Dynamics of Peccei-Quinn Scalar Revisited

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Based on 14xx.yyyy
with T. Moroi, K. Nakayama, M. Takimoto

Introduction

Introduction

■ Strong CP problem

◆QCD θ-term violates CP: $\mathscr{L}_\theta = \theta \frac{\alpha_s}{8\pi} F^{a\mu\nu} \tilde{F}^a_{\mu\nu}$

➡Tightly constrained by neutron **E**lectric **D**ipole **M**oment: $|\theta| \lesssim 10^{-10}$

Why is it so small??

■ Peccei-Quinn (PQ) Mechanism [Peccei, Quinn, '77; Weinberg, '78; Wilczek, '78]

◆Introduce $\mathbf{U(1)_{PQ}}$ so that $U(1)_{PQ}$-$SU(3)_C$-$SU(3)_C$ becomes anomalous.

◆Spontaneous breaking of $\mathbf{U(1)_{PQ}}$ at a scale $f_a = \sqrt{2}\langle|\Phi|\rangle$. [Φ: PQ-Scalar]

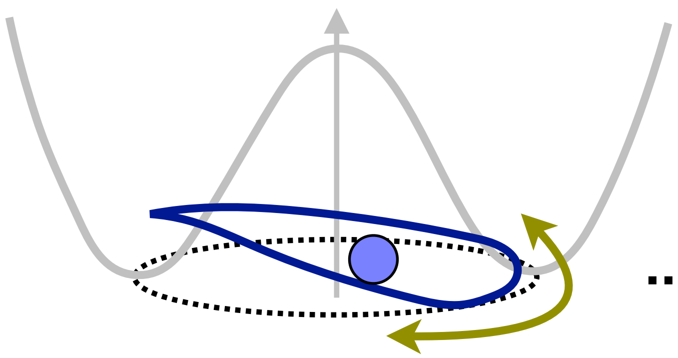
◆θ becomes dynamical as a Pseudo NG-boson: "**Axion**". **➡θ = 0 at Vacuum.**

Cold Dark Matter (CDM) candidate

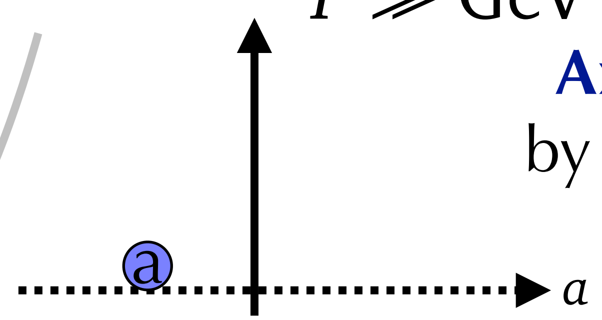
Introduction

■ Axion Cold Dark Matter (3 contributions)

◆Coherent oscillation:

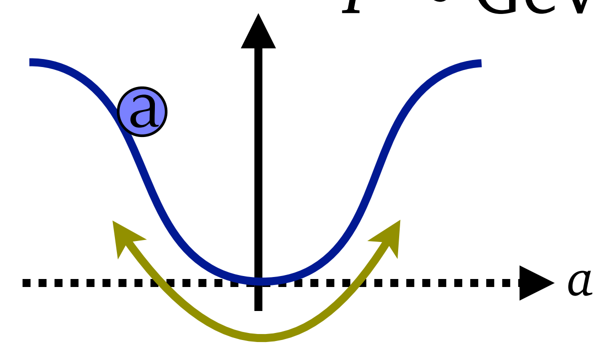


$T \gg$ GeV

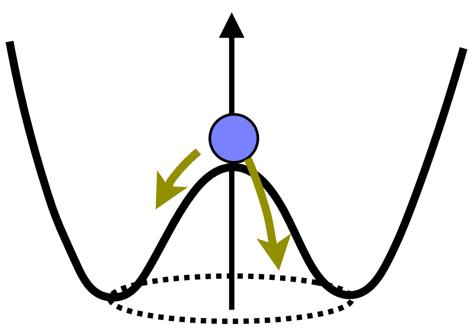


Axion acquires mass
by the instanton effect.

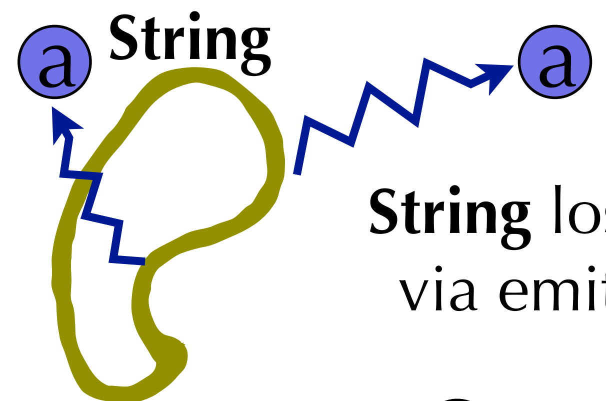
$T \sim$ GeV



◆Axionic string radiation: $T \gg$ GeV

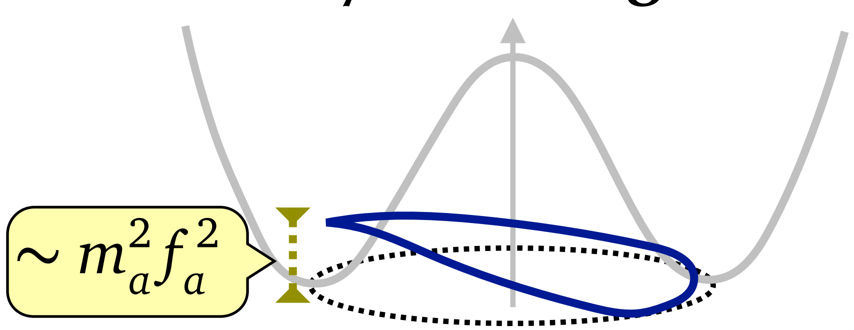


String formation due
to SSB of $U(1)_{PQ}$

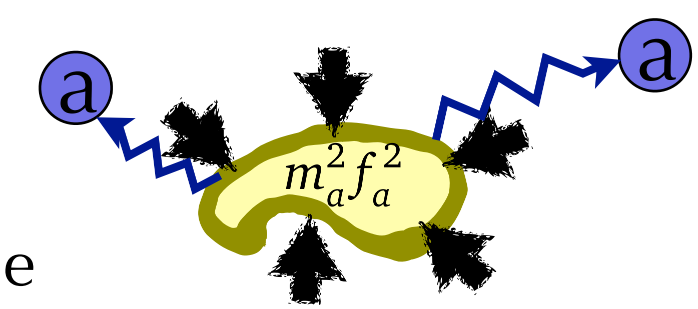


String loses its energy
via emitting **axions.**

◆Decay of string-wall system: $T \sim$ GeV



String-wall system becomes unstable
due to the instanton effect.

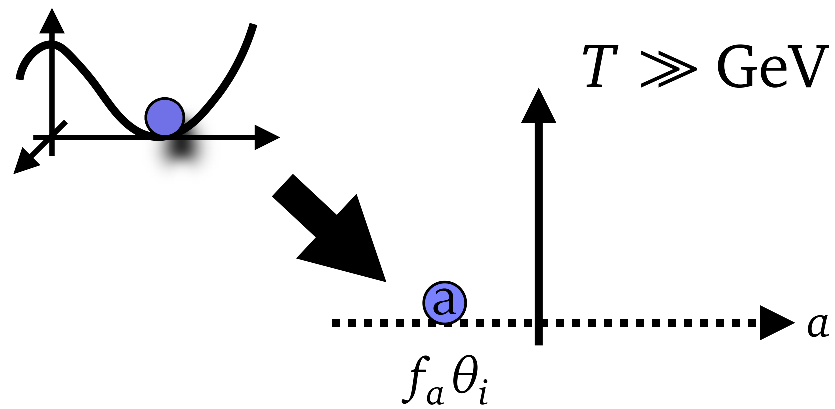


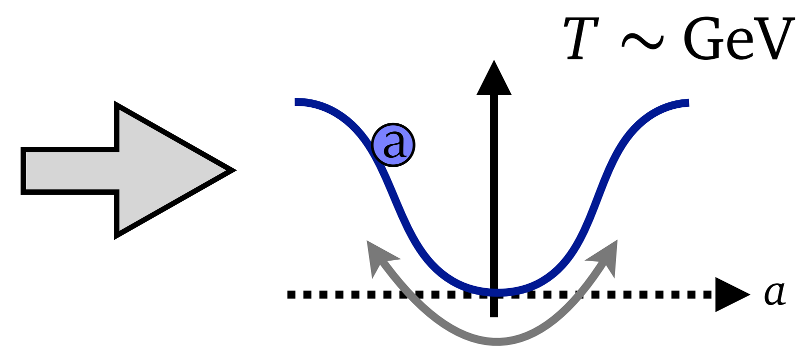
Collapse and
produce **axions.**
(N_{DW}=1)

Introduction

■ Axion v.s. High Scale Inflation

◆If ~~$U(1)_{PQ}$~~ during the inflation,





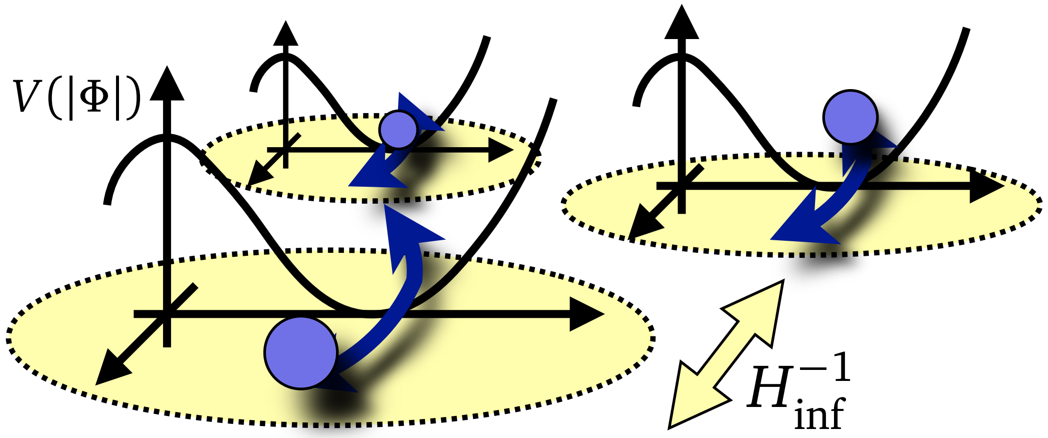
➡**CDM density:** $r \equiv \dfrac{\Omega_a}{\Omega_c} \simeq 1.5 \left(\theta_i^2 \quad \right) \left(\dfrac{f_a}{10^{12}\,\mathrm{GeV}} \right)^{1.19} \left(\dfrac{\Lambda}{400\,\mathrm{MeV}} \right)$

✦Homogenous value~O(1)

Introduction

■ Axion v.s. High Scale Inflation

- If ~~U(1)~~$_{PQ}$ during the inflation, the axion acquires **quantum fluctuations**.



$$|\delta\theta| = \frac{H_{\rm inf}}{2\pi\varphi_{\rm inf}}; \quad \varphi_{\rm inf} \equiv \sqrt{2}\,\langle|\Phi|\rangle_{\rm inf}$$

➡**CDM density:** $r \equiv \frac{\Omega_a}{\Omega_c} \simeq 1.5\left(\theta_i^2 + \delta\theta^2\right)\left(\frac{f_a}{10^{12}\,\mathrm{GeV}}\right)^{1.19}\left(\frac{\Lambda}{400\,\mathrm{MeV}}\right)$

✦Homogenous value~O(1) ✦**Fluctuations**

[Kawasaki, Nakayama, '13]

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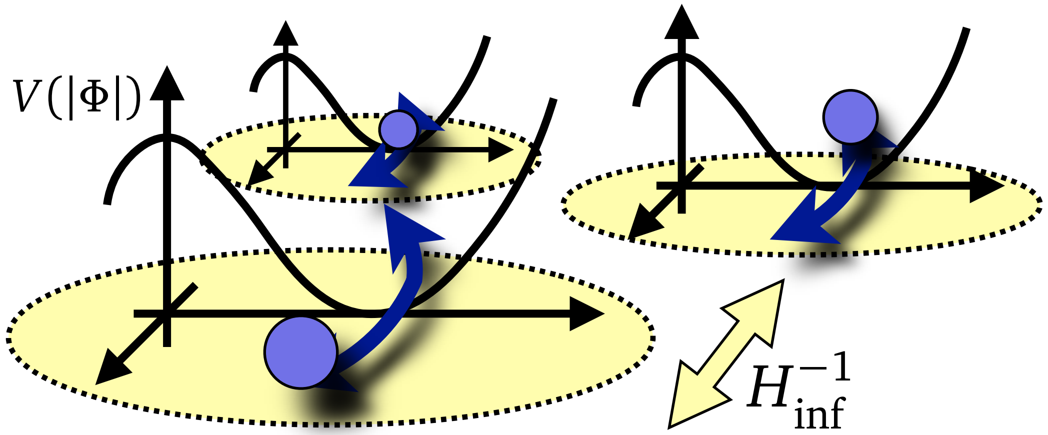

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✦Homogenous value~O(1) ✦**Fluctuations**

[Kawasaki, Nakayama, '13]

➡**Axion Isocuvature:** tightly constrained by **CMB** observation. [Planck, '13]

$$\mathcal{P}_{S_{\rm CDM}} = 4r^2 \frac{\delta\theta^2}{\theta_i^2 + \delta\theta^2} \lesssim 10^{-10}$$

[Linde, Lyth, '90]

This constraint can be avoided
if U(1)$_{PQ}$ is restored during (after) inflation.

Outline

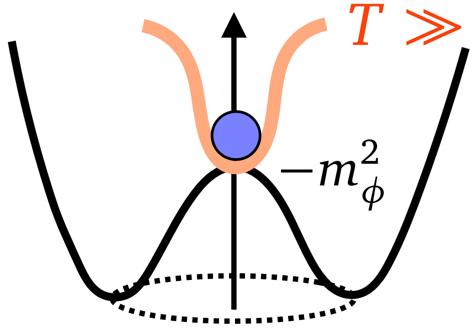
- Introduction
- Thermal History
- Dynamics of PQ Scalar
- Conclusion

Thermal History

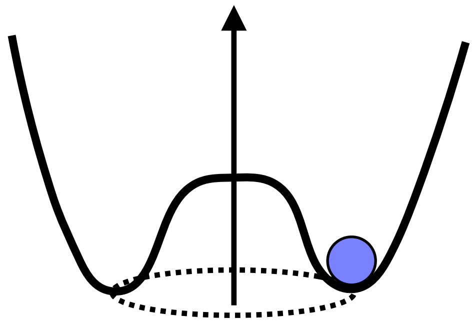
Thermal History

■ Rough Sketch of Thermal History

End of Inflation

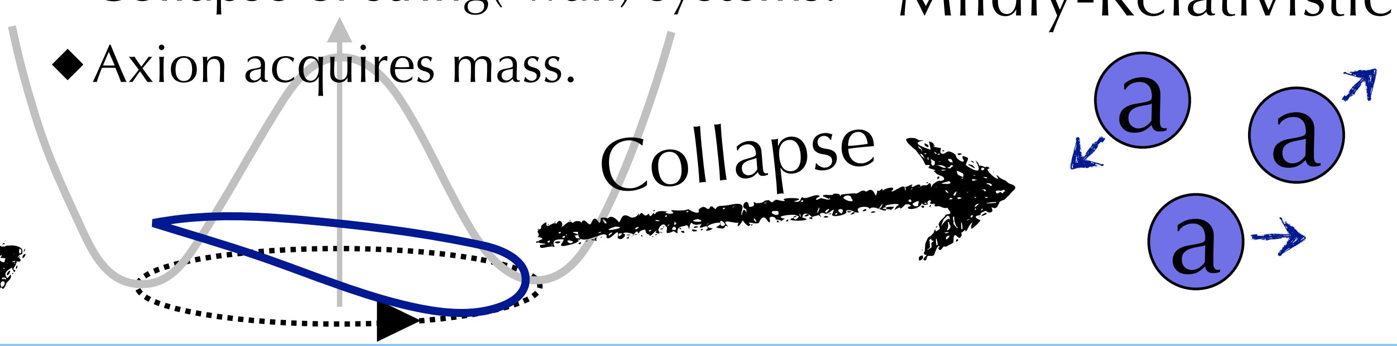


$T \sim m_\phi/\lambda$ **Spontaneous Breaking of $U(1)_{PQ}$; string formation**



$T \sim \mathrm{GeV}$ **QCD Instanton Effect**

- Collapse of string(-wall) systems.
- Axion acquires mass.



Dark Matter

- Dominated by string decay.

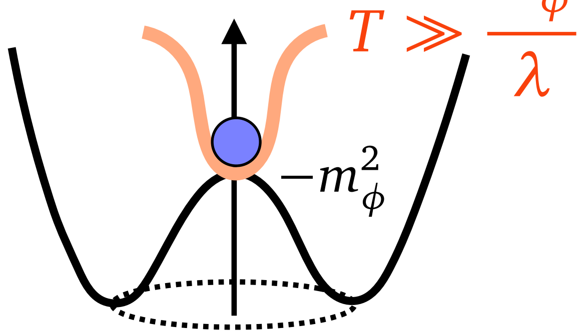
$f_a \lesssim (2.0-3.8)\times 10^{10}\ \mathrm{GeV}$

[Hiramatsu et al.,'12]

Thermal History

■ Rough Sketch of Thermal History

End of Inflation



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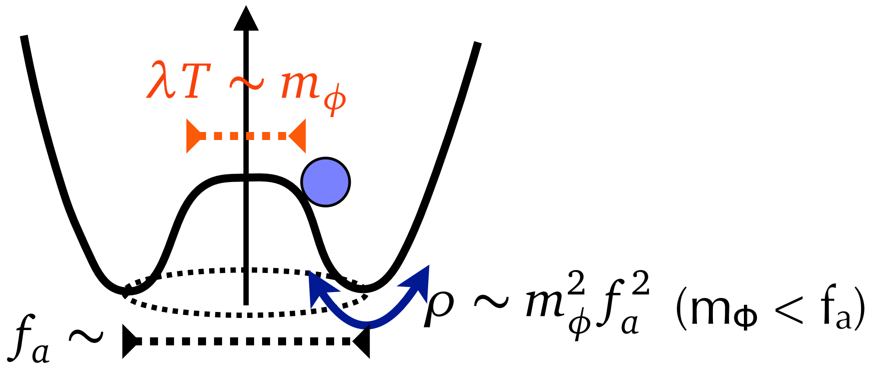

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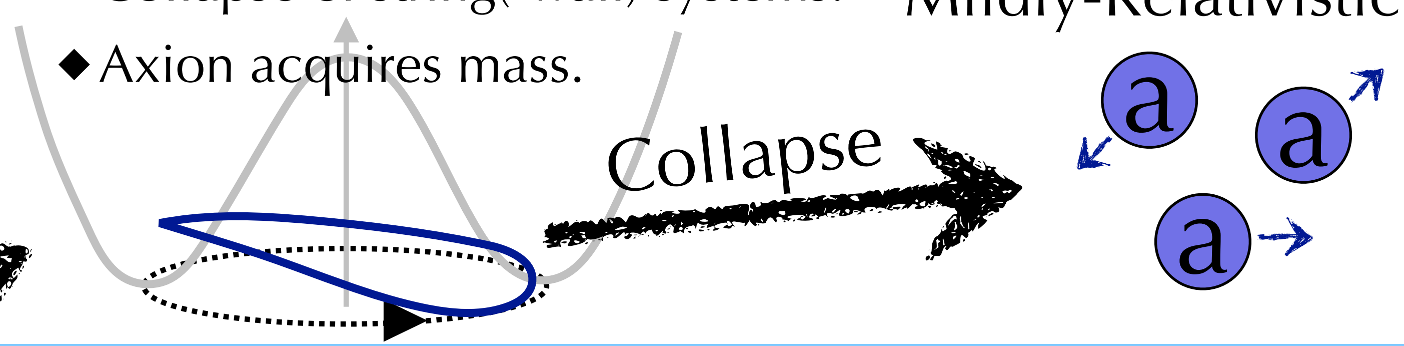
Mildly-Relativistic **Axions**



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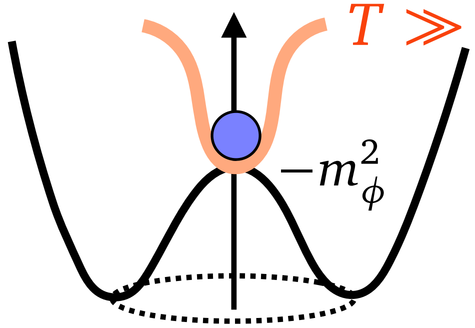

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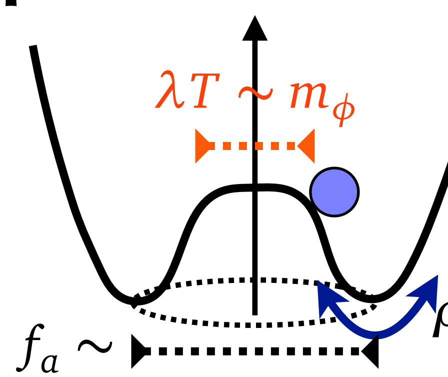
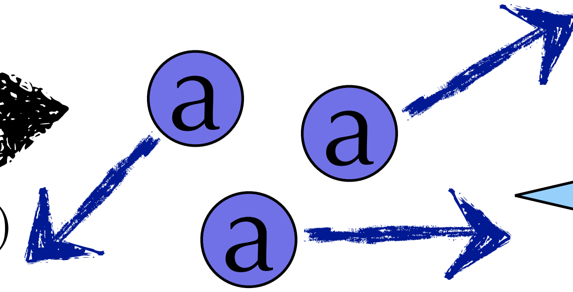


Ultra-Relativistic **Axions**



Dark Radiation

- Additional relativistic d.o.f.
- Constrained by **CMB**.

$N_{\rm eff} = 3.36^{+0.68}_{-0.64}$ (2σ)

($N_{\rm eff} \simeq 3 + \Delta N_{\rm eff}$)

$T \sim$ GeV

QCD Instanton Effect

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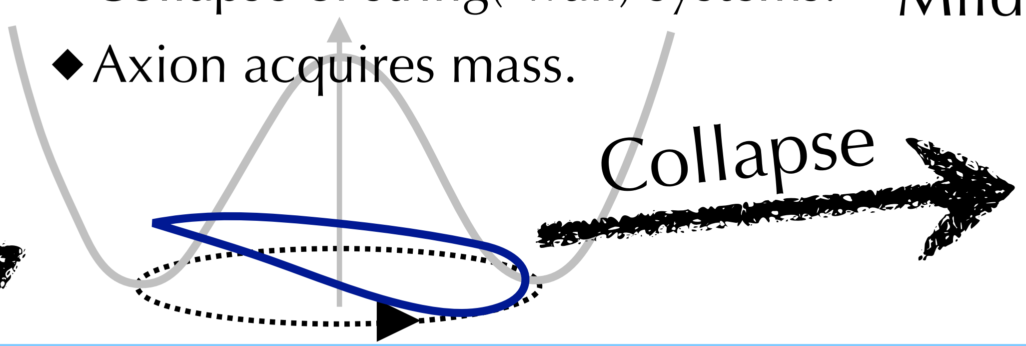

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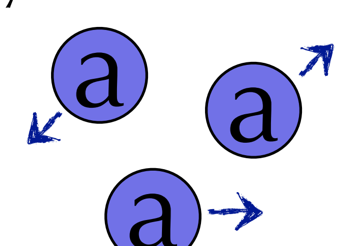

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Thermal History

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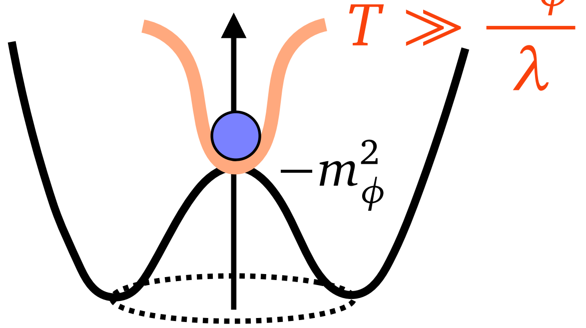


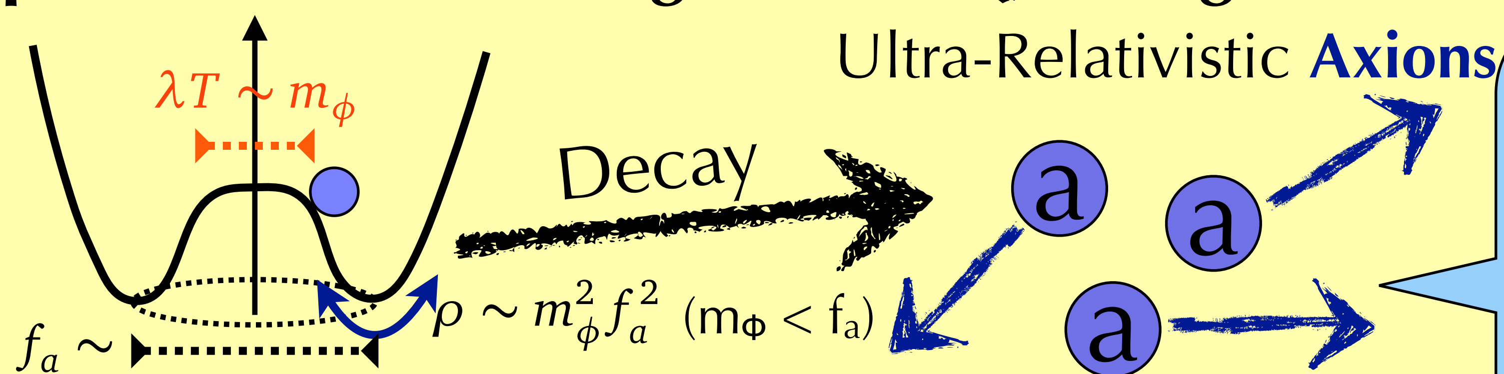
Interaction with Thermal Plasma

◆ **PQ scalar Φ** interacts w/ thermal plasma via a **f_a-suppressed** term.

➡ Reduce the amount of **Axion Dark Radiation**.

$T \sim m_\phi/\lambda$

Spontaneous Breaking of $U(1)_{PQ}$; string formation



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$$N_{\rm eff} = 3.36^{+0.68}_{-0.64} \ (2\sigma)$$

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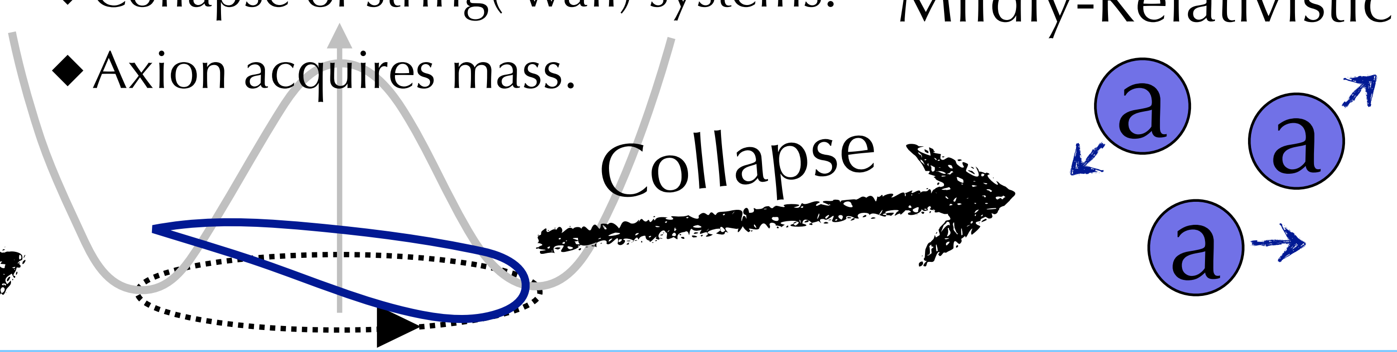

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$f_a \lesssim (2.0 - 3.8) \times 10^{10}$ GeV

[Hiramatsu et al.,'12]

Dynamics of PQ Scalar

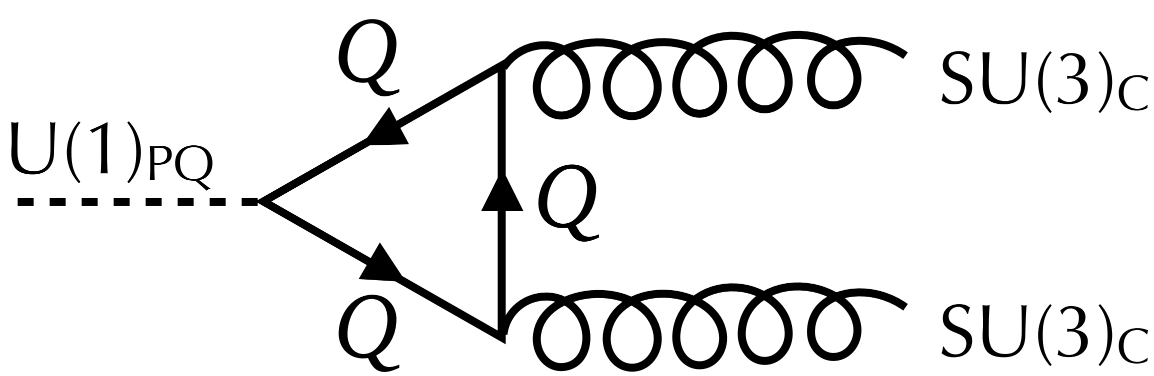
Dynamics of PQ Scalar

- Let us consider a simple **hadronic axion model**

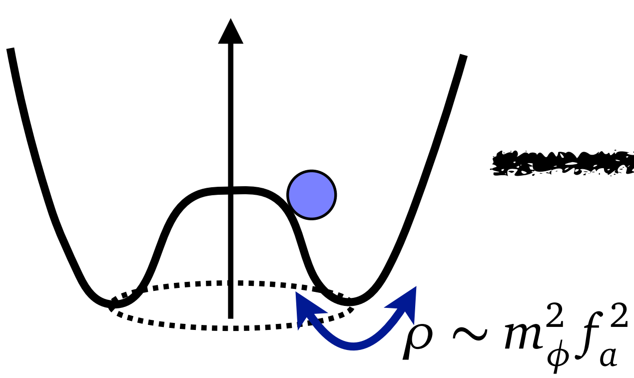
$$\mathcal{L} = |\partial\Phi|^2 - (\lambda\Phi Q\bar{Q} + \mathrm{h.c.}) - \frac{m_\phi^2}{2f_a^2}\left(|\Phi|^2 - \frac{f_a^2}{2}\right)^2 \; ; \quad \Phi = \frac{1}{\sqrt{2}}(f_a + \varphi)\, e^{ia/f_a}$$

◆Charge assignment:

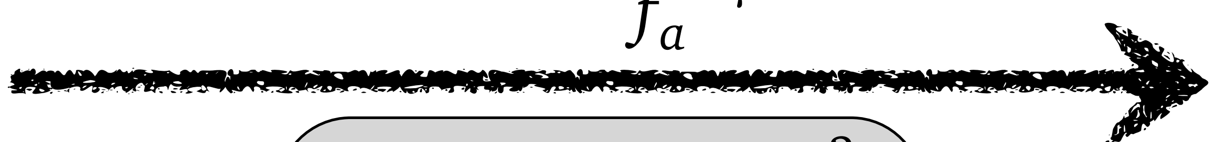
	Φ	Q	$\bar{Q}$
$\mathrm{U}(1)_{\mathrm{PQ}}$	$+1$	-1	0
$\mathrm{SU}(3)_{\mathrm{C}}$	1	3	$\bar{3}$



◆Decay of $\varphi \to 2a$:

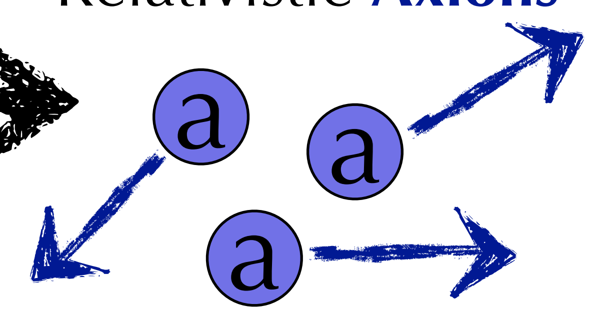


Decay via $\frac{\varphi}{f_a}\partial_\mu a \partial^\mu a$



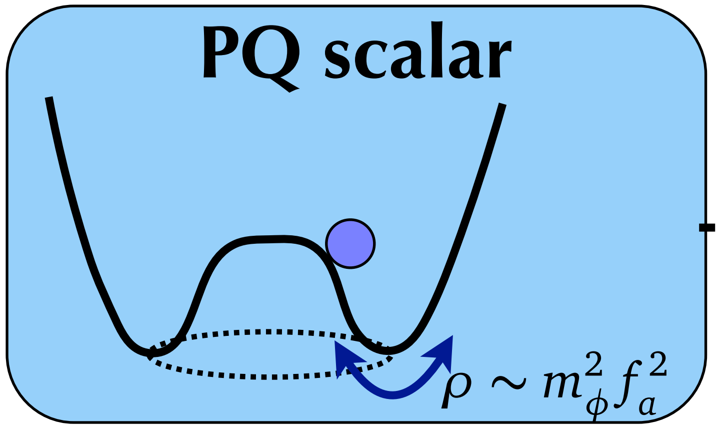
$$\Gamma_{\varphi\to 2a} = \frac{1}{32\pi}\frac{m_\varphi^3}{f_a^2}$$

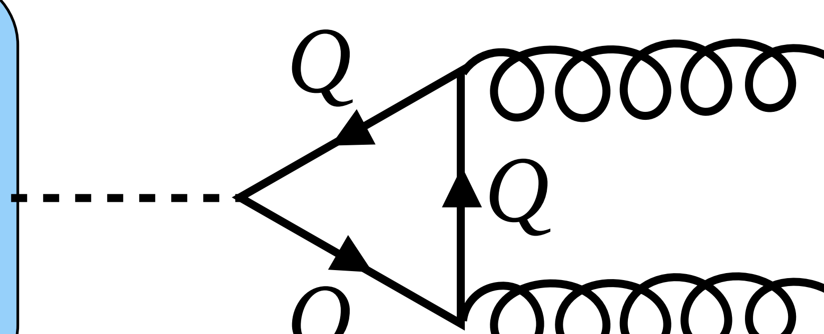
Relativistic **Axions**



Dynamics of PQ Scalar

- **Thermal Dissipation** of PQ scalar





Thermal Plasma

- Many relativistic d.o.f.
- Standard Model:

$$g_* = 106.75$$

$$\sim \frac{\alpha_s}{f_a} \begin{cases} \varphi F^a_{\mu\nu} F^{a\mu\nu} & \text{for } \varphi \\ a F^a_{\mu\nu} \tilde{F}^{a\mu\nu} & \text{for } a \end{cases}$$

$$\Gamma^{(\text{dis})} = \left.\frac{\rho_O(P)}{2p_0}\right|_{p_0=\omega_p}, \quad \rho_O(P) = C^2 \frac{\alpha_s^2}{f_a^2} \int_P e^{iP\cdot x} \langle [O(x), O(0)] \rangle ; \quad O = \begin{cases} F^a_{\mu\nu} F^{a\mu\nu} & \text{for } \varphi \\ F^a_{\mu\nu} \tilde{F}^{a\mu\nu} & \text{for } a \end{cases}$$

- Dissipation of φ ($p_0 = m_\Phi$, $p = 0$)

$$\Gamma_\varphi^{(\text{dis})} \simeq \frac{b\alpha_s^2 T^3}{f_a^2} \times \begin{cases} 1 & \text{for } m_\phi \ll g_s^4 T \\ \sqrt{\frac{g_s^4 T}{m_\phi}} & \text{for } g_s^4 T \ll m_\phi \ll g_s^2 T \end{cases}$$

- Dissipation of a (relativistic: $p_0 = p$)

$$\Gamma_a^{(\text{dis})} \simeq \frac{b\alpha_s^2 T^3}{f_a^2} \frac{p^2}{g_s^4 T^2} \times \begin{cases} 1 & \text{for } p \ll g_s^4 T \\ \left(\frac{g_s^4 T}{p}\right)^2 & \text{for } g_s^4 T \ll p \ll g_s^2 T \end{cases}$$

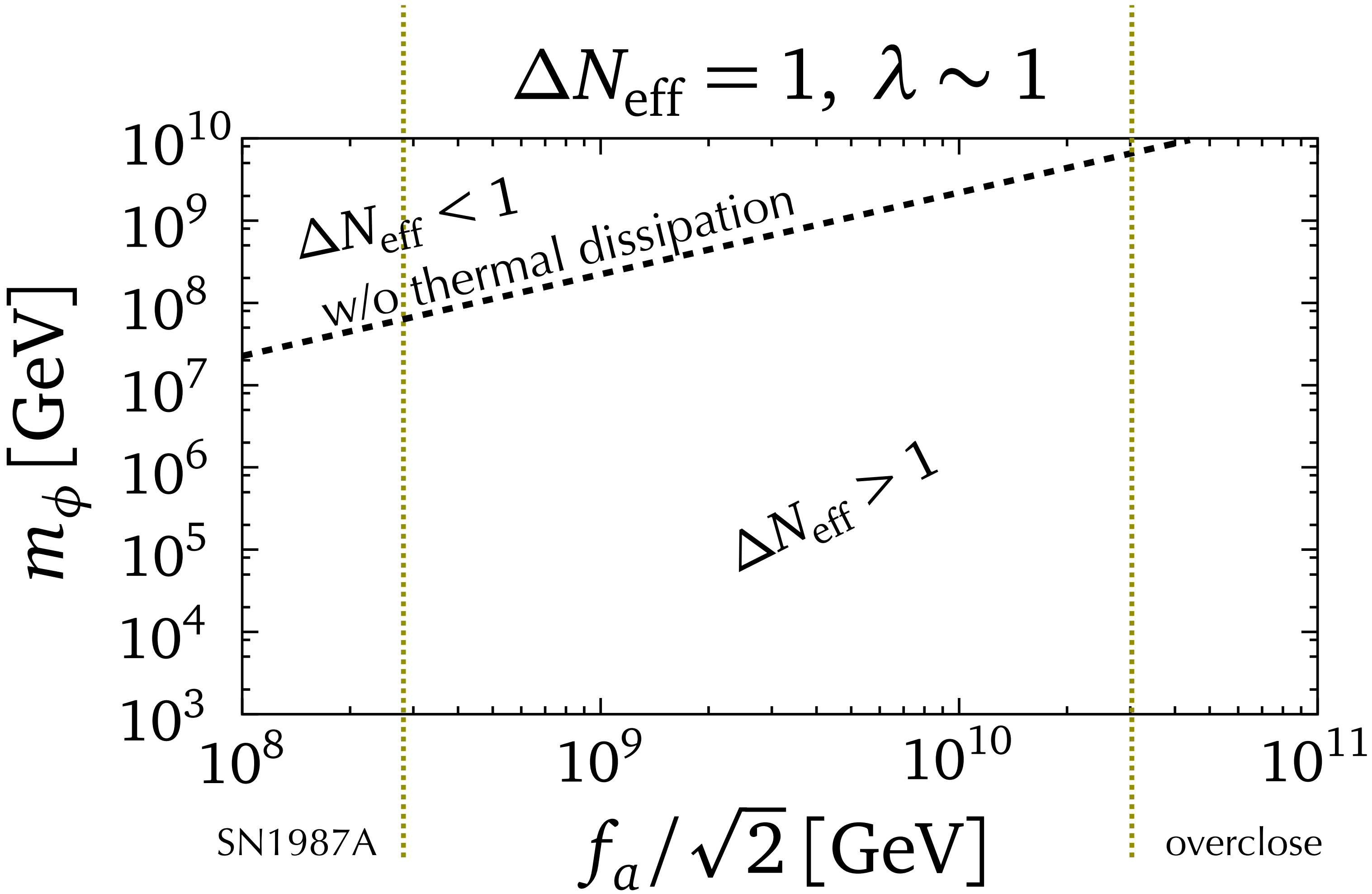
➡ Related w/ the **bulk viscosity**. [Bodeker, '06; Laine, '10]

✦ For $m_\Phi > g^2 T$, the cut contribution becomes important. HTL results for $m_\Phi > gT$ are given in [Graf, Steffen, '11,'13].

Dynamics of PQ Scalar

- Contour plot of **ΔN_{eff} = 1** as a function of **m_Φ** and **f_a**.

  - **Axion Dark Radiation:** $\Delta N_{\rm eff} \equiv N_{\rm eff} - N_{\rm eff}^{\rm (std)}$.



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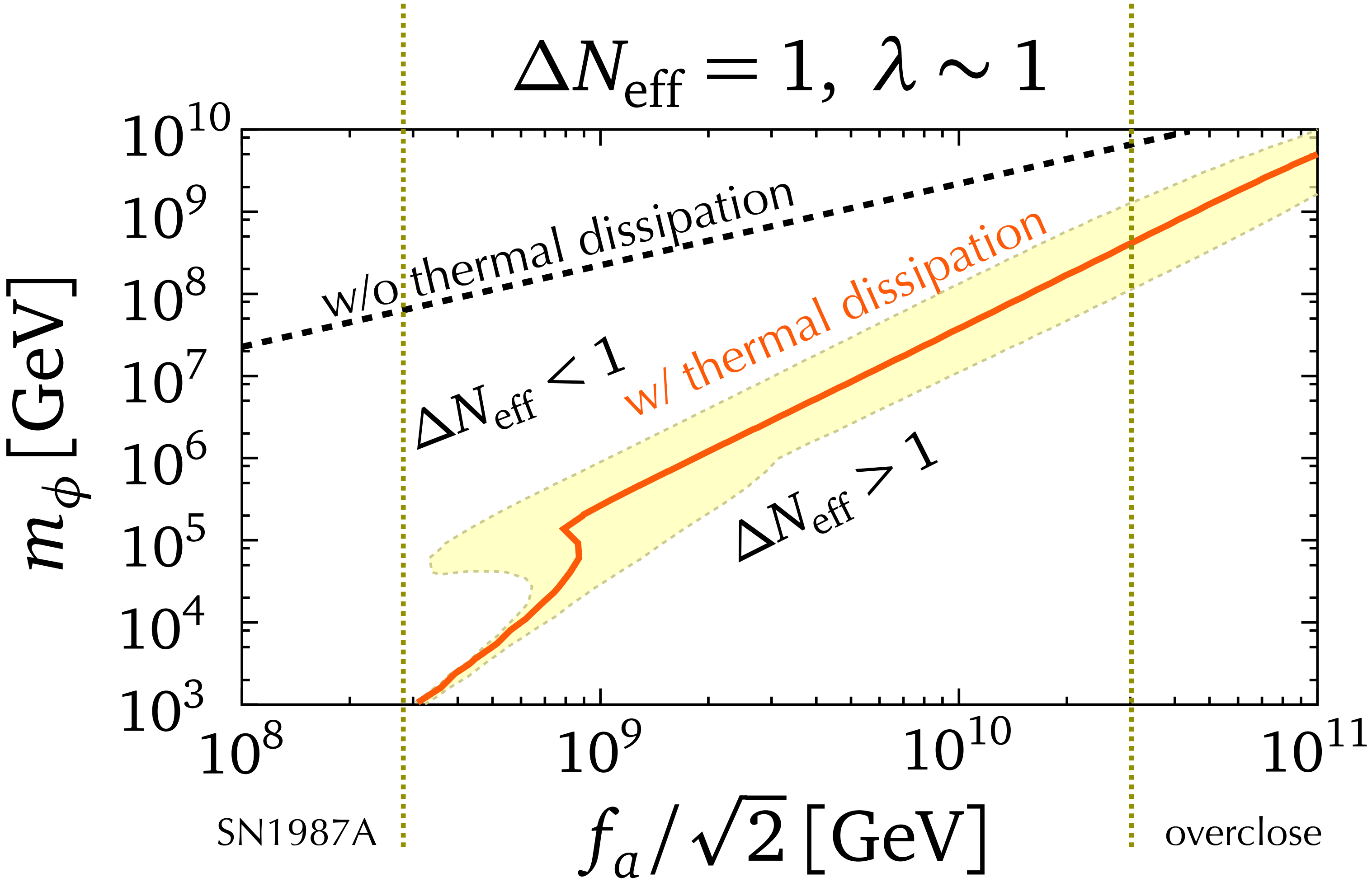
◆**Interaction between PQ scalar and thermal plasma plays crucial roles!**



Dynamics of PQ Scalar

- Contour plot of **$\Delta N_{eff} = 1$** as a function of **m_ϕ** and **f_a**.

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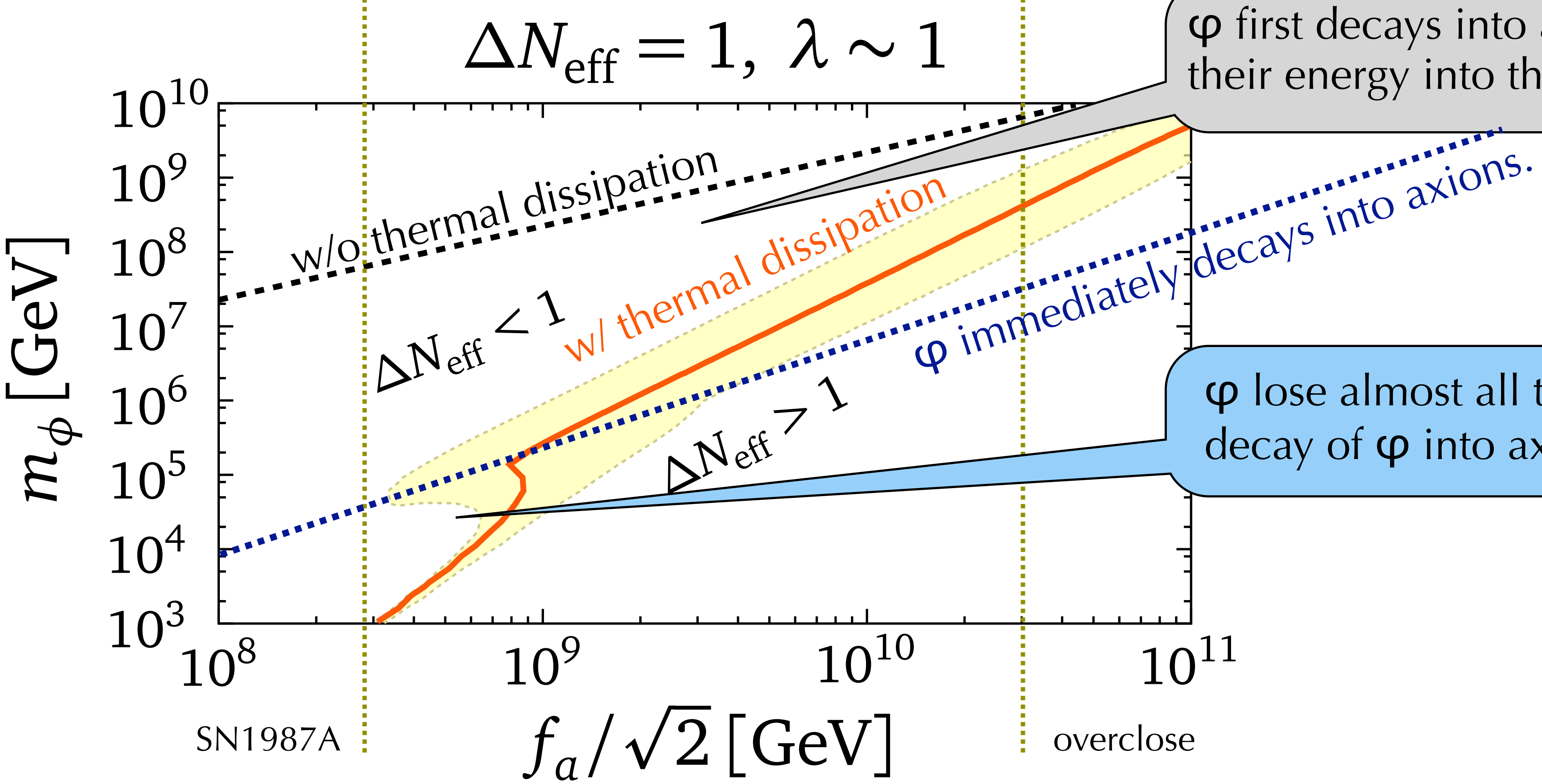


φ first decays into axions. Then axions lose their energy into thermal plasma.

φ lose almost all the energy before the decay of φ into axions becomes efficient.

Conclusion

Conclusion

- We have revisited the dynamics of **PQ scalar** after the PQ phase transition for $m_\Phi < f_a$, paying attention to the **axion dark radiation** constraint.

- It is shown that **interactions with thermal plasma** play crucial roles in **reducing the axion dark radiation**.

- Typical example of $m_\Phi < f_a \rightarrow$ SUSY axion model. In this case, we have to avoid the **axino overproduction** simultaneously.

➡ Strongly model dependent (e.g., R_p-violation, mass spectrum, dilution by another PQ scalar etc)

Back Up

Bulk Viscosity

- The **dissipation rate of φ** is related with the **bulk viscosity**.

[Bodeker, '06; Laine, '10]

- **Dissipation rate of φ** is given by

$$\Gamma^{(\mathrm{dis})} = \left.\frac{\rho_O(P)}{2p_0}\right|_{p_0=\omega_p}, \; \rho_O(P) = C^2 \frac{\alpha_s^2}{f_a^2} \int_P e^{iP\cdot x} \langle [O(x), O(0)] \rangle \,; \quad O = F^a_{\mu\nu} F^{a\mu\nu}.$$

Trace Anomaly: $T^\mu_\mu \simeq \frac{B}{2} F^a_{\mu\nu} F^{a\mu\nu} = \frac{B}{2} O$

- **Bulk viscosity** is given by

$$\zeta = \left.\frac{\rho_{T^\mu_\mu}(p_0, \mathbf{0})}{2p_0}\right|_{p_0\to 0}, \; \rho_{T^\mu_\mu}(p_0, \mathbf{0}) = \frac{1}{9} \int_P e^{i\omega t} \left\langle [T^\mu_\mu(x), T^\nu_\nu(0)] \right\rangle.$$

➡LO weak coupling expression: $\zeta \simeq \dfrac{B^2 g_s^4 T^3}{4\ln(\alpha_s^{-1})}$ [Arnold, Dogan, Moore, '06]

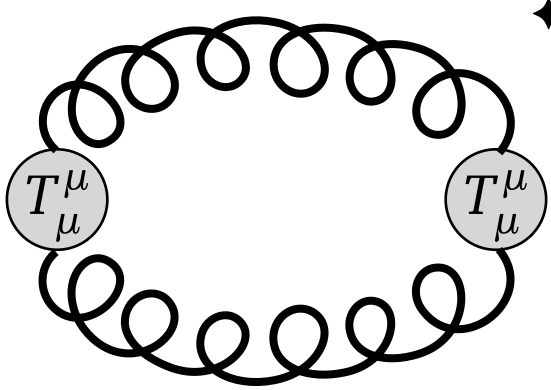
➡**Dissipation rate of φ** can be expressed as

$$\Gamma^{(\mathrm{dis})} \simeq \frac{C^2\alpha_s^2}{f_a^2} \frac{4\times 9\zeta}{B^2} \simeq \frac{(12\pi C)^2 \alpha_s^2 T^3}{\ln(\alpha_s^{-1})}.$$

One-Loop Estimation

- Though one needs to resum many diagrams to obtain complete LO results, one may estimate its order by one-loop computations.

◆ **Bulk Viscosity:**

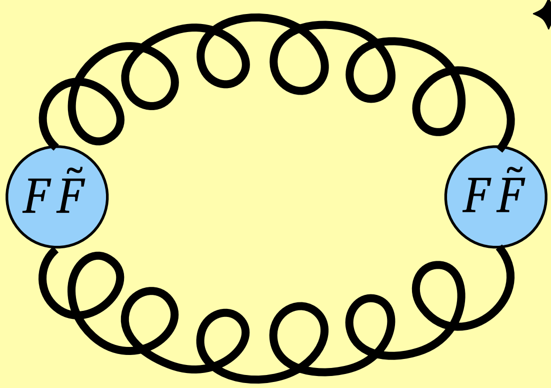


✦ Regulated by the **width,** $\Gamma_p \sim g_s^4 T^3/p^2$, which comes from thermal interactions.

$$\left.\frac{\rho_{T^\mu_\mu}(\omega,\mathbf{0})}{2\omega}\right|_{\omega \ll \Gamma} \sim \int \frac{\Gamma_p}{\omega^2+\Gamma_p^2}\frac{p^2 dp}{E_p^2 T} f_{\mathrm{B}}(E_p)(1+f_{\mathrm{B}}(E_p)) \left[\left(\frac{1}{3}-v_s^2\right)p^2 + \frac{\beta(g)m_D^2}{2g^2}\right]^2$$

[Moore, Saremi, ‘08]

◆ **Axion Dissipation:**



✦ Regulated by the **width,** $\Gamma_p \sim g_s^4 T^3/p^2$, which comes from thermal interactions.

$$\left.\frac{\rho_{F\tilde{F}}(\omega,k)}{2\omega}\right|_{\omega = k \ll \Gamma} \sim \frac{\alpha_s^2 T^3}{f_a^2}\int \frac{T\Gamma_p}{\omega^2+\Gamma_p^2}\frac{p^2 dp}{T E_p^2}\frac{1}{T^2}\left[p^2 + \frac{1}{3}E_p^2\right]$$