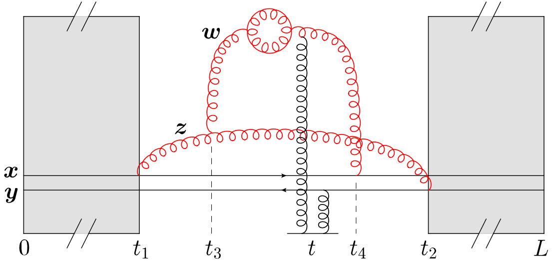
The non–linear evolution of jet quenching

Edmond Iancu
IPhT Saclay & CNRS

arXiv: 1403.1996



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closely related work:

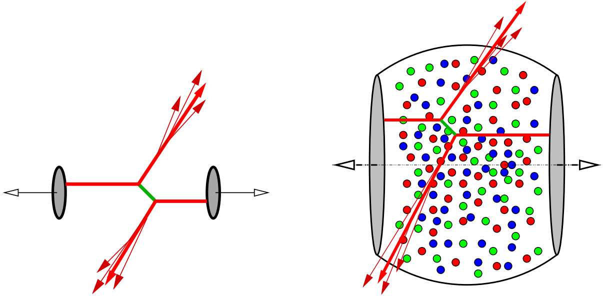
Liou, Mueller, Wu (arXiv:1304.7677)

Blaizot, Mehtar-Tani (arXiv:1403.2323)

E.I., Triantafyllopoulos (arXiv:1405.3525)

Hard probes in heavy ion collisions

- Hard particle production in nucleus–nucleus collisions (RHIC, LHC) can be modified by the surrounding medium ('quark–gluon plasma')



- The ensemble of these modifications : 'jet quenching'

  ▷ energy loss, transverse momentum broadening, di–jet asymmetry ...

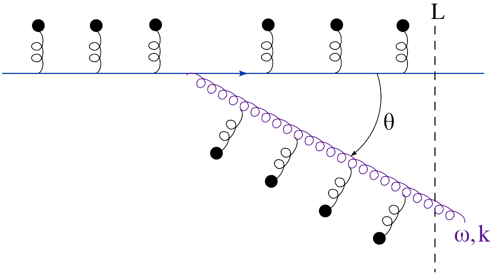
  ▷ *cf. the review talks by Federico Antinori and Jean–Paul Blaizot*

- Assuming the coupling to be weak, can one understand these phenomena from first principles (perturbative QCD) ?

A ubiquitous transport coefficient

- In pQCD, all such phenomena find a common denominator:

  ▷ incoherent multiple scattering off the medium constituents



  - random kicks leading to Brownian motion in $k_\perp$: $\langle k_\perp^2 \rangle \simeq \hat{q}\,\Delta t$
  - acceleration causing medium induced radiation (BDMPSZ, LPM)
  - multiple branchings leading to many soft quanta at large angles

- At leading order in α_s, only one transport coefficient :

  ▷ the jet quenching parameter $\hat{q}$

A ubiquitous transport coefficient

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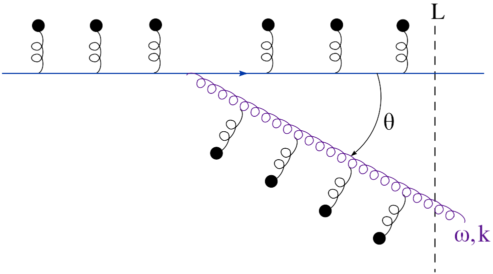
  ▷ incoherent multiple scattering off the medium constituents



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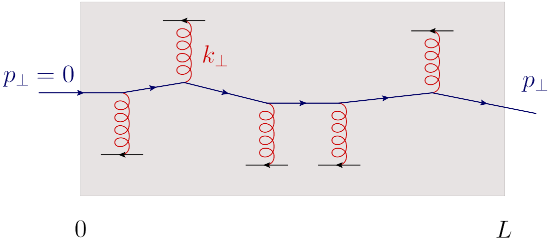
  - acceleration causing medium induced radiation (BDMPSZ, LPM)
  - multiple branchings leading to many soft quanta at large angles

- Will this universality survive the quantum ('radiative') corrections ?

  ▷ if so, how will these corrections affect the value of $\hat{q}$?

- An energetic quark acquires a transverse momentum $p_\perp$ via collisions in the medium, after propagating over a distance L



- Quark energy $E \gg$ typical $p_\perp \Longrightarrow$ small deflection angle $\theta \ll 1$

Transverse momentum broadening

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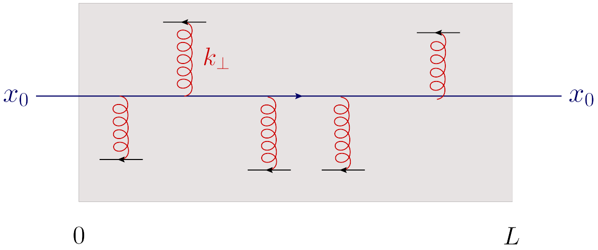


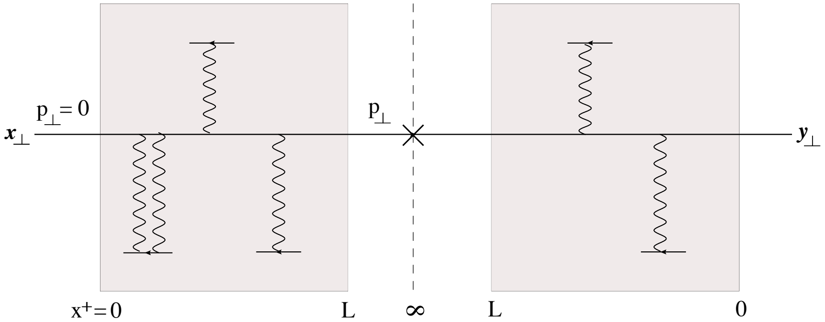
- Quark energy $E \gg$ typical $p_\perp \Longrightarrow$ small deflection angle $\theta \ll 1$
- The quark transverse position is unchanged: eikonal approximation

$$V(\boldsymbol{x}) = \mathrm{P}\exp\left\{ ig \int \mathrm{d}x^+ \, A_a^-(x^+, \boldsymbol{x}) t^a \right\}$$

- The quark is a 'right mover' : $x^+ \equiv (t+z)/\sqrt{2} \simeq \sqrt{2}t$ is its LC time

Transverse momentum broadening (2)

- Direct amplitude (DA) $\times$ Complex conjugate amplitude (CCA) :



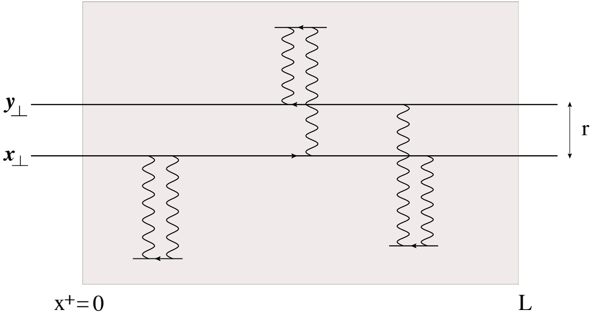
- The $p_\perp$–spectrum of the quark after crossing the medium $(\boldsymbol{r} = \boldsymbol{x} - \boldsymbol{y})$

$$\frac{\mathrm{d}N}{\mathrm{d}^2\boldsymbol{p}} = \frac{1}{(2\pi)^2} \int_{\boldsymbol{r}} \mathrm{e}^{-\mathrm{i}\boldsymbol{p}\cdot\boldsymbol{r}} \langle S_{\boldsymbol{x}\boldsymbol{y}} \rangle \,, \qquad S_{\boldsymbol{x}\boldsymbol{y}} \equiv \frac{1}{N_c} \mathrm{tr}\big(V_{\boldsymbol{x}} V_{\boldsymbol{y}}^\dagger\big)$$

- Average over A_a^- (the distribution of the medium constituents)

Dipole picture

- Formally, $\langle S_{\boldsymbol{xy}} \rangle$ is the average S–matrix for a $q\bar{q}$ color dipole



  - ▷ 'the quark at $\boldsymbol{x}$' : the physical quark in the DA
  - ▷ 'the antiquark at $\boldsymbol{y}$' : the physical quark in the CCA

- Quark cross–section $\longleftrightarrow$ dipole amplitude
- The dipole S–matrix also controls the rate for medium–induced gluon branching (energy loss, jet fragmentation)

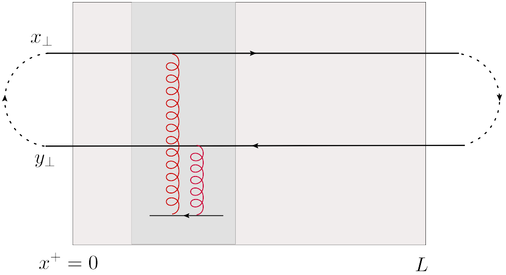
The tree–level approximation

- At zeroth order, $\langle S_{\boldsymbol{xy}} \rangle$ is fully specified by one parameter: $\hat{q}_0$
- Weakly coupled medium $\Rightarrow$ quasi independent color charges

  ▷ Gaussian distribution for the color fields A^-, local in time (x^+)

  ▷ multiple scattering series exponentiates (Glauber, McLerran–Venugopalan)

$$\langle S_{\boldsymbol{xy}} \rangle = \mathrm{e}^{-T_{2g}} \simeq \exp\left\{-\frac{1}{4}\, L\hat{q}_0(1/r^2)\, \boldsymbol{r}^2\right\}$$



▷ T_{2g} : scattering amplitude via two–gluon exchange (single scattering)

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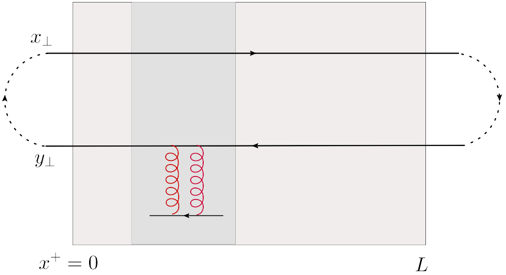

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The tree–level jet quenching parameter

$$\hat{q}_0(Q^2) \equiv n \int^{Q^2} \frac{\mathrm{d}^2\boldsymbol{k}}{(2\pi)^2}\, \boldsymbol{k}^2\, \frac{g^4 C_F}{(\boldsymbol{k}^2 + m_D^2)^2} \simeq 4\pi\alpha_s^2 C_F n \ln \frac{Q^2}{m_D^2}$$

$\triangleright$ n : density of the medium constituents; m_D : Debye mass

- The cross–section for $p_\perp$–broadening :

$$\frac{\mathrm{d}N}{\mathrm{d}^2\boldsymbol{p}} = \frac{1}{(2\pi)^2} \int_{\boldsymbol{r}} \mathrm{e}^{-\mathrm{i}\boldsymbol{p}\cdot\boldsymbol{r}}\; \mathrm{e}^{-\frac{1}{4} L\hat{q}_0(1/r^2)\, \boldsymbol{r}^2} \simeq \frac{1}{\pi Q_s^2}\, \mathrm{e}^{-\boldsymbol{p}^2/Q_s^2}$$

- The saturation momentum : exponent of $\mathcal{O}(1)$ when $r \sim 1/Q_s$

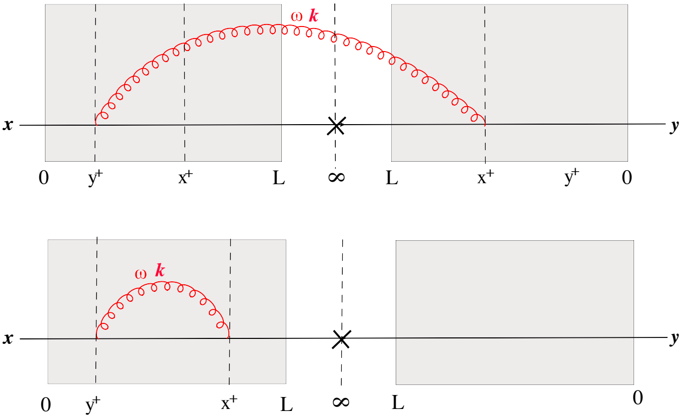
$$Q_s^2 = L\hat{q}_0(Q_s^2) = 4\pi\alpha_s^2 C_F n L \ln \frac{Q_s^2}{m_D^2} \propto L \ln L$$

- The physical jet quenching parameter : $\hat{q}_0(Q_s^2) \propto \ln L$

- N.B. $p_\perp$–broadening probes the dipole S–matrix near unitarity

Radiative corrections to $p_\perp$–broadening

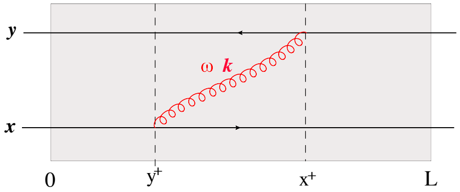
- The quark 'evolves' by emitting a gluon ('real' or 'virtual')

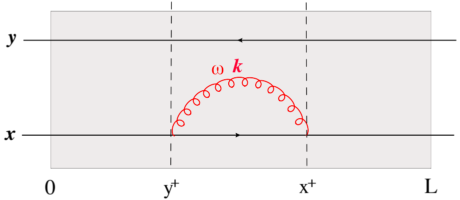


- The 'evolution' gluon is not measured: one integrates over ω and $\boldsymbol{k}$
- All partons undergo multiple scattering: non–linear evolution

Dipole evolution

- Alternatively depicted as the evolution of the dipole S–matrix:





- Exchange graphs between q and $\bar{q}$, or self–energy graphs
- This evolution needs not be restricted to a change in $\hat{q}$

  ▷ quantum corrections can change the functional form of $\langle S(\boldsymbol{r})\rangle$

The phase space

- The radiative corrections are suppressed by powers if α_s ...

  ... but can be enhanced by the phase–space for gluon emissions
- A 'naive' argument: bremsstrahlung in the vacuum

$$\mathrm{d}P = \frac{\alpha_s C_R}{\pi^2} \frac{\mathrm{d}\omega}{\omega} \frac{\mathrm{d}^2\boldsymbol{k}}{\boldsymbol{k}^2}$$

- The emission requires a formation time $\tau \simeq 2\omega/k_\perp^2$
- For our present purposes, better use τ instead of ω

The phase space

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$$\mathrm{d}P = \frac{\alpha_s C_R}{\pi} \frac{\mathrm{d}\tau}{\tau} \frac{\mathrm{d}k_\perp^2}{k_\perp^2}$$

- τ can take all the values between $\lambda \sim 1/T$ and L
- For a given τ, $k_\perp^2$ should be larger than $\hat{q}\tau$ (multiple scattering) but smaller than $Q_s^2 = \hat{q}L$ (dipole resolution $r \sim 1/Q_s$)

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$$\Delta P(L) = \frac{\alpha_s C_R}{\pi} \int_\lambda^L \frac{\mathrm{d}\tau}{\tau} \int_{\hat{q}\tau}^{\hat{q}L} \frac{\mathrm{d}k_\perp^2}{k_\perp^2} = \frac{\alpha_s C_R}{\pi} \frac{1}{2} \ln^2 \frac{L}{\lambda}$$

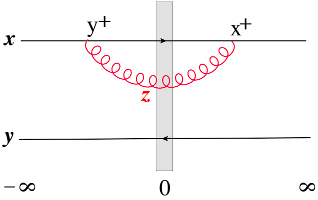
▷ large, double–logarithmic, correction

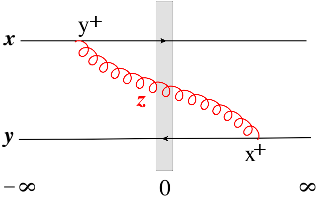
▷ $\Delta P(L) \sim \mathcal{O}(1)$ for $L = 5$ fm, $T = 500$ MeV, $\alpha_s = 0.3$

Non–linear evolution

- The previous argument is 'naive' as it ignores multiple scattering
- Non–linear evolution is well understood for a shock–wave target

  ▷ proton–nucleus collisions at RHIC or the LHC





- Lifetime $\tau = x^+ - y^+ \gg$ target width $L \Longrightarrow$ eikonal approx.

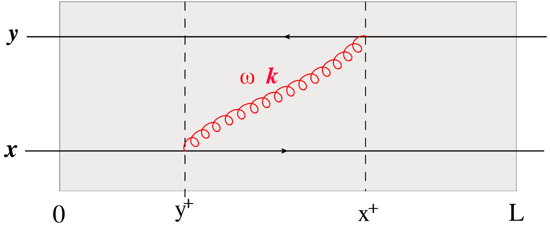
  ▷ the 'evolution' gluon interacts at a fixed transverse coordinate $\boldsymbol{z}$

- Non–linear equations for correlators of Wilson lines, like $\langle S_{\boldsymbol{xy}} \rangle$: Balitsky, JIMWLK, BK (large N_c)

  ▷ the functional form of $\langle S(\boldsymbol{r}) \rangle$ for $r \sim 1/Q_s$ changes indeed

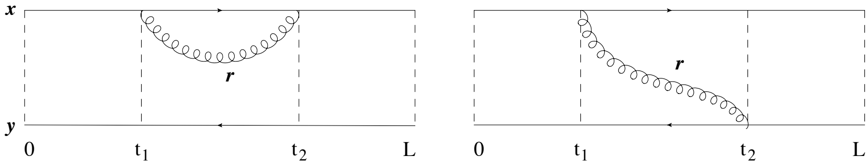
Beyond the eikonal approximation

- The eikonal approximation fails for gluon emissions inside the medium
  - ▷ the fluctuation can scatter at any time t during its lifetime: $y^+ < t < x^+$



- One needs to consider the transverse diffusion of the gluon fluctuations
  - ▷ $D = 2 + 1$ quantum mechanical problem in a random background field
  - ▷ formal solution in the form of a path integral
- Generalization of the JIMWLK (or BK) equations to an extended target ('medium') *(E.I., arXiv: 1403.1996)*

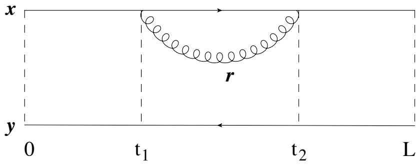
The BK equation for jet quenching

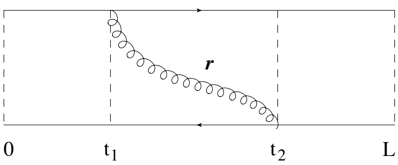


$$\frac{\partial \mathcal{S}_{L,0}(\boldsymbol{x},\boldsymbol{y})}{\partial \omega} =$$

$$\times \Big[\mathcal{S}_{L,t_2}(\boldsymbol{x},\boldsymbol{y})\, \mathcal{S}_{t_2,t_1}(\boldsymbol{x},\boldsymbol{r}(t))\, \mathcal{S}_{t_2,t_1}(\boldsymbol{r}(t),\boldsymbol{y}) \mathcal{S}_{t_1,0}(\boldsymbol{x},\boldsymbol{y}) \; - \; \mathcal{S}_{L,0}(\boldsymbol{x},\boldsymbol{y}) \Big]$$

The BK equation for jet quenching

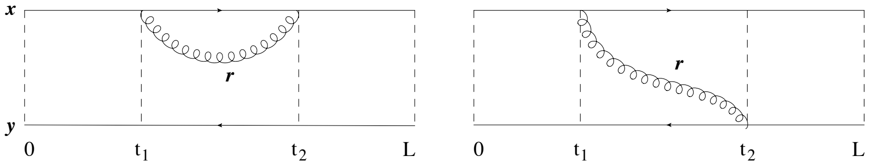




$$\frac{\partial \mathcal{S}_{L,0}(\boldsymbol{x},\boldsymbol{y})}{\partial \omega} = \partial^i_{\boldsymbol{r}_1}\partial^i_{\boldsymbol{r}_2}\int_{\boldsymbol{r}_1,\boldsymbol{r}_2}\left[\mathcal{D}\boldsymbol{r}(t)\right]\,\mathrm{e}^{\,\mathrm{i}\,\frac{\omega}{2}\int_{t_1}^{t_2}\mathrm{d}t\,\dot{\boldsymbol{r}}^2(t)}$$

$$\times\Big[\mathcal{S}_{L,t_2}(\boldsymbol{x},\boldsymbol{y})\,\mathcal{S}_{t_2,t_1}(\boldsymbol{x},\boldsymbol{r}(t))\,\mathcal{S}_{t_2,t_1}(\boldsymbol{r}(t),\boldsymbol{y})\mathcal{S}_{t_1,0}(\boldsymbol{x},\boldsymbol{y})\,-\,\mathcal{S}_{L,0}(\boldsymbol{x},\boldsymbol{y})\Big]$$

The BK equation for jet quenching



$$\frac{\partial \mathcal{S}_{L,0}(\boldsymbol{x},\boldsymbol{y})}{\partial \omega} = -\frac{\alpha_s N_c}{2\omega^3}\int_0^L \mathrm{d}t_2 \int_0^{t_2} \mathrm{d}t_1\; \partial^i_{\boldsymbol{r}_1}\partial^i_{\boldsymbol{r}_2} \int_{\boldsymbol{r}_1,\boldsymbol{r}_2} \left[\mathcal{D}\boldsymbol{r}(t)\right]\, \mathrm{e}^{\mathrm{i}\,\frac{\omega}{2}\int_{t_1}^{t_2}\mathrm{d}t\,\dot{\boldsymbol{r}}^2(t)}$$

$$\times\Big[\mathcal{S}_{L,t_2}(\boldsymbol{x},\boldsymbol{y})\,\mathcal{S}_{t_2,t_1}(\boldsymbol{x},\boldsymbol{r}(t))\,\mathcal{S}_{t_2,t_1}(\boldsymbol{r}(t),\boldsymbol{y})\mathcal{S}_{t_1,0}(\boldsymbol{x},\boldsymbol{y}) \;-\; \mathcal{S}_{L,0}(\boldsymbol{x},\boldsymbol{y})\Big]$$

- A functional equation : path integral for $\boldsymbol{r}(t)$
  - ▷ likely, too complicated to be solved in the general case
- A starting point for controlled approximations

The single scattering approximation

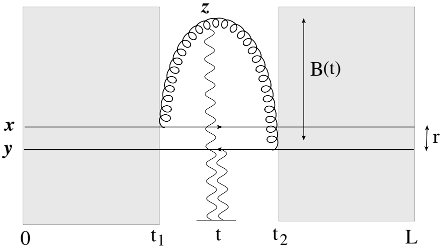
- Only one scattering during the lifetime of the fluctuation

  ▷ enhanced by the infrared & collinear 'divergences' of bremsstrahlung

$$\mathcal{S}_{t_2,t_1}(\boldsymbol{z},\boldsymbol{y}) \simeq \mathrm{e}^{-\frac{1}{4}(t_2-t_1)\,\hat{q}\,B_\perp^2}$$

$$B_\perp^2 = |\boldsymbol{z}-\boldsymbol{y}|^2 \sim 1/p_\perp^2$$

$$t_2 - t_1 \sim \tau = \omega/p_\perp^2$$



- External dipole 'near saturation' : $r \sim 1/Q_s \Longrightarrow p_\perp^2 \lesssim Q_s^2 = \hat{q}L$
- Weak scattering $\Longleftrightarrow$ small exponent $\Longrightarrow p_\perp^2 \gg \hat{q}\tau$
- Large longitudinal (energy) phase–space: $\lambda \ll \tau \ll L$

  $\Longrightarrow$ large transverse phase–space as well : $\hat{q}\tau \ll p_\perp^2 \ll \hat{q}L$

The phase–space for linear evolution

- $Q_s^2(\tau) \equiv \hat{q}\tau$: the saturation line for gluons with lifetime τ

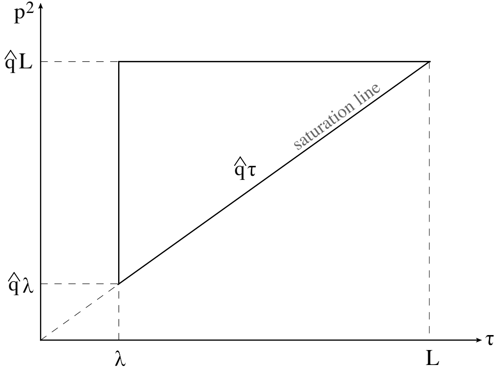
- The longitudinal phase–space:

$$\lambda \ll \tau \ll L$$

- ... and the transverse one :

$$\hat{q}\tau \ll p_\perp^2 \ll \hat{q}L$$

- ... increase equally fast !



- The conditions for a double logarithmic approximation (DLA)

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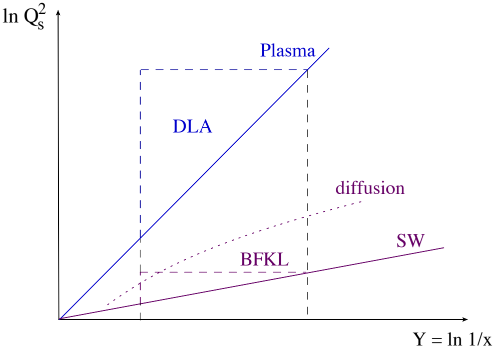

- The conditions for a double logarithmic approximation (DLA)
- Very different from the respective evolution for a shock wave: stronger dependence of Q_s^2 upon τ (or $1/x$)

  ▷ *see the talks by D. Triantafyllopoulos and K. Kutak*

The double logarithmic approximation

- To DLA, the dipole S–matrix $\mathcal{S}_L(\boldsymbol{r})$ preserves the same functional form as at tree–level, but with a renormalized $\hat{q}$:

$$\mathcal{S}_L(\boldsymbol{r}) \simeq \exp\left\{-\frac{1}{4} L\hat{q}(L)\,\boldsymbol{r}^2\right\}$$

- Universality : $\hat{q}_0(L) \to \hat{q}(L)$ in all the quantities related to $\mathcal{S}$

  ▷ $p_\perp$–broadening, radiative energy loss, jet fragmentation ...

- BK equation reduces to a relatively simple, linear, equation for $\hat{q}(L)$

$$\hat{q}(L) = \hat{q}_0 + \bar{\alpha}\int_\lambda^L \frac{\mathrm{d}\tau}{\tau}\int_{\hat{q}\tau}^{\hat{q}L}\frac{\mathrm{d}p_\perp^2}{p_\perp^2}\,\hat{q}(\tau, p_\perp^2)$$

  ▷ Liou, Mueller, Wu (arXiv: 1304.7677) [$p_\perp$–broadening]

  ▷ Blaizot, Mehtar-Tani (arXiv: 1403.2323) [radiative energy loss]

  ▷ E.I. (arXiv: 1403.1996) [evolution of the dipole S–matrix]

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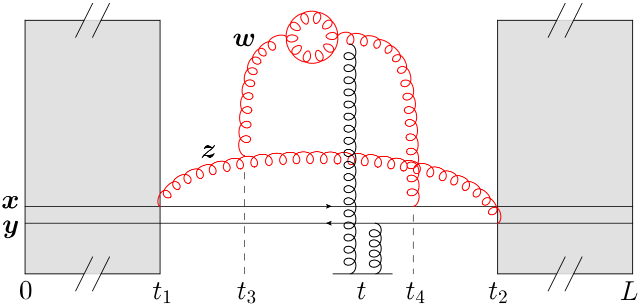
- Not the standard DLA limit of the DGLAP or BFKL eqs. : different boundary conditions (multiple scattering) $\Longrightarrow$ different solutions

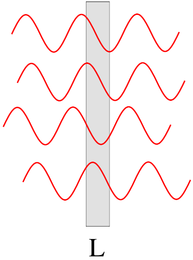
- Predicts a strong dependence of $\hat{q}$ upon the medium properties: L, T

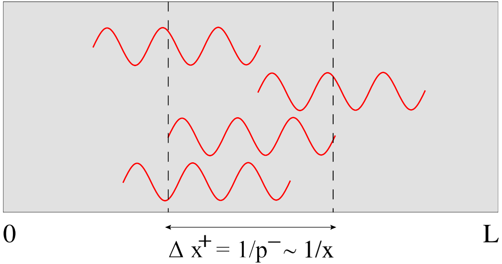
To be continued ...



- See the talk by Dionysis Triantafyllopoulos for
  - details of the solution
  - running coupling effects
  - physical implications

Gluon saturation in the medium





- Multiple scattering is tantamount to gluon saturation in the target
- $Q_s^2(x)$ is proportional to the width of the region where a gluon (with longitudinal fraction x) can overlap with its sources
  - ▷ for a shockwave, this region is the SW width L (fixed and small)
  - ▷ for a gluon in the medium, this is the gluon longitudinal wavelength:

$$\tau \equiv \Delta x^+ = 1/p^- \propto 1/x$$

- The x–dependence of $Q_s^2(x)$ is further amplified by the evolution

Fixed coupling

- Use logarithmic variables, as standard for BFKL, or BK:

  ▷ $Y \equiv \ln\frac{\tau}{\lambda}$ ('rapidity') and $\rho \equiv \ln\frac{p_\perp^2}{\hat{q}\lambda}$ ('momentum')

$$\hat{q}(Y,\rho) = \hat{q}^{(0)} + \bar{\alpha}\int_0^Y \mathrm{d}Y_1 \int_{Y_1}^{\rho} \mathrm{d}\rho_1\, \hat{q}(Y_1,\rho_1) \quad \text{with} \quad \rho \geq Y$$

- Not the standard DLA (as familiar from studies of DGLAP, or BFKL) !

  ▷ saturation boundary: $\rho_1 \geq Y_1$ (multiple scattering)

- Straightforward to solve via iterations (Liou, Mueller, Wu, 2013)

$$\hat{q}_s(Y) = \hat{q}^{(0)}\,\frac{\mathrm{I}_1\big(2\sqrt{\bar{\alpha}}\,Y\big)}{\sqrt{\bar{\alpha}}\,Y} = \hat{q}^{(0)}\,\frac{\mathrm{e}^{2\sqrt{\bar{\alpha}}\,Y}}{\sqrt{4\pi}\,(\sqrt{\bar{\alpha}}Y)^{3/2}}\Big[1+\mathcal{O}(1/\sqrt{\bar{\alpha}}Y)\Big]$$

- Rapid increase at large Y, with 'anomalous dimension' $2\sqrt{\bar{\alpha}} \sim 1$
- The standard artifact of using a fixed coupling (recall e.g. BK)

Running coupling

(E.I. Triantafyllopoulos, arXiv:1405.3525)

- One–loop QCD running coupling : $\bar{\alpha} \rightarrow \bar{\alpha}(\rho_1) \equiv \frac{b}{\rho_1+\rho_0}$

$$\hat{q}(Y,\rho) = \hat{q}^{(0)} + b\int_0^Y \mathrm{d}Y_1 \int_{Y_1}^{\rho} \frac{\mathrm{d}\rho_1}{\rho_1+\rho_0}\,\hat{q}(Y_1,\rho_1)$$

- The standard DLA with RC (no saturation boundary) would give

$$\hat{q}(Y,\rho) = \hat{q}^{(0)}\mathrm{I}_1\big(2\sqrt{bY\ln\rho}\big) \;\propto\; \mathrm{e}^{2\sqrt{bY\ln\rho}}$$

- The actual solution is very different (and much more complicated !)

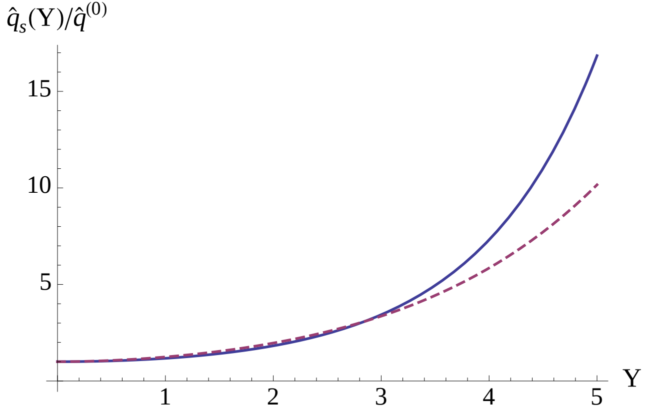
$$\ln\hat{q}_s(Y) \;=\; 4\sqrt{bY} - 3|\xi_1|(4bY)^{1/6} + \frac{1}{4}\ln Y + \kappa + \mathcal{O}\big(Y^{-1/6}\big)$$

  ▷ $\xi_1 = -2.338\ldots$ is the rightmost zero of the Airy function

- Surprisingly similar to the asymptotic expansion of $\ln Q_s^2(Y)$ for a SW (Mueller, Triantafyllopoulos, 2003; Munier, Peschanski, 2003)

Running vs. fixed coupling

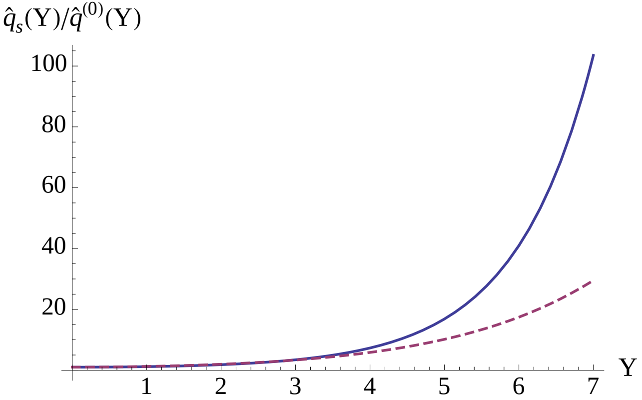
- The enhancement factor $\hat{q}_s(Y)/\hat{q}^{(0)}$ as a function of Y :



- Results are numerically similar up to $Y \simeq 3$, but for larger Y, the rise is much faster with FC

Running vs. fixed coupling

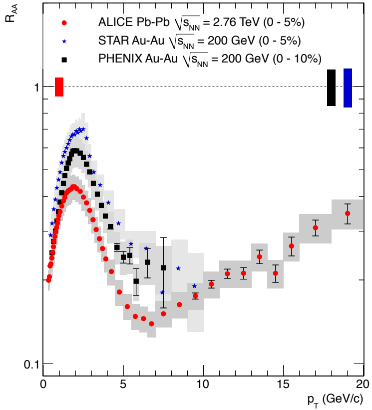
- The enhancement factor $\hat{q}_s(Y)/\hat{q}^{(0)}$ as a function of Y :

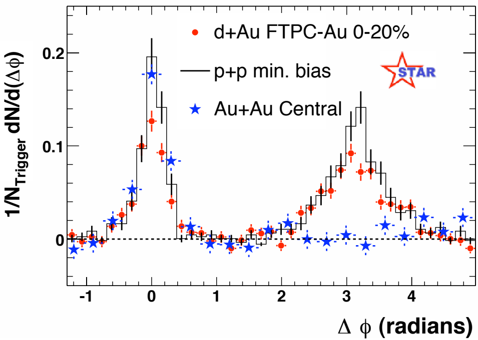


- Interestingly, the phenomenologically relevant values are $Y = 2 \div 3$ $\Longrightarrow$ enhancement $= 2 \div 3$ with both FC and RC

Jet quenching

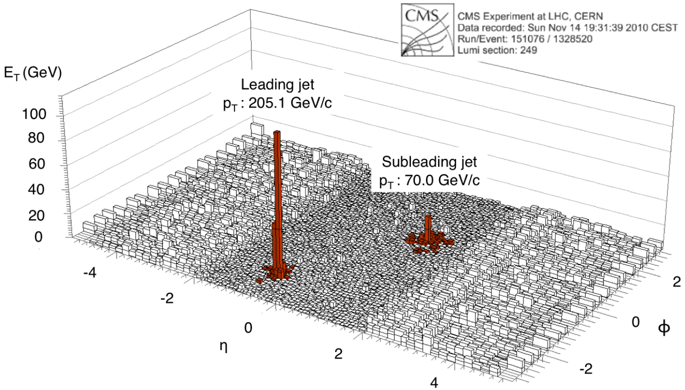
- Nuclear modification factor, di–hadron azimuthal correlations ...





- Energy loss & transverse momentum broadening by the leading particle

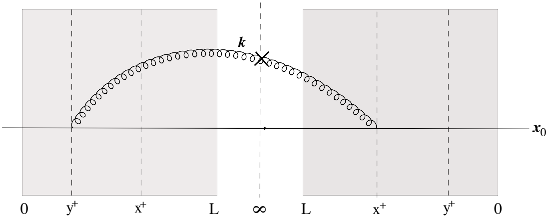
Di–jet asymmetry



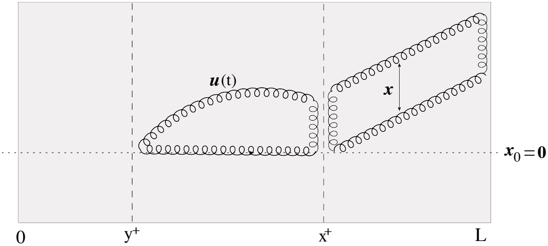
- Additional energy imbalance as compared to p+p : 20 to 30 GeV
- Compare to the typical scale in the medium: $T \sim 1$ GeV (average $p_\perp$)
- Detailed studies show that the 'missing energy' is carried by many soft ($p_\perp < 2$ GeV) hadrons propagating at large angles

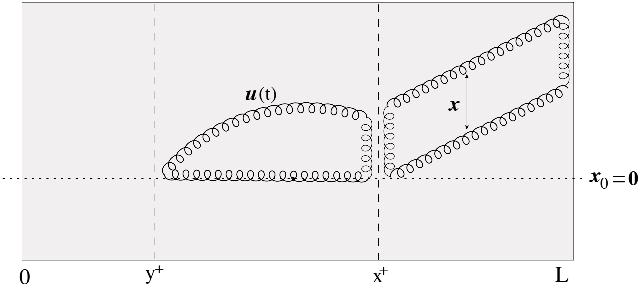
Radiative energy loss (1)

- Consider the radiation by a very energetic, eikonal, quark, for simplicity



- Once again, the cross–section can be related to (adjoint) dipoles:





$$k^+ \frac{\mathrm{d}N_g}{\mathrm{d}k^+\mathrm{d}^2\boldsymbol{k}} \propto \int_{x^+,y^+}\int_{\boldsymbol{x}} \mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}}\, \mathcal{S}^{\mathrm{adj}}_{L,x^+}(\boldsymbol{x})\, \partial^i_{\boldsymbol{x}}\partial^i_{\boldsymbol{y}}\mathcal{K}(x^+,\boldsymbol{x};y^+,\boldsymbol{y};k^+)\Big|_{\boldsymbol{y}=0}$$

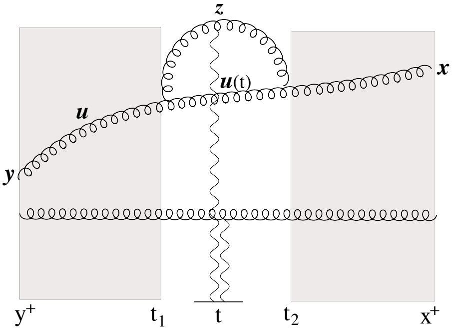
$$\mathcal{K}(x^+,\boldsymbol{x};y^+,\boldsymbol{y};k^+) = \int [\mathcal{D}\boldsymbol{u}]\, \mathrm{e}^{i\frac{k^+}{2}\int_{y^+}^{x^+}\mathrm{d}t\,\dot{\boldsymbol{u}}^2(t)}\, S^{\mathrm{adj}}_{x^+,y^+}([\boldsymbol{u}(t)],\boldsymbol{x}_0)$$

- The only difference w.r.t. $p_\perp$–broadening:

  the radiated gluon within the 1st dipole ($\mathcal{K}$) is not eikonal anymore

- However, the radiated gluon is relatively hard, $k^+ \sim \omega_c$, so the hierarchy is preserved between radiation and fluctuations: $\omega \ll k^+$

  $\triangleright$ during the relatively short lifetime $t_2 - t_1 = \tau$ of the fluctuation (ω), the radiated gluon (k^+) can be treated as eikonal



- Then the same arguments apply as in the case of $p_\perp$–broadening:
  $\hat{q}^{(0)} \rightarrow \hat{q}_{\tau_f}(k_\perp^2)$ ... *in agreement with J.-P. Blaizot and Y. Mehtar-Tani*