The Polyakov loop extension to the quark meson model at finite B and μ

William Naylor

Norwegian University of Science and Technology (NTNU)

Jens Andersen (NTNU) and Anders Tranberg (University of Stavanger)

Background

- Models (generally) predict magnetic catalysis around T_C
  - Inverse magnetic catalysis in some models [1]
- Lattice predicts INVERSE magnetic catalysis around T_C
  - Using physical quark masses [2]
- Both predict magnetic catalysis at T=0

[1] Farias et al. & Ferreira et al. (2014)   [2] Bali et al. (2012)

Polyakov-quark meson model

$$\psi = \begin{pmatrix} u \\ d \end{pmatrix} = \begin{matrix}\text{up quark} \\ \text{down quark}\end{matrix} \qquad \sigma = \begin{matrix}\text{sigma} \\ \text{meson}\end{matrix} \qquad \vec{\pi} = \begin{matrix}\text{pi} \\ \text{mesons}\end{matrix}$$

$$\mathcal{L}_{PQM} = \bar{\psi}\Big(i\gamma_\mu D^\mu + g(\sigma + i\gamma_5 \vec{\tau}\cdot\vec{\pi}) + \mu\gamma_0\Big)\psi + \mathcal{L}_{LSM} + \mathcal{L}_{glue}$$

$$\begin{aligned}\mathcal{L}_{LSM} = \frac{1}{2}\Big((\partial_\mu\sigma)^2 + (\partial_\mu\pi)^2 + 2D_\mu\pi_+ D^\mu\pi_-\Big) \\ + \frac{\mu^2}{2}\big(\sigma^2 + \vec{\pi}^2\big) - \frac{\lambda}{4}\big(\sigma^2 + \vec{\pi}^2\big)^2\end{aligned}$$

$$\mathcal{L}_{glue} = \text{Phenomenological}$$

Polyakov loop

• An order parameter for confinement
• Defined in the follow equation

$$\Phi = \frac{1}{N_c} Tr_c \ \exp\left(ig' \int_0^{\beta} d\tau \, t_a {A_a}^0 \right)$$

• Suppresses quark degrees of freedom at temperatures lower than the deconfinement transition
• Glue potential reproduces pure glue lattice results around T_c

[1] Fukushima (2004)

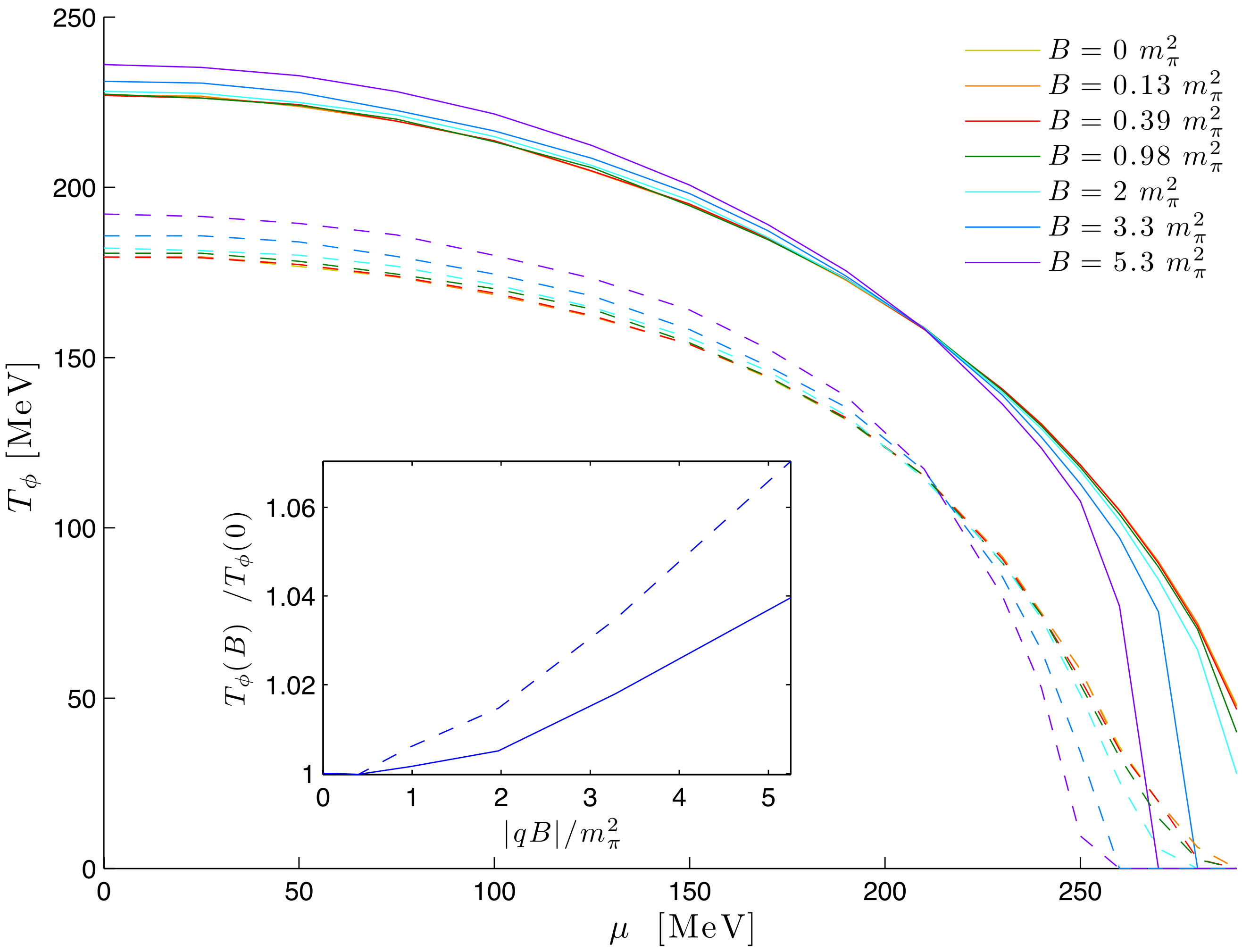
What we do?

- Use FRG to obtain a DE which we solve (numerically) giving us a potential, U_{RG}, in Φ, $\underline{\Phi}$, σ; T, B, μ space

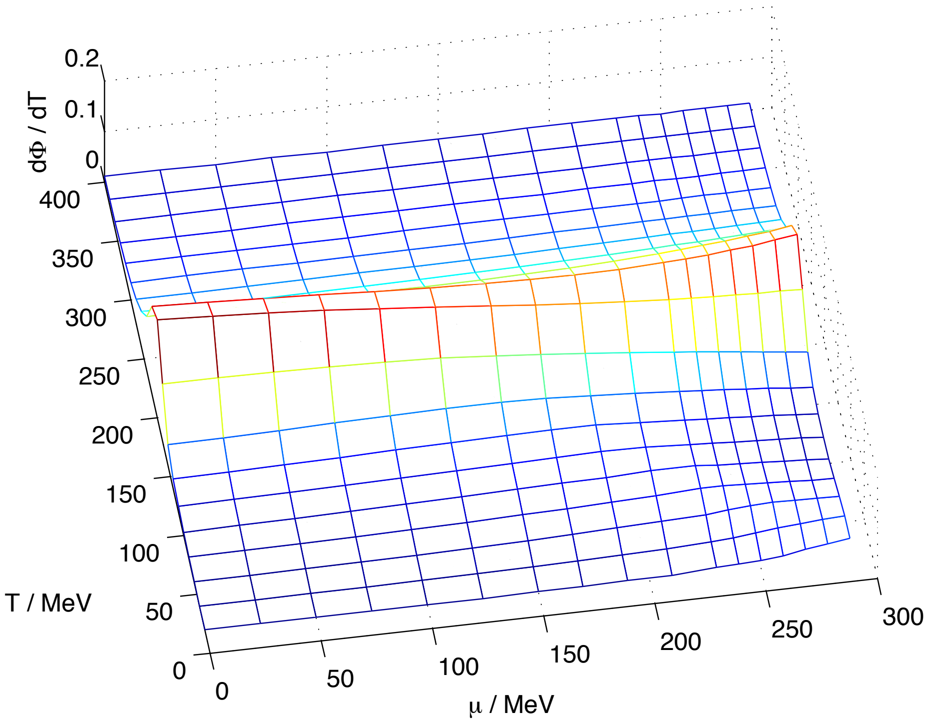
- U_{RG} doesn't include the pure-glue potential, U_{Glue}, so we create our purely phenomenological potential for this

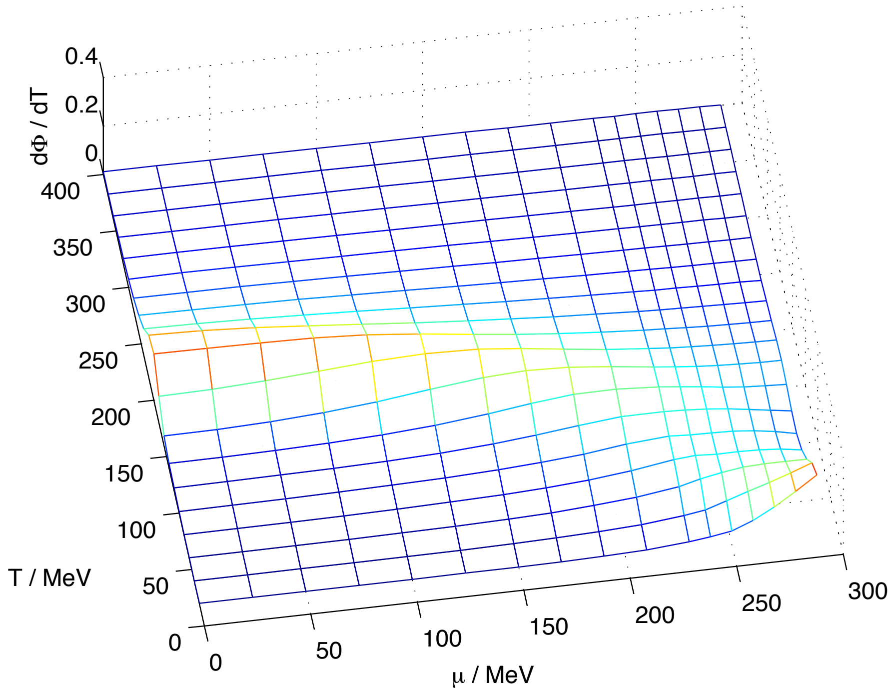
- Then we minimise $U_{RG} + U_{Glue}$ find the expectation values of Φ, $\underline{\Phi}$, σ

Polyakov loop helps!



Must compensate for finite μ



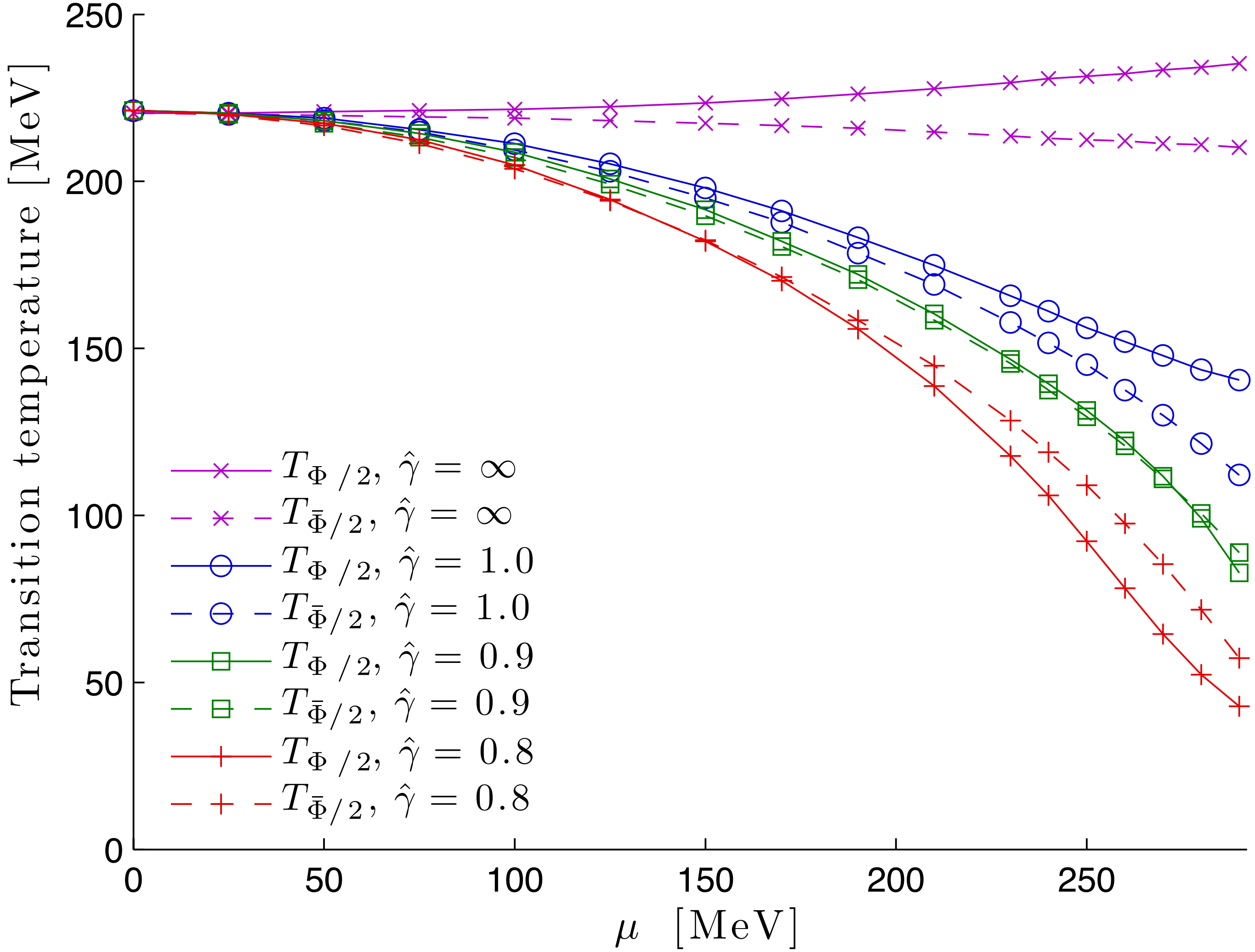


$$T_0 = T_\tau e^{-1/(\alpha_0\, b(N_f,\mu))}$$

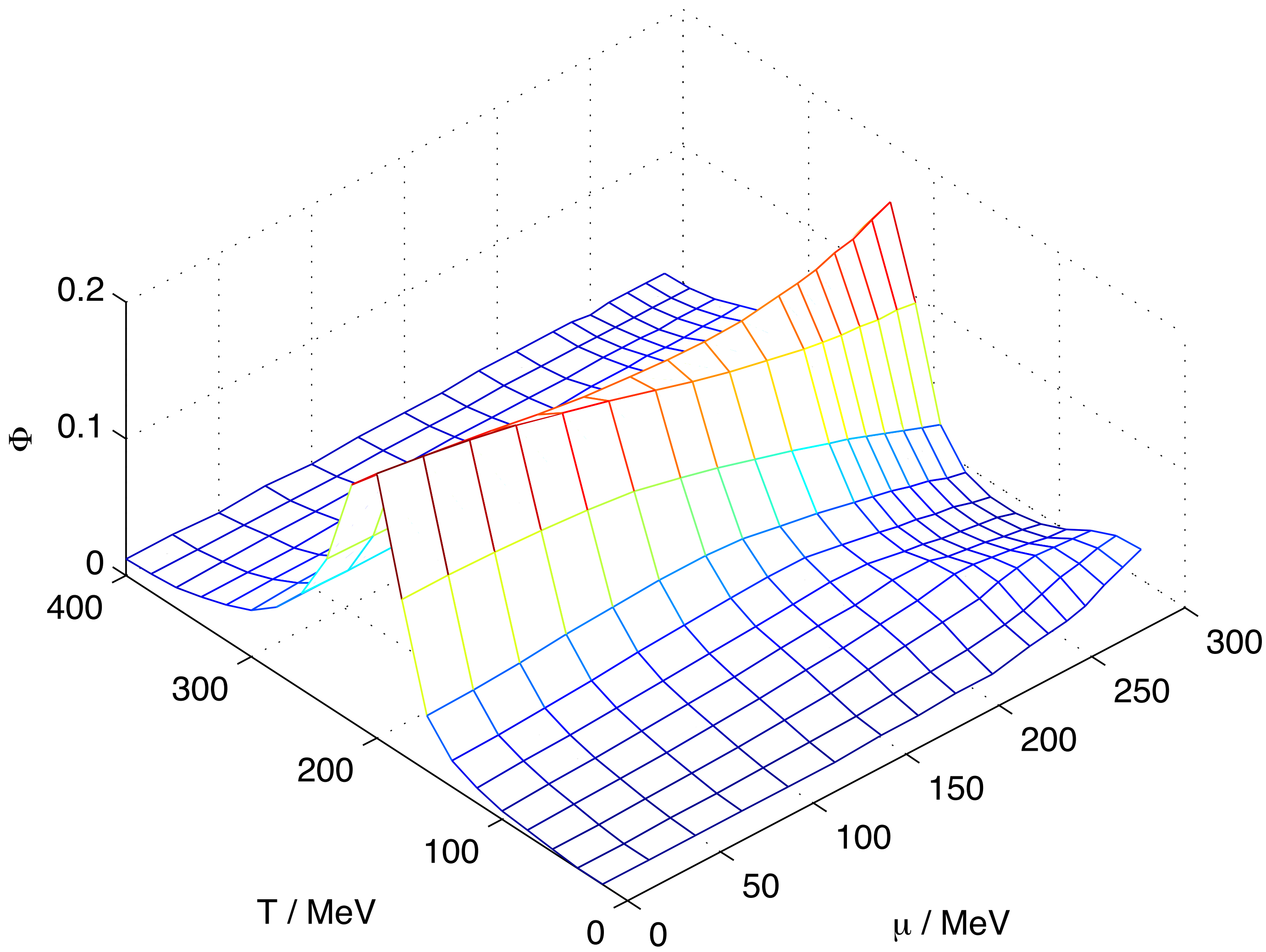
$$b(N_f,\mu) = \frac{1}{6\pi}(11N_c - 2N_f) - \frac{16}{\pi} N_f \frac{\mu^2}{(\hat{\gamma}\, T_\tau)^2}$$

[1] Herbst, Pawlowski, Schaefer (2011)

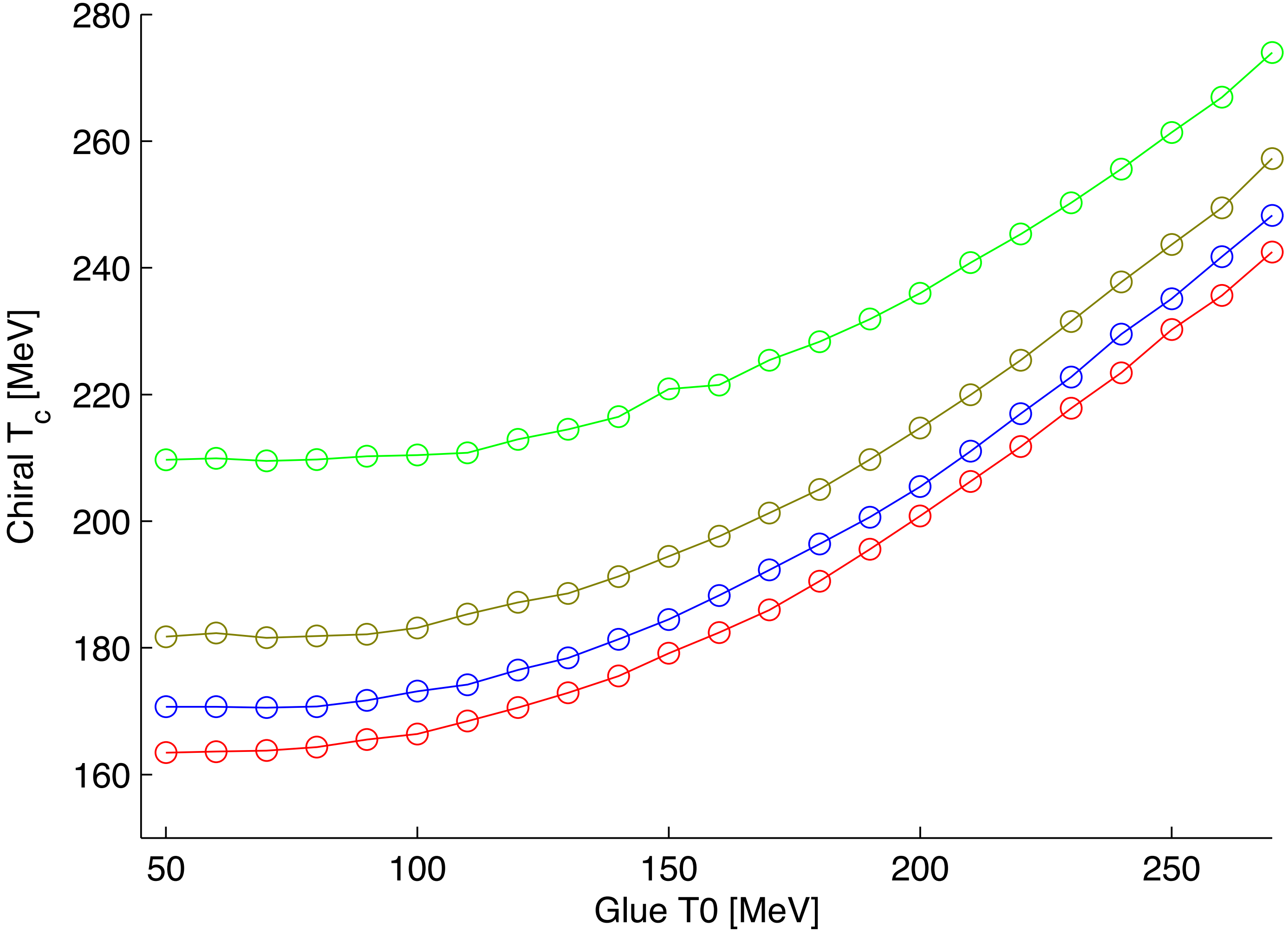
Must compensate for finite μ



Must compensate for finite μ



Varying T_0 doesn't give inverse magnetic catalysis



[1] Fraga, Mintz, Schaffner-Bielich (2014)

Summary and Outlook

- Polyakov loop does add to our chiral phase diagram
- Gluonic potential is very important to deconfinement transition
- Gluonic potential does not effects the chiral transition enough to give inverse magnetic catalysis

- Look for inverse magnetic catalysis in QM model, not Polyakov loop

Why use the QM at finite B/μ?

- Non central heavy ion collisions
  - $|qB| \sim m_\pi^2$
- Magnetars
  - Possibly similar orders of magnitude in star cores
- Understanding models of QCD
  - Additional handle on calculations
- Because we can!

Polyakov loop

- An order parameter for confinement

$$n_q = \frac{1 + 2\bar{\Phi}e^{(E_q-\mu)/T} + \Phi e^{2(E_q-\mu)/T}}{1 + 3\bar{\Phi}e^{(E_q-\mu)/T} + 3\Phi e^{2(E_q-\mu)/T} + e^{3(E_q-\mu)/T}}$$

- When Φ = 0 we have

$$n_q = \frac{1}{1 + e^{3(E_q-\mu)/T}}$$

- When Φ = 1 we have

$$n_q = \frac{1}{1 + e^{(E_q-\mu)/T}}$$

[1] Fukushima (2004)

Coupling to B field

- Constant homogenous background field

$$\vec{p}^{\,2} \to p_z^2 + (2m+1)|qB|$$

$$\int \frac{d^3p}{(2\pi)^3} \to \frac{|qB|}{2\pi} \sum_{m=0}^{\mathrm{inf}} \int \frac{dp_z}{2\pi}$$

- Fermions have spin dependence
- Additional symmetry breaking due to charges of the fermions
  – Different mass terms for neutral and charged mesons
  – These set to equal (true in mean field)

Polyakov loop

$$\partial^\mu \rightarrow D^\mu = \partial^\mu - ig' t_a A_a^0$$

$$\Phi = \frac{1}{N_c} Tr_c \ \exp\left(ig' \int_0^\beta d\tau \, t_a {A_a}^0 \right)$$

- In pure glue Φ is an order parameter for confinement
  - Φ=1, deconfinement
  - Φ=0, confinement

- We'll also have to add a gluonic potential

FRG approach

- We create (using the LPA) a differential equation, which when solved gives us an effective potential for our fields
- The equation includes diagrams of all orders
- An exact equation would give exact results
- No FRG for glue potential, simply added to Polyakov-QM potential

RG approach

$$\partial_k \Gamma_k[\,\langle\phi\rangle, \langle\psi\rangle\,] = \frac{1}{2}\mathrm{Tr}\Big[\partial_k R_{kB}\big(\Gamma_k^{(2,0)} + R_{kB}\big)^{-1}\Big]$$
$$-\mathrm{Tr}\Big[\partial_k R_{kB}\big(\Gamma_k^{(0,2)} + R_{kB}\big)^{-1}\Big]$$

- We can create (using approximations) a differential equation, which when solved gives us an effective potential for our fields
- The equation includes diagrams of all orders
- An exact equation would give exact results

What we do?

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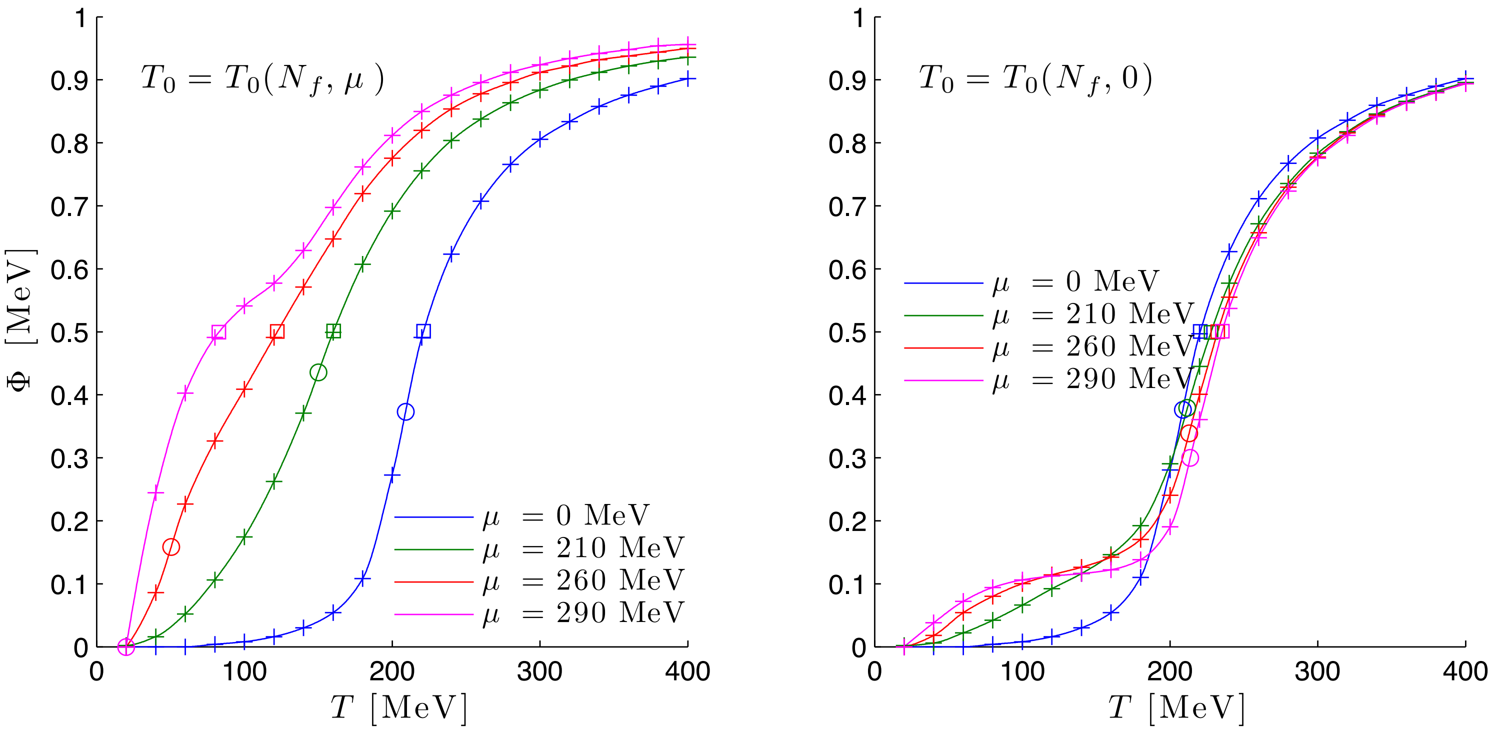

$T_0 = T_0(N_f, \mu)$
Φ [MeV]
T [MeV]
μ = 0 MeV
μ = 210 MeV
μ = 260 MeV
μ = 290 MeV
$T_0 = T_0(N_f, 0)$
T [MeV]
μ = 0 MeV
μ = 210 MeV
μ = 260 MeV
μ = 290 MeV