Light meson phenomenology and Bose-Einstein condensation in the 2-loop 2PI framework

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Abstract and Motivation

We present accurate FFT based numerical investigations done in Euclidean space using the 2-loop level truncation of the 2PI effective potential. The theoretical and numerical tools needed for a phenomenological application of the Phi-derivable method beyond the leading Hartree-Fock approximation were developed in [Markó et al., PRD **86** 085031 (2012)]. Our primary interest here is to study the thermodynamics of the chiral phase transition and the bounds triviality poses on the predictive power of the model [Markó et al., PRD **87** 105001 (2013)].

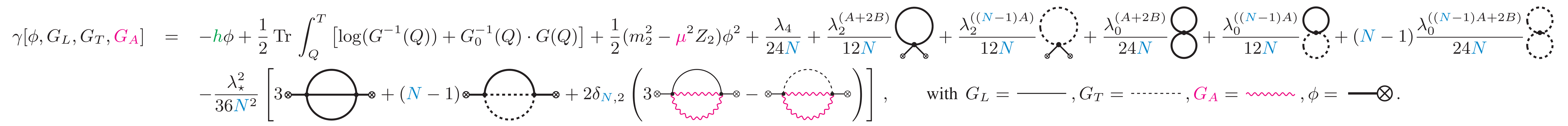
We also investigate the relation between spontaneous symmetry breaking and the forming of a Bose-Einstein condensate within the charged scalar model at finite chemical potential, where we discuss the silver blaze phenomenon [Markó et al., in preparation].

Due to the presence of the Landau pole (Λ_p), an important question when dealing with effective scalar models is whether a physical parametrization is possible, as in order to retain the predictability of the model, one has to keep the physical scales well below the scale of the Landau pole. We show that a reasonable parametrization of the O(4) model at vanishing isospin chemical potential is possible using the pion and sigma curvature masses and the pion decay constant.

As a first, however simple, application at finite chemical potential we study the charged scalar model, in order to learn how to cope with some problems specific in a 2PI setting. Later we aim to extend our investigations to assess the role of pion condensation similarly as in [Andersen, PRD **75** 065011 (2007)].

The 2PI effective potential

The 2-loop truncated 2PI effective potential reads, at non-vanishing chemical potential (μ), explicit symmetry breaking source term h and flavor number N with the propagator matrices $G = \begin{pmatrix} G_L & G_A \\ -G_A & G_T \end{pmatrix}$ and $G_0^{-1} = \begin{pmatrix} Z_0 Q^2 + m_0^2 - Z_0\mu^2 & -2Z_0\mu\omega \\ 2Z_0\mu\omega & Z_0Q^2 + m_0^2 - Z_0\mu^2 \end{pmatrix}$ in the $N=2$ case, while $G = \mathrm{diag}\,(G_L, G_T, G_T, G_T)$ and $G_0^{-1} = \mathbb{1} \times (Z_0 Q^2 + m_0^2)$ in the $N=4$ case



$$\gamma[\phi, G_L, G_T, G_A] = -h\phi + \frac{1}{2}\,\mathrm{Tr}\int_Q^T \left[\log(G^{-1}(Q)) + G_0^{-1}(Q)\cdot G(Q)\right] + \frac{1}{2}(m_2^2 - \mu^2 Z_2)\phi^2 + \frac{\lambda_4}{24N} + \frac{\lambda_2^{(A+2B)}}{12N} + \frac{\lambda_2^{((N-1)A)}}{12N} + \frac{\lambda_0^{(A+2B)}}{24N} + \frac{\lambda_0^{((N-1)A)}}{12N} + (N-1)\frac{\lambda_0^{((N-1)A+2B)}}{24N}$$

$$-\frac{\lambda_\star^2}{36N^2}\left[3 + (N-1) + 2\delta_{N,2}\left(3 - \right)\right],$$

with $G_L =$ ——, $G_T =$ ·········, $G_A =$ ∿∿∿, $\phi =$ —⊗.

Equations

The field and gap equations determining the field expectation value ($\bar\phi$) and the physical propagators ($\bar G_{L,T,A}$) at arbitrary ϕ are obtained from the stationarity conditions:

$$0 = \frac{\delta\gamma[\phi, G_L, G_T, G_A]}{\delta\phi}\bigg|_{\bar\phi,\bar G_L,\bar G_T,\bar G_A} = \frac{\delta\gamma[\phi, G_L, G_T, G_A]}{\delta G_L}\bigg|_{\phi,\bar G_L,\bar G_T,\bar G_A}$$
$$= \frac{\delta\gamma[\phi, G_L, G_T, G_A]}{\delta G_T}\bigg|_{\phi,\bar G_L,\bar G_T,\bar G_A} = \frac{\delta\gamma[\phi, G_L, G_T, G_A]}{\delta G_A}\bigg|_{\phi,\bar G_L,\bar G_T,\bar G_A}.$$

Inverting the propagators we can define the gap masses

$$\bar M^2_{L,T}(Q) = \frac{\bar G_{T,L}}{\bar G_L \bar G_T + \bar G_A^2} + \mu^2 - Q^2 \quad , \quad \bar M_A^2(Q) = -\frac{\bar G_A}{\bar G_L\bar G_T + \bar G_A^2} + 2\mu\omega$$

and at the extremum $\bar\phi$, the curvature masses are determined from

$$\frac{\bar\phi_a\bar\phi_b}{\bar\phi^2}\hat M_L^2 + \left(\delta_{ab} - \frac{\bar\phi_a\bar\phi_b}{\bar\phi^2}\right)\hat M_T^2 = \frac{\delta^2\gamma[\phi, \bar G_L, \bar G_T, \bar G_A]}{\delta\phi_a\delta\phi_b}\bigg|_{\phi=\bar\phi} + \delta_{ab}\mu^2.$$

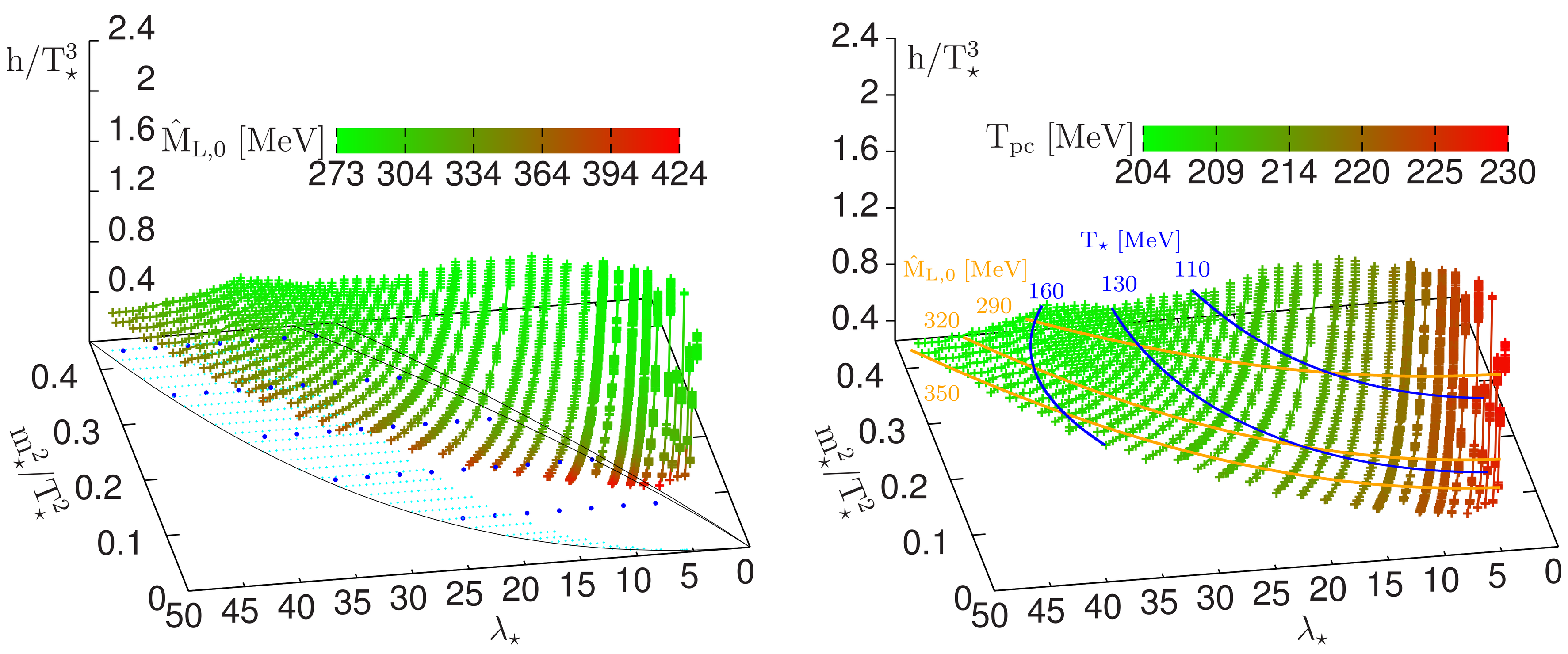
Renormalization

- Bare couplings are fixed by imposing conditions on n-point functions at $T = T_\star$ and $\mu = \bar\phi = 0$.
- Ambiguity in the definition of n-point functions (e.g. $\bar M^2$ and $\hat M^2$) is a truncation artefact. Convergence to the right theory, where no ambiguity is present, is ensured, by imposing consistency conditions, that lift the ambiguities at the renormalization point.
- 3 renormalization + 6 consistency conditions fix 9 counterterms as a function of only 2 renormalized parameters ($m_\star^2$, $\lambda_\star$) and a renormalization scale ($T_\star$).
- The bare couplings diverge positively at the cutoff value $\Lambda_p \sim m_\star\, e^{c/\lambda_\star}$, which is the Landau pole. For $\Lambda > \Lambda_p$ the bare couplings turn negative and the theory loses its meaning due to the loss of stability.

Temperature evolution

- High temperature - symmetry restoration: $\bar\phi = 0$, $\bar M_L^2 = \bar M_T^2 = \bar M^2_{\phi=0}$, while $\bar M_A^2 = 0$ and $\hat M_L^2 = \hat M_T^2 = \hat M^2_{\phi=0}$.
- Critical temperature(s): $\hat M^2_{\phi=0}(T_c) = 0$ and $\bar M^2_{\phi=0}(\bar T_c) = 0$, with $\bar T_c < T_c$. Monitoring the potential shows that a 2nd order PT occurs at T_c.
- Low temperature - broken phase: $\bar\phi \neq 0$. Goldstone theorem is fulfilled by the curvature masses, but not by the gap masses ⇒ curvature masses are used for parametrization in the $O(4)$ case.
- Coupled field and gap equations are solved iteratively.
- 3d rotation invariance ⇒ 2d p-ω grid to store the propagators $G_{L,T}$ and G_A/ω.
- Convolutions appearing in the bubble type diagrams, are computed using FFT.

Parameter space in the $O(4)$ case



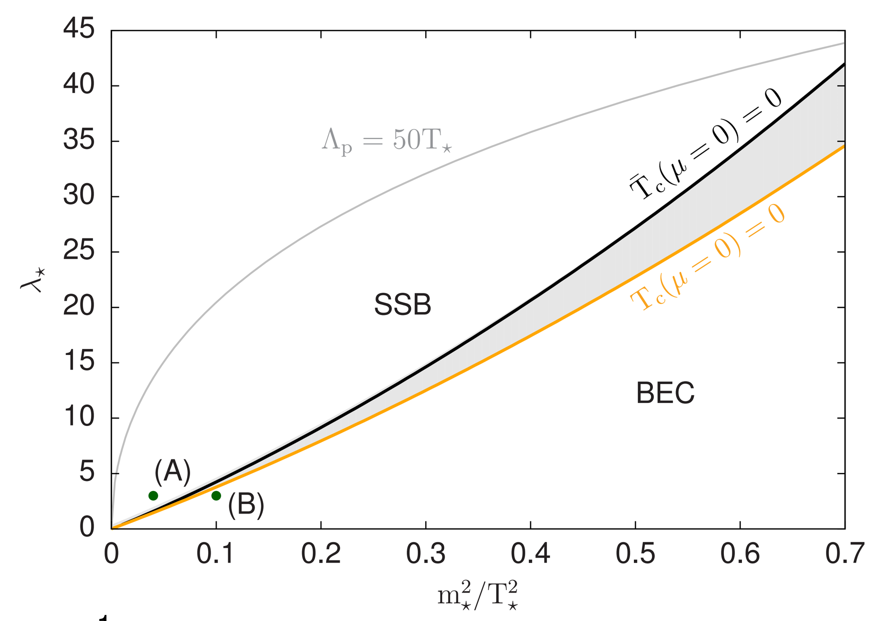
The parameter space ($m_\star^2$, $\lambda_\star$, h) is scanned, $\hat M_L \equiv m_\sigma$, $\hat M_T \equiv m_\pi$ and T_{pc} (pseudocritical temperature) are measured in units of $T_\star$ and $T_\star$[MeV] is determined from $\bar\phi(m_\star^2/T_\star^2, \lambda_\star, h/T_\star^3, T_\star = 1) \stackrel{!}{=} f_\pi/T_\star$. Those points are retained in the parameter space, where $m_\pi = 138$ MeV±1% and $m_\sigma > 2m_\pi$.

Left: The value of the sigma mass as a function of the parameters. In the examined region the value of the Landau pole, Λ_p is large, however m_σ is smaller than the physical value (450-550 MeV) [Peláez, PoS ConfinementX 019 (2012)]. The black curves are (left to right): $\Lambda_p/T_\star = 50$, $\bar T_c = 0$, $T_c^{h=0} = 0$.

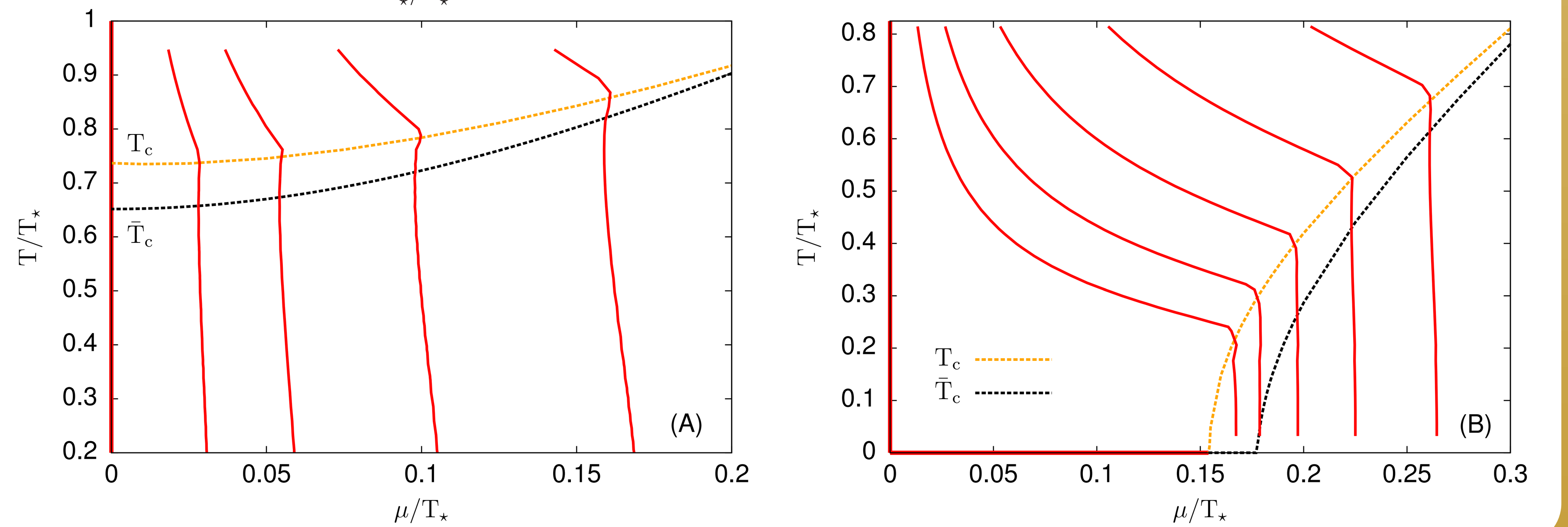
Right: The predicted value of the pseudo-critical temperature as a function of the parameters. Some iso-m_σ and iso-$T_\star$ lines are denoted with orange and blue respectively. The value of T_{pc} is less sensitive to the change of parameters than the sigma mass.

Note: The surfaces on both plots are obtained in a further simplified approximation, where the gap equations are replaced by their Hartree-Fock level variants, in order to save computer time. The dark-blue points of the left plot denote those parameter values, where for several choices of h it has been checked that the difference between the full 2-loop results and this further approximation is less than 3%.

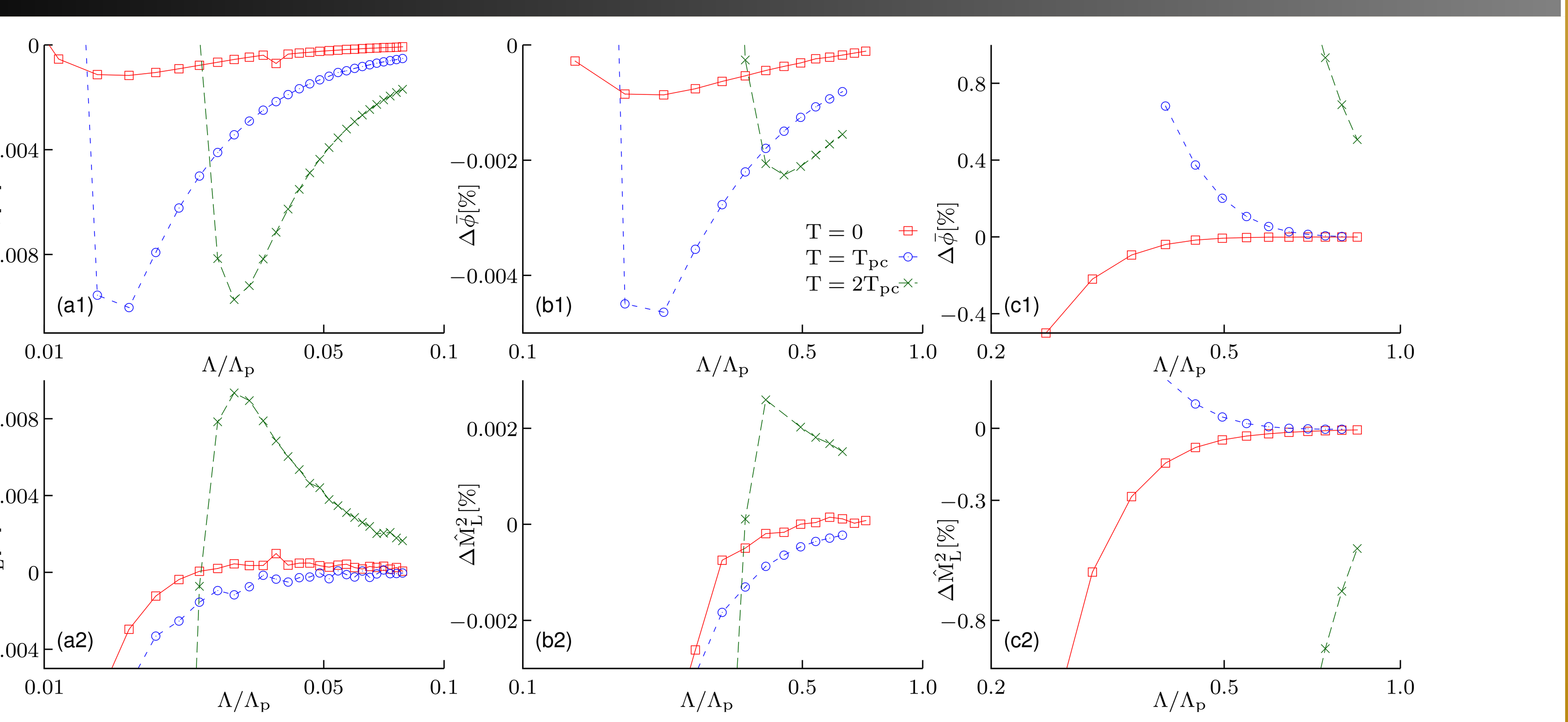
Parameter space in the $O(2)_\mu$ case



Top left: The existence of critical temperature(s) at $\mu = 0$ decides whether spontaneous symmetry breaking or Bose-Einstein condensation occurs.
Bottom both: Iso-density lines on the $T - \mu$ plane, with ρ increasing from left to right.
Bottom left: In the SSB case $T_c(\mu = 0) > 0$ therefore the $\rho = 0$ line is such that $\bar\phi(T < T_c) > 0$.
Bottom right: In the BEC case $T_c(\mu = 0) < 0$, however there exists μ_c such that $T_c(\mu_c) = 0$ and $T_c(\mu > \mu_c) > 0$. Here the $\rho = 0$ iso-density line is extended on the $T = 0$ axis up to $\mu = \mu_c$.

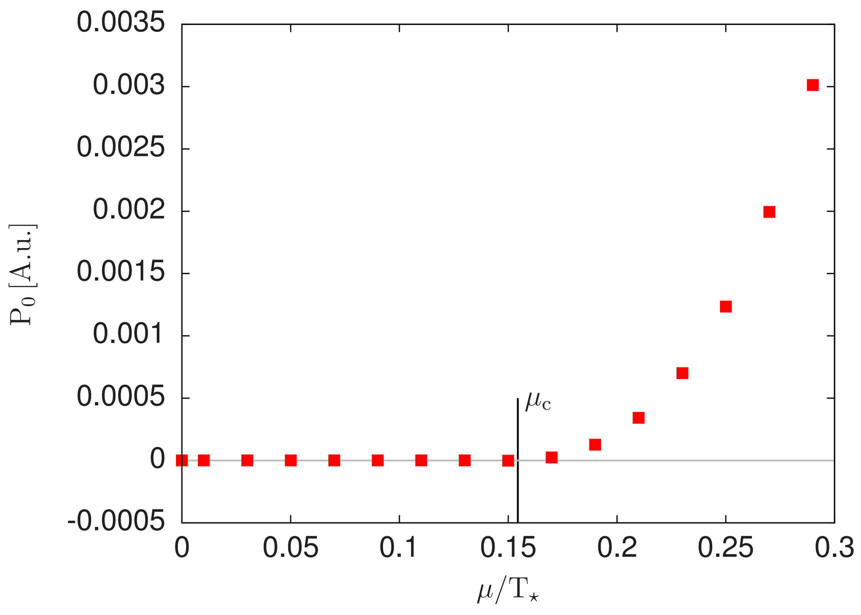


Realistic sigma mass vs. triviality bounds



The cutoff (Λ) dependence of the relative change of $\bar\phi$ and $\hat M_L^2$ at $T = \{0, T_c, 2T_c\}$, for three different sets of parameters. From left to right: m_σ[MeV] $\approx \{280, 360, 465\}$, while Λ_p[GeV] $\approx \{186, 16.2, 3.35\}$.

Silver blaze



The pressure extrapolated to $T = 0$. There is no chemical potential dependence in the symmetric phase, and the pressure starts to grow from $\mu = \mu_c$, as expected in a theory with the silver blaze property [Gattringer et al., Nucl. Phys. **B869** 65-73 (2013)].

Conclusions & Outlook

- Renormalization program and numerical method succesfully extended to $N \neq 1$ and $\mu \neq 0$.
- Phenomenological parametrization of the $O(4)$ model is possible, however cutoff independence decreases.
- Parameter space divided into SSB and BEC regions in the $O(2)_\mu$ case.
- The used truncation preserves the silver blaze property in the $O(2)_\mu$ model.
- At high T and μ the solution is lost in the $O(2)_\mu$ case, further investigations are needed.
- Merge the two projects to investigate pion condensation.