Spontaneous generation of local CP-violation & Inverse magnetic catalysis

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Content

I. Inverse magnetic catalysis

II. Sphaleron transition


III. Instanton-anti-instanton condensation


IV. Conclusion and discussion
MAGNETIC FIELDS

• Inside *compact stars*
  – $10^{10}$ to $10^{15}$ Gauss

• Non-central HIC
  – $10^{18}$ to $10^{19}$ Gauss

• *Early Universe*
  – up to $10^{24}$ Gauss

\[
1 \text{ MeV}^2 = 1.44 \times 10^{13} \text{ Gauss} \\
\frac{m_{\pi}^2}{c^2} \sim 2.8 \times 10^{17} \text{ Gauss}
\]
QCD phase diagram under strong magnetic field

- CME
- CVE
- Inverse Magnetic Catalysis
- Magnetic Catalysis
- Vacuum SC
- (Inverse) Magnetic Catalysis?
- Magnetar
- Early Universe
- Quark-Gluon-Plasma
- Inhomogeneous phase
- Quarkonic phase
- Neutron Stars
- Colour Superconductor
- Nuclei
- Temperature (MeV)
- Density
Magnetic catalysis at zero temperature

S.P. Klevansky and R. H. Lemmer (’89); H. Suganuma and T. Tatsumi (’91); V. P. Gusynin, V. A. Miransky and I. A. Shovkovy (’94, ’95, ’96, ...)

\[ \mathcal{L} = \bar{\Psi} i\gamma^\mu D_\mu \Psi + \frac{G}{2} \left[ (\bar{\Psi}\Psi)^2 + (\bar{\Psi}i\gamma^5\Psi)^2 \right] \]

\[ D_\mu = \partial_\mu - ieA_\mu^{\text{ext}}, \quad A^{\text{ext}} = (0, Bx^1, 0) \]

\[ m = G\text{tr}[S(x,x)] \approx \frac{Gm}{(2\pi)^2} \left( \Lambda^2 + |eB| \ln \frac{|eB|}{\pi m^2} + O(m^2) \right) \]

\[ m \propto \exp\left(-\frac{2\pi^2}{G|eB|}\right) \]

nonzero mass for arbitrary small G
Magnetic catalysis at zero temperature

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V. P. Gusynin, V. A. Miransky and I. A. Shovkovy ('94, '95, '96,...)

attractive channel: spin-0 flavor-diagonal states

\[
\begin{align*}
\bar{u}u &
\end{align*}
\]

\[
\begin{align*}
\bar{d}d &
\end{align*}
\]

\[
\begin{align*}
\langle \bar{u}u \rangle &
\end{align*}
\]

\[
\begin{align*}
\langle \bar{d}d \rangle &
\end{align*}
\]

enhances chiral symmetry breaking
Magnetic catalysis at zero temperature

Bali et al. arXiv:1206.4205 [hep-lat]
Inverse Magnetic catalysis at nonzero temperature

Surprise ! Puzzle!
Some important information is missing in our Understanding of chiral phase transition, which is enhanced by magnetic field!

Bali et.al. arXiv:1206.4205 [hep-lat]
How to understand Inverse Magnetic catalysis?


   Contribution from neutral pions

2) Contribution from sea quarks
   Bruckmann et.al. arXiv:1303.3972

3) Polyakov holomoly  Nowak et.al. arXiv:1304.6020

4) Chiral imbalance
   Sphaleron transition
   Instanton-anti-instanton pairing condensate
Theta vacuum, instanton and sphaleron:

QCD vacuum has non-trivial topological structure characterized by an integer valued Chern-Simons number $N_{cs}$.

\[ \Delta N_{cs} = \frac{g^2}{32\pi^2} \int d^4x \ Tr[F_{a\mu\nu}\tilde{F}^{a\mu\nu}] \]

Induce chiral imbalance:

\[ (N_R - N_L)_{t=+\infty} - (N_R - N_L)_{t=-\infty} = -2N_f\Delta N_{cs} \]
Chiral Magnetic Effect

Fukushima, Kharzeev, Warringa 2008

Visual picture:

1. $P$-invariant plasma of quarks (and gluons)
2. Sphaleron (instanton) = topological charge
3. $P$-odd plasma of quarks (and gluons)

Red: momentum
Blue: spin

Electric charges:
- u-quark: $q = +2e/3$
- d-quark: $q = -e/3$

Role of topology:
- $u_L \rightarrow u_R$
- $d_L \rightarrow d_R$
Chiral chemical potential $\mu_5$:

\[ \mu_5 \bar{\psi} \gamma^0 \gamma^5 \psi \quad \mathcal{N}_5 = \bar{\psi} \gamma^0 \gamma^5 \psi = \psi_R^\dagger \psi_R - \psi_L^\dagger \psi_L \]

\[ \omega_{R\pm} = \pm p_3 - \mu_5, \]
\[ \omega_{L\pm} = \mp p_3 + \mu_5. \]

Destroys chiral condensate:

\[ \langle \bar{\psi} \psi \rangle = \langle \bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L \rangle \]

FIG. 1: Spectrum of massless Dirac fermions with right- and left-handed chirality in the presence of an chiral chemical potential $\mu_5$. The subscript $\pm$ denotes the eigenvalue of the spin in the $z$-direction. The chiral chemical potential induces a nonzero density of right-handed particles and left-handed anti-particles.
Why chiral imbalance destroys chiral condensate?

\[ n_q = 0 = n_R - n_{\bar{R}} + n_L - n_{\bar{L}} \]

\[ n_R = n_{\bar{R}}, \quad n_L = n_{\bar{L}} \]

\[ n_5 = n_R - n_L >> 0 \rightarrow n_{\bar{R}} >> n_L \]

\[ \langle \bar{\psi} \psi \rangle = \langle \bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L \rangle \]

Local chiral imbalance

\[ < n_5 > = 0 \]

\[ n_5 > 0 \]

\[ n_5 < 0 \]
Local imbalance of chirality:

\[ n_5 = \frac{\mu_5^3}{3\pi^2} + \frac{\mu_5 T^2}{3} \]

\[ \langle n_5 \rangle = 0 \text{ but } \langle n_5^2 \rangle \neq 0. \]

Time evolution for chiral quark density

\[ \frac{\partial n_5}{\partial t} = (4N_f)^2 \frac{\Gamma_{ss}}{T} \frac{\partial F}{\partial n_5} \]

Sphaleron diffusion rate from AdS/CFT

\[ \mu_5 = \sqrt{3\pi} \left( \frac{320 N_f^2 \Gamma_{ss}}{T^2} - \frac{T^2}{3} \right)^{\frac{1}{2}} \]

\[ \Gamma_{ss}(B, T) = \frac{(g_s^2 N_c)^2}{384\sqrt{3}\pi^5} \left(eBT^2 + 15.9T^4 \right) \]

G. Basar and D. E. Kharzeev, Phys. Rev. D 85, 086012

Sphaleron diffusion rate at finite $T$: variation of topological number per unit time and per unit volume


\[
\Gamma_{ss} \sim T^4
\]

\[
\Gamma_{ss} \sim (g^2 T)^4
\]

\[
\Gamma_{ss} \sim g^4 \log(1/g^2) T (g^2 T)^3
\]

Sphaleron transition rate under magnetic field:

**Debye mass for longitudinal gluons** \[ g(T + c\sqrt{eB}) \]

\[
\Gamma_{ss} \sim (T^4 + c^2 eBT^2) \quad eB \gg T^2
\]

\[
\mu_5 \sim \begin{cases} 
\kappa_1(g)(T + c_1 e^2 B^2 / T^3) & \text{for } \sqrt{eB} \lesssim T; \\
\kappa_2(g)(\sqrt{eB} + c_2 T^2 / \sqrt{eB}) & \text{for } \sqrt{eB} \gtrsim T,
\end{cases}
\]
Chiral phase transition induced by chiral anomaly

\[ \mathcal{L} = \bar{\psi} \left( i\gamma_\mu D^\mu + \mu \gamma^0 + \mu_5 \gamma^0 \gamma^5 \right) \psi + G \left[ (\bar{\psi} \psi)^2 + (\bar{\psi} i\gamma^5 \tau \psi)^2 \right], \]

Jingyi Chao, Pengcheng Chu, Mei Huang, arXiv:1305.1100
Inverse magnetic catalysis at $\mu$?

Behavior at high $B$ is fake due to the cut-off!

Avancini, Menezes, et al.  

Jingyi Chao, Pengcheng Chu, Mei Huang, arXiv:1305.1100
- CEP under strong magnetic field

Avancini, Menezes, et al.  

Jingyi Chao, Pengcheng Chu, Mei Huang,  
arXiv:1305.1100
Chiral imbalance induced by instanton anti-instanton molecule pairing:


\[
\mathcal{L}_{mol\ sym} = G \left\{ \frac{2}{N_c^2} \left[ (\bar{\psi} \tau^a \psi)^2 - (\bar{\psi} \tau^a \gamma^5 \psi)^2 \right] \\
- \frac{1}{2N_c^2} \left[ (\bar{\psi} \gamma^\mu \gamma^5 \psi)^2 + (\bar{\psi} \gamma^\mu \gamma^5 \psi)^2 \right] \\
+ \frac{2}{N_c^2} (\bar{\psi} \gamma^\mu \gamma^5 \psi)^2 \right\} + \mathcal{L}_8,
\]

\[
T \gtrsim T_c \quad G_S = \frac{2G}{N_c^2}, \quad G_V = \frac{G}{2N_c^2}, \quad G_A = -\frac{3G}{2N_c^2}
\]

\[
\mathcal{L} = \bar{\psi} i \gamma_\mu D^\mu \psi + G_S \left[ (\bar{\psi} \psi)^2 + (\bar{\psi} i \gamma^5 \tau \psi)^2 \right] \\
- G_V (\bar{\psi} \gamma^\mu \psi)^2 - G_A (\bar{\psi} \gamma^\mu \gamma^5 \psi)^2.
\]
Mean-field approximation:

\[
\mathcal{L} = -\frac{\sigma^2}{4G_S} + \frac{\tilde{\mu}_5^2}{4G_A} + \bar{\psi} \left( i\gamma_\mu D^\mu - \sigma + \tilde{\mu}_5 \gamma^0 \gamma^5 \right) \psi
\]

\[
\sigma = -2G_S \langle \bar{\psi} \psi \rangle \quad \tilde{\mu}_5 = -2G_A \langle \bar{\psi} \gamma^0 \gamma^5 \psi \rangle
\]

\[
\Omega = \frac{\sigma^2}{4G_S} - \frac{\tilde{\mu}_5^2}{4G_A}
\]

\[
\sum_{f=u,d} \frac{|q_fB|}{2\pi} \sum_{s,k} \alpha_{sk} \int_{-\infty}^{\infty} \frac{dp_z}{2\pi} f^2_A(p) \omega_{sk}(p)
\]

\[
-2N_c T \sum_{f=u,d} \frac{|q_fB|}{2\pi} \sum_{s,k} \alpha_{sk} \int_{-\infty}^{\infty} \frac{dp_z}{2\pi} \times \ln \left( 1 + e^{-\beta \omega_{sk}} \right),
\]

\[r_A = \frac{G_A}{G_S}\]
Spontaneous generation of CP-violation above Tc even at zero magnetic field!

(a) $\sigma$ and $\tilde{\mu}_5$ at $eB = 0$ for different values of $r_A$.

Magnetic field catalyzes local CP-violation!

Inverse magnetic catalysis

FIG. 2. (Color online) $T_c$ as a function of $eB$ for $r_A=0,-0.5,-0.75,-0.85$ and -1.0.

Conclusion

1. Inverse magnetic catalysis around $T_c$ indicates that some important information is missing in our understanding of chiral phase transition, which is enhanced by magnetic field!

2. We suggest that the inverse magnetic catalysis can be naturally induced by chiral imbalance, which is related to topological structure of QCD vacuum, i.e. Sphaleron transitions or instanton-anti-instanton pairing condensate.
QCD vacuum and matter under strong magnetic field II

15-17 October 2014, IHEP, CAS

Invited speakers:

Dr. Gokce Basar (Stony Brook Uni., USA),
Prof. Maxim Chernodub (Tours Uni., France),
Prof. Kenji Fukushima (Tokyo Uni., Japan),
Prof. Huanzhong Huang (UCLA, USA)
Prof. Jinfeng Liao (Indiana Uni., USA),
Prof. Andreas Schafer (Regensburg Uni., Germany)
Prof. Igor Shovkovy (Arizona State Uni., USA)
Thanks for your attention!