Validating the 2PI: The Bloch-Nordsieck example

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Introduction to the B-N model

Bloch-Nordsieck model ≡ IR limit of the QED

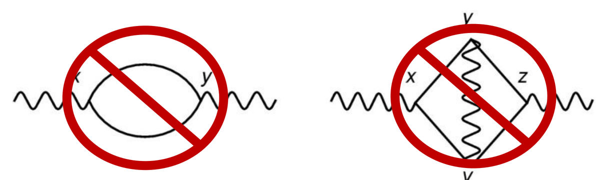
This approximation only takes into account the ultrasoft photon interactions:

- No pair creation
- No spin flip

$$\mathcal{L} = \psi^\dagger \left(i u^\mu \partial_\mu - m - e u^\mu A_\mu \right) \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$\gamma \leftrightarrow u$, where u is the 4-velocity of the fermion

No spinor structure → No radiative correction to vacuum polarization



The Bloch-Nordsieck model:

- describes the low energy region of the QED efficiently
- the full infrared fermion propagator can be calculated *exactly* (Bloch and Nordsieck 1937, Ref.1.)

A great opportunity to benchmark non-perturbative calculational techniques!

The dressed fermion

T=0

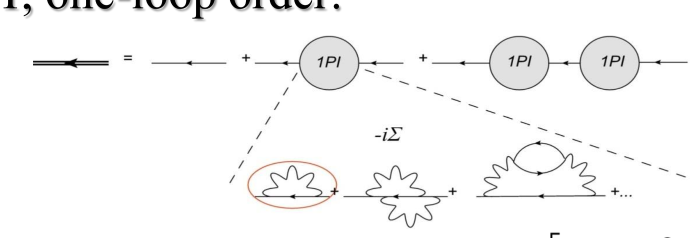
1 Using the B-N method:

$$u^\mu \left(i\partial_\mu + eA_\mu(x) \right) - m] G(x, y|A) = \delta(x-y)$$

$$G(p) = \frac{1}{(up) - m} \left(\frac{(up)}{m} - 1 \right)^{-\frac{e^2(3-\xi)}{8\pi^2}} \quad (1937)$$

- Resummed IR contribution (Ref.1.)

2 PT, one-loop order:



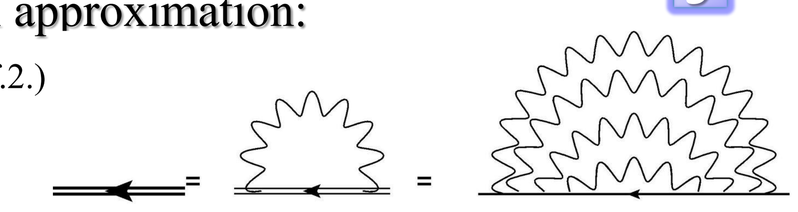
(Ref.2.)

$$G_{1-loop}(p^0) = \frac{1}{p^0 - m - \Sigma_r} = \frac{1}{p^0 - m} \left[1 - \frac{e^2}{4\pi^2} \ln \left(\frac{p^0 - m}{\lambda} \right) \right]$$

IR singularities, PT breaks down.

3 2PI approximation:

(Ref.2.)

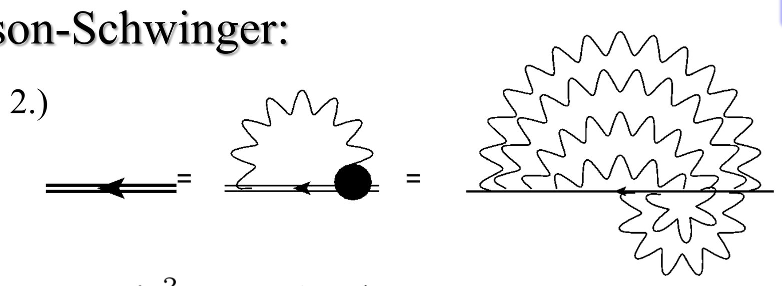


$$G[\Sigma] = \frac{1}{G_0^{-1} - \Sigma} \longleftrightarrow \Sigma[G] = \frac{-ie^2}{(2\pi)^4} \int dk^4 \frac{G(p^0 - k^0)}{k^2 + i\epsilon}$$

- Self-consistent equations, numerical solution
- Works in IR, but not a good approximation (see Ref. 2.)

4 Dyson-Schwinger:

(Ref. 2.)



$$\Sigma(p) = \frac{-ie^2}{(2\pi)^4} \int dk^4 \frac{1}{k^2 + i\epsilon} G(p-k) u_\mu \Gamma^\mu(k; p-k, p)$$

+ Ward-Takahashi

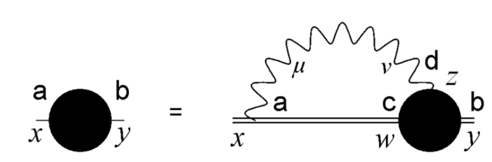
$$k_0 \Gamma^0(p, p-k, k) = G^{-1}(p) - G^{-1}(p-k)$$

Reproduces the B-N result!

T≠0

4 Exact solution at finite temperature:

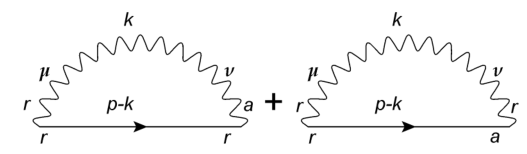
Using D-S with finite T Ward-Takahashi



$$\rho(w) = \frac{N_\alpha \beta \sin(\alpha) e^{\beta w/2}}{\cosh(\beta w) - \cos(\alpha)} \frac{1}{\left| \Gamma \left(1 + \frac{\alpha}{2\pi} + i \frac{\beta w}{2\pi} \right) \right|^2}$$

(Ref. 3.)

2 PT, one-loop order at finite temperature:

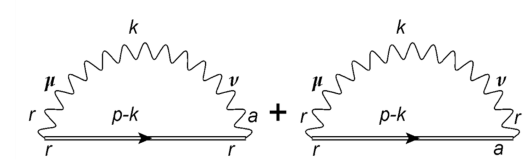


Inserting free fermion propagator

$$\underset{p_0}{\mathrm{Disc}}\, \Sigma_{ar}(p) = \frac{e^2}{2\pi} \Theta(pu - m)(pu - m) + \frac{Te^2}{4\pi u} \ln \left(\frac{1 - e^{-\beta \frac{pu - m}{u - u_0}}}{1 - e^{-\beta \frac{pu - m}{u + u_0}}} \right)$$

(Ref. 4.)

3 2PI at finite temperature:



Inserting full fermion propagator

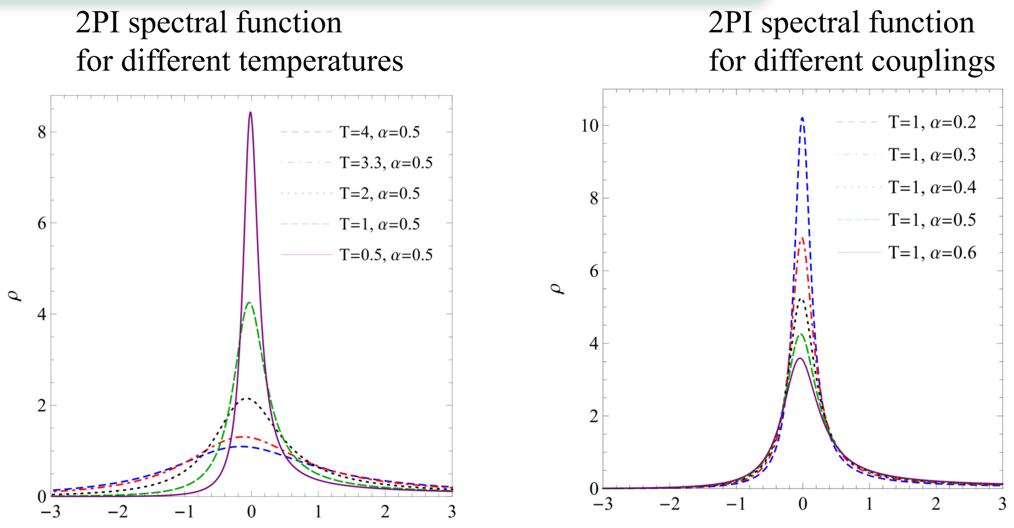
$$\underset{p_0}{\mathrm{Disc}}\, \Sigma_{ar}(w) = \frac{\alpha}{2\pi} \int_{-\infty}^{\infty} dz \bar{\rho}_f(z) \frac{T}{u} \ln \frac{1 - e^{-\beta \frac{z - w}{u_0 - u}}}{1 - e^{-\beta \frac{z - w}{u_0 + u}}}$$

(Ref. 4.)

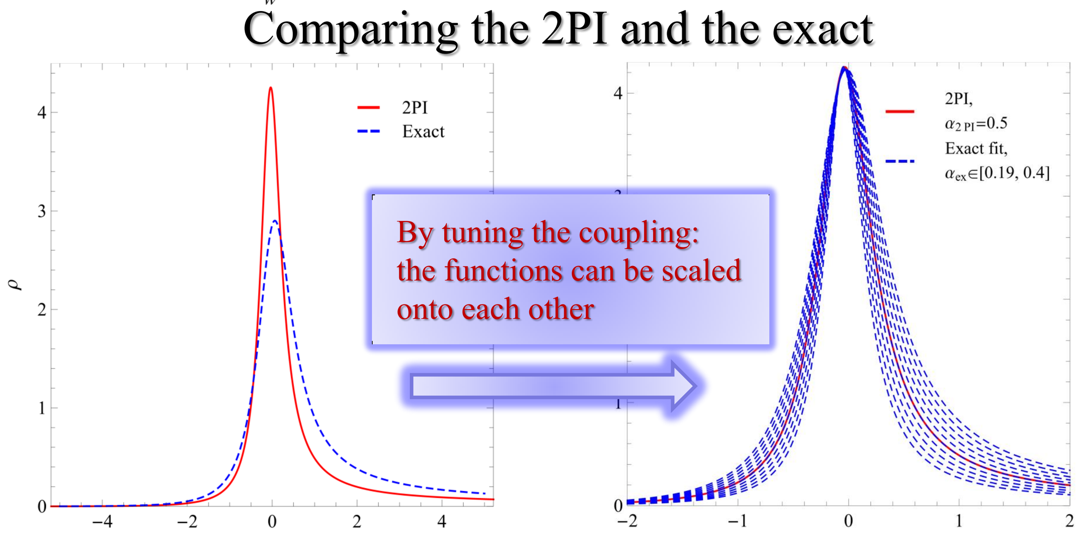
Remarks:

- to cancel UV divergencies we used T=0 renormalization procedure
- all the finite temperature calculations consistent with the T=0 results
- from DiscΣ (or ImΣ) ReΣ can be constructed using the Kramers-Kronig relations

Spectral functions at finite T



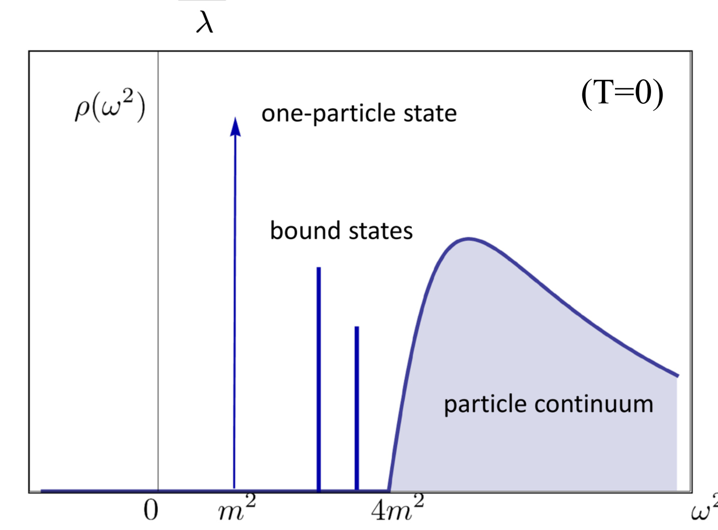
Comparing the 2PI and the exact



Källén-Lehmann spectral representation

$$\langle 0 | T\phi(x)\phi(y) | 0 \rangle = \int_0^\infty \frac{d\omega^2}{2\pi} \rho(\omega^2) G_F(x - y; \omega^2)$$

$$\rho(\omega^2) = \sum_\lambda \delta(\omega^2 - m_\lambda^2) |\langle 0 | \phi(0) | \lambda_0 \rangle|^2$$



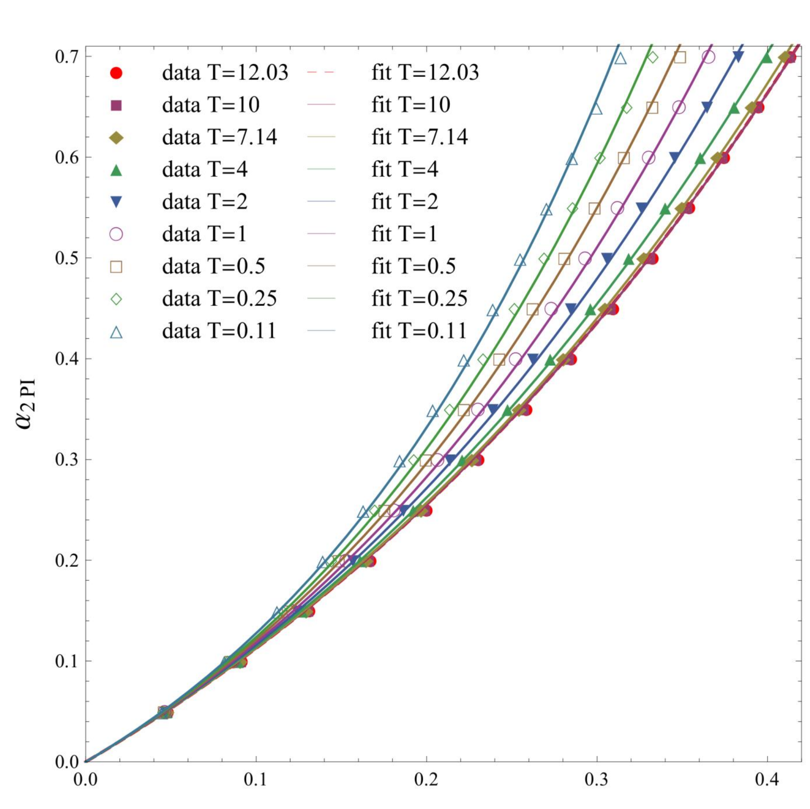
From PT (Σ is the self-energy):

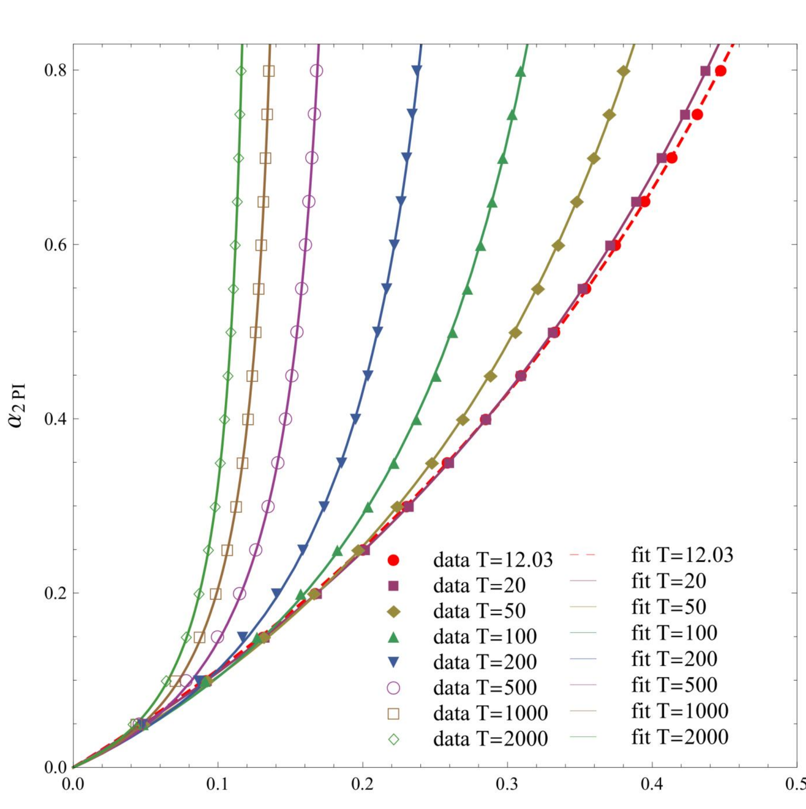
$$\rho(p^2) = \frac{2\mathrm{Im}\Sigma(p^2)}{(p^2 - m^2 - \mathrm{Re}\Sigma(p^2))^2 + (\mathrm{Im}\Sigma(p^2))^2}$$

- at T≠0 the peaks broaden
- in B-N: there is no mass-gap

Matching between the couplings

For different temperatures the relation between the 2PI and the exact couplings:

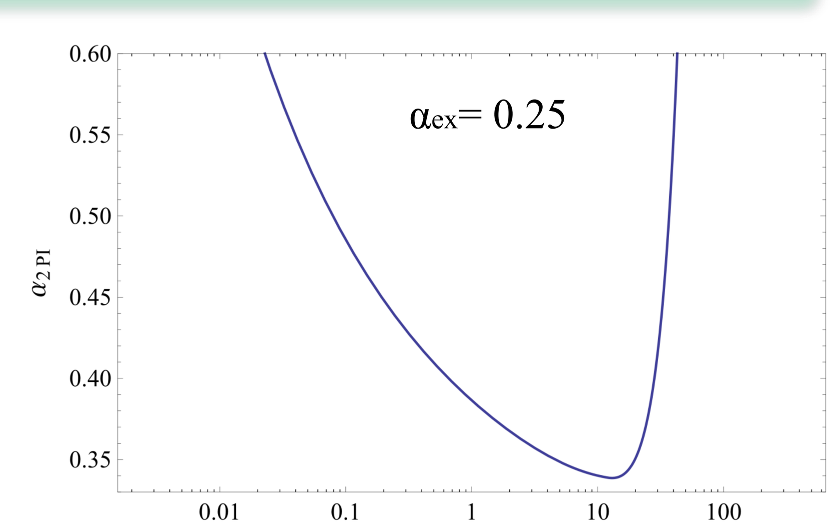




$$\alpha_{2PI} = A_T (e^{\frac{\alpha_{ex}}{A_T}} - 1) \quad (T \leq 12.03) \qquad \alpha_{2PI} = \frac{\alpha_{ex}}{B_T - C_T \alpha_{ex}} \quad (T > 12.03)$$

There are two qualitatively different mappings between the couplings separated by a (dimensionless) treshold temperature T_c=12.03 (the scale is set by B_T/C_T). A_T, B_T and C_T are fit parameters.

Running of the coupling



The temperature running of the 2PI coupling can be obtained through the mappings. Here one can observe:

- a fixed point at T_c=12.03 dim.less temperature,
- a singularity specified by B_T and C_T; this corresponds to the Landau-pole which can be seen in QED, too

Conclusion & References

We studied the Bloch-Nordsieck model (which can be considered as the IR limit of QED) at finite temperature using 2PI techniques. We found that the 2PI fermionic spectrum can be mapped on the exact one at finite temperature with a very high precision. This non-trivial mapping provides a non-peturbative temperature running of the 2PI coupling constant which shows a Landau-pole at a specific energy scale, just like in QED.

References:

1. F. Bloch and A. Nordsieck, Phys. Rev.52(1937) 54.
2. A. Jakovac and P. Mati, Phys. Rev. D85(2012) 085006 [arXiv:1112.3476 [hep-ph]]
3. A. Jakovac and P. Mati, Phys. Rev. D87(2013) 125007 [arXiv:1301.1803 [hep-ph]]
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