Approach to equilibrium in weakly coupled nonabelian plasmas

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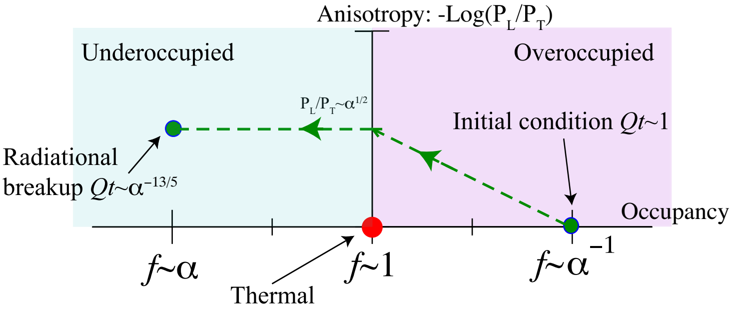
1401.3751 with M. Abraao York, E. Lu, and G. Moore (McGill)
1405.6318 with E. Lu

1107.5050, 1108.4684, 1209.4091, 1207.1663 with Moore

- What: Thermalization in $\alpha \ll 1$ nonabelian gauge theory
- How: Using combination of classical field theory and kinetic theory
- New: Smooth shift of d.o.f from fields to particles,
  first numerical estimates of bottom-up thermalization

Motivation: Bottom-up thermalization



- CGC: Initial condition overoccupied

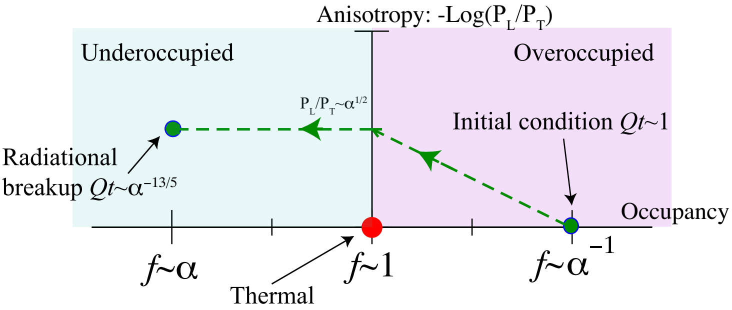
$$f(Q) \sim 1/\alpha$$

- Expansion makes system underoccupied before thermalizing

Baier et. al hep-ph/0009237, AK, Moore 1108.4684

$$f(Q) \ll 1$$

Motivation: Bottom-up thermalization



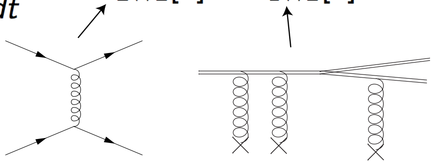
- Degrees of freedom:
  - Overoccupied: Classical field theory, $f \gg 1$
  - Underoccupied: (Semi-)classical particles, eff. kinetic theory, $f \ll 1/\alpha$

Talk by Schlichting

- Full description: Need change of d.o.f. from fields to particles
- Overlapping domain of validity $1 \ll f \ll 1/\alpha$: Field-particle duality

Effective kinetic theory of Arnold, Moore and Yaffe hep-ph/0209353

$$\frac{df}{dt} = -C_{2\leftrightarrow 2}[f] - C_{1\leftrightarrow 2}[f]$$



- Soft and collinear divergences lead to nontrivial matrix elements

soft: screening, Hard-loop; collinear: LPM, ladder resum

- No free parameters; LO accurate in the $\alpha \to 0$, $\alpha f \to 0$ limit.
- Used (in linearized form) *e.g.* for LO transport coefficients in QCD

Arnold, Moore, Yaffe hep-ph/0302165

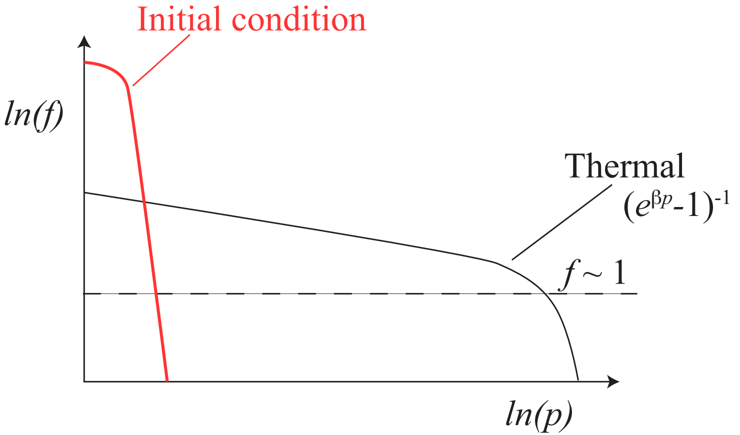
Outline:

- Isotropic overoccupied system, field-particle duality
- Isotropic underoccupied system, radiational breakup
- Application to Bottom-up of BMSS

Overoccupied cascade

Abraao York, AK, Lu, Moore 1401.3751

What happens if you have too many soft gluons, $f \sim 1/\alpha$.
No longitudinal expansion.



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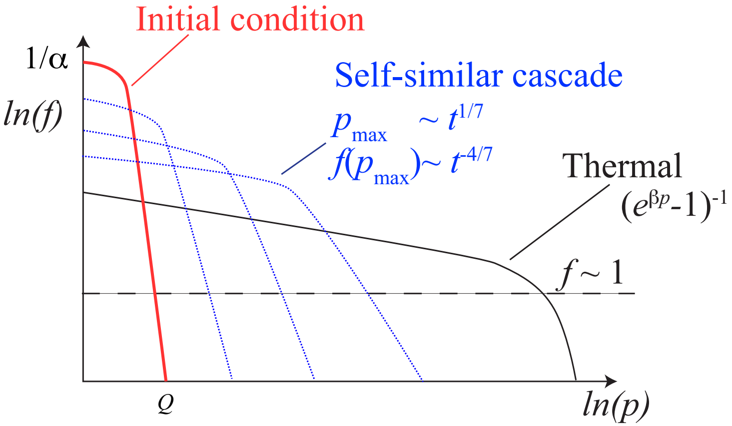
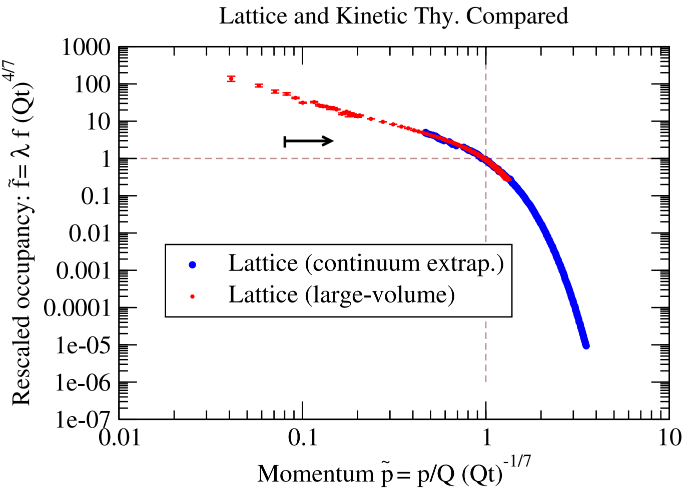


Overoccupied cascade

Abraao York, AK, Lu, Moore 1401.3751



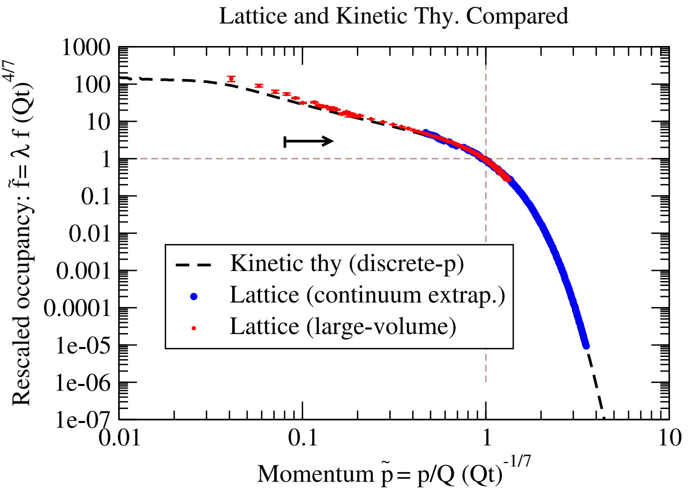
Form of cascade from classical lattice simulation,

$$1 \ll f \lesssim 1/\alpha$$

Large-volume: (Qa)=0.2, (QL)=51.2, Cont. extr.: down to (Qa)=0.1, (QL)=25.6, Qt=2000, $\tilde{m} = 0.08$

Overoccupied cascade

Abraao York, AK, Lu, Moore 1401.3751



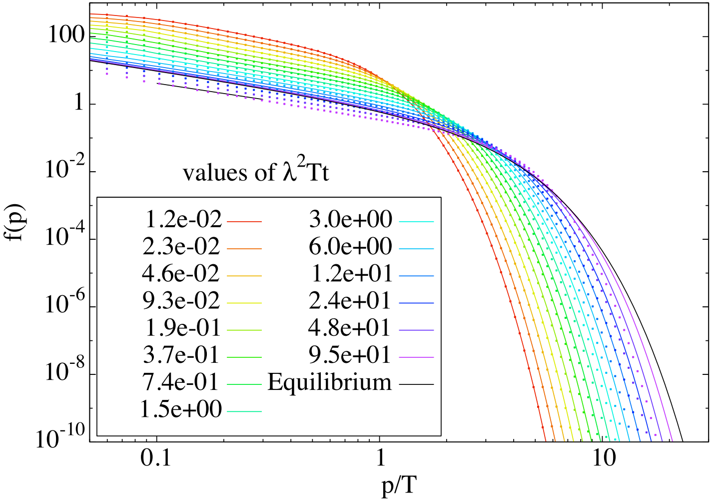
Same system, very different degrees of freedom

$$1 \lesssim f \ll 1/\alpha$$

Sensitive to the details of the collision terms

Overoccupied cascade

AK, Lu 1405.6318



Thermal equilibrium reached once $f \sim 1$ (or $t \sim \frac{1}{\alpha^2 T}$).

Overoccupied cascade

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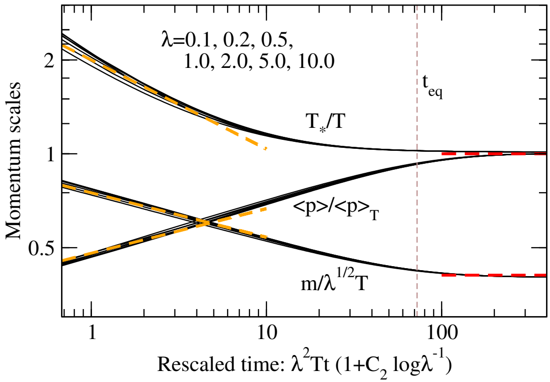


$$m^2 = \lambda \int_{\mathbf{p}} \frac{f(p)}{p}$$

$$T_* = \frac{\lambda}{2} \int_{\mathbf{p}} f(p)[1+f(p)]/m^2$$

$$\langle p\rangle = \frac{1}{n} \int_{\mathbf{p}} p\, f(p)$$

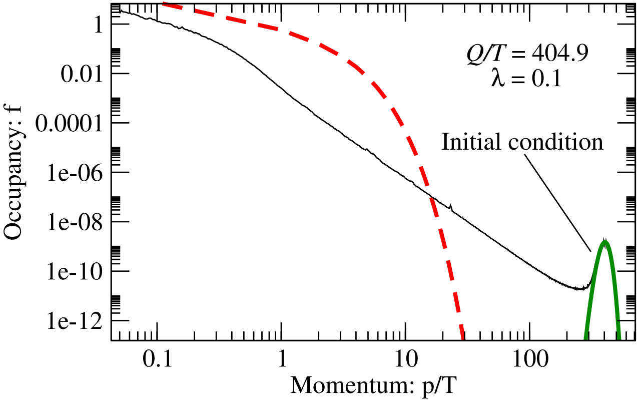
Therm. time through the approach of $\langle p\rangle - \langle p\rangle_T \sim \exp(-t/t_{\rm eq})$

$$t_{\rm eq} \approx \frac{72.}{1+0.12\log\lambda^{-1}}\,\frac{1}{\lambda^2 T}, \qquad \lambda = 4\pi N_c \alpha.$$

Underoccupied cascade

AK, Lu 1405.6318

Isotropic, underoccupied initial conditions, initial scale $\langle p^2 \rangle = Q^2$



Thermalization time parametrically given by stopping time of jet of momentum Q:

AK, Moore 1107.5050

$$t_{\mathrm{eq}} \approx \left(\frac{Q}{T}\right)^{1/2} \frac{1}{\lambda^2 T}$$

Underoccupied cascade

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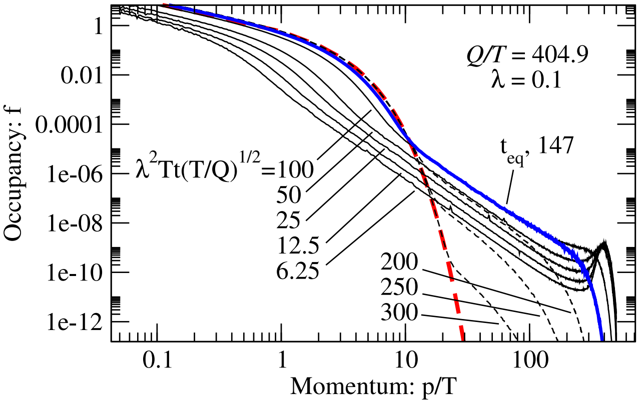

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Underoccupied cascade

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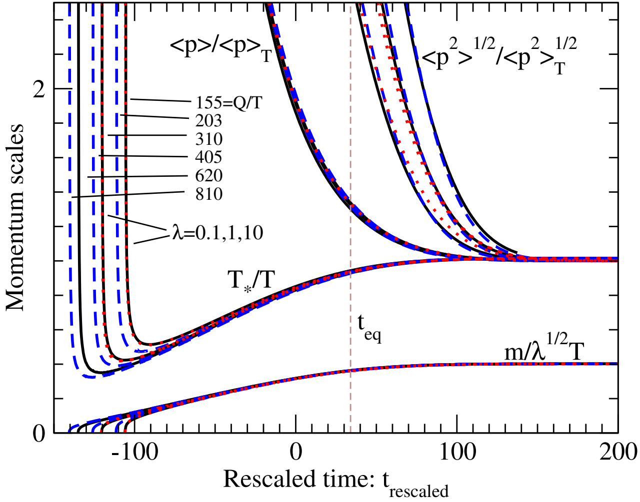
Scaling analysis with gaussian and step-cutoff initial conditions



$$t_{\rm eq} \approx \frac{34. + 21.\ln(Q/T)}{1 + 0.037\log\lambda^{-1}} \left(\frac{Q}{T}\right)^{1/2} \frac{1}{\lambda^2 T}$$

Connection to heavy-ion physics

AK, Lu 1405.6318

Bottom-up thermalization a la BMSS: Baier et. al hep-ph/0009237, AK, Moore 1108.4684

- Underoccupied cascade, but expansion reduces the target temperature

$$\tau_{\mathrm{eq}} \sim \frac{1}{\alpha^2 T}\left(\frac{Q}{T}\right)^{1/2}, \quad \epsilon \sim T^4 \sim \frac{Q^4}{\alpha(Qt)} \Rightarrow Qt \sim \alpha^{-13/5}$$

- Rough estimate: replace parametric estimate by the numerical
  - Estimate for energy density $\epsilon \approx 1.5 d_A Q^4/\pi\lambda(Qt)$ and $\alpha = 0.3$

Lappi 1105.5511

$$Qt_{\mathrm{eq}} \approx 1.5$$

Caveats: Angular dependence, no fermions, definition dependence of t_{eq}, extrapolation to $\alpha = 0.3$

Connection to heavy-ion physics

AK, Lu 1405.6318

Quantifying uncertainties:

- For $\alpha = 0.2$:

$$Qt_{\rm eq} \approx 4.0$$

- Varying ϵ by a factor of 2:

$$Qt_{\rm eq} \approx 2.5$$

.

  - with $\alpha = 0.2$: $Qt_{\rm eq} \approx 8.0$

- Replacing free streaming $(Qt)^{-1}$ by redshifting $(Qt)^{-4/3}$:

$$Qt_{\rm eq} < 4$$

For $Q_s \approx 2 GeV$, corresponds to

$$t_{\rm eq} \approx 0.1 - 1 \mathrm{fm/c}$$

Conclusions

- Combination of classical simulations and effective kinetic theory allows to follow the time evolution from highly occupied initial condition to thermal equilibrium
- Thermalization times for simple systems faster than naively expected
- Inserting the underoccupied thermalization time to bottom-up thermalization yields a rough estimate for heavy-ion collisions

$$t_{\mathrm{eq}} \lesssim 0.1 - 1\mathrm{fm/c}$$

Outlook

- Proper treatment of expansion and angular dependence
- Implementation of fermions to kinetic theory
- Inclusions of plasma unstable modes — AK, Moore 1108.4684
- NLO not inconceivable
- Applications to jets — AK, Wiedemann 1407.0293

p.s. No sign of BEC AK, Moore 1207.1663

Scaling analysis

$$f_{\mathrm{step}}(p) \propto \Theta(Q_s - p), \quad f_g(p) \propto \exp\left[-\frac{(Q_s - p)^2}{(Q_s/10)^2}\right]$$

$$Q^2 \equiv \int_{\mathbf{p}} f(p) p^2 \Big/ \int_{\mathbf{p}} f(p)$$

run	Q/T	n_H/n_T	λ	init	run	Q/T	n_H/n_T	λ	init
1	202.5	0.0134	0.1	g	4	155.1	0.01799	0.1	step
2	404.9	0.00668	0.1	g	5	310.0	0.00900	0.1	step
3	809.8	0.00334	0.1	g	6	620.0	0.00450	0.1	step
7	155.137	0.01799	1.0	step	9	310.0	0.00900	1.0	step
8	155.137	0.01799	10.0	step	10	310.0	0.00900	10.0	step

In reality many more simulations with varying parameters

Power law from of the cascade

- Low scales have time to thermalize: $1/p$ AK, Moore,1107.5050
- Non-trivial turbulent kolmogorov cascade $1/p^{4/3}$, (BEC: $1/p^{3/2}$)?

Berges et al 0811.4293