# Finite density QCD and thermodynamics with heavy quarks

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## Introduction

- We investigate QCD with heavy quarks at finite density
- This is done employing a 3d effective theory derived from full lattice QCD with Wilson fermions by a combined strong coupling and hopping parameter expansions
- The effective theory is derived by expanding the gauge action and fermionic determinant so that the spatial links can be analytically integrated out

$$Z = \int d[U_\mu]\ \det[Q]\ e^{-S_G} = \int [dU_0]\ e^{-S_{\text{eff}}}, \quad -S_{\text{eff}} = \log \int [dU_i]\ \det[Q]\ e^{-S_G}$$

- The resulting theory depends only on traces of temporal links, called Polyakov Loops: $L_{\vec{x}} = \text{Tr}W_{\vec{x}} = \text{Tr}\prod_{i=0}^{N_\tau - 1} U_0(\vec{x})$
- The effective theory can be simulated using Monte Carlo Methods at imaginary chemical potential. Real chemical potential can be simulated using reweighting or the method of Complex Langevin. Furthermore, leading orders can be calculated analytically.

## Strong Coupling Expansion

- The gauge part of the action is expanded via a strong coupling expansion around $\beta = 0$ [1]
- In the fundamental character expansion coefficient, $a_f = u(\beta) = \frac{\beta}{18} + \text{O}(\beta^2)$, the leading order is a two-point-interaction

$$S_{2,0}(\lambda) = \lambda(u, N_\tau) \sum_{<ij>} (L_i L_j^* + L_i^* L_j), \ \lambda(u, N_\tau) = u^{N_\tau}[1 + ...]$$

- The expansion is done up to order $\beta^{10}$ and reproduces the deconfinement temperature of pure Yang-Mills-Theory up to an error of $\approx 10\%$

## Hopping Parameter Expansion

- The Quark determinant is expanded using a hopping expansion around $\kappa = 0$ ($\kappa = \frac{1}{aM_q + 8}$)
- To leading order we get the static quark determinant (not showing antiquarks, which can be neglected at large $\mu$)

$$\det[Q]_{stat} = \prod_{\vec{x}} \det(1 + cW_{\vec{x}}) = \prod_{\vec{x}} (1 + c\ \text{Tr}W_{\vec{x}} + c^2\ \text{Tr}W_{\vec{x}}^* + c^3)^2$$

- First corrections are interactions between neighboring lattice sites

$$\det[Q]_{kin} = \prod_{<ij>} \left[1 - \frac{2\kappa^2 N_\tau}{N_c} \text{Tr}\frac{cW_i}{1 + cW_i} \text{Tr}\frac{cW_j}{1 + cW_j}\right] + \text{O}(\kappa^4)$$

- Inclusion of plaquettes from the expansion of the gauge action lead to gauge corrections to the coupling constants: $c(\kappa, \mu, N_\tau) \to h(\beta, \kappa, \mu, N_\tau)$ and $\frac{2\kappa^2 N_\tau}{N_c} \to h_2(\beta, \kappa, N_\tau)$
- At $N_\tau = 4$ the effective theory is valid up to $\kappa \approx 0.1$ and $\beta \leq \beta_c$

## Continuum Extrapolation of Baryon Density

- In order to extrapolate to the continuum simulations are performed at several lattice spacings [3]
- The density is expressed in terms of the baryon mass to get a dimensionless quantity
- We then fit finite lattice spacings corrections starting with $\text{O}(a)$ (Wilson fermions)

$$\frac{n_{\text{latt}}(\mu)}{m_B^3} = \frac{n_{\text{cont}}(\mu)}{m_B^3} + A(\mu)a + B(\mu)a^2 + ..., \quad m_B = -3\log(2\kappa) - 18\kappa^2 \frac{u}{1-u} + ...$$

- Other quantities such as energy density can be extrapolated in the same way

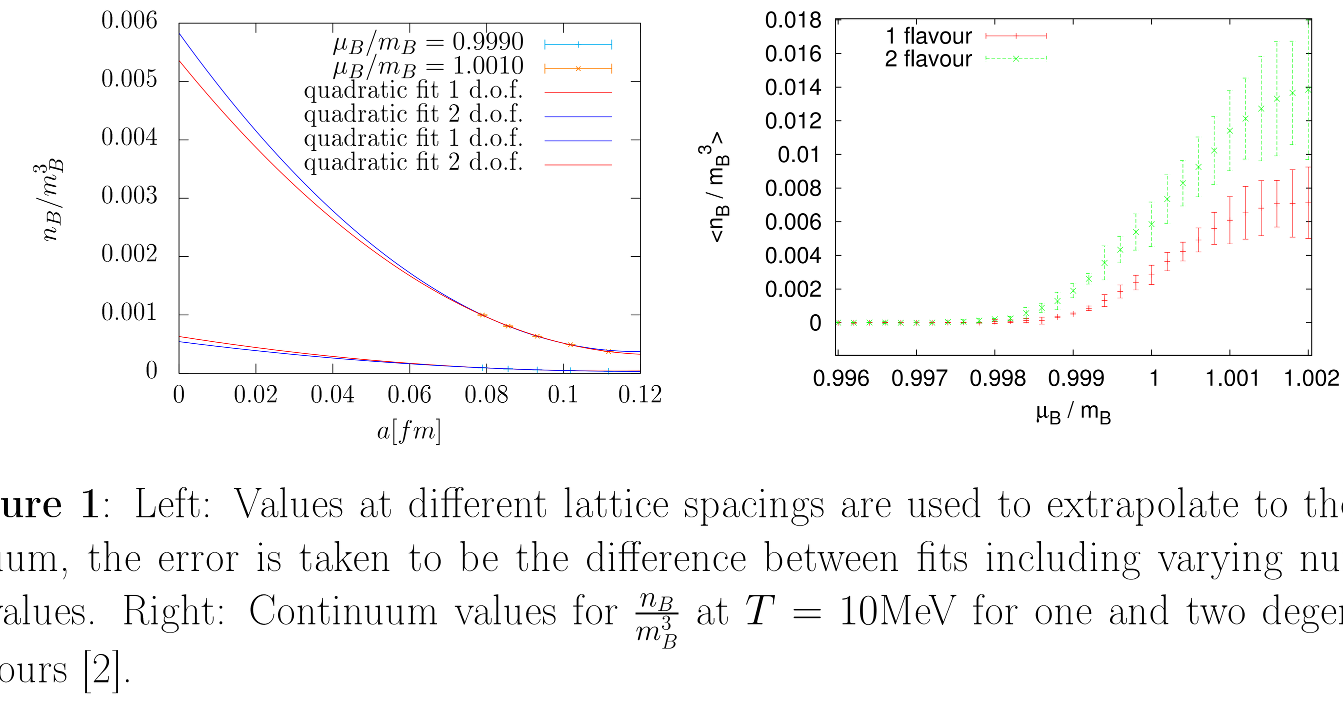


**Figure 1**: Left: Values at different lattice spacings are used to extrapolate to the continuum, the error is taken to be the difference between fits including varying numbers of values. Right: Continuum values for $\frac{n_B}{m_B^3}$ at $T = 10$MeV for one and two degenerate flavours [2].

- Errors quickly grow at larger chemical potential due to unphysical saturation on the lattice
- Approaching the continuum with fixed $\frac{m}{T}$ requires $N_\tau \to \infty$. Therefore the region of convergence of our model restricts the accessible lattice spacings
- We restrict ourselves to values of $\frac{\kappa^2 N_\tau}{N_c} < 0.007$ where the contributions of $\text{O}(\kappa^4)$ are small, this means a minimal lattice spacing of $a = 0.079fm$

## Equation of State

- The pressure of the hadron gas can be calculated from the density, $P(\mu) = \int_0^\mu d\mu' n_B(\mu')$

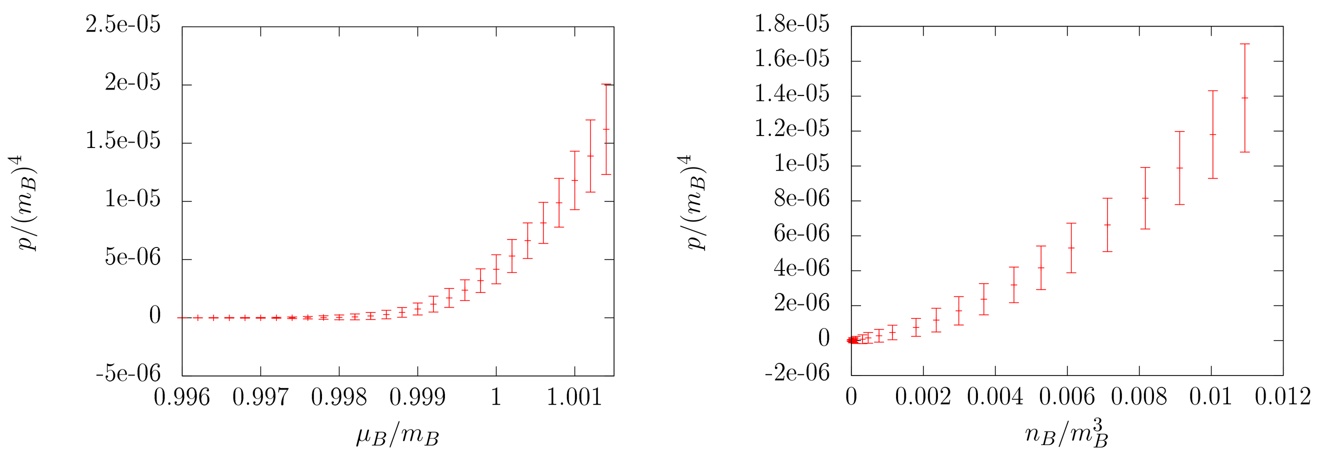


**Figure 2**: Continuum extrapolated results for pressure vs chemical potential (left) and pressure vs baryon density (right) with heavy baryons at $T = 10MeV$, $N_f = 2$ [2].

- Pressure stays zero until $\mu_B \approx m_B$ (Silver Blaze property)

## Binding Energy

- The formation of nuclear matter in form of baryons requires the existence of a negative binding energy
- To measure this we first calculate the energy density and subtract the rest energy $a^4 n_B m_B$

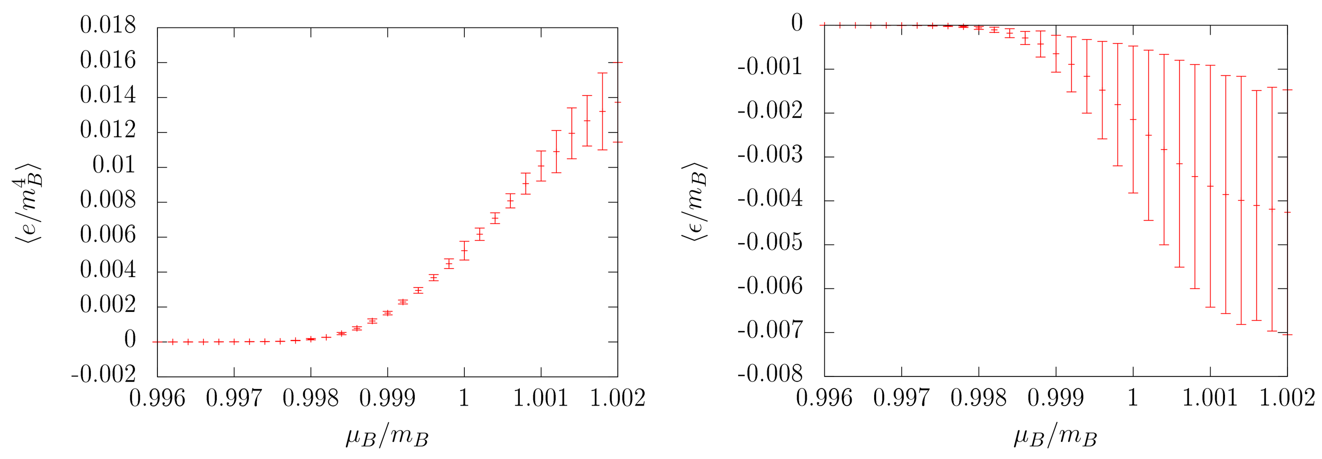


**Figure 3**: Left: Continuum extrapolated energy density for a system of heavy baryons at $T = 10MeV$, $N_f = 2$. Right: Binding energy for the same system [2].

- To leading order the binding energy decays exponentially with the pion mass, $\epsilon \propto e^{-am_\pi}$

## Chiral Condensate

- The chiral condensate is calculated as $\langle \bar{q}q \rangle = -\frac{T}{V}\frac{\partial}{\partial m_q} \log \text{Z}$. For vanishing chemical potential this can be compared to results from full QCD

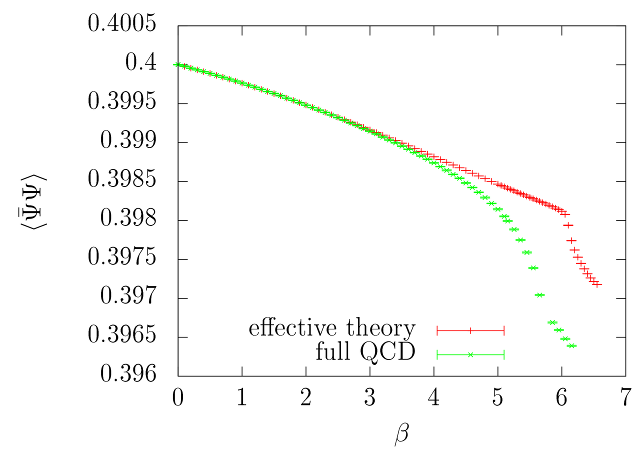


**Figure 4**: Chiral condensate at $\mu_B = 0, N_\tau = 4$

- Full QCD is reproduced up to the deconfinement transition at $\beta_c \approx 5.7$ (full QCD) resp. 6.1 (effective theory)

## References

[1] J. Langelage, S. Lottini and O. Philipsen, Centre symmetric 3d effective actions for thermal SU(N) Yang-Mills from strong couplings series

[2] J. Langelage, M. Neuman and O. Philipsen, Nuclear matter in heavy dense QCD from an effective lattice theory

[3] M. Fromm, J. Langelage, S. Lottini, M. Neuman and O. Philipsen, Onset Transition to Cold Nuclear Matter from Lattice QCD with Heavy Quarks