Chiral Transport Equation: derivation and connection with anomalous HTL/HDL

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Outline

- Foldy-Wouthuysen diagonalization
- Semi-classical equations of motion
- EFT approach to the FW diagonalization
- Chiral transport equation
- Connection with the anomalous HTL/HDLs

Foldy-Wouthuysen Diagonalization

- The Dirac eq. for a free fermion mixes particles and antiparticles d.o.f.
- FW found a representation where these can be separated, through a canonical transformation

exact for the free theory

approx. for an interacting theory

$$H\psi = i\frac{\partial \psi}{\partial t} \qquad H' = UHU^\dagger \qquad \psi' = U\psi$$

$$\mathrm{H}_0 = \alpha\cdot(\mathbf{P} - e\mathbf{A}(\mathbf{R})) + \beta m + eA_0(\mathbf{R})$$

$$\alpha_k = \begin{pmatrix} 0 & \sigma_k \\ \sigma_k & 0 \end{pmatrix}, \quad \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix},$$

At order $\mathcal{O}(\hbar^0)$

$$U = \frac{E + m + \beta\alpha\cdot(\mathbf{P} - e\mathbf{A}(\mathbf{R}))}{\sqrt{2E(E+m)}}$$

$$H_D = UH_0U^\dagger = \beta E + eA_0(\mathbf{R})$$

$$E \equiv \sqrt{(\mathbf{P} - e\mathbf{A}(\mathbf{R}))^2 + m^2}.$$

At order $\mathcal{O}(\hbar)$ $[R_i, P_j] = i\hbar\delta_{ij}$

Gosselin, Berard and Mohrbach 2007

Give a prescription to deal with products of R, P

Keep unitarity; project over the diagonal

Rotate all operators

$$\mathbf{r} = \mathcal{P}[U(\mathbf{P},\mathbf{R})\,\mathbf{R}\,U^\dagger(\mathbf{P},\mathbf{R})] = \mathbf{R} + \mathcal{P}(\mathcal{A}_R)\ ,$$
$$\mathbf{p} = \mathcal{P}[U(\mathbf{P},\mathbf{R})\,\mathbf{P}\,U^\dagger(\mathbf{P},\mathbf{R})] = \mathbf{P} + \mathcal{P}(\mathcal{A}_P)$$

$$\mathcal{P}(\mathcal{A}_{R_i}) = -\hbar\frac{E[\mathbf{\Sigma}\times(\mathbf{P}-e\mathbf{A})]_i}{2E^2(E+m)}\ , \qquad \mathcal{A}_{P^i} = e\,\nabla_{R^i}A_k(\mathbf{R})\mathcal{A}_{R^k}$$

$$\Sigma_k = \begin{pmatrix} \sigma_k & 0 \\ 0 & \sigma_k \end{pmatrix}$$

In terms of the rotated variables

$$H_D = \beta \left(E - \frac{e\hbar \boldsymbol{\Sigma} \cdot \mathbf{B}}{2E} - \frac{e\mathbf{L} \cdot \mathbf{B}}{E} \right) + eA_0(\mathbf{r})$$

$$E = \sqrt{(\mathbf{p} - e\mathbf{A}(\mathbf{r}))^2 + m^2}$$

$$\mathbf{L} = \tilde{\mathbf{p}} \times \mathcal{P}(\mathcal{A}_{\mathbf{R}}) = \hbar \, \frac{\tilde{\mathbf{p}} \times (\tilde{\mathbf{p}} \times \boldsymbol{\Sigma})}{2E(E+m)} \qquad \tilde{\mathbf{p}} \equiv \mathbf{p} - e\mathbf{A}(\mathbf{r})$$

Gauge invariance kept at order of accuracy

The new variables are non canonical

$$\begin{aligned} [r_i, r_j] &= i\hbar^2 G_{ij} = -i\hbar^2 \epsilon_{ijk} G_k \\ [\tilde{p}_i, \tilde{p}_j] &= ie\hbar F_{ij} + ie^2\hbar^2 F_{ik} F_{jm} G_{km} \ , \\ [r_i, \tilde{p}_j] &= i\hbar\delta_{ij} + ie\hbar^2 F_{jk} G_{ik} \end{aligned}$$

$$\mathbf{G}(\tilde{\mathbf{p}}) = \frac{1}{2E^3} \left(m\boldsymbol{\Sigma} + \frac{(\boldsymbol{\Sigma} \cdot \tilde{\mathbf{p}})\tilde{\mathbf{p}}}{E+m} \right)$$

Massless fermions

$$\tilde{\mathbf{p}} \to \mathbf{p}$$

$$\mathbf{G} = \lambda \boldsymbol{\Omega} \,, \qquad \boldsymbol{\Omega} = \frac{\mathbf{p}}{2p^3} \qquad \lambda = \frac{\Sigma \cdot \mathbf{p}}{p}$$

Fermion dispersion law in an B field is modified

$$\epsilon^{\pm}_{\mathbf{p}} = \pm p \left(1 - e\hbar\, \lambda \frac{\mathbf{B} \cdot \mathbf{p}}{2p^3} \right)$$

Semiclassical equations of motion (e.g. right-handed)

$$\begin{aligned} \dot{\mathbf{p}} &= -\frac{\partial \epsilon^{+}_{\mathbf{p}}}{\partial \mathbf{r}} + e(\mathbf{E} + \dot{\mathbf{r}} \times \mathbf{B}) \,, \\ \dot{\mathbf{r}} &= \frac{\partial \epsilon^{+}_{\mathbf{p}}}{\partial \mathbf{p}} - \hbar(\dot{\mathbf{p}} \times \boldsymbol{\Omega}) \end{aligned}$$

The chiral transport equation recently proposed can be deduced simply by computing (for m=0) the first quantum corrections to the classical eqs. of motion

↓

Semiclassical chiral transport equation

EFT approach to the FW diagonalization - OSEFT

$\hbar = 1$

Separating fermion/antifermion d.o.f. within QFT (HQET, NRQED, LEET, HDET, ...)

Describing physics for an almost on-shell m=0 fermion

$$q^\mu = p v^\mu + k^\mu \qquad v^\mu = (1, \mathbf{v})$$

k^μ → residual momentum

$$\psi_v(x) = e^{-ipv\cdot x}\left(P_{+v}\chi_v(x) + P_{-v}H_v^1(x)\right)$$

P_{+v}, P_{-v} → particle/antiparticle projectors

Integrate out the off-shell antiparticles

$$\mathcal{L}_f = \sum_{\mathbf{v}} \left(\chi^\dagger_{+v}(x) iv\cdot D \chi_{+v}(x) - \chi^\dagger_{+v}(x) \frac{(\not{D}_\perp)^2}{2p} \chi_{+v}(x) \right)$$

This produces the same FW Hamiltonian we obtained before for fermions!

(on-shell antifermions can be treated equally)

Some advantages:

NLO corrections easier to obtain

Feynman diagram computations for corrections of different quantities, etc

in preparation

Chiral Transport Equation

Son and Yamamoto, '12; Stephanov and Yin, '12

In a collisionless case

$$\frac{\partial f_p}{\partial t} + (1+e\hbar\mathbf{B}\cdot\mathbf{\Omega})^{-1}\left\{\left[\tilde{\mathbf{v}}+e\hbar\,\tilde{\mathbf{E}}\times\mathbf{\Omega}+e\hbar\,\mathbf{B}(\tilde{\mathbf{v}}\cdot\mathbf{\Omega})\right]\cdot\frac{\partial f_p}{\partial \mathbf{r}} + e\left[\tilde{\mathbf{E}}+\tilde{\mathbf{v}}\times\mathbf{B}+e\hbar\mathbf{\Omega}\,(\tilde{\mathbf{E}}\cdot\mathbf{B})\right]\cdot\frac{\partial f_p}{\partial \mathbf{p}}\right\}=0$$

where

$$\tilde{\mathbf{E}} = \mathbf{E}-\frac{1}{e}\frac{\partial \epsilon^{+}_{\mathbf{p}}}{\partial \mathbf{r}}$$

$$\tilde{\mathbf{v}} = \frac{\partial \epsilon^{+}_{\mathbf{p}}}{\partial \mathbf{p}}$$

One can reproduce the chiral anomaly equation

$$\frac{\partial n}{\partial t} + \nabla \cdot \mathbf{j} = -e^2 \hbar \int \frac{d^3 p}{(2\pi\hbar)^3} \left(\mathbf{\Omega} \cdot \frac{\partial f_p}{\partial \mathbf{p}} \right) \mathbf{E} \cdot \mathbf{B}$$

In a thermal plasma: take into account both particles/antiparticles to correctly reproduce the chiral anomaly

$$\begin{aligned} f_p^{R,L} &= \frac{1}{\exp\left[\frac{1}{T}\left(p \mp e\hbar \frac{\mathbf{B}\cdot\mathbf{p}}{2p^2} - \mu_{R,L}\right)\right] + 1} \\ \bar{f}_p^{L,R} &= \frac{1}{\exp\left[\frac{1}{T}\left(p \pm e\hbar \frac{\mathbf{B}\cdot\mathbf{p}}{2p^2} + \mu_{R,L}\right)\right] + 1} \end{aligned}$$

$$\partial_\mu j_A^\mu = \frac{e^2}{2\pi^2 \hbar^2} \mathbf{E} \cdot \mathbf{B} \qquad \partial_\mu j_V^\mu = 0$$

Linear response analysis

Electromagnetic current obtained in a thermal plasma, with chiral misbalance

$$J^\mu(k) = \Pi_+^{\mu\nu}(k) A_\nu(k) + \Pi_-^{\mu\nu}(k) A_\nu(k)$$

$$\Pi_+^{\mu\nu}(k) = -m_D^2 \left(\delta^{\mu 0}\delta^{\nu 0} - \omega \int_v \frac{v^\mu v^\nu}{v \cdot k} \right)$$

$$\Pi_-^{\mu\nu}(k) = \frac{c_E e^2}{2\pi^2} i\epsilon^{\mu\nu\alpha\beta} k^2 k_\beta \int_v \frac{v_\alpha}{(v \cdot k)^2}$$

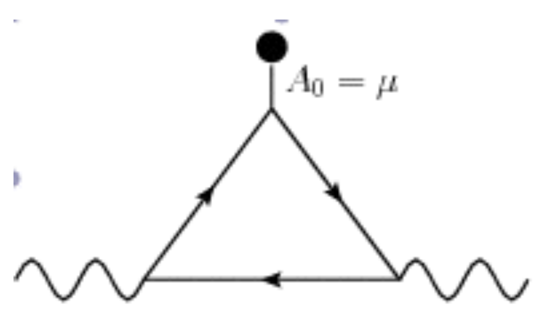
$$m_D^2 = e^2 \left(\frac{T^2}{3} + \frac{\mu_R^2 + \mu_L^2}{2\pi^2} \right) \qquad c_E = -\mu_5/2 \qquad \mu_5 = \mu_R - \mu_L$$

Both pieces (+/-) agree with the non-anomalous/anomalous Feynman diagrams computed in the HTL/HDL approximation

Laine, `05

$$S^{+}_{\rm HTL} = -\frac{m_D^2}{4}\int_{x,v} F_{\alpha\mu}(x)\,\frac{v^\alpha v^\beta}{(v\cdot\partial)^2}\,F_\beta{}^\mu(x)$$

$$S^{-}_{\rm HTL} = \frac{c_E e^2}{4\pi^2}\int_{x,v} \widetilde{F}_{\alpha\mu}(x)\,\frac{v^\alpha v^\gamma}{(v\cdot\partial)^3}\,F_\gamma{}^\mu(x)$$



Kinetic theory provides a framework to treat in a local way also the anomalous HTL effects (energy density, etc ...)

In the static limit

$$\mathbf{J}(x) = \frac{e^2 \mu_5}{4\pi^2}\,\mathbf{B}(x)$$

Chiral Magnetic Effect

$$\partial_\mu F^{\mu\nu} = J^\nu$$

The system exhibits magnetic instabilities

Joyce and Shaposnikov, `97, Laine, `05; Akamatsu and Yamamoto,‘13

Transport theory provides a perfect framework to study the dynamical evolution of the system

Conclusions

- The recent chiral transport equation can be obtained after computing the first quantum corrections to classical physics (here done with a FWD and with a EFT approach)
- The resulting transport approach describes also the anomalous HTL/HDL diagrams, and the chiral anomaly
- In presence of chiral imbalance there are magnetic instabilities