Excitations of finite temperature QCD: hadrons, partons and continuum

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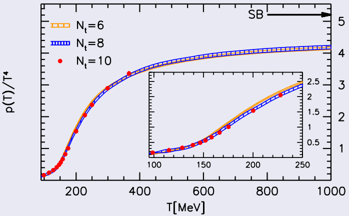
Goal: describe QCD equation of state

- "measurement": Monte Carlo simulations

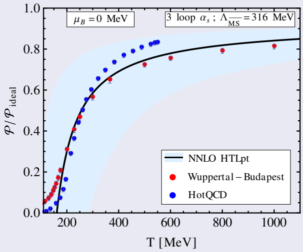
EoS from MC



(Sz. Borsanyi et al, JHEP 1011 (2010) 077)

Goal: describe QCD equation of state

HTL 3-loops



(N. Haque, *et.al.*, e-Print: arXiv:1402.6907)

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- at high T: QGP with 8 gluon + 3 quark; valid for $T \gtrsim 250 - 300\,\mathrm{MeV}$.

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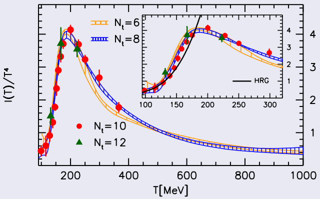
HRG thermodynamics



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  free hadron resonance gas (HRG) with real masses $T \lesssim 150 - 180\,\mathrm{MeV}$.

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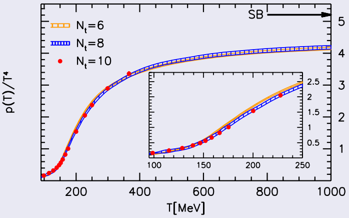

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- at high T: QGP with 8 gluon + 3 quark; valid for $T \gtrsim 250 - 300\,\mathrm{MeV}$.
- at low T: hadrons
  free hadron resonance gas (HRG) with real masses $T \lesssim 150 - 180\,\mathrm{MeV}$.
- between: continuous crossover
  "T_c" $= 156$ MeV
  is it a nonperturbative regime?
  hadronic matter also seems to be nonperturbative from the point of view of QGP

Key for understanding the crossover regime

what happens with the QCD excitations above T_c?

- no change of ground state (1st or 2nd order phase transition)
  $\Rightarrow$ hadrons do not disappear at once

  (J. Liao, E.V. Shuryak PRD73 (2006) 014509 [hep-ph/0510110])

- MC: hadronic states are observable even at $T \sim 1.2\text{-}1.5\, T_c$!

  (AJ., P. Petreczky, K. Petrov, A. Velytsky, PRD75 (2007) 014506)

- MC measurements: no uncorrelated quasiparticles at $T \in [150, 250]$ MeV: no usual HRG, QGP

  (P. Petreczky, J. Phys. Conf. Ser. **402**, 012036 (2012) [arXiv:1204.4414 [hep-lat]])
  (R. Bellwied, S. Borsanyi, Z. Fodor, S. D. Katz and C. Ratti, [arXiv:1305.6297 [hep-lat]])

Approach the question from the point of view of thermodynamics.

Gas of bound states

How do the excitations contribute to QCD pressure?

1st approach

bound states $\equiv$ energy & momentum eigenstates

Partition function and partial pressure:

$$Z = \sum_n g_n e^{-\beta E_n} \quad \longrightarrow \quad P = \sum_\ell \mp T \int \frac{d^3p}{(2\pi)^3} \ln\left(1 \mp e^{-\beta E_p^{(\ell)}}\right)$$

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Successful

- with the ground states describes chemistry
- with hadron spectrum: QCD at low T
- mathematical background: Beth-Uhlenbeck formula

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Successful

- with the g
- with hadr
- mathemat
  formula

Needs improvement!

- H-atom Balmer-spectrum (Coulomb problem):
  $E_n = \frac{E_0}{n^2}, \; g_n = n^2 \quad \Rightarrow \quad Z$ is not convergent!
- QCD hadrons (Hagedorn-spectrum): $\varrho_H \sim e^{-m/T_H}$
  $\Rightarrow \quad Z$ is not convergent for any $T > T_H$!

Gas of bound states

2nd approach

particle (bound) states $\equiv$ peaks in the spectrum

- Quasiparticles? finite lifetime $\Rightarrow$ $\hat{H} \to \hat{H} - i\gamma$ $\Rightarrow$ loss of unitarity!
- 2PI-approach: treat the complete spectrum!

  (Ward, Luttinger, Phys.Rev. 118 (1960) 1417; J. Berges and J. Cox, Phys. Lett. B 517 (2001) 369)

  physics: quasiparticle $\nexists$ independently of environment.
- From where should we take the spectrum?
  - perturbatively: use self-consistent equations (e.g. 2PI or SD equations)
  - nonperturbatively: use experimental information to reproduce spectral functions (e.g. hadron masses, widths)

    $\Rightarrow$ we will follow this way

Lagrangian formalism for general spectral functions

(AJ and T.S. Biro arXiv:1405.5471, AJ. Phys.Rev. D86 (2012) 085007; Phys.Rev. D88 (2013) 065012)

Strategy:

- ϱ spectral function input
- construct a free model representing this ϱ
  tree level: no interactions, correlation through spectrum
- e.g. for scalar field

$$\mathcal{L} = \frac{1}{2}\Phi^*(p)\mathcal{K}(p)\Phi(p) \quad \Rightarrow \quad G_r = \mathcal{K}^{-1}\Big|_{p_0+i\varepsilon} \quad \Rightarrow \quad \varrho = \mathrm{Disc}\, iG_r$$

- defines a consistent field theory: (just like in 2PI case)
  unitary, causal, Lorentz-invariant, $E, \mathbf{p}$ conserving

Thermodynamics

Technically:

→ energy-momentum tensor from Noether currents

→ energy density $\varepsilon = \frac{1}{Z} \operatorname{Tr} e^{-\beta \hat{H}} \hat{T}_{00}$

→ averaging with KMS relations

→ free energy, pressure from thermodynamical relations

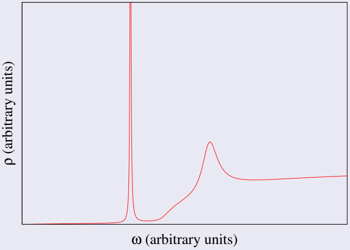
Result

$$P = \mp T \int \frac{d^4 p}{(2\pi)^4} \frac{E(p)}{p_0} \ln\left(1 \mp e^{-\beta p_0}\right) \varrho(p), \qquad E(p) = p_0 \frac{\partial \mathcal{K}}{\partial p_0} - \mathcal{K}$$

- get back free gas pressure for Dirac-delta spectrum
- generally: ϱ dependence: directly and through $E(p)$!
  ⇒ nonlinear
- e.g. P does not depend on the normalization of ϱ.

Qualitative analysis

A typical spectral function



$\varrho(p_0, p, T, \dots)$

- quasiparticle (qp) peak(s)
- continuum of multiparticle states
- qp width $\sim$ continuum height

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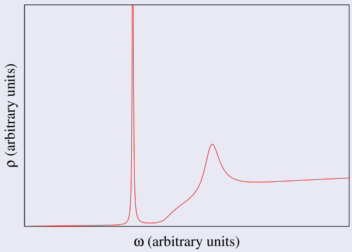

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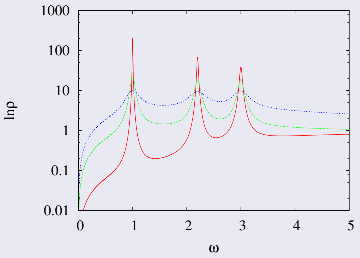
What can change with T?

- qp position (thermal mass)
- continuum height $\Rightarrow$ qp width and qp wave function renormalization
- continuum shape. . . will not be discussed

Illustrative example

We can examine different realistic spectra:

Three-peak spectra



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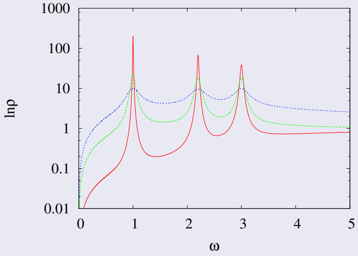
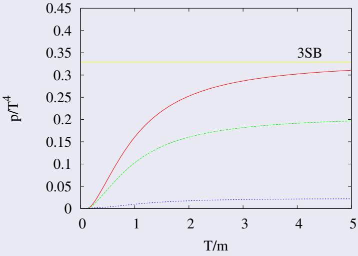


Three-peak pressure



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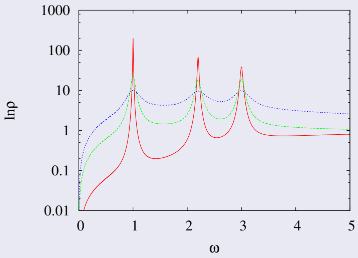

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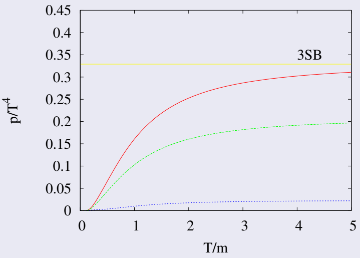
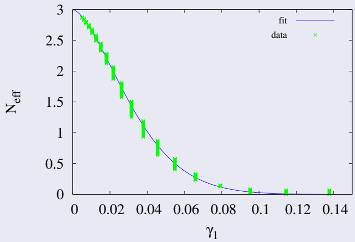


- Characterize the spectrum with 1st peak width γ
- pressure vanishes for $\gamma \to \infty$!
- In lot of cases P factorizes: $P(T, \gamma) = N_{eff}(\gamma) P_0(T)$.

Effective number of degrees of freedom

Robust result: N_{eff} vs. qp width γ

Effective # of dof



- vertical extent: temperature variation
- pressure vanishes $N_{eff}(\gamma) \xrightarrow{\gamma \to 0} 0$
- fit function stretched exponential: $N_{eff}(\gamma) = e^{-a\gamma^b}$ typically $b \sim 1.5 - 2$.

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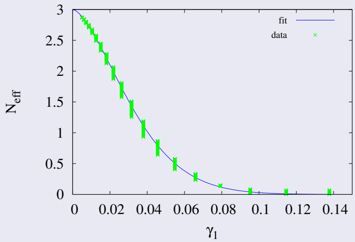

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Physically: **Melting of bound states!**

Application to QCD

An oversimplified realization of these ideas for QCD

- free particle excitations: hadrons and partons (quark and gluon quasiparticles) $\Rightarrow$ free pressure P_0
- hadrons: Hagedorn-spectrum up to a certain mass ($m \lesssim 3\,\mathrm{GeV}$)
- assume common suppression factor for all hadrons: $N_{hadr}(\gamma)$ in a form of stretched exponential, while $\gamma \sim T$
- partonic suppression factor depends on the available hadronic resonances: $N_{part}(N_{hadr})$

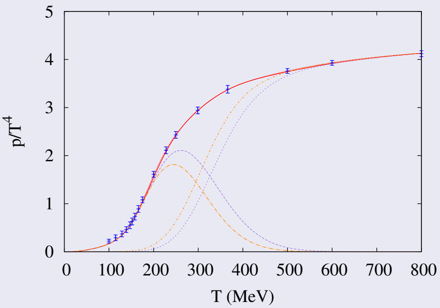
Total pressure $P = P_{hadr} + P_{QGP}$

$$P_{hadr}(T) = N_{eff}^{(hadr)} \sum_{n \in \mathrm{hadrons}}^{N} P_0(T, m_n), \qquad \ln N_{eff}^{(hadr)} = -(T/T_0)^b,$$

$$P_{QGP}(T) = N_{eff}^{(part)} \sum_{n \in \mathrm{partons}} P_0(T, m_n), \qquad \ln N_{eff}^{(part)} = G_0 - c(N_{eff}^{(hadr)})^d.$$

Matter content of QCD

QCD EoS matter content



- total pressure is well reproduced
- hadrons do not vanish at T_c: they just start to melt there.
- hadrons dominate the pressure until $\sim 2T_c$
- pure QGP only for $T \gtrsim 3T_c$
- different fits yield similar results

Conclusion

QCD phase transition at the physical point may be governed by

hadron melting

Gribov; chemistry with excited states

- experimental hint: observable hadronic states at $T > T_c$
- QCD: hadrons start to melt at $T \sim T_c$, dominate pressure for $T \lesssim 2T_c$ and vanish at $T \sim 3T_c$
- interpretation of (de)confinement, chiral dynamics: hadronic/partonic qp states cease to exist
- mechanism: hadrons, partons are not independent of their continuum: melting $\equiv$ qp peak merges with the continuum
- not particle-like excitations! $\Rightarrow$ transport, correlations...
- perturbative field theory? besides QCD dof we need all hadrons!