Spontaneous generation of local CP-violation & Inverse magnetic catalysis

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II. Sphaleron transition

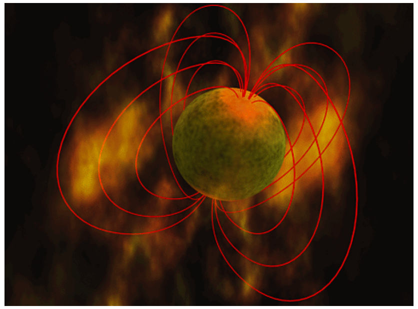
Jingyi Chao, Pengcheng Chu, MH,
PRD88(2013), arXiv:1305.1100,

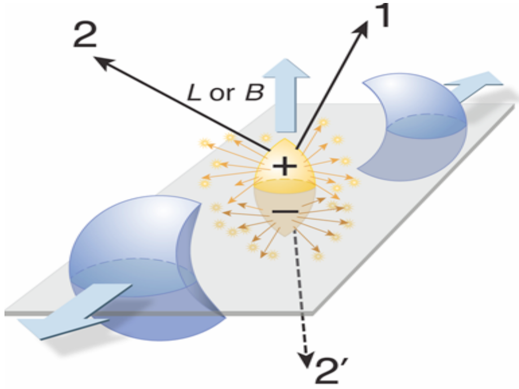
III. Instanton-anti-instanton condensation

Lang Yu, Hao Liu, MH, arXiv:1404.6969

IV. Conclusion and discussion

MAGNETIC FIELDS

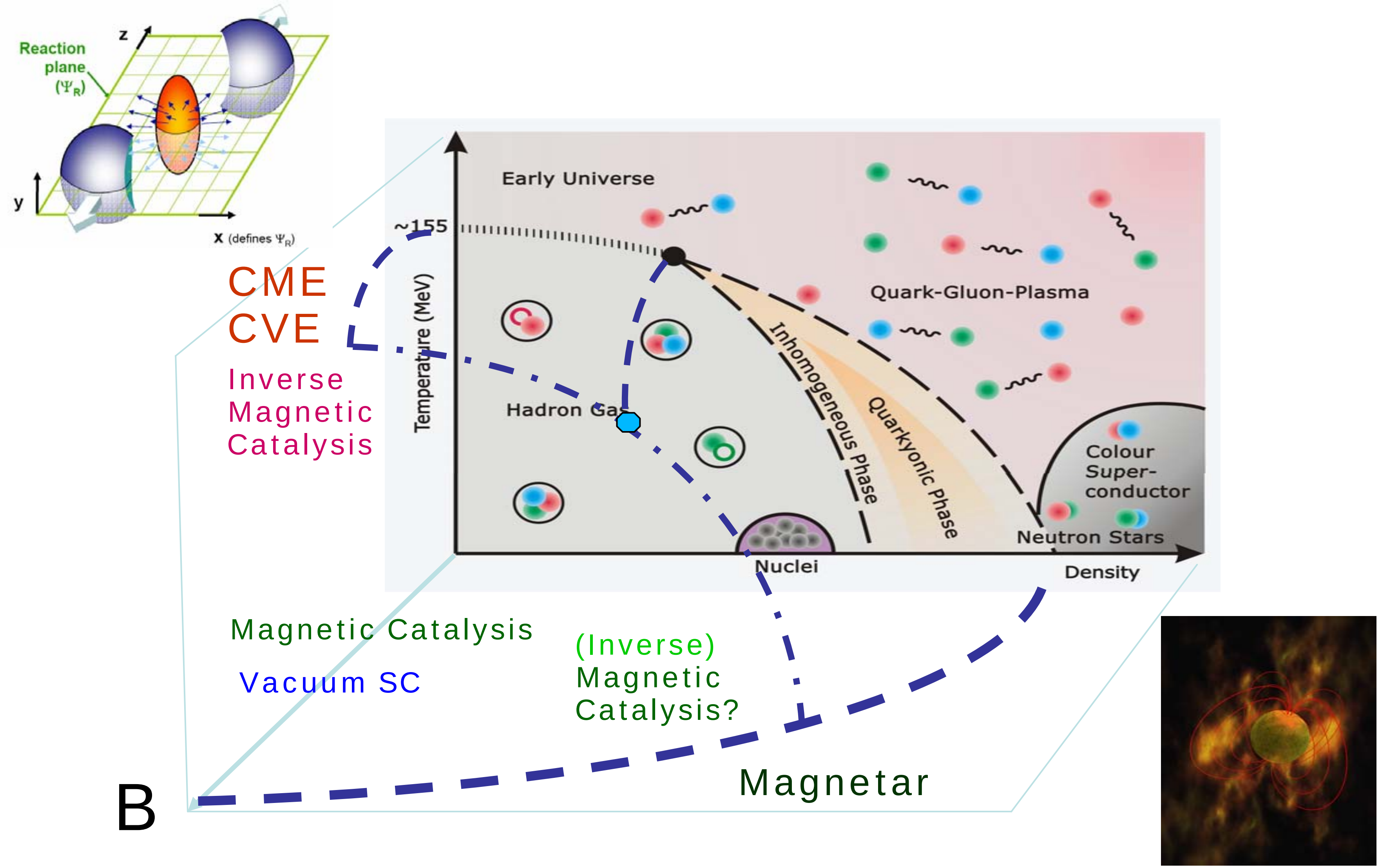




- Inside *compact stars*
  - **10^{10} to 10^{15} Gauss**
- Non-central HIC
  - **10^{18} to 10^{19} Gauss**
- *Early Universe*
  - **up to 10^{24} Gauss**

$$\boxed{\begin{aligned} 1\,\mathrm{MeV}^2 &= 1.44\times 10^{13}\,\mathrm{Gauss} \\ m_\pi^2 &\sim 2.8\times 10^{17}\,\mathrm{Gauss} \end{aligned}}$$

QCD phase diagram under strong magnetic field



CME

CVE

Inverse Magnetic Catalysis

Magnetic Catalysis

Vacuum SC

(Inverse) Magnetic Catalysis?

Magnetar

B

Magnetic catalysis at zero temperature

S.P. Klevansky and R. H. Lemmer ('89); H. Suganuma and T. Tatsumi ('91);
V. P. Gusynin, V. A. Miransky and I. A. Shovkovy ('94, '95, '96,...)

$$\mathscr{L} = \bar{\Psi} i\gamma^{\mu} D_{\mu} \Psi + \frac{G}{2}\left[(\bar{\Psi}\Psi)^2 + (\bar{\Psi} i\gamma^5 \Psi)^2\right]$$

$$D_{\mu} = \partial_{\mu} - ieA_{\mu}^{\text{ext}} \qquad \mathbf{A}^{\text{ext}} = (0, Bx^1, 0)$$

$$m = G\,\mathrm{tr}\left[S(x,x)\right] \approx \frac{Gm}{(2\pi)^2}\left(\Lambda^2 + |eB| \ln\frac{|eB|}{\pi m^2} + O\left(m^2\right)\right)$$

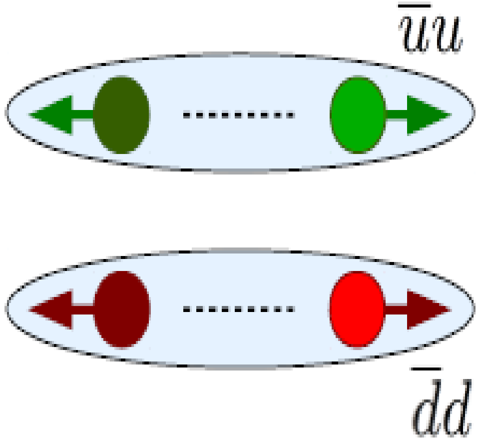
$$m \propto \exp\left(-\frac{2\pi^2}{G|eB|}\right)$$

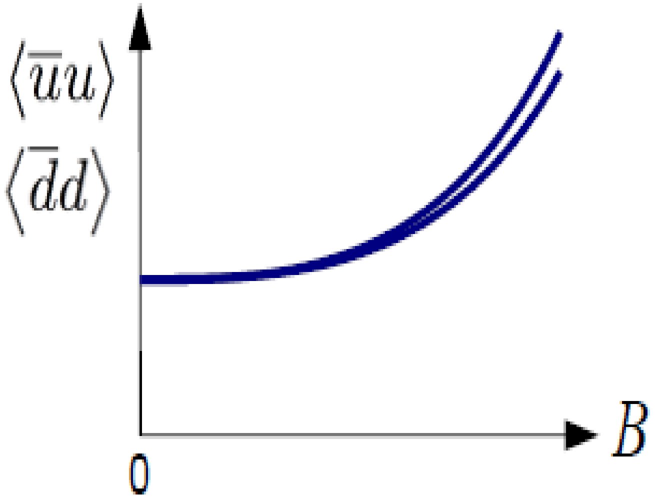
nonzero mass for arbitrary small G

Magnetic catalysis at zero temperature

S.P. Klevansky and R. H. Lemmer ('89); H. Suganuma and T. Tatsumi ('91);
V. P. Gusynin, V. A. Miransky and I. A. Shovkovy ('94, '95, '96,...)

attractive channel: spin-0 flavor-diagonal states

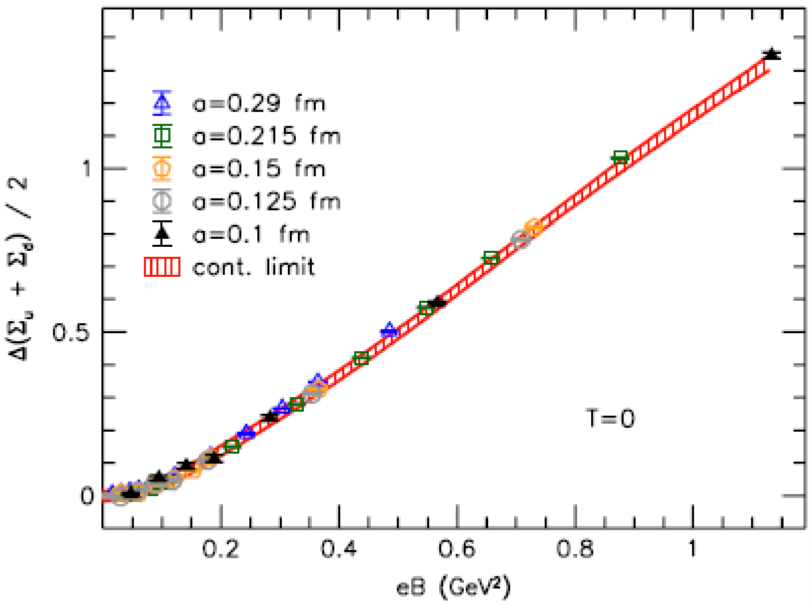


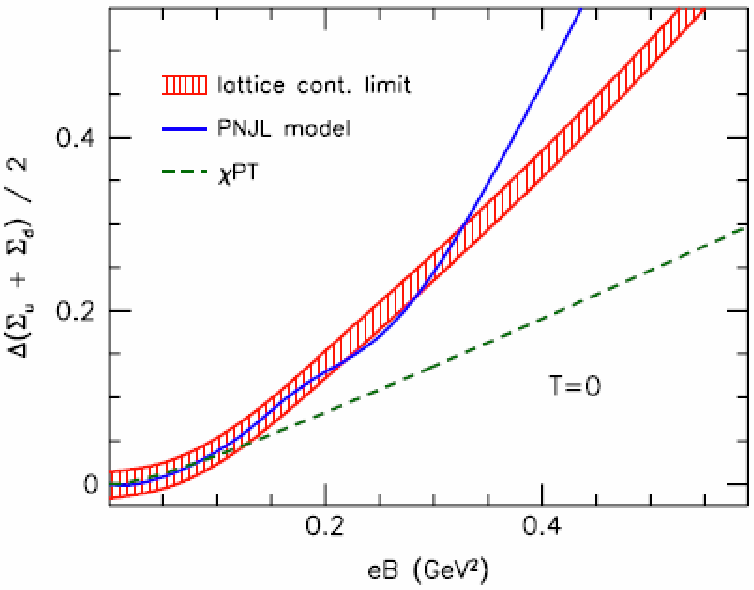


enhances chiral
symmetry breaking

Magnetic catalysis at zero temperature

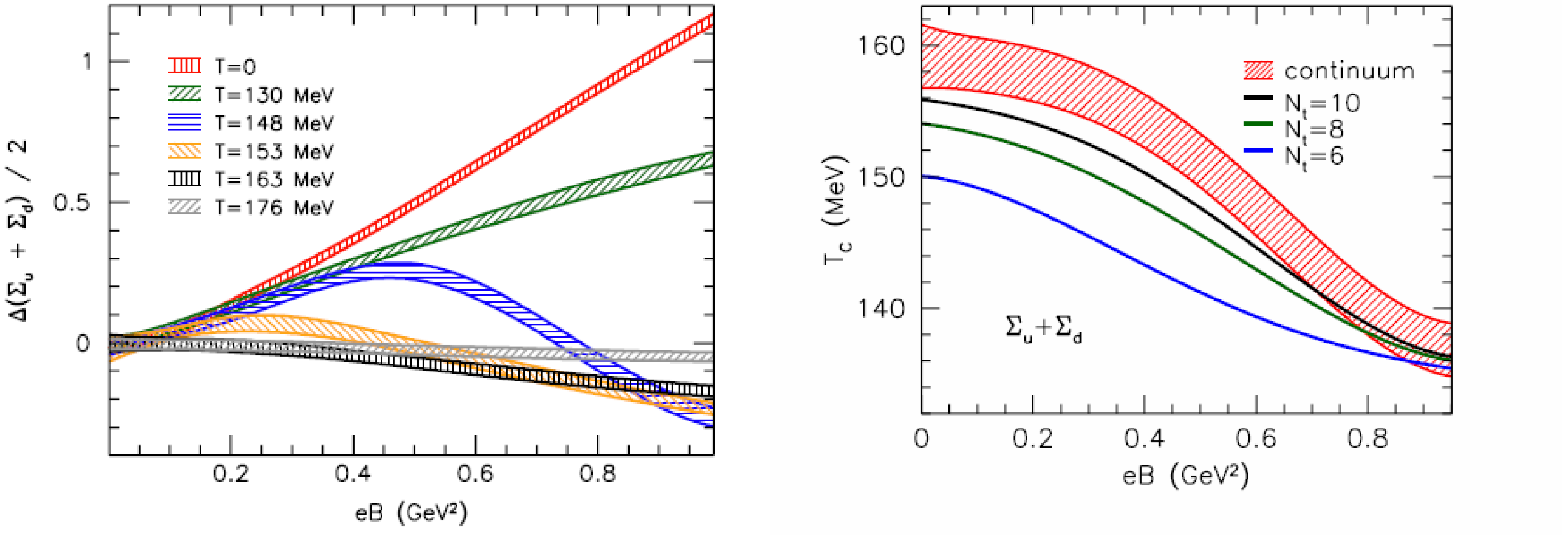
Bali et.al. arXiv:1206.4205 [hep-lat]





Inverse Magnetic catalysis at nonzero temperature

Bali et.al. arXiv:1206.4205 [hep-lat]



Surprise ! Puzzle!

Some important information is missing in our Understanding of chiral phase transition, which is enhanced by magnetic field!

How to understand Inverse Magnetic catalysis ?

1) Magnetic inhibition K. Fukushima, Y. Hidaka, PRL 110, 031601 (2013)

Contribution from neutral pions

2) Contribution from sea quarks

Bruckmann et.al. arXiv:1303.3972

3) Polyakov holomoly Nowak et.al. arXiv:1304.6020

4) Chiral imbalance

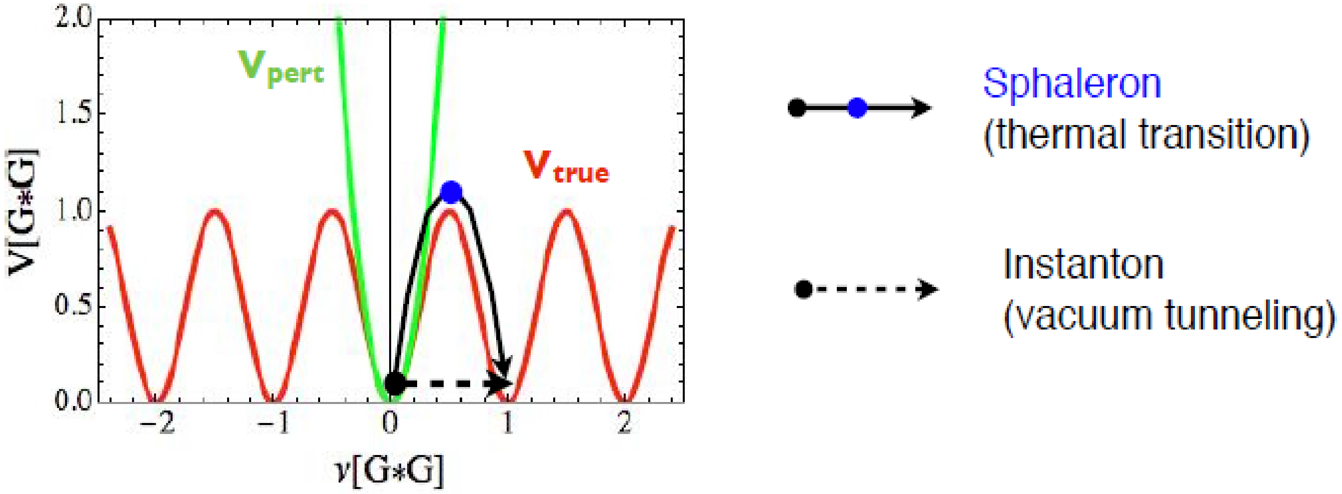
Sphaleron transition Jingyi Chao, Pengcheng Chu, MH,
PRD88(2013), arXiv:1305.1100,

Instanton-anti-instanton pairing condensate

Lang Yu, Hao Liu, MH, arXiv:1404.6969

Theta vacuum, instanton and sphaleron:

QCD vacuum has non-trivial topological structure characterized by an integer valued Chern-Simons number N_{cs}



$$\Delta N_{cs} = \frac{g^2}{32\pi^2}\int d^4x \,\mathrm{Tr}[F_{a\mu\nu}\tilde{F}^{a\mu\nu}]$$

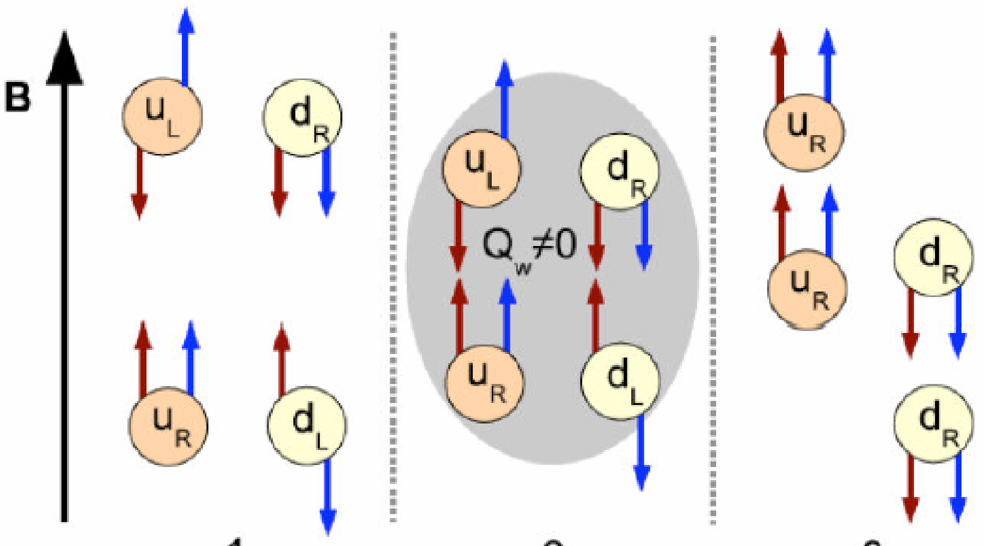
Induce chiral imbalance:

$$(N_R - N_L)_{t=+\infty} - (N_R - N_L)_{t=-\infty} = -2N_f\Delta N_{cs}$$

Chiral Magnetic Effect

Fukushima,Kharzeev,Warringa 2008

Visual picture:



P-invariant plasma of quarks (and gluons)

Sphaleron (instanton) = topological charge

P-odd plasma of quarks (and gluons)

Red: momentum
Blue: spin

Electric charges:
u-quark: $q=+2e/3$
d-quark: $q= - e/3$

Role of topology:
$u_L \rightarrow u_R$
$d_L \rightarrow d_R$

Chiral chemical potential mu_5:

$$\mu_5 \bar{\psi}\gamma^0\gamma^5\psi \qquad \mathcal{N}_5 = \bar{\psi}\gamma^0\gamma^5\psi = \psi_R^\dagger\psi_R - \psi_L^\dagger\psi_L$$

$$\omega_{R\pm} = \pm p_3 - \mu_5,$$
$$\omega_{L\pm} = \mp p_3 + \mu_5.$$

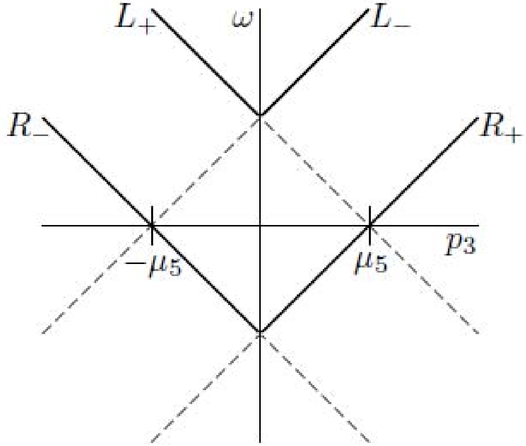


FIG. 1: Spectrum of massless Dirac fermions with right- and left-handed chirality in the presence of an chiral chemical potential μ_5. The subscript ± denotes the eigenvalue of the spin in the z-direction. The chiral chemical potential induces a nonzero density of right-handed particles and left-handed anti-particles.

Destroys chiral condensate:

$$\langle\bar{\psi}\psi\rangle = \langle\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L\rangle$$

Why chiral imbalance destroys chiral condensate?

$$n_q = 0 = n_R - n_{\bar{R}} + n_L - n_{\bar{L}}$$

$$n_R = n_{\bar{R}}, n_L = n_{\bar{L}}$$

$$n_5 = n_R - n_L >> 0 \rightarrow n_{\bar{R}} >> n_L$$

$$\langle \bar{\psi}\psi \rangle = \langle \bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L \rangle$$

Local chiral imbalance $\boxed{< n_5 >= 0}$

$n_5 > 0$

$n_5 < 0$

Local imbalance of chirality:

$$n_5 = \frac{\mu_5^3}{3\pi^2} + \frac{\mu_5 T^2}{3}$$

$$\langle n_5 \rangle = 0 \text{ but } \langle n_5^2 \rangle \neq 0.$$

Time evolution for chiral quark density

$$\frac{\partial n_5}{\partial t} = (4N_f)^2 \frac{\Gamma_{ss}}{T} \frac{\partial F}{\partial n_5}$$

S. Y. .Khlebnikov and M. E. Shaposhnikov, Nucl. Phys. B **308**, 885 (1988).
G. D. Moore, C. -r. Hu and B. Muller, Phys. Rev. D **58**, 045001 (1998); G. D. Moore and M. Tassler, JHEP **1102**, 105 (2011).

$$\mu_5 = \sqrt{3}\pi \left(\frac{320 N_f^2 \Gamma_{ss}}{T^2} - \frac{T^2}{3} \right)^{\frac{1}{2}}$$

Sphaleron diffusion rate from AdS/CFT

$$\boxed{\mu_5 \simeq c\sqrt{eB}}$$

$$\Gamma_{ss}(B,T) = \frac{(g_s^2 N_c)^2}{384\sqrt{3}\pi^5} \left(eBT^2 + 15.9T^4 \right)$$

G. Basar and D. E. Kharzeev, Phys. Rev. D **85**, 086012

Sphaleron diffusion rate at finite T: variation of topological number per unit time and per unit volumn

Khlebnikov, Shaposhnikov 1988, Bodeker,1998, Son, Starinets 2002

$$\Gamma_{ss} \sim T^4$$

$$\Gamma_{ss} \sim (g^2 T)^4$$

$$\Gamma_{ss} \sim g^4 \log(1/g^2) T (g^2 T)^3$$

Sphaleron transition rate under magnetic field:

Debye mass for longitudinal gluons $g(T + c\sqrt{eB})$

$$\Gamma_{ss} \sim (T^4 + c^2 eBT^2) \qquad eB \gg T^2$$

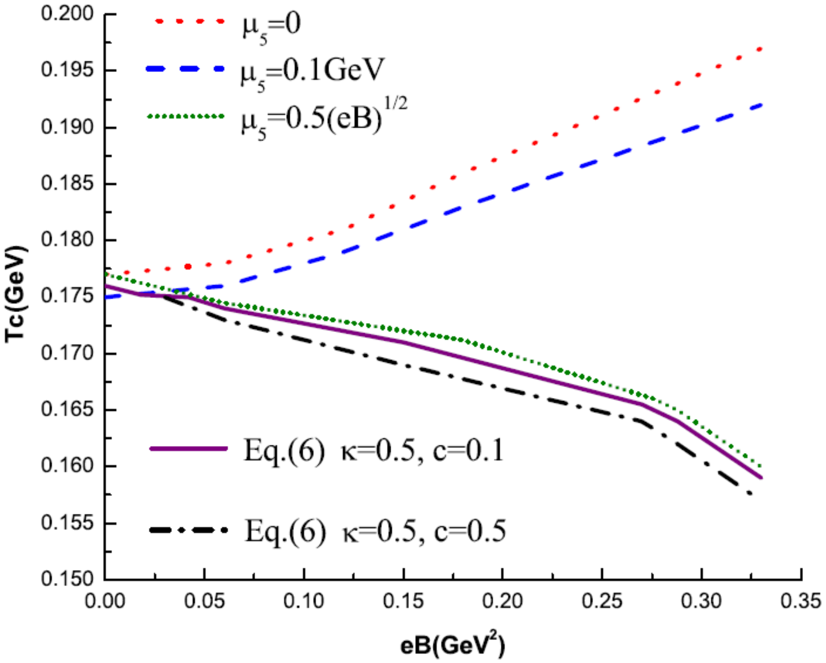
$$\mu_5 \simeq \begin{cases} \kappa_1(g)(T + c_1 e^2 B^2 / T^3) & \text{for} \quad \sqrt{\mathrm{eB}} \lesssim \mathrm{T}; \\ \kappa_2(g)(\sqrt{eB} + c_2 T^2 / \sqrt{eB}) & \text{for} \quad \sqrt{\mathrm{eB}} \gtrsim \mathrm{T}, \end{cases}$$

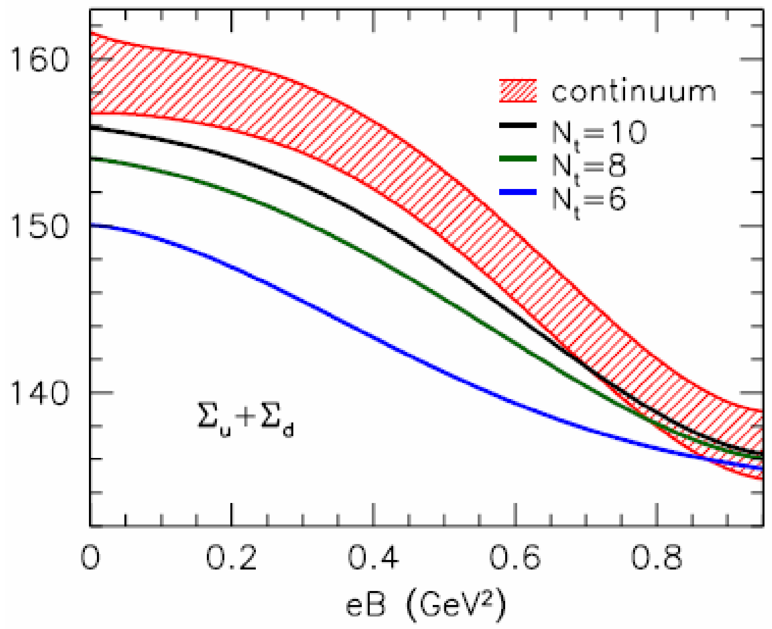
Chiral phase transition induced by chiral anomaly

$$\mathcal{L} = \bar{\psi}\left(i\gamma_\mu D^\mu + \mu\gamma^0 + \mu_5\gamma^0\gamma^5\right)\psi + G\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma^5\tau\psi\right)^2\right] ,$$

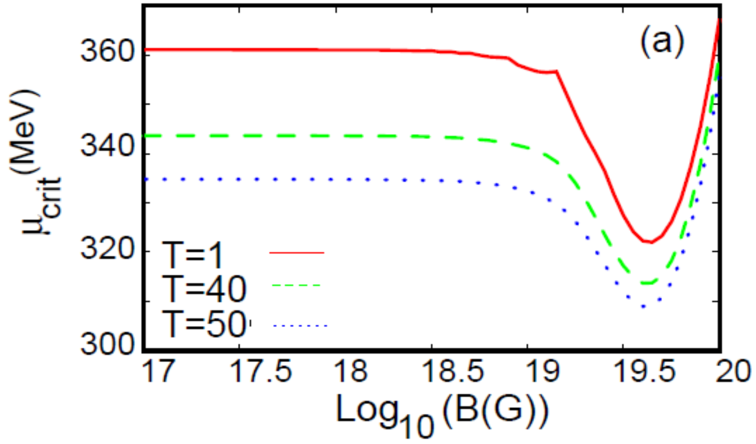
$$\Omega = \frac{\sigma^2}{4G} - N_c \sum_{f=u,d} \frac{|q_f B|}{2\pi} \sum_{s,k} \alpha_{sk} \int_{-\infty}^{\infty} \frac{dp_z}{2\pi} \omega_s(p) - T N_c \sum_{f=u,d} \frac{|q_f B|}{2\pi} \sum_{s,k} \alpha_{sk} \int_{-\infty}^{\infty} \frac{dp_z}{2\pi} \times \left(\ln[1+e^{-\beta(\omega_s+\mu)}] + \ln[1+e^{-(\beta\omega_s-\mu)}]\right) .$$

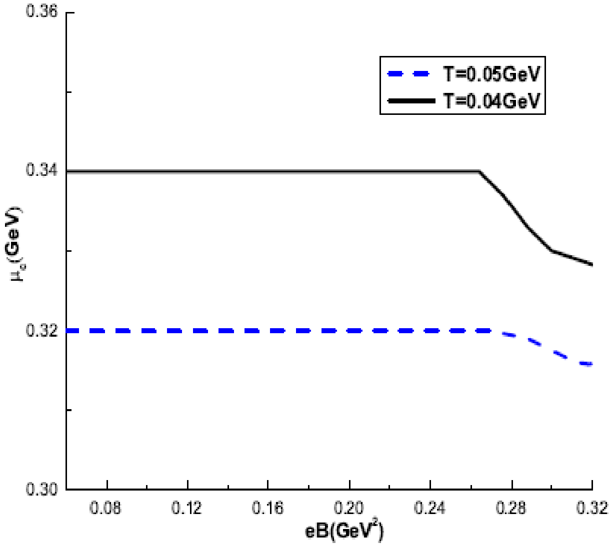
Jingyi Chao, Pengcheng Chu, Mei Huang, arXiv:1305.1100





- Inverse magnetic catalysis at mu?



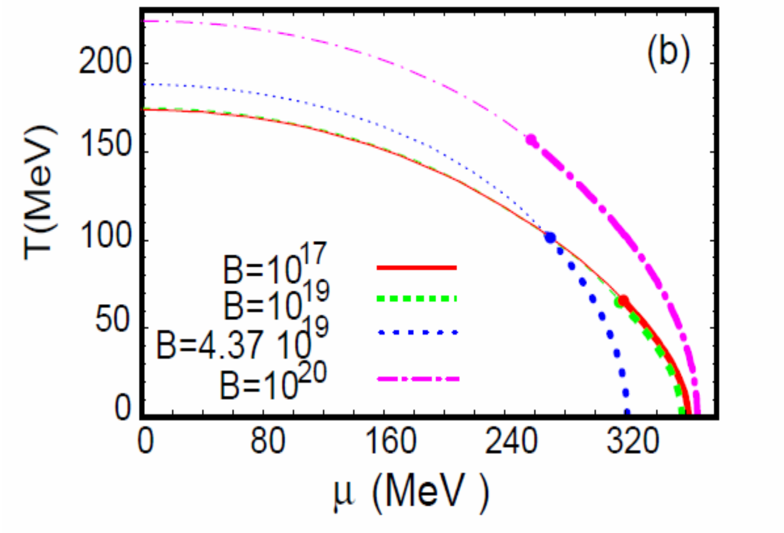


Behavior at high B is fake due to the cut-off !

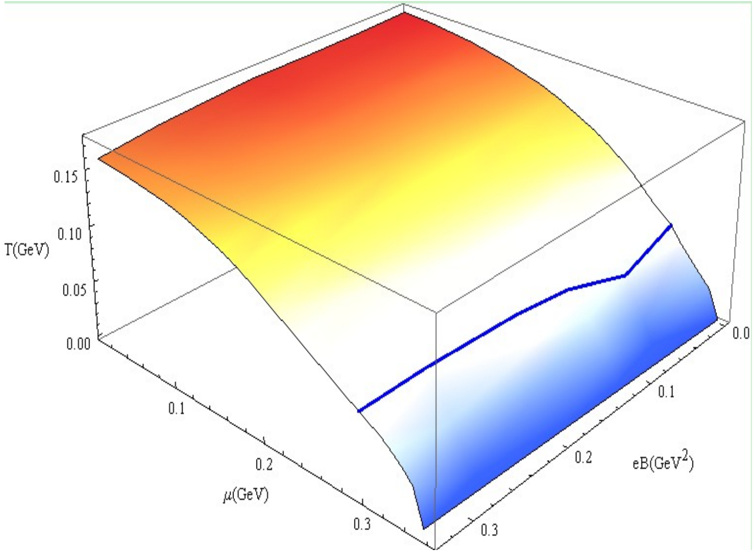
Avancini,Menezes,et.al.
Phys.Rev. D85 (2012) 091901

Jingyi Chao, Pengcheng Chu,
Mei Huang, arXiv:1305.1100

■ CEP under strong magnetic field



Avancini,Menezes,et.al.
Phys.Rev. D85 (2012) 091901



Jingyi Chao, Pengcheng Chu,
Mei Huang, arXiv:1305.1100

Chiral imbalance induced by instanton anti-instanton molecule pairing:

T. Schafer, E. V. Shuryak and J. J. M. Verbaarschot, Phys. Rev. D **51**, 1267 (1995) [hep-ph/9406210].

$$
\begin{aligned}
\mathcal{L}_{mol\ sym} = G\Bigg\{ & \frac{2}{N_c^2}\left[\left(\bar\psi\tau^a\psi\right)^2 - \left(\bar\psi\tau^a\gamma^5\psi\right)^2\right] \\
& -\frac{1}{2N_c^2}\left[\left(\bar\psi\tau^a\gamma^\mu\psi\right)^2 + \left(\bar\psi\tau^a\gamma^\mu\gamma^5\psi\right)^2\right] \\
& +\frac{2}{N_c^2}\left(\bar\psi\gamma^\mu\gamma^5\psi\right)^2 \Bigg\} + \mathcal{L}_8\ ,
\end{aligned}
$$

$T \gtrsim T_c$

$$\boxed{G_S = \frac{2G}{N_c^2},\ G_V = \frac{G}{2N_c^2},\ G_A = -\frac{3G}{2N_c^2}}$$

$$
\begin{aligned}
\mathcal{L} = & \bar\psi i\gamma_\mu D^\mu\psi + G_S\left[\left(\bar\psi\psi\right)^2 + \left(\bar\psi i\gamma^5\boldsymbol{\tau}\psi\right)^2\right] \\
& -G_V\left(\bar\psi\gamma^\mu\psi\right)^2 - G_A\left(\bar\psi\gamma^\mu\gamma^5\psi\right)^2\ .
\end{aligned}
$$

Mean-field approximation:

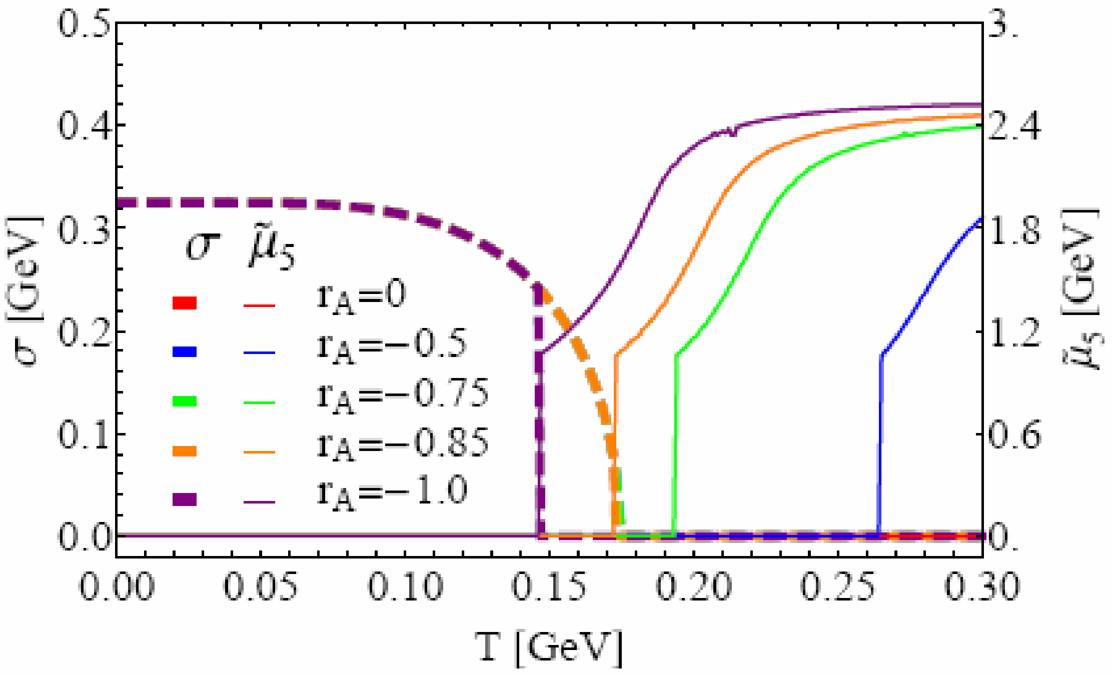
$$\mathcal{L} = -\frac{\sigma^2}{4G_S} + \frac{\tilde{\mu}_5^2}{4G_A} + \bar{\psi}\left(i\gamma_\mu D^\mu - \sigma + \tilde{\mu}_5\gamma^0\gamma^5\right)\psi$$

$$\sigma \;=\; -2G_S\langle\bar{\psi}\psi\rangle \qquad \tilde{\mu}_5 \;=\; -2G_A\langle\bar{\psi}\gamma^0\gamma^5\psi\rangle$$

$$\boxed{r_A = G_A/G_S}$$

$$\begin{aligned}\Omega = & \frac{\sigma^2}{4G_S} - \frac{\tilde{\mu}_5^2}{4G_A} \\ & -N_c \sum_{f=u,d} \frac{|q_f B|}{2\pi} \sum_{s,k} \alpha_{sk} \int_{-\infty}^{\infty} \frac{dp_z}{2\pi}\, f_\Lambda^2(p)\, \omega_{sk}(p) \\ & -2N_c T \sum_{f=u,d} \frac{|q_f B|}{2\pi} \sum_{s,k} \alpha_{sk} \int_{-\infty}^{\infty} \frac{dp_z}{2\pi} \qquad\qquad (16) \\ & \times \ln\left(1 + e^{-\beta\omega_{sk}}\right),\end{aligned}$$

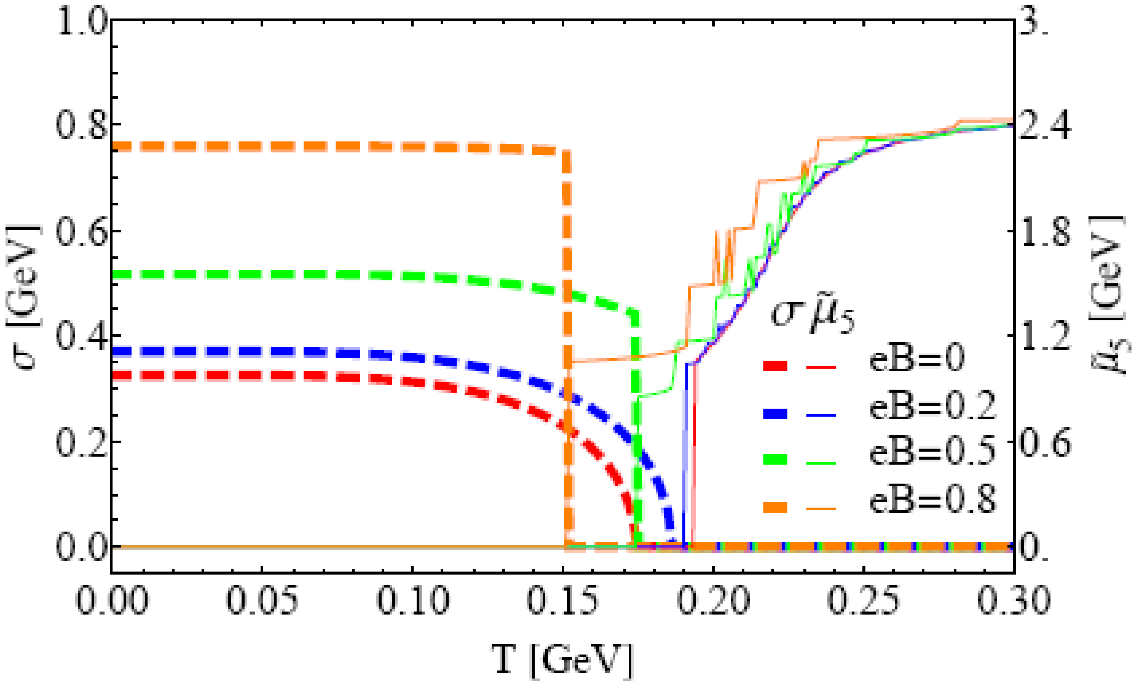
Spontaneous generation of CP-violation above Tc even at zero magnetic field !



(a) σ and $\tilde{\mu}_5$ at $eB = 0$ for different values of r_A.

Lang Yu, Hao Liu, MH, arXiv:1404.6969

Magnetic field catalyzes local CP-violation!



(b) σ and $\tilde{\mu}_5$ at $r_A = -0.75$ for different values of eB.

Lang Yu, Hao Liu, MH, arXiv:1404.6969

Inverse magnetic catalysis

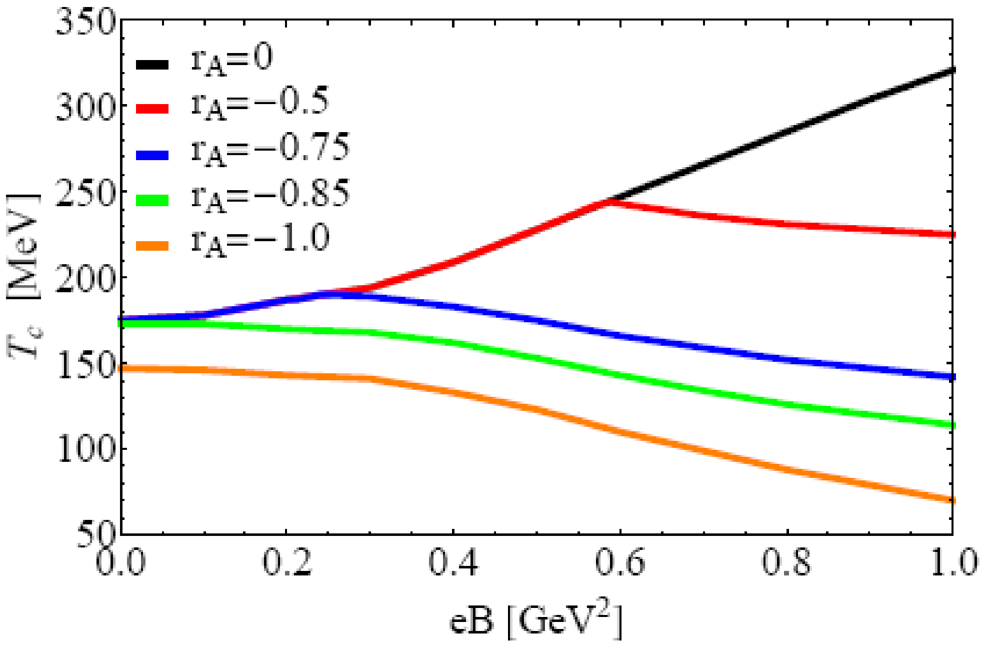


FIG. 2. (Color online) T_c as a function of eB for r_A=0,-0.5, -0.75, -0.85 and -1.0.

Lang Yu, Hao Liu, MH, arXiv:1404.6969

Conclusion

1, Inverse magnetic catalysis around Tc indicates that some important information is missing in our understanding of chiral phase transition, which is enhanced by magnetic field!

2, We suggest that the inverse magnetic catalysis can be naturally induced by chiral imbalance, which is related to topological structure of QCD vacuum, i.e. Sphaleron transitions or instanton-anti-instanton pairing condensate.

http://indico.ihep.ac.cn/conferenceDisplay.py?confId=4321

QCD vacuum and matter under strong magnetic field Ⅱ

15-17 October 2014, IHEP,CAS

Invited speakers:

Dr. Gokce Basar (Stony Brook Uni., USA),
Prof. Maxim Chernodub (Tours Uni., France),
Prof. Kenji Fukushima (Tokyo Uni., Japan),
Prof. Huanzhong Huang (UCLA, USA)
Prof. Jinfeng Liao (Indiana Uni., USA),
Prof. Andreas Schafer (Regensburg Uni., Germany)
Prof. Igor Shovkovy (Arizona State Uni., USA)

Thanks for your attention!