The QCD critical line at finite chemical potentials

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Strong and ElectroWeak Matter

Based on an ongoing study in collaboration with:
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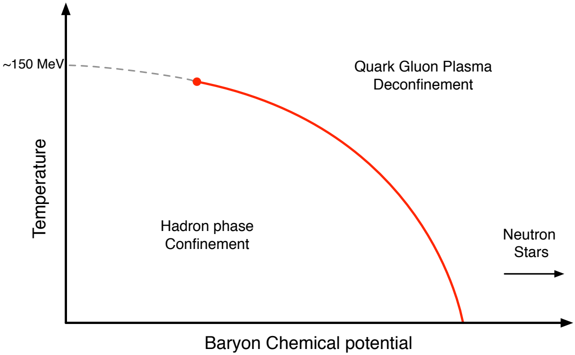
Outline

- The QCD phase diagram in the $T - \mu$ plane
- The analytic continuation method
- Lattice QCD and Observables
- Preliminary results
- Conclusions and outlook

The QCD phase diagram in the $T - \mu$ plane

A minimal version of the phase diagram: mostly based on conjectures.



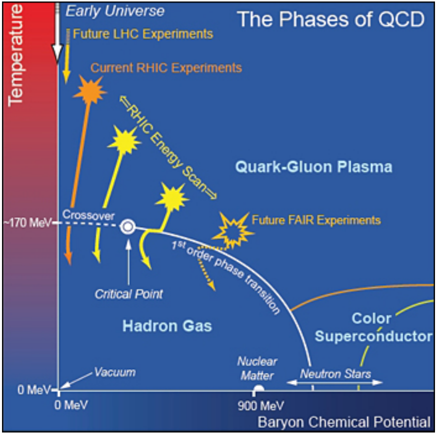
Lattice QCD approach

$\longrightarrow$ NP technique to study strong interactions from first principles.

Sign Problem at nonzero baryon chemical potential!

The QCD phase diagram in the $T - \mu$ plane

The HIC experiments are investigating the existence of the high-T QGP phase and of the critical endpoint.



The freeze-out curve can be determined from experimental data [Cleymans et al., '06; Beccattini et al., '13].

What we can do is to try to estimate the pseudo-critical line and compare the two.

In a collision event we expect $\mu_u = \mu_d \neq 0$ and $\mu_s = 0$, because the net strangeness of the initial state is zero.

The analytic continuation method

At lowest order in μ^B, we can parameterize the critical line as

$$\frac{T_{pc}(\mu^B)}{T_{pc}(\mu^B = 0)} = 1 - \kappa \left(\frac{\mu^B}{T_{pc}} \right)^2$$

Anyhow, the *sign problem* hinders direct lattice QCD simulations at

$\mu^B \neq 0$.

We avoid it by assuming the theory to be analytical in μ^B and studying the phase diagram in the $T - \mu_I^B$ plane.
Hence, we consider the theory in the presence of an imaginary quark chemical potential $\mu_B = i\mu_I^B$.

$$\frac{T_{pc}(\mu_I^B)}{T_{pc}(\mu_I^B = 0)} = 1 + \kappa' \left(\frac{\mu_I^B}{T_{pc}} \right)^2$$

Assuming analyticity means assuming $\kappa = \kappa'$

Observables

In order to characterize and identify the confined and the deconfined phases of QCD, we compute 2 different observables:

- $\langle\overline{\psi}\psi\rangle_r$, Renormalized Chiral Condensate
- $\Delta_r\chi_{\overline{\psi}\psi}$, Renormalized Chiral Condensate Susceptibility

They are associated to the partial restoration of chiral symmetry above T_{pc}.

We identify T_{pc} respectively as:

- The inflection point of $\langle\overline{\psi}\psi\rangle_r$
- The peak of $\Delta_r\chi_{\overline{\psi}\psi}$

The transition is known to be a broad cross-over; hence different observable may lead to different transition temperatures.

Observables: details 1/2

We consider $N_f = 2 + 1$ QCD at the physical point:

$$\mathcal{Z} = \exp(-f/T) = \int \mathcal{D}U \ \exp(-S_{YM}[U]) \ (\det(M_u)\det(M_d)\det(M_s))^{1/4}$$

The chiral condensate is defined as:

$$\langle\overline{\psi}_f\psi_f\rangle = \frac{1}{V_4}\frac{\partial \log \mathcal{Z}}{\partial m_f} = \frac{N_f}{4V_4}\left\langle Tr(M_f^{-1})\right\rangle .$$

It can be properly renormalized by removing additive and multiplicative divergencies [M. Cheng et al., PRD '08]:

$$\langle\overline{\psi}\psi\rangle_r = \frac{\langle\bar{\ell}\ell\rangle(T) - \frac{2m_{ud}}{m_s}\langle\bar{s}s\rangle(T)}{\langle\bar{\ell}\ell\rangle(0) - \frac{2m_{ud}}{m_s}\langle\bar{s}s\rangle(0)}$$

Observables: details 2/2

The susceptibility of the chiral condensate reads:

$$\chi_{\overline{\psi}\psi} = \frac{1}{V_4}\frac{\partial^2 \log \mathcal{Z}}{\partial m_f^2} = \frac{1}{V_4}\left(\frac{N_f}{4}\right)^2 \left[\left\langle Tr(M_f^{-1})^2\right\rangle - \left\langle Tr(M_f^{-1})\right\rangle^2\right] + \\ + \frac{N_f}{4V_4}\left\langle Tr(M_f^{-2})\right\rangle$$

It can be properly renormalized by defining [Y. Aoki et al, JHEP '06]:

$$\Delta_r \chi_{\overline{\psi}\psi} = m_{ud}^2 \left(\chi_{\overline{\psi}\psi}(T) - \chi_{\overline{\psi}\psi}(0)\right)$$

Numerical Setup

We adopt a state-of-art discretization of $N_f = 2 + 1$ QCD:

- Fermionic Sector: stout smearing improved staggered fermions.
- Gauge Sector: tree level improved Symanzik action.
- Bare Parameters: chosen according to [Aoki et al., '09]; we are on a line of constant physics at the physical point.

For the observable we use stochastic estimators, with 8 random vectors per flavour.

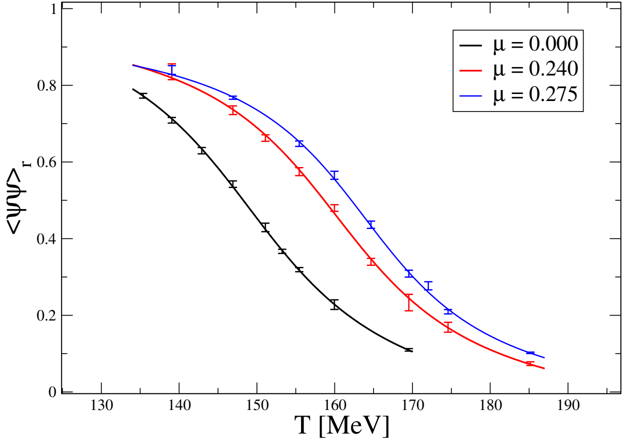
We are running simulations on the BG/Q machine at CINECA (Italy).

Simulations on 4 lattices:
24^3x6 and 32^3x8 for $T \neq 0$ and 24^4 and 32^4 for $T = 0$.

For each lattice we performed simulations at several chemical potentials, exploring a range of temperatures close to the transition.
We explore both the $(\mu_{ud} \neq 0;\ \mu_s = 0)$ and the $(\mu_{ud} = \mu_s \neq 0)$ cases.

Preliminary Results: $T_{pc}(\mu_I)$ from $\langle\overline{\psi}\psi\rangle_r$

Lattice: 24^3x6

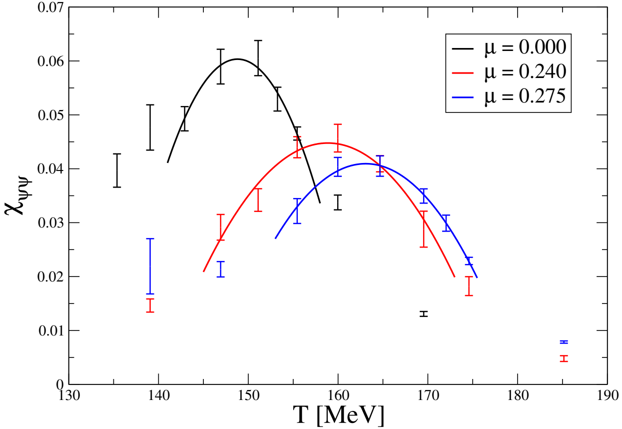


We fit the renormalized chiral condensate with

$$\langle\overline{\psi}\psi\rangle_r = a \cdot \mathrm{atan}(b \cdot (T - T_{pc})) + c$$

Preliminary Results: $T_{pc}(\mu_I)$ from $\Delta_r \chi_{\overline{\psi}\psi}$

Lattice: $24^3 x6$

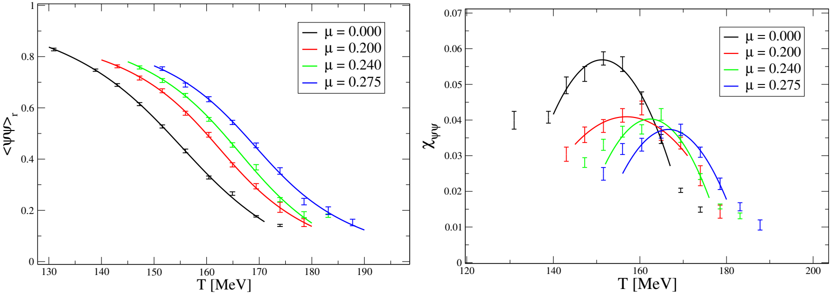


We fit the peaks of the renormalized chiral susceptibility with

$$\Delta_r \chi_{\overline{\psi}\psi} = a \cdot (T - T_{pc})^2 + b$$

Preliminary Results: $T_{pc}(\mu_I)$ from both the observables

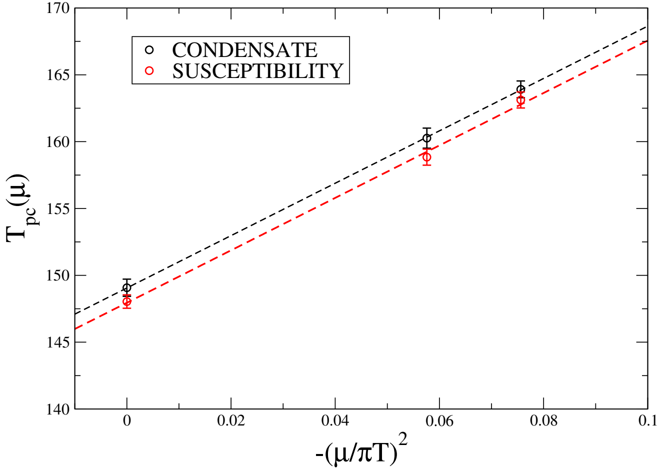
Lattice: 32^3x8



We fit data according to the previous functions.
We are going to finer lattices (at fixed physical volume) in order to approach the continuum limit.

Critical Lines on $24^3 x6$

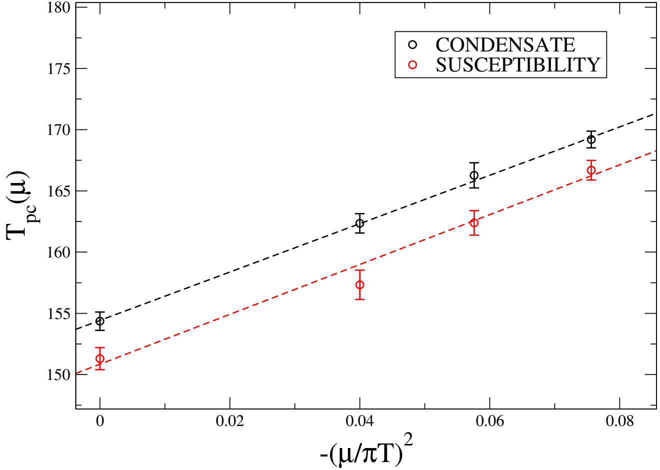
We perform a linear fit in μ^2 to extract the critical line curvature.



From the chiral condensate we get: $\kappa = 0.0148(7)$

From the susceptibility we get: $\kappa = 0.0149(8)$

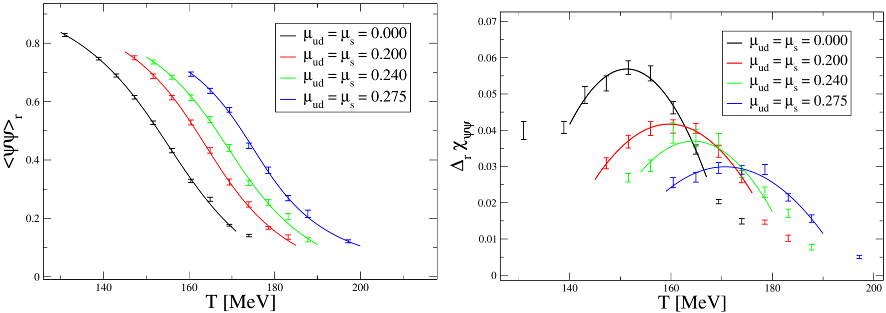
Critical Lines on $32^3 x8$



From the chiral condensate we get: $\kappa = 0.0144(10)$
From the susceptibility we get: $\kappa = 0.0152(12)$

The case at nonzero strange chemical potential

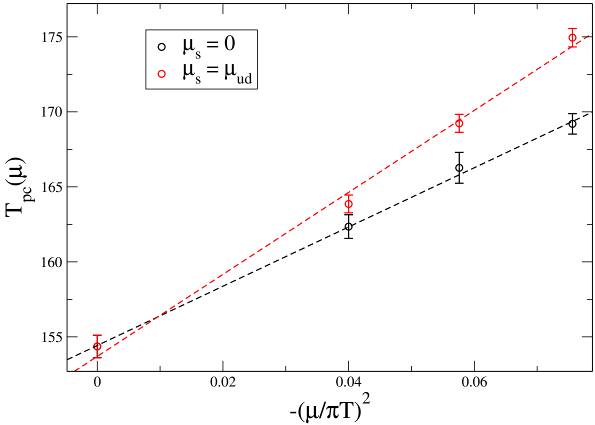
We want to understand what is the influence of μ_s on the curvature of the critical line.
Hence, we consider now the setup with $\mu_{ud} = \mu_s \neq 0$.



The strange quark chemical potential contribute substantially to the displacement of T_{pc}.

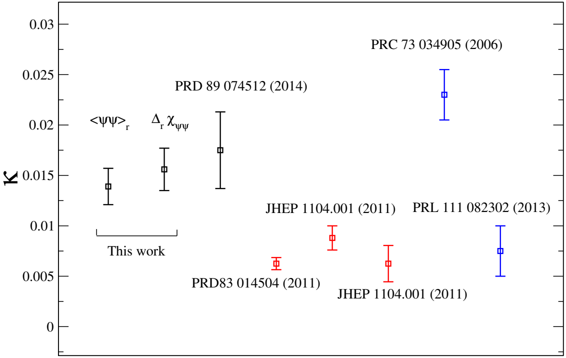
The case at nonzero strange chemical potential

Both curves are obtained from the chiral condensate.



From the $\mu_s = 0$ we get: $\kappa = 0.0144(10)$
From the $\mu_s = \mu_{ud}$ we get: $\kappa = 0.0190(12)$
The ratio is $\simeq$ 1.32 $\Longrightarrow$ Enhancement of κ due to μ_s!

Comparisons



1. This work, condensate.
2. This work, susceptibility.
3. [Cea et al, '14] HISQ action, $\mu_s = \mu_{ud}$, susceptibility.
4. [Kaczmarek et al, '10] P4 action, condensate.

5&6. [Endrodi et al, '10] STOUT action, strange quark numb. susceptibility & condensate.

7. [Cleymans et al, '06] Chemical Freeze-out.
8. [Beccattini et al, '06] Chemical Freeze-out.

- Black points: LQCD, Analytic cont.
- Red points: LQCD, Taylor exp.
- Blue points: Freeze-out data

Conclusions and outlook

- Computation of two observables sensitive to the transition.
- Determination of the critical line curvature at 2 lattice spacings.
- Our result for κ is larger than previous Taylor expansion-based determinations (but we still need the continuum limit).
- We got a larger curvature in the case $\mu_s = \mu_{ud}$.

Future perspectives:

- Compare with further observables.
- Perform simulations at $N_t = 10$ to perform the continuum limit.