\Omega_R^{-1} \equiv \det[q^2 - M^2 - \Pi_R]^{-1} \approx \frac{Z}{(q_0^2 - q_{0,1}^2)(q_0^2 - q_{0,2}^2)}\,.$$

In terms of the zeros $q_{0,I}$ of the denominator the effective masses and widths are given by

$$q_{0,I} = \pm\,\omega_I - \frac{i}{2}\Gamma_I\,,\quad \omega_I = (\mathbf{q}^2 + M_I^2)^{\frac{1}{2}}\,.$$

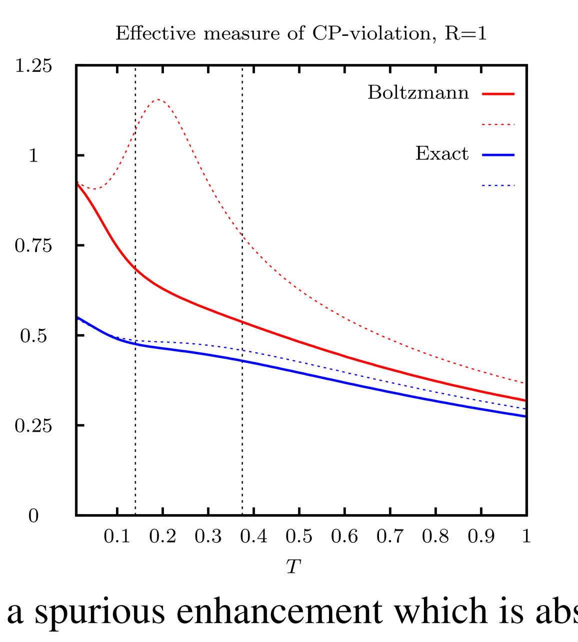


Depending on the values of the couplings the difference of the pole masses $\Delta\mathbf{M}$ can increase with temperature ('runaway' regime) or first decrease, reach a minimum and increase again with temperature ('level crossing' regime). The analysis of the full and diagonal spectral functions demonstrates that in the vicinity of the level crossing temperature the peaks of the diagonal and full propagators do not coincide. Furthermore, the off-diagonal component has only one well pronounced peak and cannot describe two quasiparticle excitations. Therefore, in the vicinity of the level crossing the quasiparticle picture breaks down and an analysis based on a more general approach is needed. If the mass-eigenvalues actually cross is a matter of definition. The evolution of the asymmetry is not affected by the choice.

Improved approximations

It is possible to evaluate the results in the Boltzmann-limit - on the cost of omitting quantum effects such as the finite widths or oscillations between the two states ψ_i. The exact solution may however also be used to obtain improved and quantifiable approximations. Simplifications are possible if the spectral functions are approximated, off-shell effects are neglected and if the late time limit $t \to \infty$ is considered. E.g. in the quasi-degenerate limit:

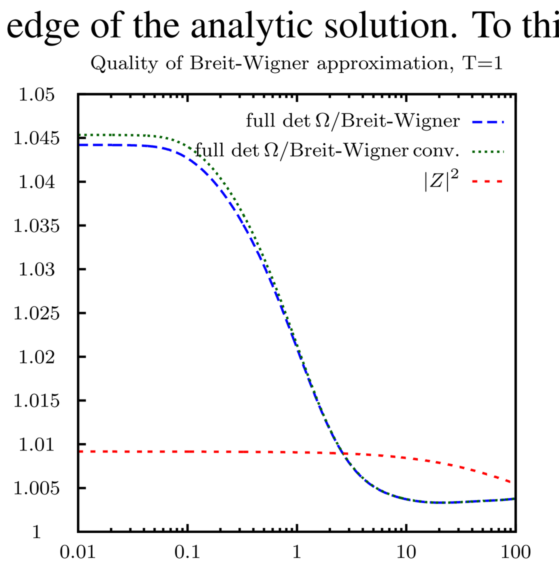
$$\mathrm{tr}\,\eta_\infty(\mathbf{q}) = -\frac{J}{\det M}\frac{|Z|^2}{|q_{0,1}^2 - q_{0,2}^2|^2}\frac{\Pi_\rho^2(\bar{\omega}_\mathbf{q},\mathbf{q})}{(2\bar{\omega}_\mathbf{q})^2}\left[\sum_{I=1,2}\frac{1}{\Gamma_I} - 2\mathrm{Re}\frac{1}{i(\omega_1 - \omega_2) + \frac{1}{2}(\Gamma_1 + \Gamma_2)}\right]$$



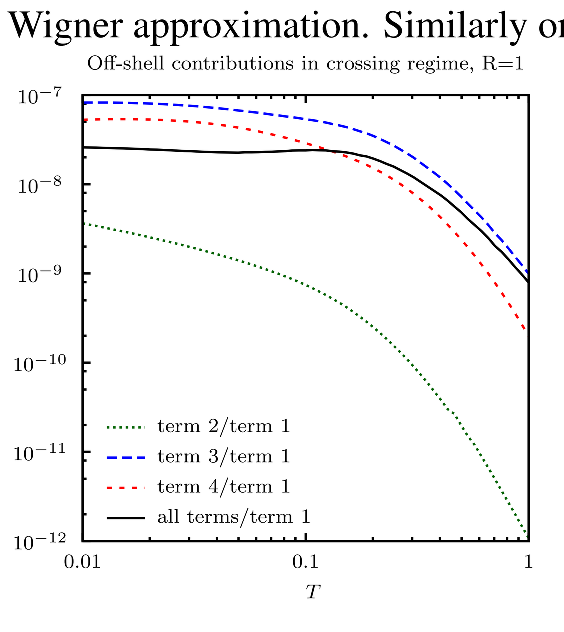
The result is proportional to J which contains the basic parameters of the Lagrangian and therefore reflects the CP-properties of the theory. The dotted lines represent a choice of parameters for which the mass-eigenvalues cross at a certain temperature. In this case a simple Boltzmann-like approximation features a spurious enhancement which is absent in the improved approximation which takes coherent oscillations into account. The quality of the Boltzmann approximation generally improves at high temperatures because the overlap of the spectral functions decreases due to increasing mass-splitting. For the same reason the Boltzmann limit works better in the runaway case (solid lines).

Testing approximations

These assumptions can be tested numerically thanks to the knowledge of the analytic solution. To this end the quadrature of the



multi-dimensional integrals (with possibly rapidly oscillating and sharply peaked integrands) must be performed. The computation simplifies a bit for the late time limit considered here. One can for instance study the corrections due to the approximation of the full $\det\Omega$ (shape of the spectral functions). Shown is the ratio of $\mathrm{tr}\,\eta_\infty(\mathbf{q})$ with and without Breit-Wigner approximation. Similarly one can integrate the off-shell



contributions numerically and compare to the size of the on-shell contributions. Remarkably one finds that the effects of the former are completely negligible. Such that the obtained analytic approximations based on these assumptions are quite accurate (at least in the late time limit).

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