Universität Bielefeld

Continuum Results of the Heavy Quark Momentum Diffusion Coefficient κ

Olaf Kaczmarek

University of Bielefeld

in collaboration with

A.Francis, M. Laine, M.Müller, T.Neuhaus, H.Ohno

[arXiv:1311.3759 and arXiv:1109.3941]

Strong and Electroweak Matter 2014

Lausanne

17.07.2014

Transport Coefficients are important ingredients into **hydro/transport models for the evolution of the system.**

Usually determined by matching to experiment (see right plot)

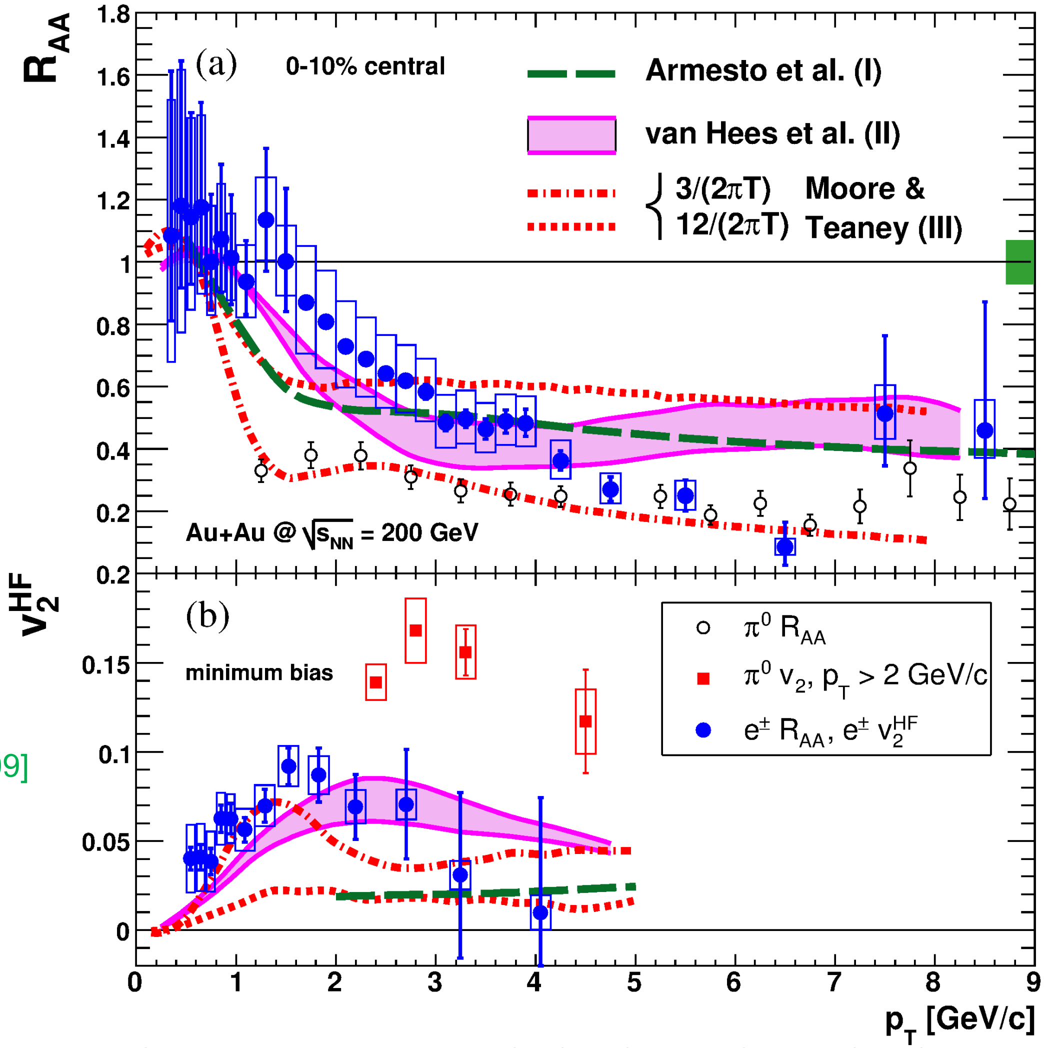
Need to be determined from QCD using first principle lattice calculations!

here heavy flavour:

- Heavy Quark Diffusion Constant D
  [H.T.Ding, OK et al., PRD86(2012)014509]
- Heavy Quark Momentum Diffusion κ

or for light quarks:

- Light quark flavour diffusion
- Electrical conductivity
  [A.Francis, OK et al., PRD83(2011)034504]



[PHENIX Collaboration, Adare et al.,PRC84(2011)044905 & PRL98(2007)172301]

Transport coefficients from Lattice QCD – Flavour Diffusion

Transport coefficients usually calculated using correlation function of conserved currents

$$G(\tau,\mathbf{p},T) = \int_0^\infty \frac{d\omega}{2\pi}\rho(\omega,\mathbf{p},T)K(\tau,\omega,T)$$

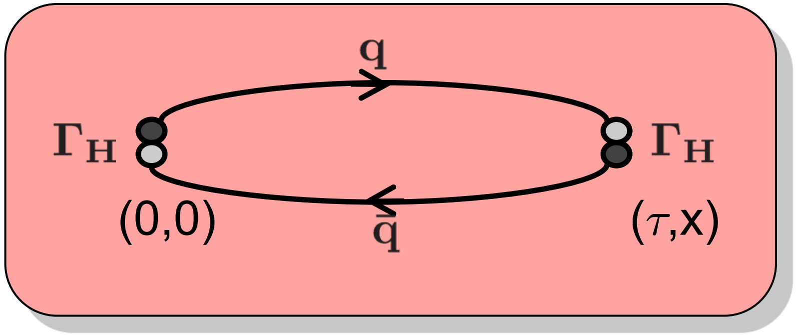
$$K(\tau,\omega,T) = \frac{\cosh\left(\omega(\tau-\frac{1}{2T})\right)}{\sinh\left(\frac{\omega}{2T}\right)}$$

Lattice observables:

$$G_{\mu\nu}(\tau,\vec{x}) = \langle J_\mu(\tau,\vec{x})J_\nu^\dagger(0,\vec{0})\rangle$$

$$J_\mu(\tau,\vec{x}) = 2\kappa Z_V\bar{\psi}(\tau,\vec{x})\Gamma_\mu\psi(\tau,\vec{x})$$

$$G_{\mu\nu}(\tau,\vec{p}) = \sum_{\vec{x}} G_{\mu\nu}(\tau,\vec{x})e^{i\vec{p}\vec{x}}$$



related to a conserved current

only correlation functions calculable on lattice but

Transport coefficient determined by slope of spectral function at ω=0 (Kubo formula)

$$D = \frac{\pi}{3\chi_{00}}\lim_{\omega\to 0}\frac{\rho_{ii}(\omega,\vec{p}=0,T)}{\omega T}$$

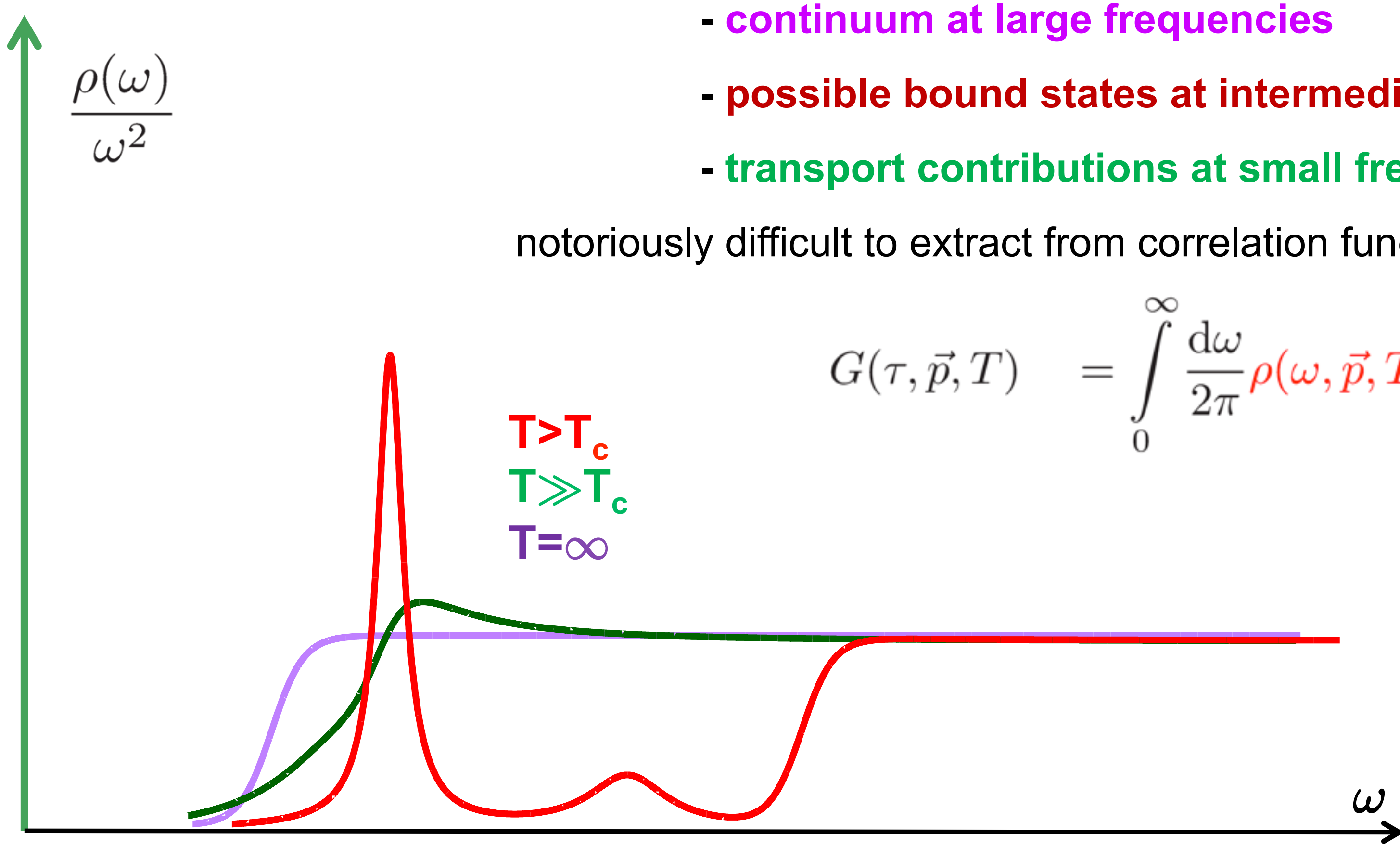
Quarkonium spectral function – hard to separate different scales

Different contributions and scales enter in the spectral function

- **continuum at large frequencies**
- **possible bound states at intermediate frequencies**
- **transport contributions at small frequencies**

notoriously difficult to extract from correlation functions

$$G(\tau,\vec{p},T) = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi} \rho(\omega,\vec{p},T) K(\tau,\omega,T)$$



+ zero-mode contribution at ω=0: $\rho(\omega) = 2\pi\chi_{00}\, \omega\delta(\omega)$

+ (narrow) transport peak at small ω: $\rho(\omega \ll T) = 2\chi_{00} \frac{T}{M} \frac{\omega\eta}{\omega^2+\eta^2}\,, \qquad \eta = \frac{T}{MD}$

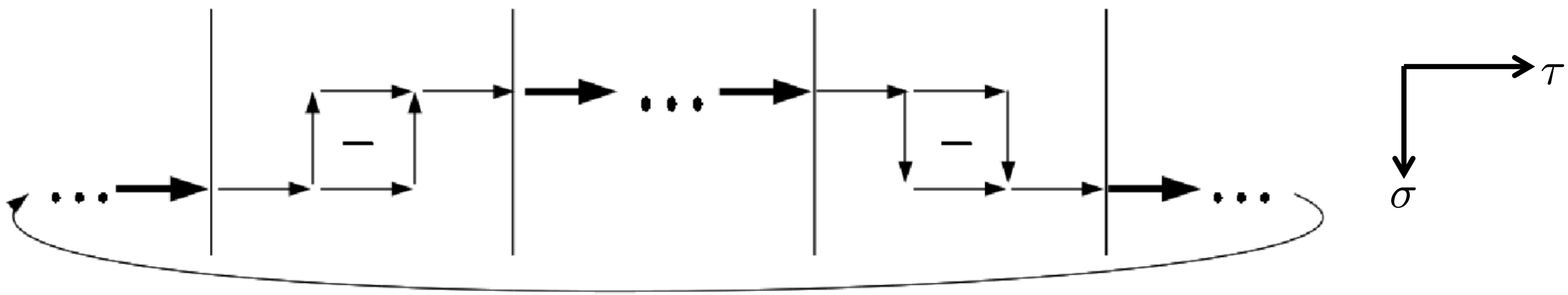
Heavy Quark Momentum Diffusion Constant – Single Quark in the Medium

Heavy Quark Effective Theory (HQET) in the large quark mass limit

for a single quark in medium

leads to a (pure gluonic) "color-electric correlator"

[J.Casalderrey-Solana, D.Teaney, PRD74(2006)085012,
S.Caron-Huot,M.Laine,G.D. Moore,JHEP04(2009)053]



$$G_E(\tau) \equiv -\frac{1}{3}\sum_{i=1}^{3} \frac{\left\langle \mathrm{Re\,Tr}\left[U(\tfrac{1}{T};\tau)\, gE_i(\tau,\mathbf{0})\, U(\tau;0)\, gE_i(0,\mathbf{0})\right]\right\rangle}{\left\langle \mathrm{Re\,Tr}[U(\tfrac{1}{T};0)]\right\rangle}$$

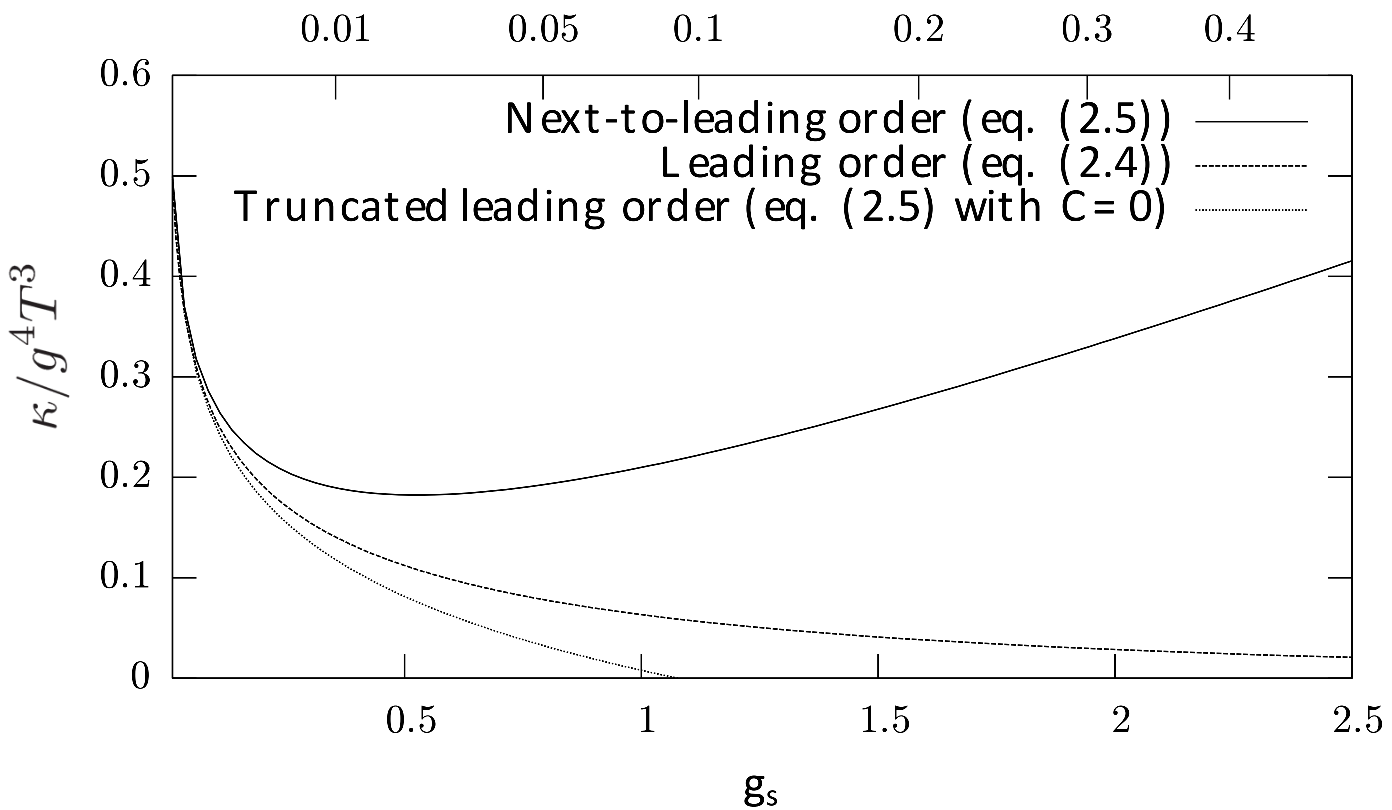
Heavy quark (momentum) diffusion:

$$\kappa = \lim_{\omega\to 0} \frac{2T\rho_E(\omega)}{\omega} \qquad D = \frac{2T^2}{\kappa}$$

Heavy Quark Momentum Diffusion Constant – Perturbation Theory

can be related to the thermalization rate: $$\eta_D = \frac{\kappa}{2M_{kin}T}\left(1 + O\left(\frac{\alpha_s^{3/2}T}{M_{kin}}\right)\right)$$

NLO in perturbation theory: [Caron-Huot, G.Moore, JHEP 0802 (2008) 081]

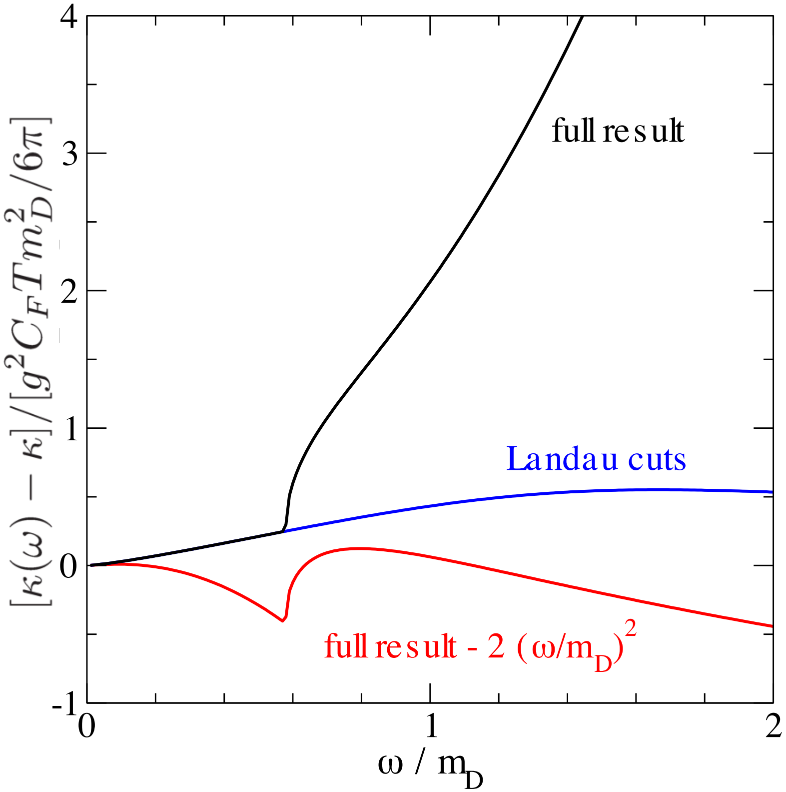


very poor convergence

→ **Lattice QCD study required in the relevant temperature region**

Heavy Quark Momentum Diffusion Constant – Perturbation Theory

NLO spectral function in perturbation theory: [Caron-Huot, M.Laine, G.Moore, JHEP 0904 (2009) 053]



in contrast to a narrow transport peak, from this a smooth limit

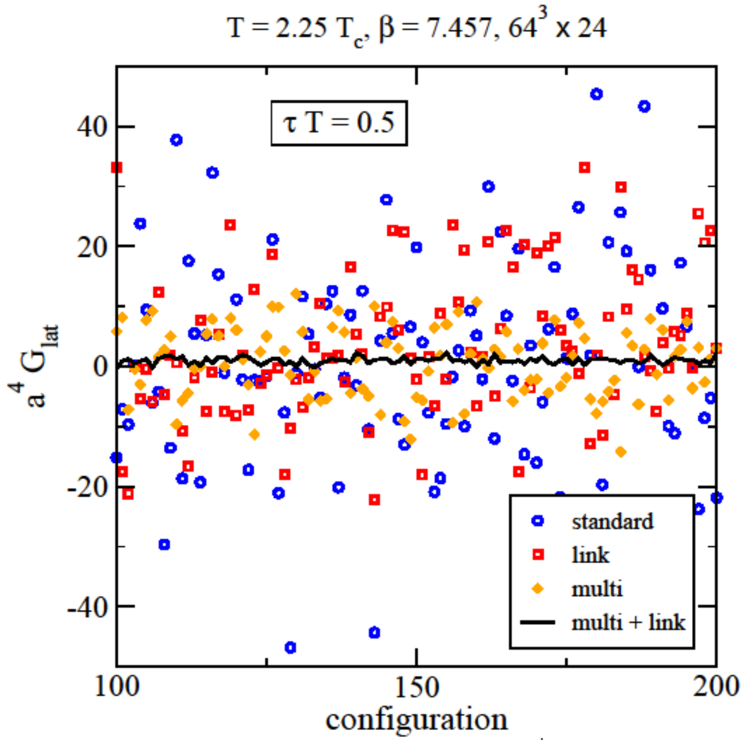
$$\kappa/T^3 = \lim_{\omega\to 0} \frac{2T\rho_{\mathrm{E}}(\omega)}{\omega}$$

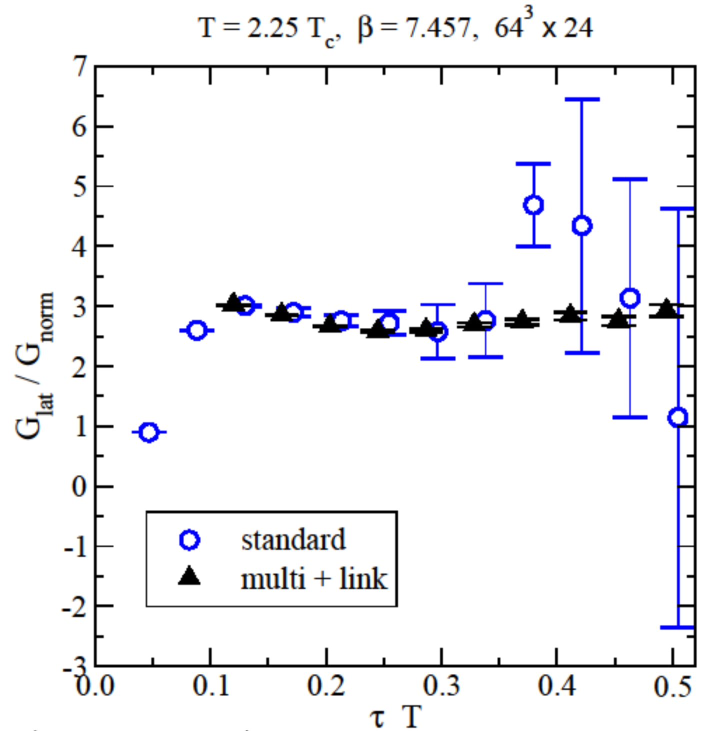
is expected

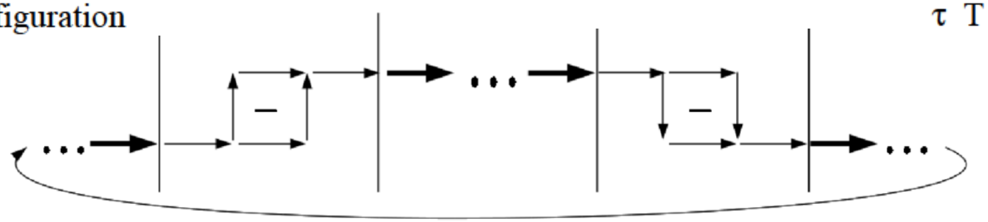
Qualitatively similar behaviour also found in AdS/CFT [S.Gubser, Nucl.Phys.B790 (2008)175]

Heavy Quark Momentum Diffusion Constant – Lattice algorithms

[A.Francis,OK,M.Laine,J.Langelage, arXiv:1109.3941 and arXiv:1311.3759]







due to the gluonic nature of the operator, signal is extremely noisy

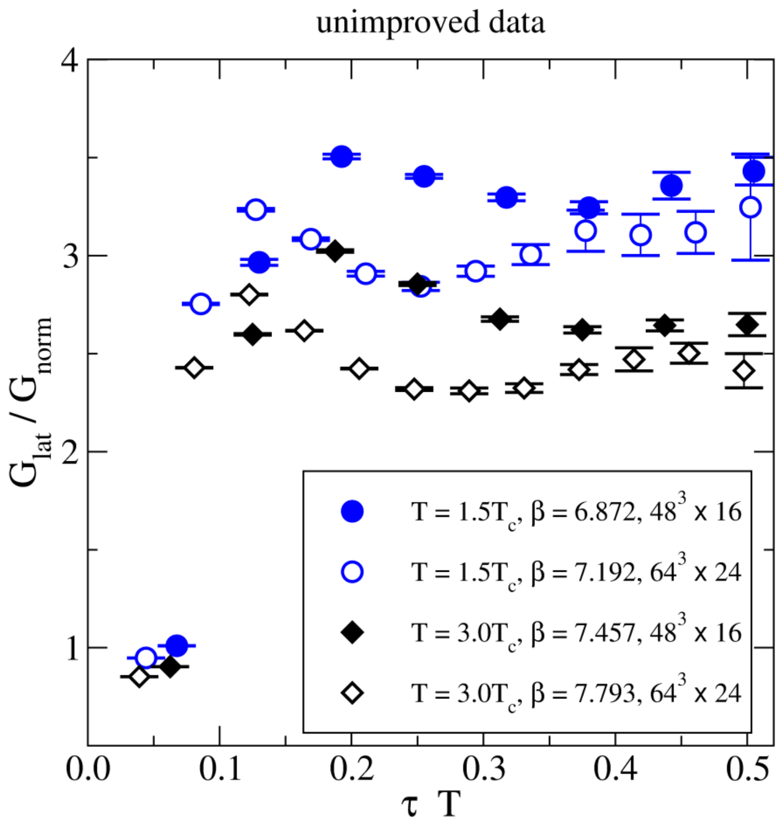
→ **multilevel combined with link-integration techniques to improve the signal**

[Lüscher,Weisz JHEP 0109 (2001)010 and H.B.Meyer PRD (2007) 101701]

[Parisi,Petronzio,Rapuano PLB 128 (1983) 418, and de Forcrand PLB 151 (1985) 77]

Heavy Quark Momentum Diffusion Constant – Tree-Level Improvement

[A.Francis,OK,M.Laine,J.Langelage, arXiv:1109.3941 and arXiv:1311.3759]



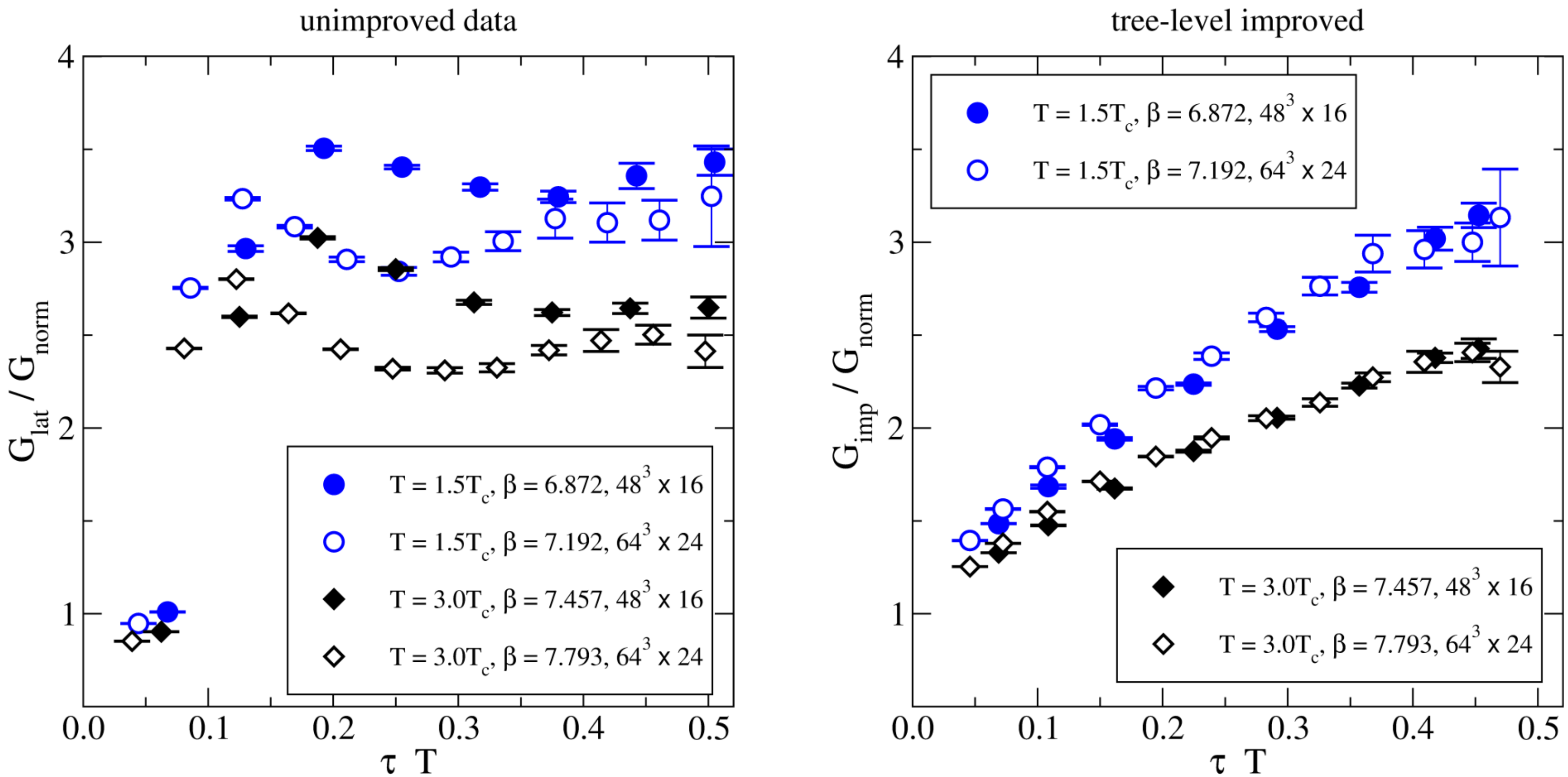
normalized by the LO-perturbative correlation function:

$$G_{\rm norm}(\tau T) \equiv \frac{G^{\rm LO}_{\rm cont}(\tau T)}{g^2 C_F} = \pi^2 T^4 \left[\frac{\cos^2(\pi\tau T)}{\sin^4(\pi\tau T)} + \frac{1}{3\sin^2(\pi\tau T)}\right] \qquad C_F \equiv \frac{N_c^2-1}{2N_c}$$

and renormalized using NLO renormalization constants $Z(g^2)$

Heavy Quark Momentum Diffusion Constant – Tree-Level Improvement

[A.Francis,OK,M.Laine,J.Langelage, arXiv:1109.3941 and arXiv:1311.3759]



lattice cut-off effects visible at small separations (left figure)

→ **tree-level improvement** (right figure) to reduce discretization effects

$$G^{\rm LO}_{\rm cont}(\overline{\tau T}) = G^{\rm LO}_{\rm lat}(\tau T)$$

leads to an effective reduction of cut-off effect for all τT

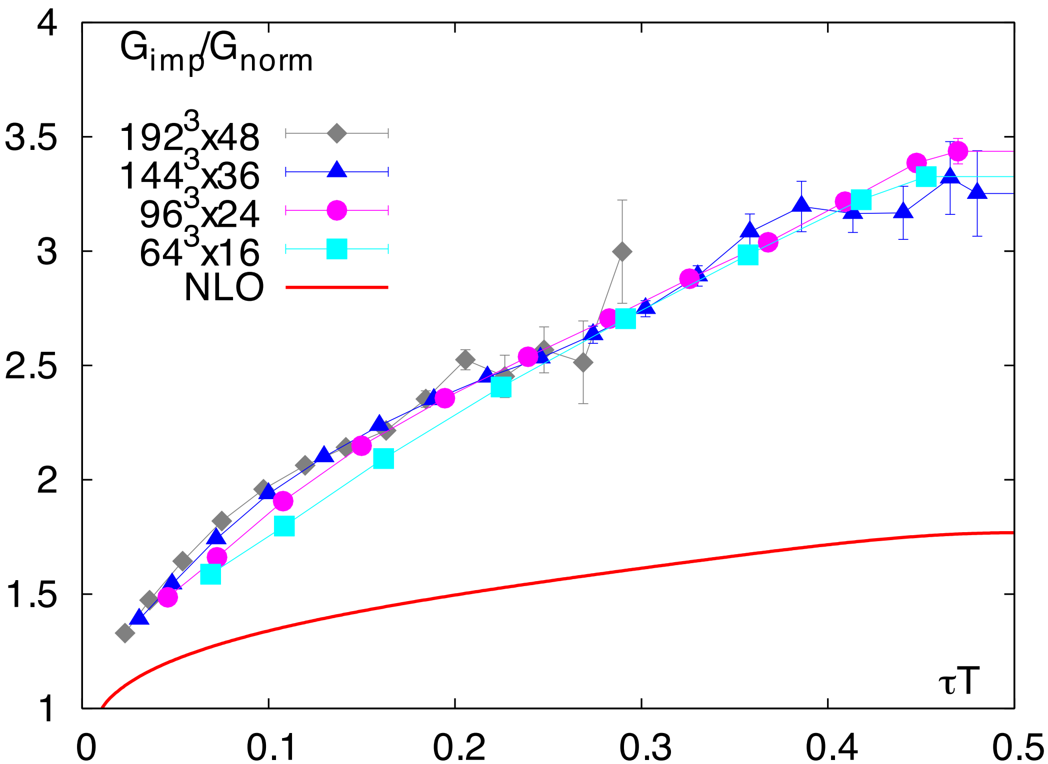
Heavy Quark Momentum Diffusion Constant – Lattice results

Quenched Lattice QCD on large and fine isotropic lattices at $T \simeq 1.4\ T_c$

- standard Wilson gauge action
- algorithmic improvements to enhance signal/noise ratio
- fixed aspect ration $N_s/N_t = 4$, i.e. fixed physical volume $(2\text{fm})^3$
- perform the continuum limit, $a \to 0 \;\leftrightarrow\; N_t \to \infty$
- determine κ in the continuum using an Ansatz for the spectral fct. $\rho(\omega)$

N_σ	N_τ	β	$1/a$[GeV]	a[fm]	#Confs
64	16	6.872	7.16	0.03	100
96	24	7.192	10.4	0.019	160
144	36	7.544	15.5	0.013	362
192	48	7.793	20.4	0.010	223

Heavy Quark Momentum Diffusion Constant – Lattice results



finest lattices still quite noisy at large τT

but only

small cut-off effects at intermediate τT

cut-off effects become visible at small τT

need to extrapolate to the continuum

perturbative behavior in the limit τT $\to$ 0

N_σ	N_τ	β	$1/a$[GeV]	a[fm]	#Confs
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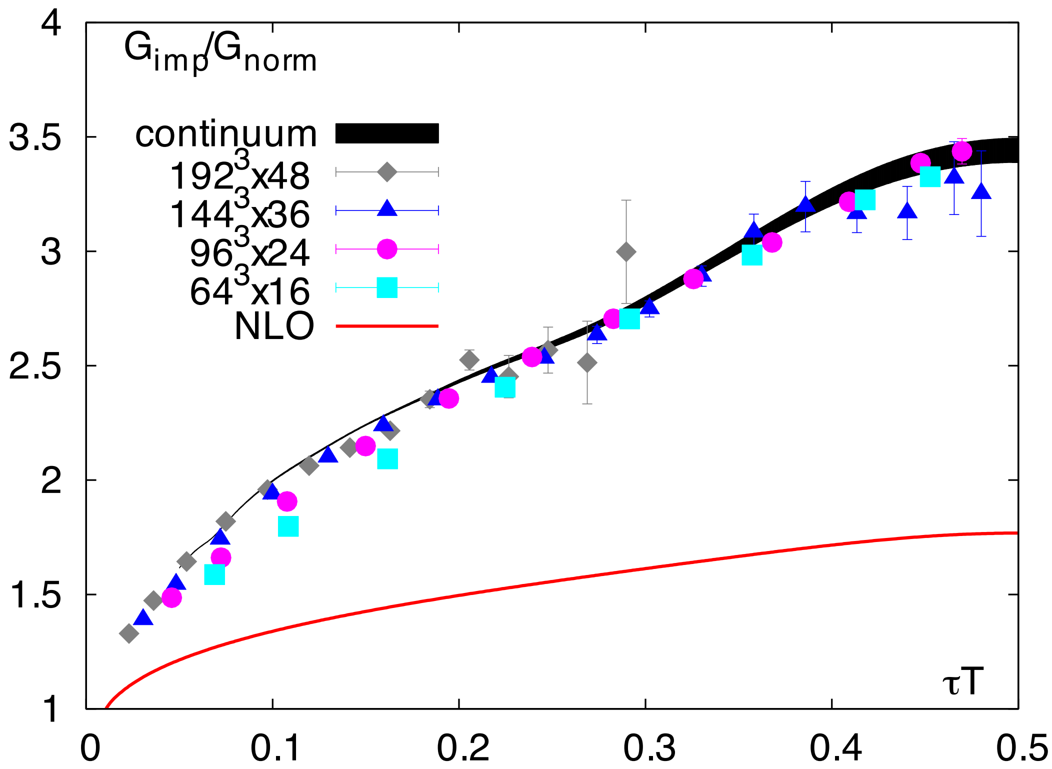
allows to perform continuum extrapolation, a$\to$ 0 $\leftrightarrow$ $N_t \to \infty$ **, at fixed** T=1/a N_t

Heavy Quark Momentum Diffusion Constant – Continuum extrapolation



finest lattices still quite noisy at large τT

but only

small cut-off effects at intermediate τT

cut-off effects become visible at small τT

need to extrapolate to the continuum

perturbative behavior in the limit $\tau T \to 0$

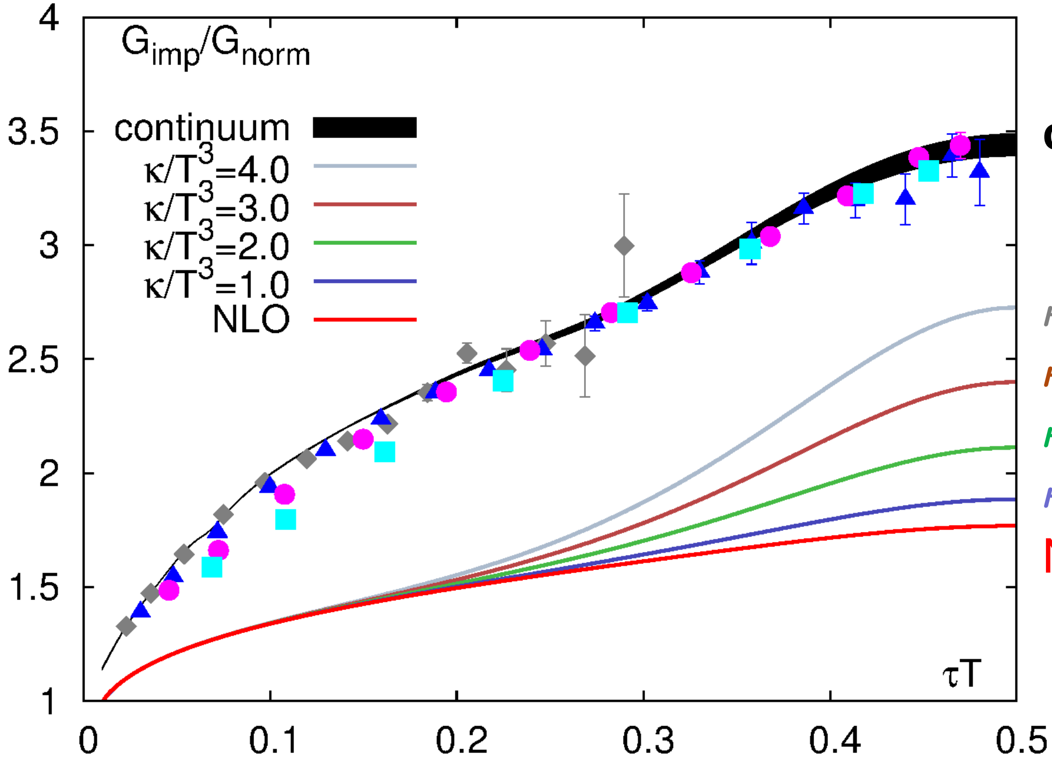
well behaved continuum extrapolation for $0.05 \leq \tau T \leq 0.5$

finest lattice already close to the continuum

coarser lattices at larger τT close to the continuum

how to extract the spectral function from the correlator?

Heavy Quark Momentum Diffusion Constant – Model Spectral Function



continuum extrapolation

$\kappa/T^3 = 4.0$

$\kappa/T^3 = 3.0$

$\kappa/T^3 = 2.0$

$\kappa/T^3 = 1.0$

NLO perturbation theory

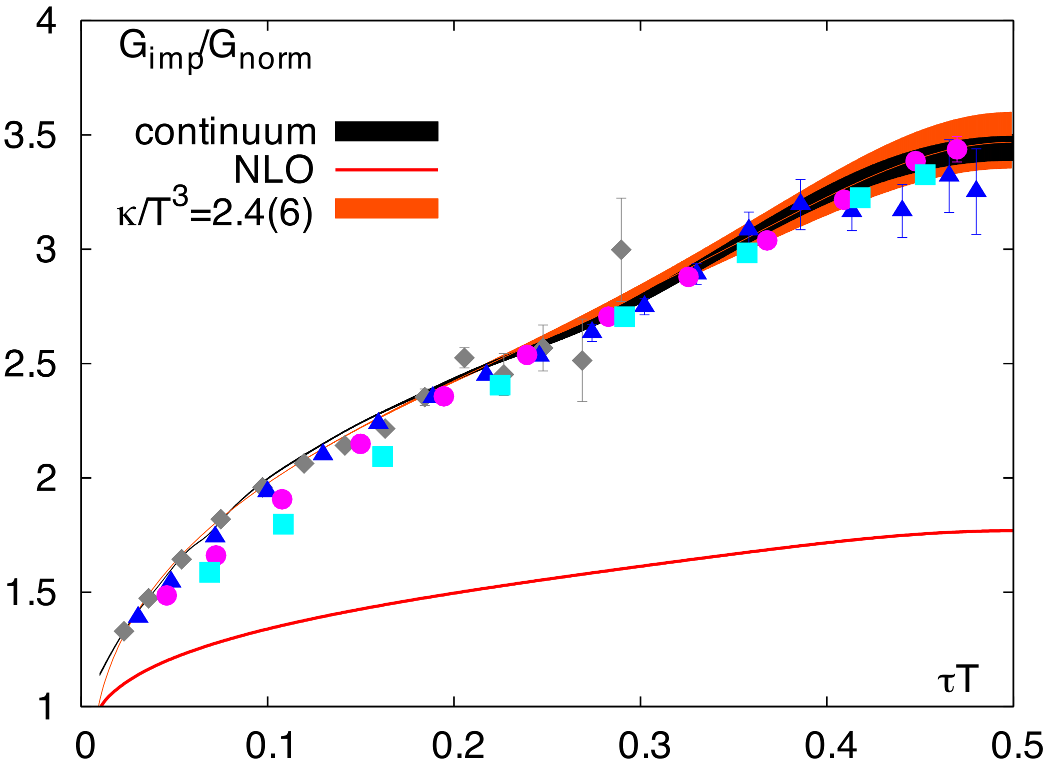
Model spectral function: transport contribution + NLO [Y.Burnier et al. JHEP 1008 (2010) 094)]

$$\rho_{\text{model}}(\omega) \equiv \max\left\{\rho_{\text{NLO}}(\omega), \frac{\omega\kappa}{2T}\right\}$$

$$G_{\text{model}}(\tau) \equiv \int_0^\infty \frac{d\omega}{\pi} \rho_{\text{model}}(\omega) \frac{\cosh\left(\frac{1}{2} - \tau T\right)\frac{\omega}{T}}{\sinh\frac{\omega}{2T}}$$

some contribution at intermediate distance/frequency seems to be missing

Heavy Quark Momentum Diffusion Constant – Model Spectral Function



result of the fit to $\rho_{model}(\omega)$

with three parameters: κ,A,B

Model spectral function: transport contribution + NLO + correction

$$\rho_{\rm model}(\omega) \equiv \max\left\{ A\rho_{\rm NLO}(\omega) + B\omega^3, \frac{\omega\kappa}{2T} \right\}$$

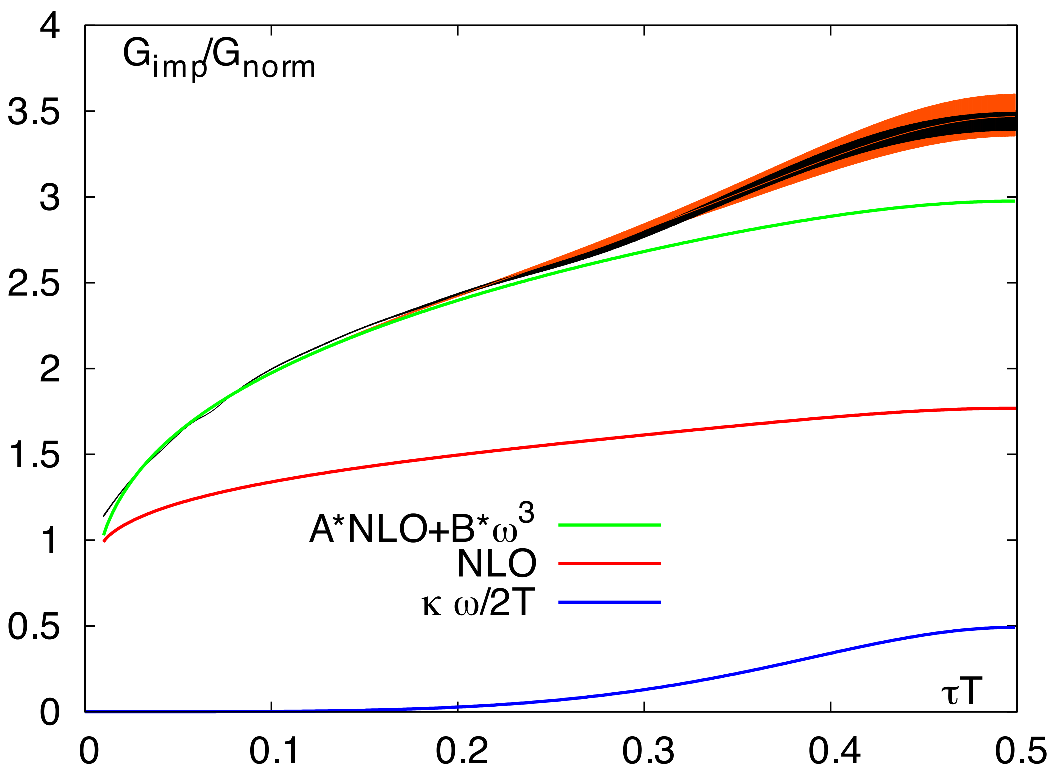
$$G_{\rm model}(\tau) \equiv \int_0^\infty \frac{{\rm d}\omega}{\pi} \rho_{\rm model}(\omega) \frac{\cosh\left(\frac{1}{2} - \tau T\right)\frac{\omega}{T}}{\sinh\frac{\omega}{2T}}$$

used to fit the continuum extrapolated data

→ **first continuum estimate of κ : (still preliminary)**

$$\kappa/T^3 = \lim_{\omega\to 0} \frac{2T\rho_{\rm E}(\omega)}{\omega} \simeq 2.4(6)$$

Heavy Quark Momentum Diffusion Constant – Model Spectral Function



result of the fit to $\rho_{model}(\omega)$

A $\rho_{NLO}(\omega)$ + B w^3

NLO perturbation theory

$\frac{\omega\kappa}{2T}$ small but relevant contribution at τT > 0.2 !

Model spectral function: transport contribution + NLO + correction

$$\rho_{\rm model}(\omega) \equiv \max\left\{A\rho_{\rm NLO}(\omega) + B\omega^3, \frac{\omega\kappa}{2T}\right\} \qquad G_{\rm model}(\tau) \equiv \int_0^\infty \frac{{\rm d}\omega}{\pi}\rho_{\rm model}(\omega)\frac{\cosh\left(\frac{1}{2}-\tau T\right)\frac{\omega}{T}}{\sinh\frac{\omega}{2T}}$$

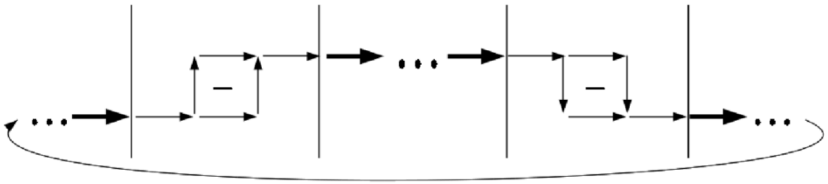
used to fit the continuum extrapolated data

→ **first continuum estimate of κ : (still preliminary)**

$$\kappa/T^3 = \lim_{\omega\to 0}\frac{2T\rho_{\rm E}(\omega)}{\omega} \simeq 2.4(6)$$

Conclusions and Outlook

$$G_E(\tau) \equiv -\frac{1}{3}\sum_{i=1}^{3} \frac{\left\langle \mathrm{Re\,Tr}\left[U(\tfrac{1}{T};\tau)\, gE_i(\tau,\mathbf{0})\, U(\tau;0)\, gE_i(0,\mathbf{0})\right]\right\rangle}{\left\langle \mathrm{Re\,Tr}[U(\tfrac{1}{T};0)]\right\rangle}$$



Continuum extrapolation for the color electric correlation function

extracted from Quenched Lattice QCD

- using noise reduction techniques to improve signal
- and an Ansatz for the spectral function

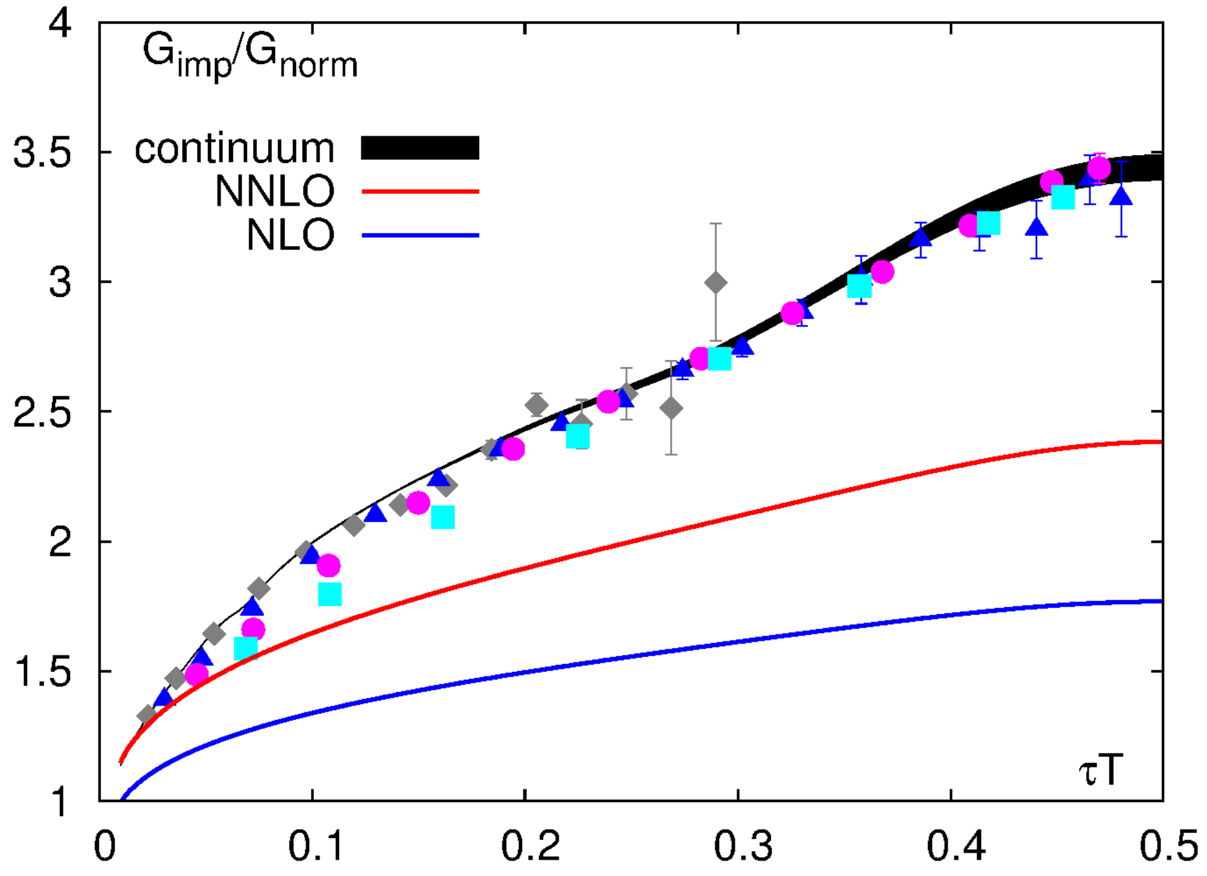
→ first continuum estimate for the Heavy Quark Momentum Diffusion Coefficient κ

More detailed analysis of the systematic uncertainties needed

- Different Ansätze for the spectral function
- Other techniques to extract the spectral function

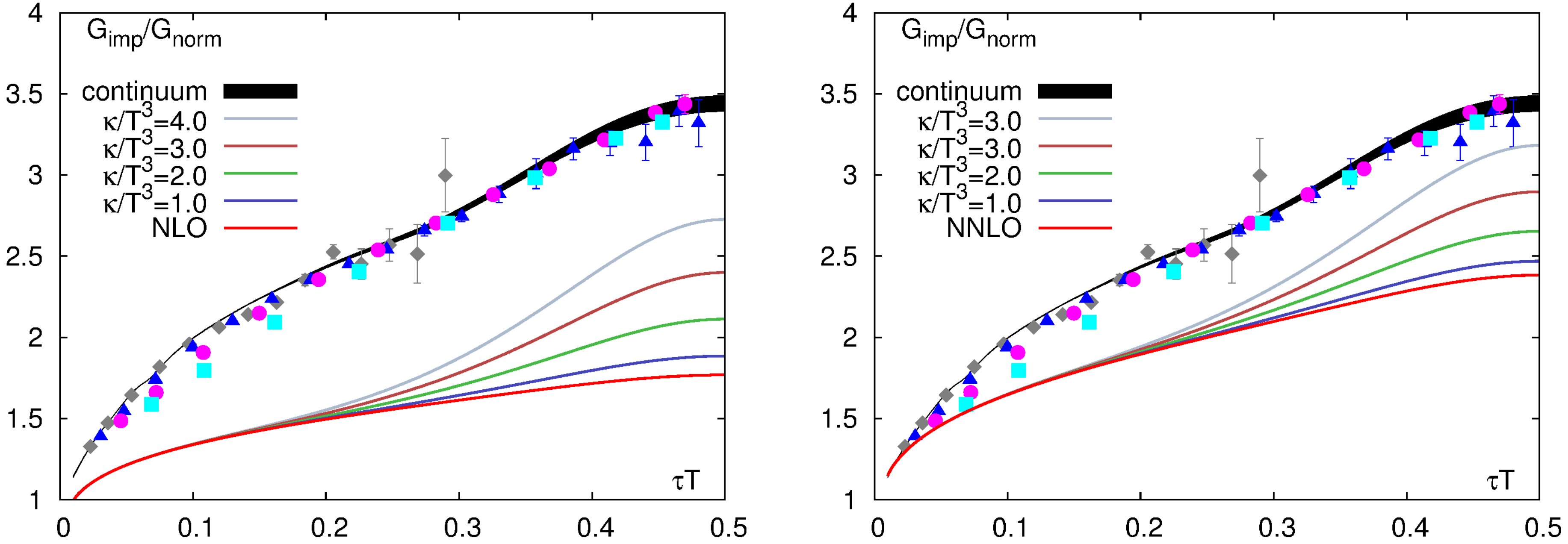
Other Transport coefficients from Effective Field Theories?

Heavy Quark Momentum Diffusion – NLO vs. NNLO



NNLO gives more contribution at small and large distances

Heavy Quark Momentum Diffusion – NLO vs. NNLO



$$\rho_{\rm model}(\omega) \equiv \max\left\{\rho_{\rm NLO}(\omega), \frac{\omega\kappa}{2T}\right\}$$

$$\rho_{\rm model}(\omega) \equiv \max\left\{\rho_{\rm NNLO}(\omega), \frac{\omega\kappa}{2T}\right\}$$

$$G_{\rm model}(\tau) \equiv \int_0^\infty \frac{{\rm d}\omega}{\pi}\rho_{\rm model}(\omega)\frac{\cosh\left(\frac{1}{2}-\tau T\right)\frac{\omega}{T}}{\sinh\frac{\omega}{2T}}$$

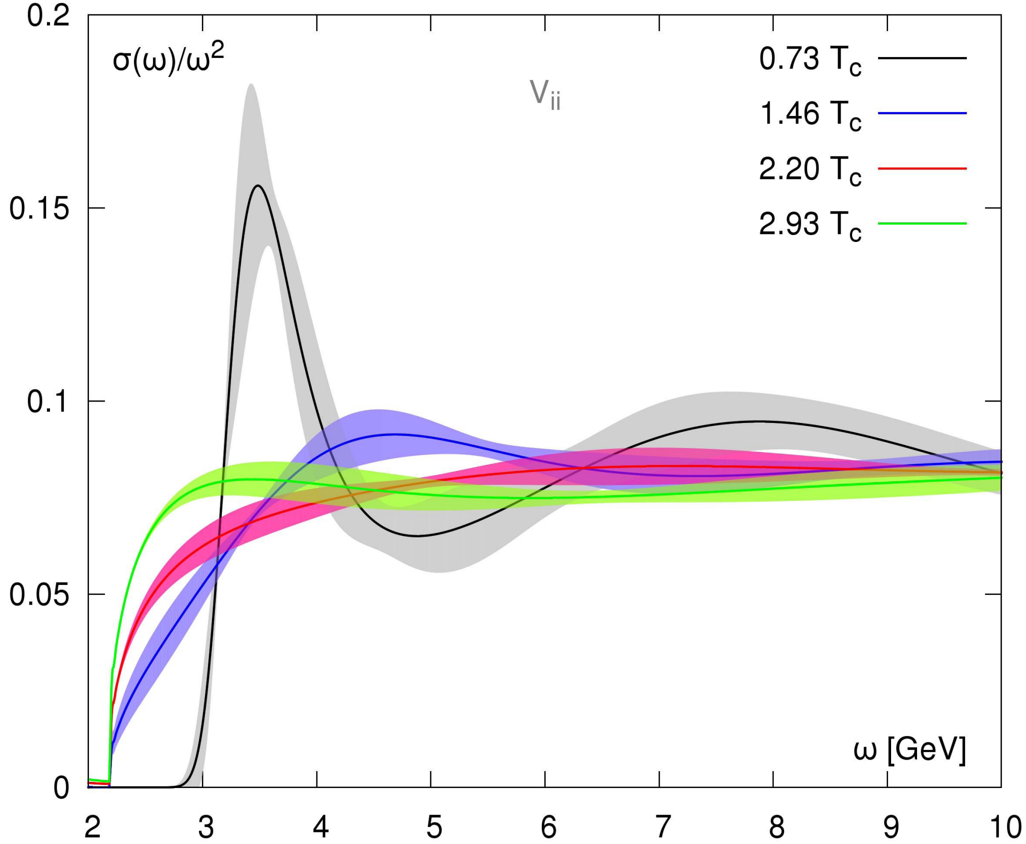
NNLO gives more contribution at small and large distances, but some contribution at intermediate distance/frequency still missing

→ improve the model spf or use more clever techniques to extract spf

Charmonium Spectral function

[H.T.Ding, OK et al., PRD86(2012)014509]

from Maximum Entropy Method analysis on a fine but finite lattice:



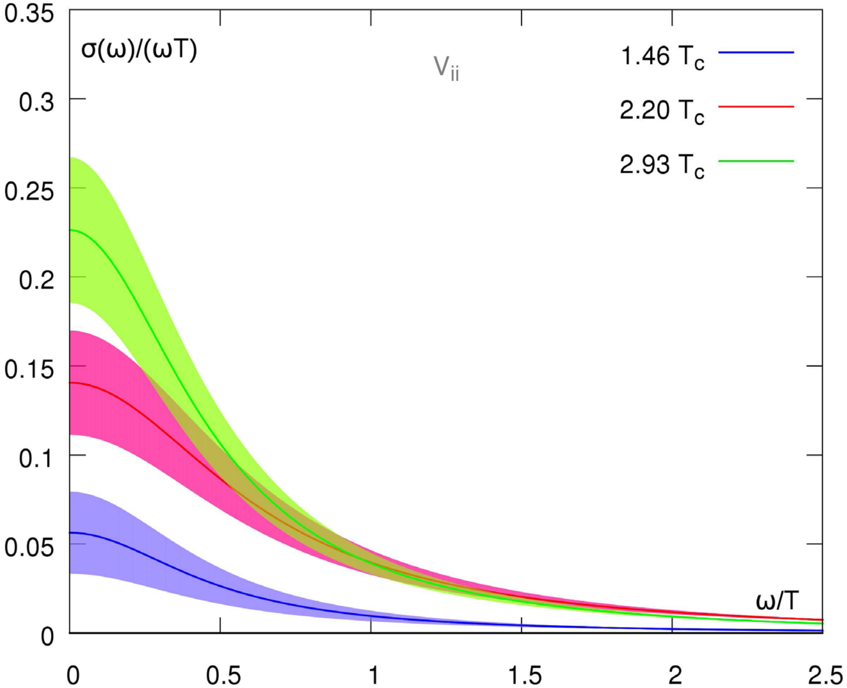
statistical error band from Jackknife analysis

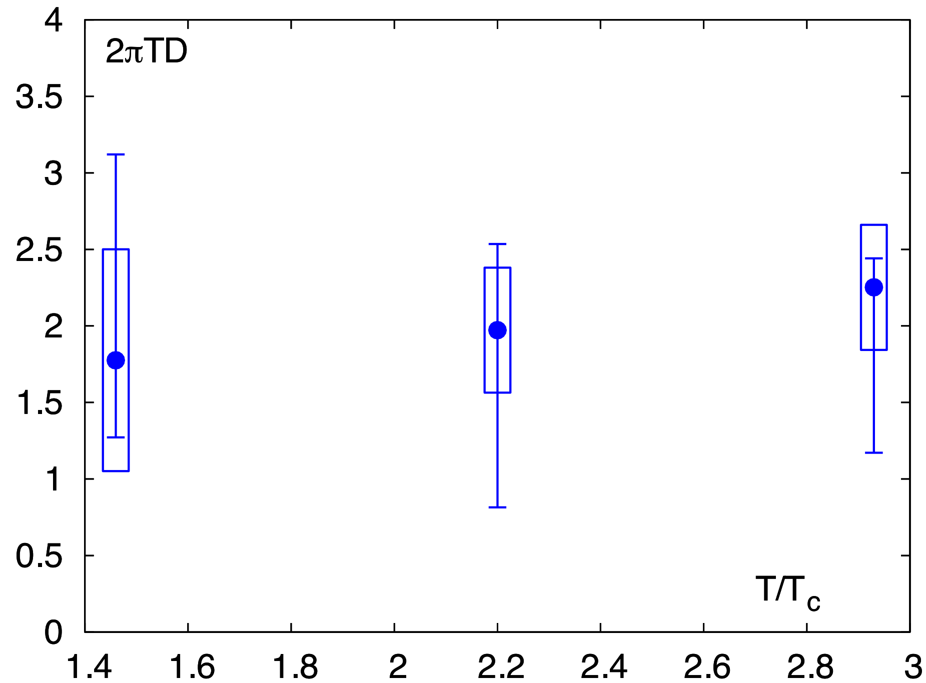
no clear signal for bound states at and above 1.46 T_c

study of the continuum limit and quark mass dependence on the way!

Charmonium Spectral function – Transport Peak

[H.T.Ding, OK et al., PRD86(2012)014509]





$$D = \frac{\pi}{3\chi_{00}} \lim_{\omega \to 0} \frac{\rho_{ii}(\omega, \vec{p}=0, T)}{\omega T}$$

Perturbative estimate (α_s~0.2, g~1.6):

LO: $2\pi TD \simeq 71.2$
NLO: $2\pi TD \simeq 8.4$

[Moore&Teaney, PRD71(2005)064904,
Caron-Huot&Moore, PRL100(2008)052301]

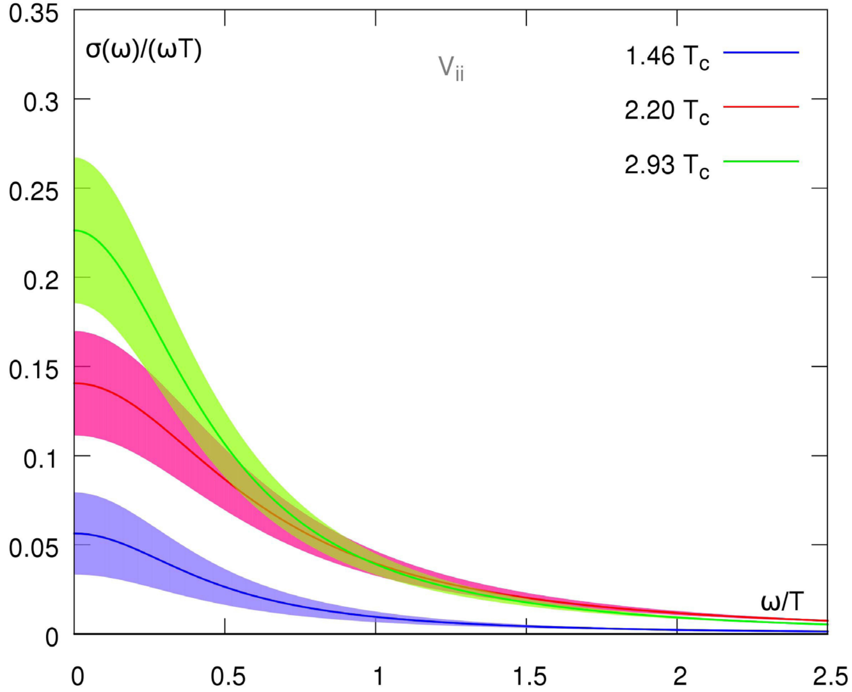
Strong coupling limit:

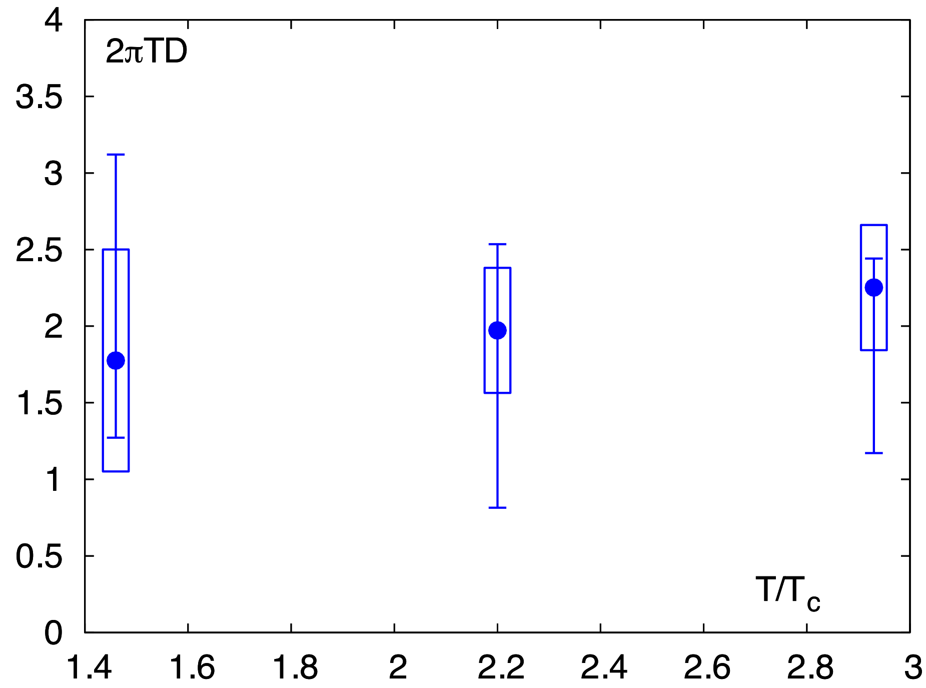
$2\pi TD = 1$

[Kovtun, Son & Starinets,JHEP 0310(2004)064]

Charmonium Spectral function – Transport Peak

[H.T.Ding, OK et al., PRD86(2012)014509]





Still large systematic uncertainties

- how to extract the spectral function
- cut-off effects become larger with increasing m_q
- quark mass dependence → bottomonium
- continuum limit needed

Is there a better observable that is more sensitive to transport properties?