The QCD critical line at finite chemical potentials

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Strong and ElectroWeak Matter

Based on an ongoing study in collaboration with: C. Bonati, M. D’Elia, M. Mariti, M. Mesiti, F. Sanfilippo.
Outline

- The QCD phase diagram in the $T - \mu$ plane
- The analytic continuation method
- Lattice QCD and Observables
- Preliminary results
- Conclusions and outlook
The QCD phase diagram in the $T - \mu$ plane

A minimal version of the phase diagram: mostly based on conjectures.

Lattice QCD approach

$\rightarrow$ NP technique to study strong interactions from first principles. Sign Problem at nonzero baryon chemical potential!
The QCD phase diagram in the $T - \mu$ plane

The HIC experiments are investigating the existence of the high-T QGP phase and of the critical endpoint.

The freeze-out curve can be determined from experimental data [Cleymans et al., '06; Beccattini et al., '13].

What we can do is to try to estimate the pseudo-critical line and compare the two.

In a collision event we expect $\mu_u = \mu_d \neq 0$ and $\mu_s = 0$, because the net strangeness of the initial state is zero.
The analytic continuation method

At lowest order in $\mu^B$, we can parameterize the critical line as

$$\frac{T_{pc}(\mu^B)}{T_{pc}(\mu^B = 0)} = 1 - \kappa \left( \frac{\mu^B}{T_{pc}} \right)^2$$

Anyhow, the *sign problem* hinders direct lattice QCD simulations at $\mu^B \neq 0$.

We avoid it by assuming the theory to be analytical in $\mu^B$ and studying the phase diagram in the $T - \mu_I^B$ plane.

Hence, we consider the theory in the presence of an imaginary quark chemical potential $\mu_B = i \mu_I^B$.

$$\frac{T_{pc}(\mu_I^B)}{T_{pc}(\mu_I^B = 0)} = 1 + \kappa' \left( \frac{\mu_I^B}{T_{pc}} \right)^2$$

Assuming analyticity means assuming $\kappa = \kappa'$. 
In order to characterize and identify the confined and the deconfined phases of QCD, we compute 2 different observables:

- $\langle \bar{\psi} \psi \rangle_r$, Renormalized Chiral Condensate
- $\Delta_r \chi_{\bar{\psi} \psi}$, Renormalized Chiral Condensate Susceptibility

They are associated to the partial restoration of chiral symmetry above $T_{pc}$.

We identify $T_{pc}$ respectively as:

- The inflection point of $\langle \bar{\psi} \psi \rangle_r$
- The peak of $\Delta_r \chi_{\bar{\psi} \psi}$

The transition is known to be a broad cross-over; hence different observable may lead to different transition temperatures.
We consider $N_f = 2 + 1$ QCD at the physical point:

$$Z = \exp(-f/T) = \int D U \exp(-S_{YM}[U]) (\det(M_u) \det(M_d) \det(M_s))^{1/4}$$

The chiral condensate is defined as:

$$\langle \bar{\psi}_f \psi_f \rangle = \frac{1}{V_4} \frac{\partial \log Z}{\partial m_f} = \frac{N_f}{4V_4} \langle Tr(M_f^{-1}) \rangle.$$ 

It can be properly renormalized by removing additive and multiplicative divergencies [M. Cheng et al., PRD ’08]:

$$\langle \bar{\psi} \psi \rangle_r = \frac{\langle \bar{\ell} \ell \rangle(T) - \frac{2m_{ud}}{m_s} \langle \bar{s} s \rangle(T)}{\langle \bar{\ell} \ell \rangle(0) - \frac{2m_{ud}}{m_s} \langle \bar{s} s \rangle(0)}$$
The susceptibility of the chiral condensate reads:

\[ \chi_{\overline{\psi}\psi} = \frac{1}{V_4} \frac{\partial^2 \log Z}{\partial m_f^2} = \frac{1}{V_4} \left( \frac{N_f}{4} \right)^2 \left[ \langle Tr(M_f^{-1})^2 \rangle - \langle Tr(M_f^{-1}) \rangle^2 \right] + \]

\[ + \frac{N_f}{4V_4} \langle Tr(M_f^{-2}) \rangle \]

It can be properly renormalized by defining [Y. Aoki et al, JHEP '06]:

\[ \Delta_r \chi_{\overline{\psi}\psi} = m_{ud}^2 \left( \chi_{\overline{\psi}\psi}(T) - \chi_{\overline{\psi}\psi}(0) \right) \]
Numerical Setup

We adopt a state-of-art discretization of $N_f = 2 + 1$ QCD:

- Fermionic Sector: stout smearing improved staggered fermions.
- Gauge Sector: tree level improved Symanzik action.
- Bare Parameters: chosen according to [Aoki et al., '09]; we are on a line of constant physics at the physical point.

For the observable we use stochastic estimators, with 8 random vectors per flavour.

We are running simulations on the BG/Q machine at CINECA (Italy).

Simulations on 4 lattices:
$24^3 \times 6$ and $32^3 \times 8$ for $T \neq 0$ and $24^4$ and $32^4$ for $T = 0$.

For each lattice we performed simulations at several chemical potentials, exploring a range of temperatures close to the transition. We explore both the $(\mu_{ud} \neq 0; \mu_s = 0)$ and the $(\mu_{ud} = \mu_s \neq 0)$ cases.
Preliminary Results: $T_{pc}(\mu_I)$ from $\langle \overline{\psi}\psi \rangle_r$

Lattice: $24^3 \times 6$

We fit the renormalized chiral condensate with

$$\langle \overline{\psi}\psi \rangle_r = a \cdot \text{atan}(b \cdot (T - T_{pc})) + c$$
Preliminary Results: $T_{pc}(\mu_I)$ from $\Delta_r \chi_{\bar{\psi} \psi}$

Lattice: $24^3 \times 6$

We fit the peaks of the renormalized chiral susceptibility with

$$\Delta_r \chi_{\bar{\psi} \psi} = a \cdot (T - T_{pc})^2 + b$$
Preliminary Results: $T_{pc}(\mu_I)$ from both the observables

Lattice: $32^3 \times 8$

We fit data according to the previous functions. We are going to finer lattices (at fixed physical volume) in order to approach the continuum limit.
Critical Lines on $24^3 \times 6$

We perform a linear fit in $\mu^2$ to extract the critical line curvature.

From the chiral condensate we get: $\kappa = 0.0148(7)$

From the susceptibility we get: $\kappa = 0.0149(8)$
From the chiral condensate we get: $\kappa = 0.0144(10)$
From the susceptibility we get: $\kappa = 0.0152(12)$
The case at nonzero strange chemical potential

We want to understand what is the influence of $\mu_s$ on the curvature of the critical line.
Hence, we consider now the setup with $\mu_{ud} = \mu_s \neq 0$.

The strange quark chemical potential contribute substantially to the displacement of $T_{pc}$.
The case at nonzero strange chemical potential

Both curves are obtained from the chiral condensate.

From the $\mu_s = 0$ we get: $\kappa = 0.0144(10)$

From the $\mu_s = \mu_{ud}$ we get: $\kappa = 0.0190(12)$

The ratio is $\approx 1.32$ $\implies$ Enhancement of $\kappa$ due to $\mu_s$!
Comparisons

1. This work, condensate.
2. This work, susceptibility.
3. [Cea et al, '14] HISQ action, $\mu_s = \mu_{ud}$, susceptibility.
7. [Cleymans et al, '06] Chemical Freeze-out.

- **Black points**: LQCD, Analytic cont.
- **Red points**: LQCD, Taylor exp.
- **Blue points**: Freeze-out data
Conclusions and outlook

- Computation of two observables sensitive to the transition.
- Determination of the critical line curvature at 2 lattice spacings.
- Our result for $\kappa$ is larger than previous Taylor expansion-based determinations (but we still need the continuum limit).
- We got a larger curvature in the case $\mu_s = \mu_{ud}$.

Future perspectives:
- Compare with further observables.
- Perform simulations at $N_t = 10$ to perform the continuum limit.