Nonlinear evolution at large values of coupling constant

Krzysztof Kutak

Instytut Fizyki Jądrowej
im. Henryka Niewodniczańskiego
Polskiej Akademii Nauk

Based on: Kutak, Surowka Phys. Rev. D 89, 026007 (2014)

Supported by grant: LIDER/02/35/L-2/10/NCBiR/2011

LIDER

Narodowe Centrum Badań i Rozwoju

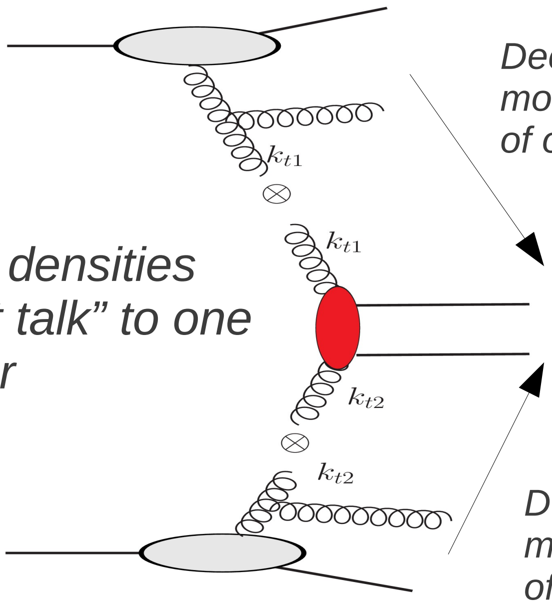
QCD at high energies (and weak coupling) – high energy factorization

$$\frac{d\sigma}{dy_1 dy_2 d^2p_{1t} d^2p_{2t}} = \sum_{a,b,c,d} \int \frac{d^2k_{1t}}{\pi}\frac{d^2k_{2t}}{\pi}\frac{1}{16\pi^2(x_1x_2S)^2}\overline{|\mathcal{M}_{ab\to cd}|}^2 \delta^2(\vec{k}_{1t}+\vec{k}_{2t}-\vec{p}_{1t}-\vec{p}_{2t})$$
$$\times\, \mathcal{F}_{a/A}(x_1,k_{1t}^2,\mu^2)\,\mathcal{F}_{b/B}(x_2,k_{2t}^2,\mu^2)\,\frac{1}{1+\delta_{cd}}$$

$$k_1^\mu = x_1 P_1^\mu + \bar{x}_1 P_2^\mu + k_{1t}^\mu \qquad k_2^\mu = x_2 P_2^\mu + \bar{x}_2 P_1^\mu + k_{2t}^\mu$$

$$\bar{x}_1 = \frac{k_1^2+\mathbf{k}^2}{Sx_1} \qquad \bar{x}_2 = \frac{k_2^2+\mathbf{k}^2}{Sx_2}$$

Parton densities "do not talk" to one another



$$|\mathcal{M}_{ab\to cd}|^2 = \frac{2x_1k_1^{\mu_1}k_1^{\nu_1}}{k_1^2}\frac{2x_2k_2^{\mu_2}k_2^{\nu_2}}{k_2^2}\mathcal{M}_{ab\to cd\,\mu_1\nu_1}\mathcal{M}^*_{ab\to cd\,\mu_2\nu_2}$$

Gribov, Levin, Ryskin '81
Ciafaloni, Catani, Hautman '93

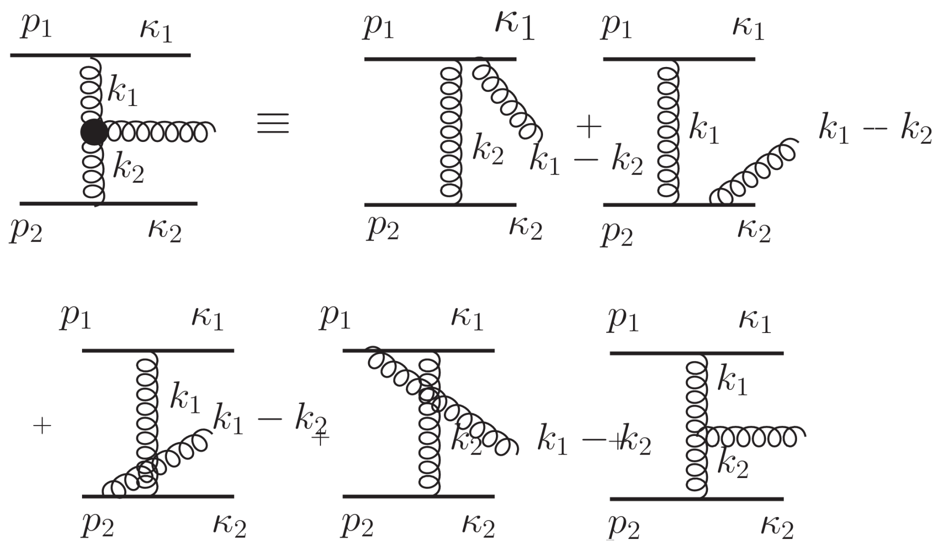
Originally derived for heavy quarks in final state.
Therefore no problem of division into density and ME
Gluons more tricky possible double counting.

Some trials to generalized to p-A
Dominguez, Huan, Marquet,Xiao '10

Does not take into account MPI as formulated in DGLAP i.e. emissions from independent chains
Does not take into account correlators of higher order like JIMWLK

The BFKL evolution

Balitsky, Fadin, Kuraev, Lipatov '77



$$1 \gg \alpha_1 \gg \alpha_2$$

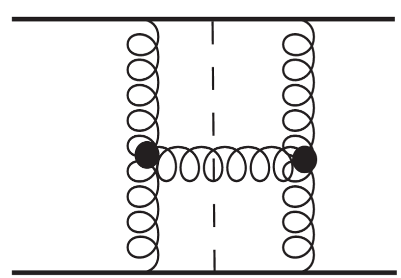
$$1 \gg |\beta_1| \gg |\beta_2|$$

$$k_1 = \alpha_1 p_1 + \beta_1 p_2 + k_{1t}$$

$$k_2 = \alpha_2 p_1 + \beta_2 p_2 + k_{2t}$$

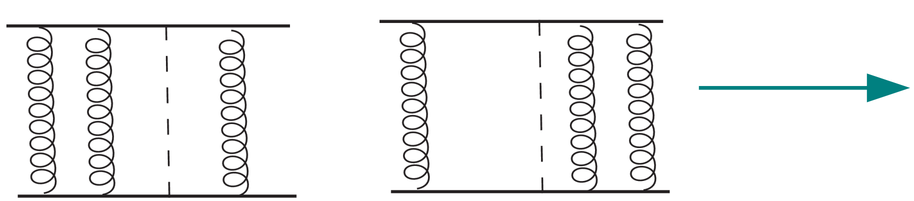
$$iA^\rho_{2\to 3} = (-2ig_2 p_1^\mu) t^a_{mj} \left(\frac{-i}{k_1^2}\right) f_{abc} g_s \Gamma^\rho_{\mu\nu}(k_1, k_2) \left(\frac{-i}{k_2^2}\right) (-2ig_s p_2^\nu) t^b_{nl}$$

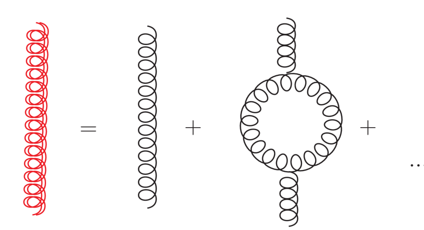
- *Known also for SM YM*
- *Studied also in context of AdS/CFT*
- *Known up to NLO*
- *No saturation*
- *No applicable to final states: "evolution without observer"*



$$J^\mu = -ig\bar{u}(p_1 + q)\gamma^\mu u(p_1) \approx -2ig_s p_1^\mu$$

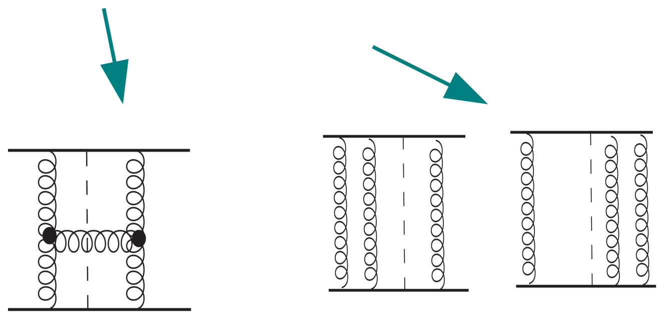
reggeized gluon

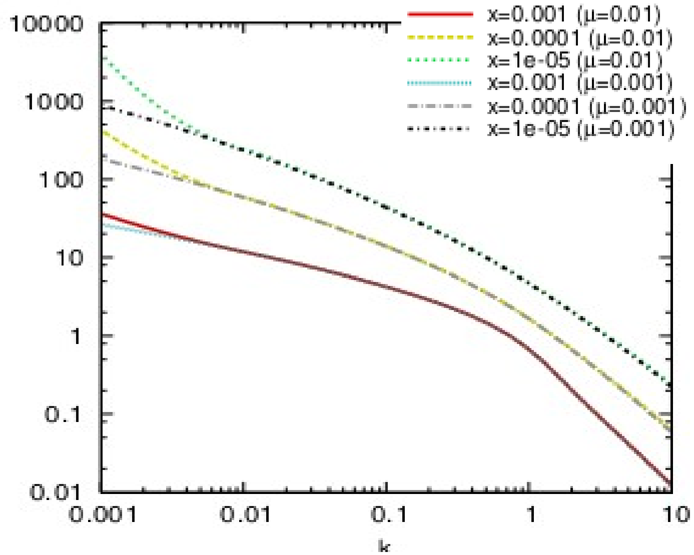


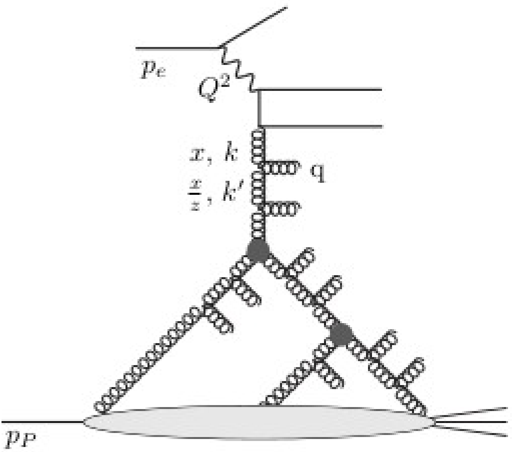


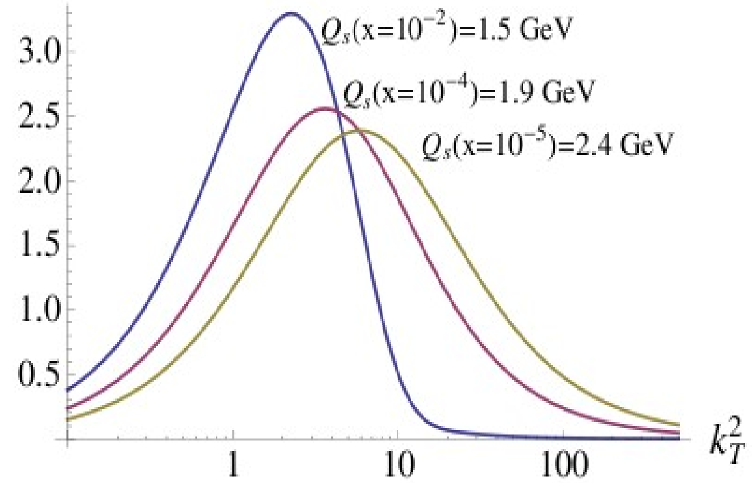
The BFKL and BK evolutions - solutions

$$\mathcal{F}(x,k^2) = \mathcal{F}_0(x,k^2) + \overline{\alpha}_s \int_{x/x_0}^{1} \frac{dz}{z} \int_0^\infty \frac{dl^2}{l^2} \left[\frac{l^2 \mathcal{F}(x/z,l^2) - k^2 \mathcal{F}(x/z,k^2)}{|k^2 - l^2|} + \frac{k^2 \mathcal{F}(x/z,k^2)}{\sqrt{(4l^4 + k^4)}} \right]$$









$$\mathcal{F}(x,k^2) = \mathcal{F}_0(x,k^2) + \overline{\alpha}_s \int_{x/x_0}^{1} \frac{dz}{z} \int_0^\infty \frac{dl^2}{l^2} \left[\frac{l^2 \mathcal{F}(x/z,l^2) - k^2 \mathcal{F}(x/z,k^2)}{|k^2 - l^2|} + \frac{k^2 \mathcal{F}(x/z,k^2)}{\sqrt{(4l^4 + k^4)}} \right]$$
$$- \frac{2\alpha_s^2 \pi}{N_c R^2} \int_{x/x_0}^{1} \frac{dz}{z} \left\{ \left[\int_{k^2}^\infty \frac{dl^2}{l^2} \mathcal{F}(x/z,l^2) \right]^2 + \mathcal{F}(x/z,k^2) \int_{k^2}^\infty \frac{dl^2}{l^2} \ln\left(\frac{l^2}{k^2}\right) \mathcal{F}(x/z,l^2) \right\}$$

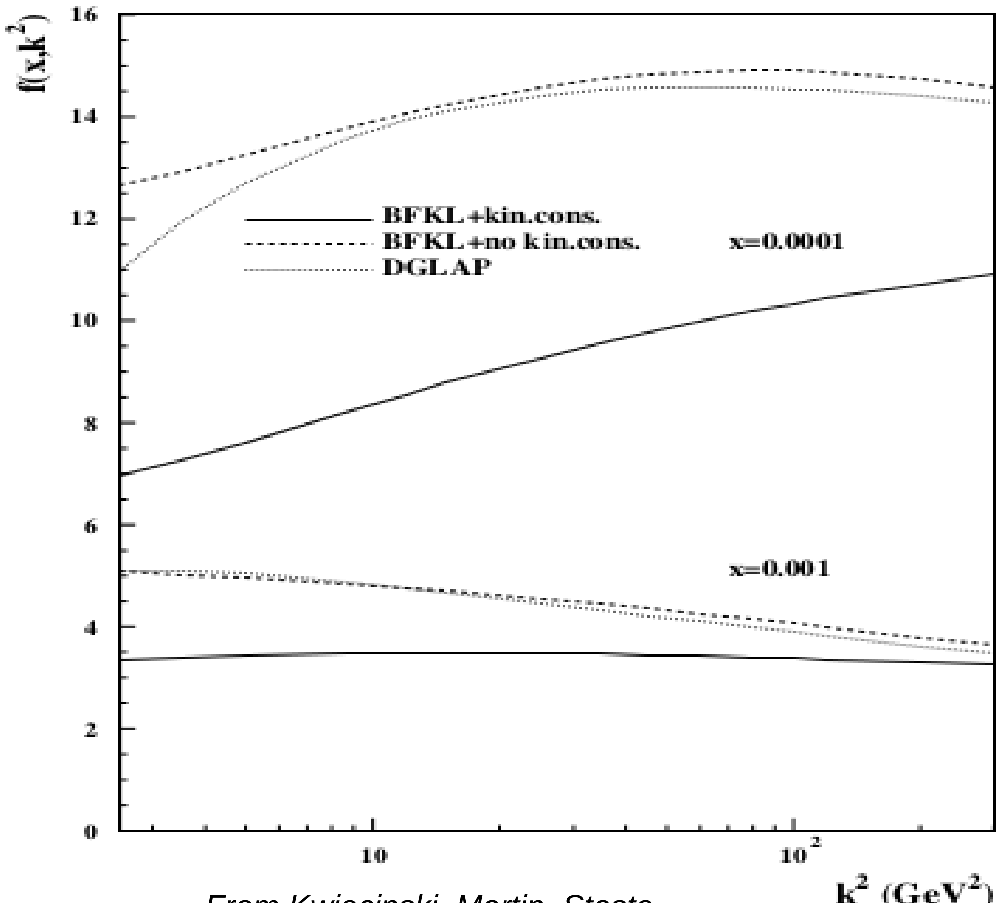
BFKL with subleading corrections
Kwiecinski, Martin, Staśto prescription

Nonsingular pieces of splitting function

Kinematical effects i.e. Momentum of gluon dominated by it's transversal component

Running coupling

In principle not applicable *to final states since no hard scale dependence*

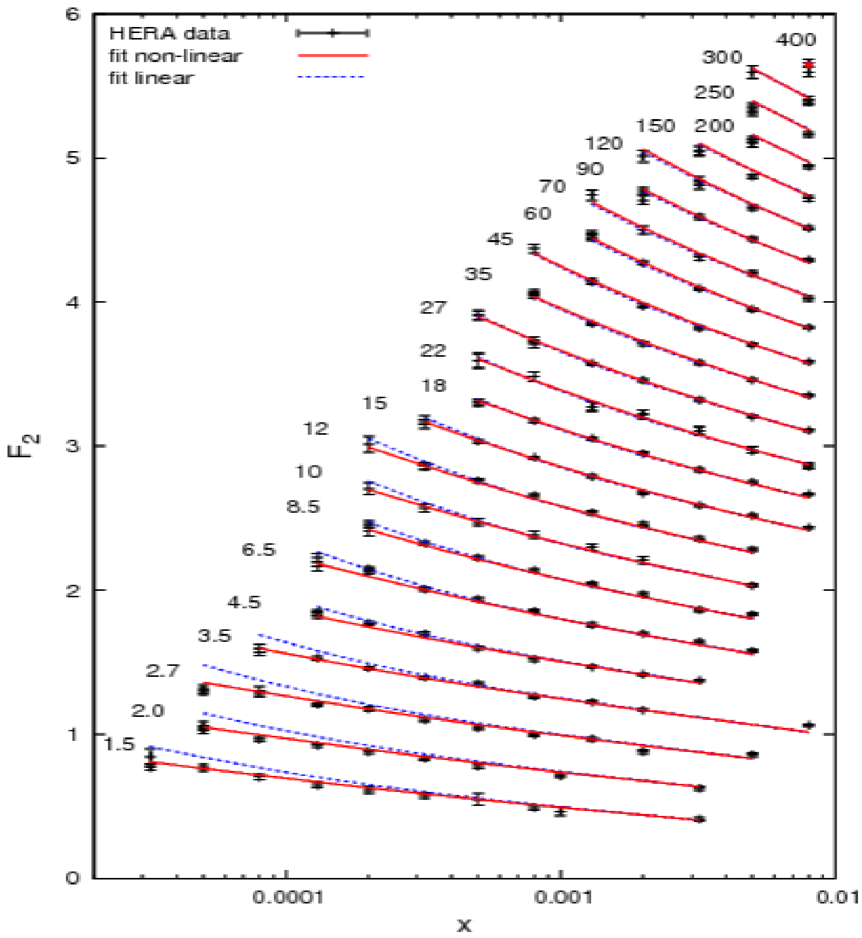


From Kwiecinski, Martin, Stasto
Phys.Rev. D56 (1997) 3991-4006

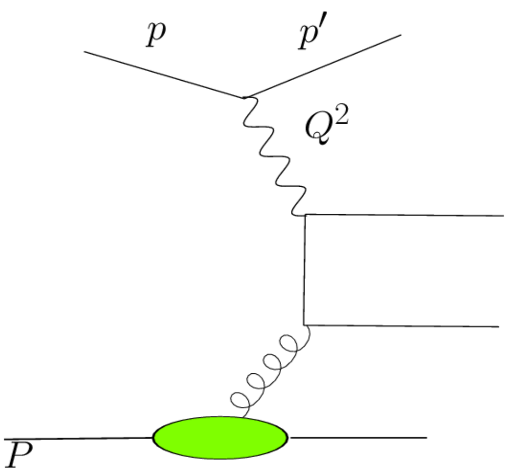
$$\mathcal{F}_p(x,k^2) \;=\; \mathcal{F}_p^{(0)}(x,k^2)$$
$$+\;\frac{\alpha_s(k^2)N_c}{\pi}\int_x^1\frac{dz}{z}\int_{k_0^2}^{\infty}\frac{dl^2}{l^2}\left\{\frac{l^2\mathcal{F}_p(\frac{x}{z},l^2)\,\theta(\frac{k^2}{z}-l^2)\;-\;k^2\mathcal{F}_p(\frac{x}{z},k^2)}{|l^2-k^2|}+\frac{k^2\mathcal{F}_p(\frac{x}{z},k^2)}{|4l^4+k^4|^{\frac{1}{2}}}\right\}$$
$$+\;\frac{\alpha_s(k^2)}{2\pi k^2}\int_x^1 dz\left(P_{gg}(z)-\frac{2N_c}{z}\right)\int_{k_0^2}^{k^2}dl^2\,\mathcal{F}_p(\frac{x}{z},l^2)$$

$$f(x,k^2)=k^2\mathcal{F}(x,k^2)$$

BFKL with higher orders applied to DIS - some recent results



Sapeta, KK '12



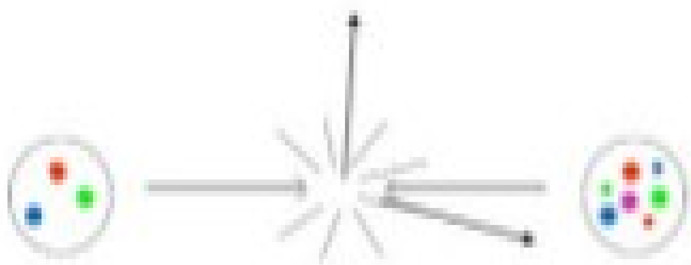
From BK equation with corrections of higher order

High energy prescription and forward-central di-jets

Deak, Jung, Hautmann Kutak
JHEP 0909:121,2009

$$\frac{d\sigma}{dy_1 dy_2 dp_{1t} dp_{2t} d\Delta\phi} = \sum_{a,c,d} \frac{p_{t1} p_{t2}}{8\pi^2 (x_1 x_2 S)^2} |\mathcal{M}_{ag\to cd}|^2 x_1 f_{a/A}(x_1,\mu^2)\, \mathcal{F}_{g/B}(x_2,k^2) \frac{1}{1+\delta_{cd}}$$

$S = 2P_1 \cdot P_2$



- *Resummation of logs of x and logs of hard scale*
- *Knowing well pdf at large x one can get information about low x physics*
- *Framework goes recently under name "hybride framework"*

$$x_1 = \frac{1}{\sqrt{S}} (p_{t1} e^{y_1} + p_{t2} e^{y_2}) \quad \xrightarrow{y_1 \sim 0,\, y_2 \gg 0} \quad \sim 1$$

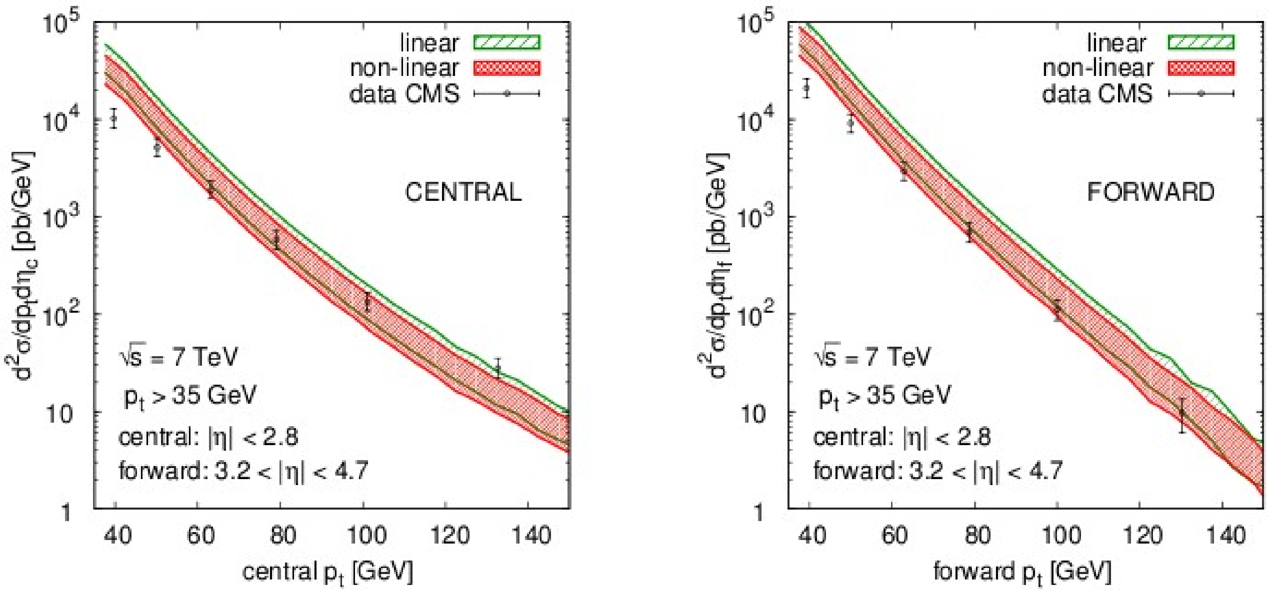
$$x_2 = \frac{1}{\sqrt{S}} (p_{t1} e^{-y_1} + p_{t2} e^{-y_2}) \qquad \ll 1$$

$$k_1^\mu = x_1 P_1^\mu$$

$$k_2^\mu = x_2 P_2^\mu + k_t^\mu$$

Di-jets p_t spectra

S.Sapeta. KK ,12

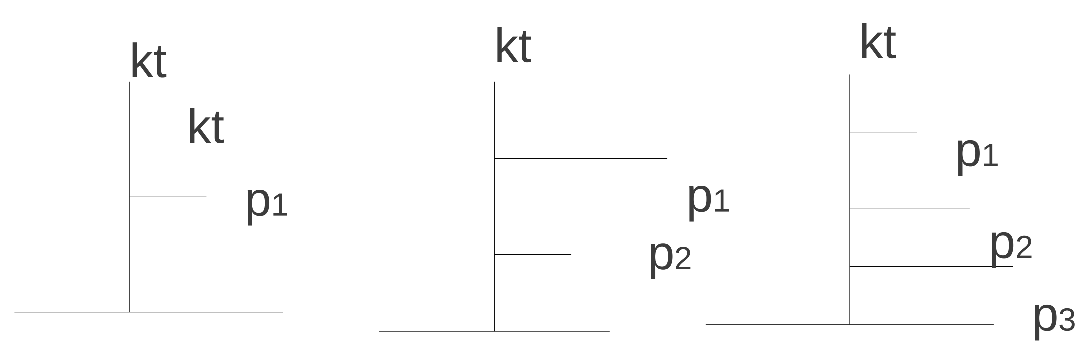


Reasonable agreement.

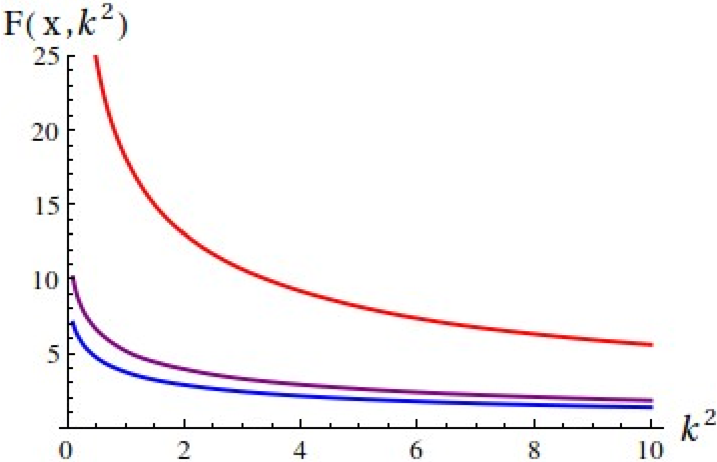
Gluon emissions are unordered in pt and udd up to $k_t = |p_1+p_2+.....p_n|$

During evolution time incoming gluon becomes off-shell

Crucial effect of higher order corrections

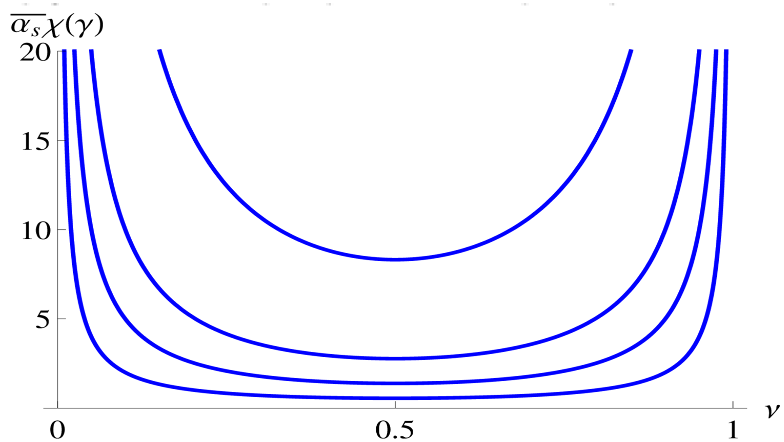


The BFKL equation and its solution

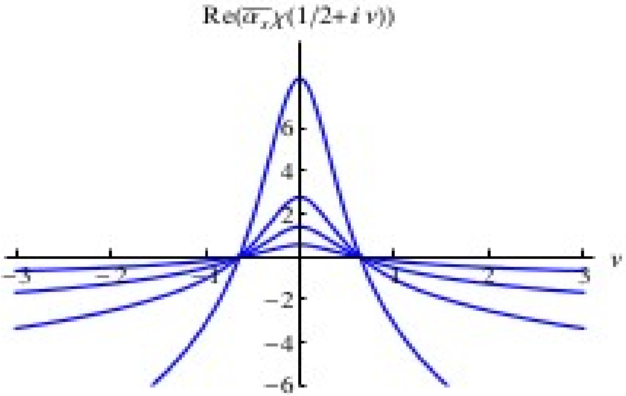


$$f(x,k^2) = f_0(x,k^2) + \bar{\alpha}_s k^2 \int_{x/x_0}^{1} \frac{dz}{z} \int_0^{\infty} \frac{dl^2}{l^2} \left[\frac{f(x/z,l^2) - f(x/z,k^2)}{|l^2 - k^2|} + \frac{f(x/z,k^2)}{\sqrt{4l^4 + k^4}} \right]$$

$$f(x,k^2) = \frac{1}{2\pi i} \int d\gamma (k^2)^{\gamma} \frac{1}{2\pi i} \int d\omega\, x^{-\omega} \frac{\omega \overline{f}_0(\omega,\gamma)}{\omega - \bar{\alpha}_s \chi_{k.c.}(\gamma,\omega,)}$$



$$\chi(\gamma) = 2\psi(1) - \psi(1-\gamma) - \psi(\gamma), \qquad \chi(1/2 + i\nu) \approx \lambda - \tfrac{1}{2}\lambda'\nu^2$$



$$\mathcal{F}(x,k^2) = \mathcal{F}(x_0,1/2) \frac{1}{\sqrt{4\pi \ln(x_0/x) 1/2\lambda'}} e^{\lambda \ln(x_0/x) - 1/2 \ln(k^2/k_0^2)} e^{\frac{-\ln(k^2/k_0^2)^2}{4 1/2\lambda' \ln(x_0/x)}}$$

$$\partial_Y \mathcal{F}(Y,\rho) = \frac{1}{2}\lambda' \partial_\rho^2 \mathcal{F}(Y,\rho) + \frac{1}{2}\lambda' \partial_\rho \mathcal{F}(Y,\rho) + (\lambda + \lambda'/8)\mathcal{F}(Y,\rho).$$

Model for resummed BFKL with kinematical constraint and DGLAP effects

Stasto, '07

$$f(x,k^2) = f_0(x,k^2) + \bar{\alpha}_s k^2 \int_x^1 \frac{dz}{z} \int_0^\infty \frac{dl^2}{l^2} \left[\frac{f(x/z,l^2)\theta(l-kz)\theta(k/z-l) - f(x/z,k^2)}{|l^2-k^2|} + \frac{f(x/z,k^2)}{\sqrt{4l^4+k^4}} \right]$$

$$\chi_{k.c.}(\gamma,\omega) = 2\psi(1) - \psi(1-\gamma+\omega/2) - \psi(\gamma+\omega/2).$$

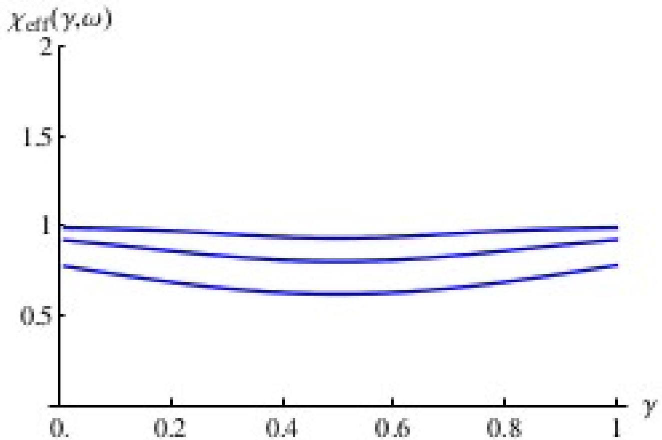
$$\chi_{eff}(\gamma,\omega) = \bar{\alpha}_s \chi_{k.c.}(\gamma,\omega)(1+A\omega)$$

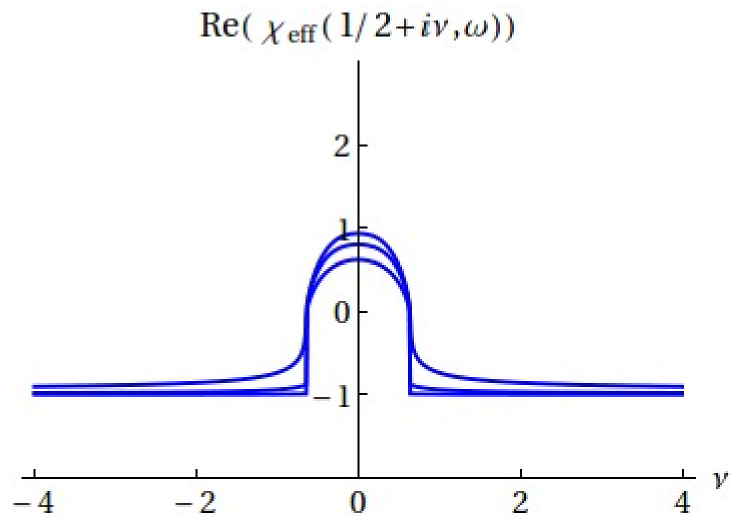
Crucial behavior vanishing eigenvalue when $\omega \to 1$

Contains DGLAP anaomalous Dimension at LO in $\ln Q^2$

$$j = 1+\omega = 2 - \frac{c_0}{\sqrt{\bar{\alpha}_s}}.$$

$$c_0 = 0.509346$$





The leading order in AdS/CFT Brower, Polchinski, Strassler

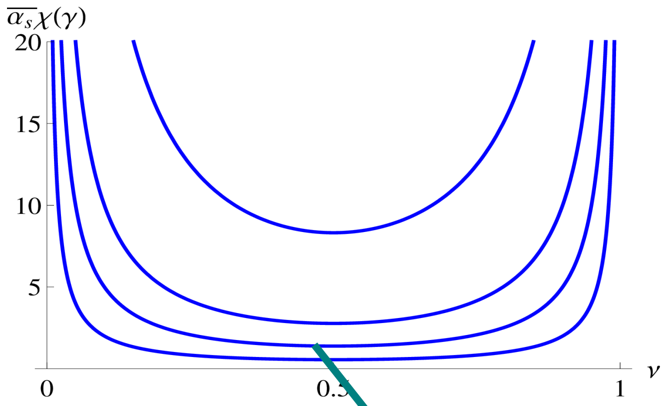
$$j = 1+\omega = 2 - \frac{c_0}{\sqrt{\bar{\alpha}_s}}, \qquad c_0 = 1/\pi$$

Higher orders:

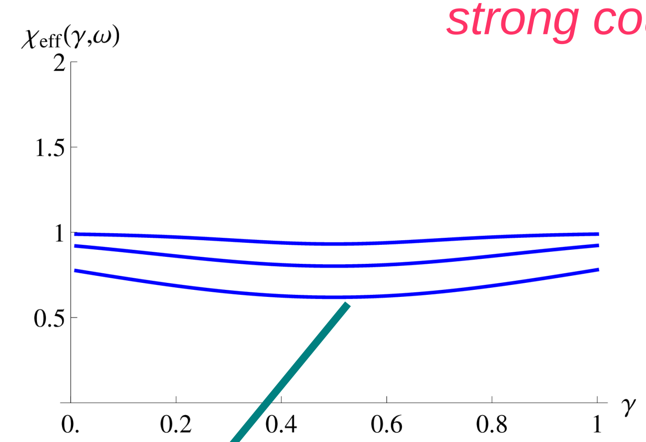
Costa, Goncalves, Penedones' 12
Kotokov, Lipatov '13
Janik '14

Strong vs. weak

weak coupling



strong coupling



weak coupling

Munier, Peschanski '03

strong coupling

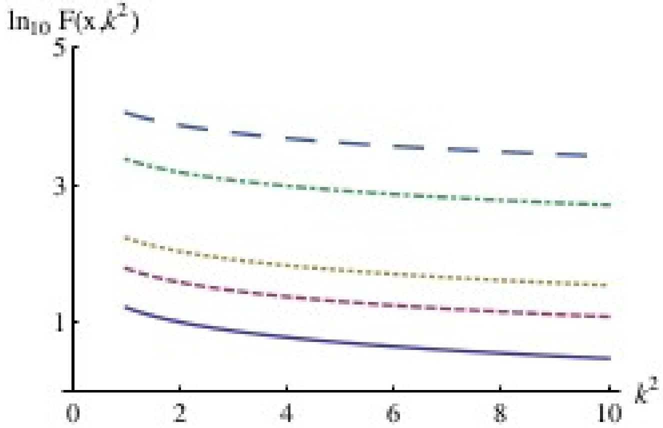
Kutak, Surowka '13

critical point dominates at large coupling

Gluon density at the large coupling values

$$\chi_{eff}(\gamma,\omega) = \bar{\alpha}_s \chi_{k.c.}(\gamma,\omega)(1 + A\omega)$$

Kutak, Surowka, '13



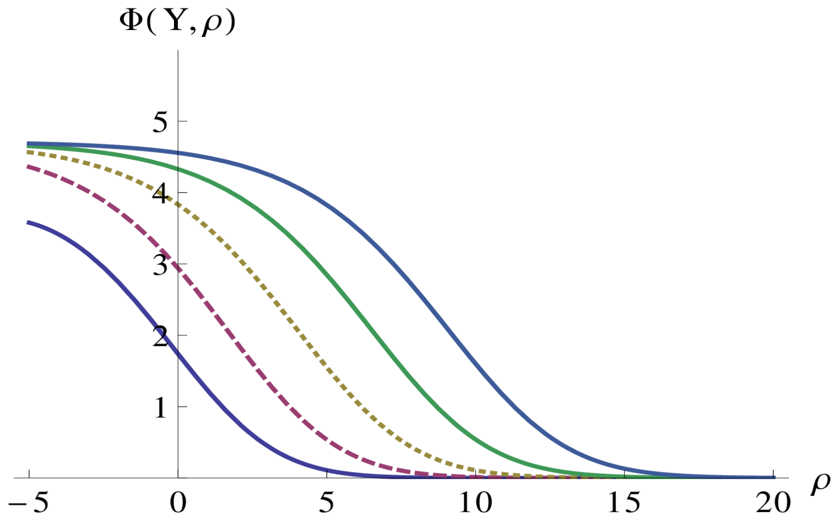
$$\chi_{eff\,\infty}(\omega, 1/2 + i\nu) = 1.02795\ - 2.04635\nu^2 \equiv \lambda_{st} - \frac{1}{2}\lambda'_{st}\nu^2$$

$$\lambda'_{st} = 4.08,\ \lambda_{st} = 1.02$$

$$\partial_Y \Phi(Y,\rho) = \frac{1}{2}\lambda'_{st}\partial^2_\rho \Phi(Y,\rho) + \frac{1}{2}\lambda'_{st}\partial_\rho \Phi(Y,\rho) + (\lambda_{st} + \lambda'_{st}/8)\Phi(Y,\rho)$$

WW density at the large coupling values

Kutak, Surowka, '13



Nonlinear nonlinear equation valid at strong coupling limit

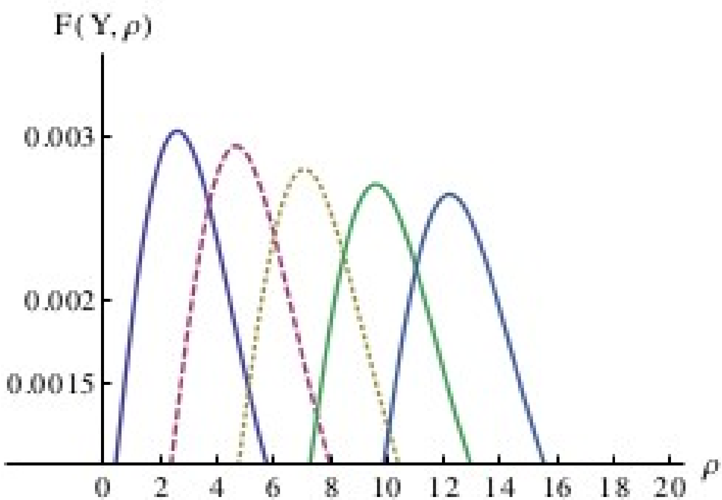
$$\partial_Y \Phi(Y,\rho) = \frac{1}{2}\lambda'_{st}\partial_\rho^2\Phi(Y,\rho) + \frac{1}{2}\lambda'_{st}\partial_\rho\Phi(Y,\rho) + (\lambda_{st} + \lambda'_{st}/8)\Phi(Y,\rho) - \frac{\bar{\alpha}_s}{\pi R^2}\Phi^2(Y,\rho)$$

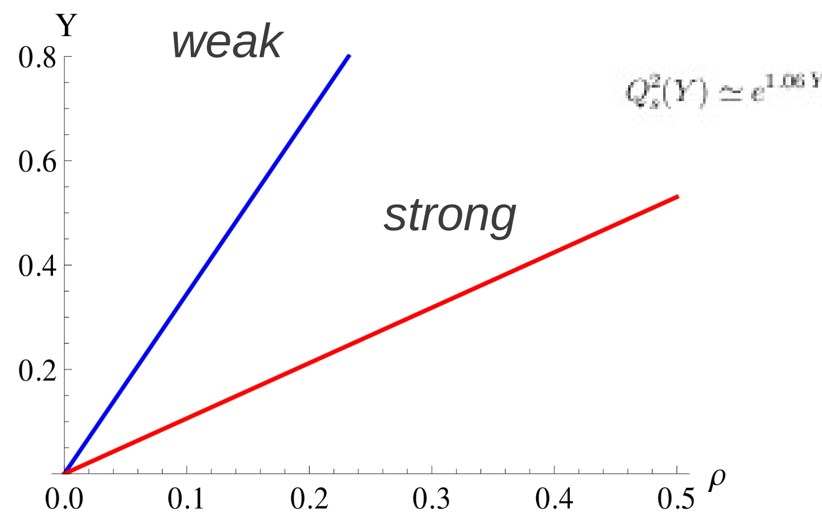
Similar equation an weak coupling (*Munier Peschanski 03*) .
The coefficients are different.

Saturation scale at large values of couplng constant

$$\mathcal{F}_{BK}(Y,\rho) = \frac{N_c}{4\pi\alpha_s}\partial^2_\rho \Phi(Y,\rho)$$

$$\partial_\rho \mathcal{F}_{BK}(Y,\rho)|_{\rho=\ln Q_s^2(Y)} = 0$$





Similar behaviour as in
Mueller, Shoshi, Xiao '10
Hatta,Iancu, Mueller' '07,

Outlook

- *Entropy at large coupling*
- *Full range in running coupling effect*
- *Just for curiosity check the cross section for inclusive production*
- *Perhaps formulate directly in momentum space*