Non-Gaussian fixed points in fermionic field theories

with no auxiliary Bose-fields

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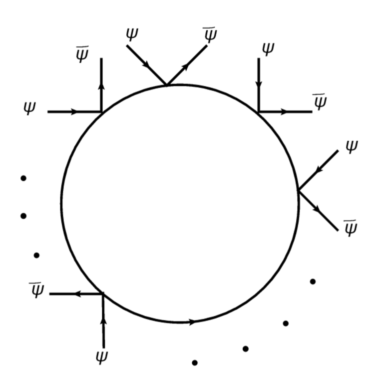
INTRODUCTION

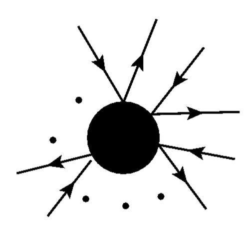
LOCAL COMPOSITE OPERATORS OF FERMIONS

- Compositeness scale: Δx
- Resolution scale: k^{-1}

- Composites of Grassmann variables $k^{-1} < \Delta x$ $(\bar\psi\psi)^n = 0$
- Composites of Grassmann variables $k^{-1} > \Delta x$ $(\bar\psi\psi)^n \neq 0$



Real Space
Coarse graining
Transformation



Effective quantum action below the compositeness scale Δx^{-1}

Wavefunction Renormalisation + Local Potential Approximation

$$\Gamma_k[\bar\psi,\psi]=\int_x\left[Z_k\bar\psi_l^\alpha(x)\partial_m\gamma_m^{\alpha\beta}\psi_l^\beta(x)+U_k(I(x))\right]$$

$I_p[\psi,\bar\psi]$: *quartic* invariants built from the Grassmann-variables

Partial bosonisation with auxiliary fields (for $U_k=cI$):

$$\Gamma_k^{aux}[\bar\psi,\psi,\sigma]=\int_x\left[Z_k\bar\psi_l^\alpha(x)\partial_m\gamma_m^{\alpha\beta}\psi_l^\beta(x)+\sigma(x)(\bar\psi\psi)-\frac{N_f}{g^2}\rho(x)\right],\qquad \rho(x)=\frac{1}{2}\sigma^2(x)$$

Fermionic Wetterich-equation

$$\partial_k\Gamma_k=\frac{1}{2}\mathrm{Str}\left[\partial_kR_k(\Gamma_k^{(2)}+R_k)^{-1}\right]=\frac{1}{2}\hat\partial_k\mathrm{Str}\log(\Gamma_k^{(2)}+R_k)$$

$$\Psi(q)=\begin{pmatrix}\psi(q)\\ \bar\psi^T(-q)\end{pmatrix}\quad \Gamma^{(2)}=\frac{\overrightarrow{\delta}}{\delta\Psi^T}\Gamma\frac{\overleftarrow{\delta}}{\delta\Psi}$$

R_k infrared regulator freezing fluctuations below k

$$\Gamma^{(2)}_{\psi^T\psi}=\frac{\overrightarrow{\delta}}{\delta\psi^T}\Gamma\frac{\overleftarrow{\delta}}{\delta\psi},\quad \Gamma^{(2)}_{\bar\psi\psi}=\frac{\overrightarrow{\delta}}{\delta\bar\psi}\Gamma\frac{\overleftarrow{\delta}}{\delta\psi},\quad \Gamma^{(2)}_{\psi^T\bar\psi^T}=\frac{\overrightarrow{\delta}}{\delta\psi^T}\Gamma\frac{\overleftarrow{\delta}}{\delta\bar\psi^T},\quad \Gamma^{(2)}_{\bar\psi\bar\psi^T}=\frac{\overrightarrow{\delta}}{\delta\bar\psi}\Gamma\frac{\overleftarrow{\delta}}{\delta\bar\psi^T}$$

Convenient factorisation:

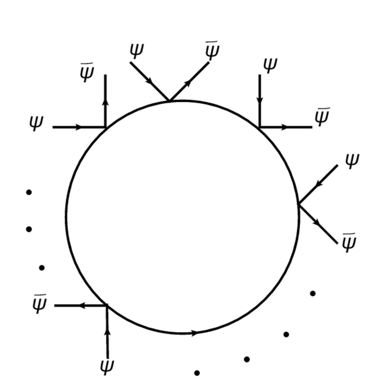
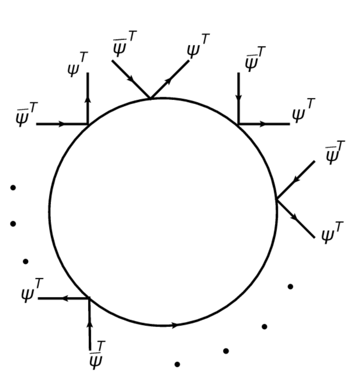
$$\Gamma^{(2)}=\frac{\overrightarrow{\delta}}{\delta\Psi^T}\Gamma\frac{\overleftarrow{\delta}}{\delta\Psi}=\begin{pmatrix}1&C_1\\0&1\end{pmatrix}\cdot\begin{pmatrix}0&\tilde\Gamma^{(2)}_{\bar\psi\psi}\\ \tilde\Gamma^{(2)}_{\psi^T\bar\psi^T}&0\end{pmatrix}\cdot\begin{pmatrix}1&C_2\\0&1\end{pmatrix}$$

$$\partial_k\Gamma_k=-\frac{1}{2}\hat\partial_k\mathrm{Tr}\left[\log\Gamma^{(2)}_{\bar\psi\psi}+\log\Gamma^{(2)}_{\psi^T\bar\psi^T}+\log\left(1-\Gamma^{(2)}_{\psi^T\psi}\Gamma^{(2)-1}_{\bar\psi\psi}\Gamma^{(2)}_{\bar\psi\bar\psi^T}\Gamma^{(2)-1}_{\psi^T\bar\psi^T}\right)\right]$$

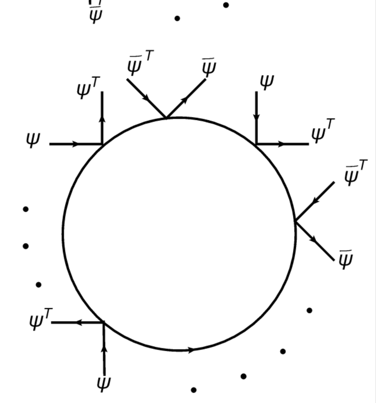
Right hand side evaluated on constant ψ_0 background

Graphical representation in momentum space:

$$\mathrm{Tr}\left(\log\Gamma^{(2)}_{\bar\psi_i\psi_p}+\Gamma^{(2)}_{\psi_i^T\bar\psi_p^T}\right)=$$

=  + 

$$\mathrm{Tr}\log\left(\delta_{in}-\Gamma^{(2)}_{\psi_i^T\psi_k}\Gamma^{(2)-1}_{\bar\psi_k\psi_l}\Gamma^{(2)}_{\bar\psi_l\bar\psi_m^T}\Gamma^{(2)-1}_{\psi_m^T\bar\psi_n^T}\right)=$$



CONCLUSION

LPA for the 3d Gross-Neveu model in pure fermionic formulation of RGE [1,2] reproduces the UV-safe fixed point earlier obtained with large-N saddle point solution [3] in the auxiliary formulation and in the NLO approximation of the gradient expansion of the exact RGE [4].

THE GROSS-NEVEU MODEL

$$\Gamma_k[\bar\psi,\psi]=\int_x\left[Z_k\bar\psi_l^\alpha(x)\partial_m\gamma_m^{\alpha\beta}\psi_l^\beta(x)+U_k(I(x))\right]$$

$$I(x)=(\bar\psi\psi)^2\equiv(\bar\psi_l^\alpha(x)\psi_l^\alpha(x))^2$$

Symmetry:

$$\psi\to-\gamma_5\psi,\qquad \bar\psi\to\bar\psi\gamma_5$$

Physical range for *I* and *U(I)* from large N_f argument

Saddle point solution of the partially bosonized theory:

$$\langle\sigma\rangle=M,\qquad \langle(\bar\psi\psi)\rangle=MN_f/g^2,\qquad \frac{g^2}{2N_f}I=M<(\bar\psi\psi)>-\frac{N_f}{2g^2}M^2\to I>0$$

Generalisation to $U_{aux}(\rho)\leftrightarrow U_{GN}(I)$ *correspondence*

$$\langle(\bar\psi\psi)\rangle=\sigma U'_{aux}(\rho),\qquad U_{GN}(I)=N_f^2(2\rho U'_{aux}(\rho)-U_{aux}(\rho))$$

$$-U_{aux}\sim a_n\rho^n,\quad a_n>0\to U_{GN}\sim-(2n-1)a_nI^n$$

Conclusion

if $U'_{aux}(\rho)>0$, for $\rho\to\infty$

then $U'_{GN}(I)<0$ for $I\to\infty$

LPA solution of the Wetterich-equation

$$\partial_k\Gamma_k=-\frac{1}{2}\hat\partial_k\mathrm{Tr}_x\left[\log G_k^{-1}+\log G_k^{(T)-1}-\log\left(1+(\bar\psi G_k\tilde U\psi)+(\psi^TG_k^{(T)}\tilde U\bar\psi^T)\right)\right]$$

(after the trace over flavor and bispinor indices has been performed)

G_k fermion propagator on *constant* fermion background

$$m_\psi(x)=2U'(I(x))(\bar\psi(x)\psi(x)),\qquad \tilde U(x)=2U'(I(x))+4IU''(I(x))$$

$$\mathrm{Tr}\left(\log\Gamma^{(2)}_{\bar\psi_i\psi_p}+\Gamma^{(2)}_{\psi_i^T\bar\psi_p^T}\right)=N_f\mathrm{Tr}_{D,q}\left(\log G^{(0)-1}+\log G^{(T0)-1}\right)-\int_q\log\frac{\left[(q^2+m_\psi^2+\tilde Um_\psi\bar\psi\psi)^2+\tilde U^2(\bar\psi\not{q}\psi)^2\right]}{(q^2+m_\psi^2)^2}$$

$$\mathrm{Tr}\log\left(\delta_{in}-\Gamma^{(2)}_{\psi_i^T\psi_k}\Gamma^{(2)-1}_{\bar\psi_k\psi_l}\Gamma^{(2)}_{\bar\psi_l\bar\psi_m^T}\Gamma^{(2)-1}_{\psi_m^T\bar\psi_n^T}\right)=-\int_q\log\frac{(q^2+m_\psi^2+\tilde Um_\psi\bar\psi\psi)^2-2\tilde U^2m_\psi^2(\bar\psi\psi)^2}{\left[(q^2+m_\psi^2+\tilde Um_\psi\bar\psi\psi)^2+\tilde U^2(\bar\psi\not{q}\psi)^2\right]}$$

Only $(\bar\psi\psi)^2$ appears in the sum

$$\partial_k\Gamma_k=-\frac{1}{2}\int_q\hat\partial_k\left[(4N_f+1)\log(q^2+4U'^2I_{GN})-\log(q^2+4U'(U'+\tilde U)I_{GN})\right]$$

Linear IR regularisation $\quad r_\psi=\left(\frac{k}{\sqrt{q^2}}-1\right)\Theta(k^2-q^2)$

$$\partial_k\Gamma_k=-\frac{k^{d+1}S_d}{d(2\pi)^d}\left[(4N_f+1)\frac{1}{k^2+4U'^2I_{GN}}-\frac{1}{k^2+4U'(U'+\tilde U)I_{GN}}\right]$$

Scaling solution $\quad \overline{I}=k^{2(1-d-\eta)}I\qquad x=(4Q_dN_f)^{-2}\overline{I}$

$$\overline{U}=k^{-d}U(I)|_{I=k^{-2(1-d)+2\eta}\overline{I}}\qquad y_k=(4Q_dN_f)^{-1}\overline{U}_k$$

$$\partial_ty_k(x)=-dy_k+2(d-1)xy'_k-\left(1+\frac{1}{4N_f}\right)\frac{1}{1+4y_k'^2(x)x}+\frac{1}{4N_f}\frac{1}{1+12y_k'^2(x)x+16y'_k(x)y''_k(x)x^2}$$

Fixed point potential

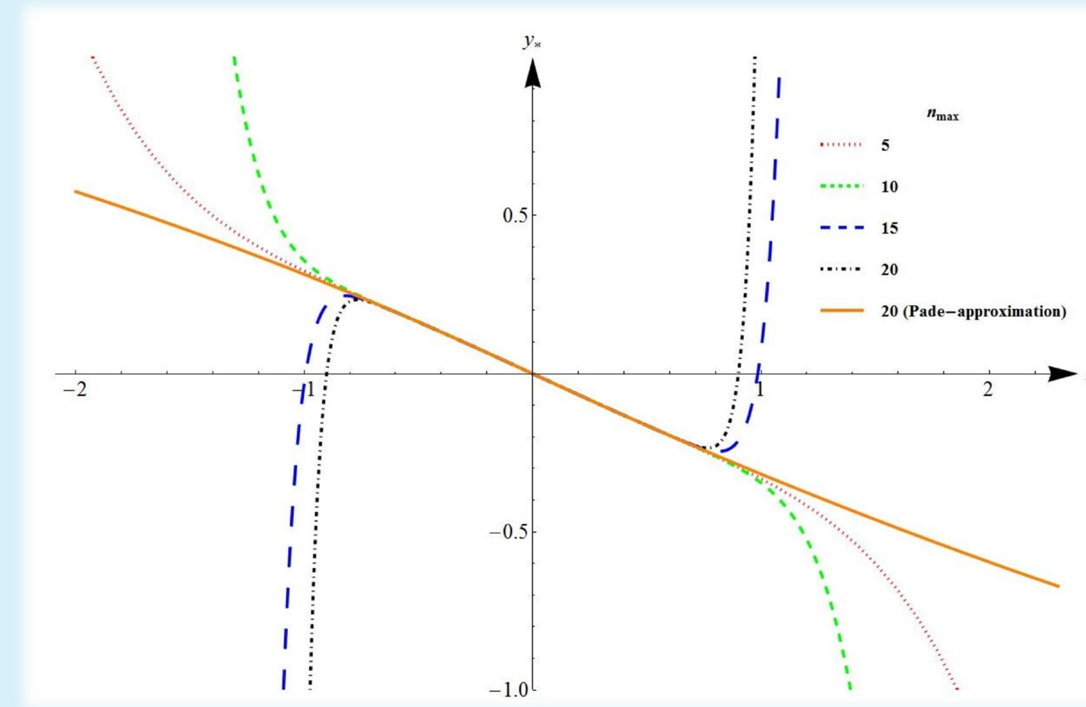
$$y_*(x)=\sum_{n=1}^{n_{max}}\frac{1}{n}l_{n*}x^n$$

Ensuring the right asymptotic behavior

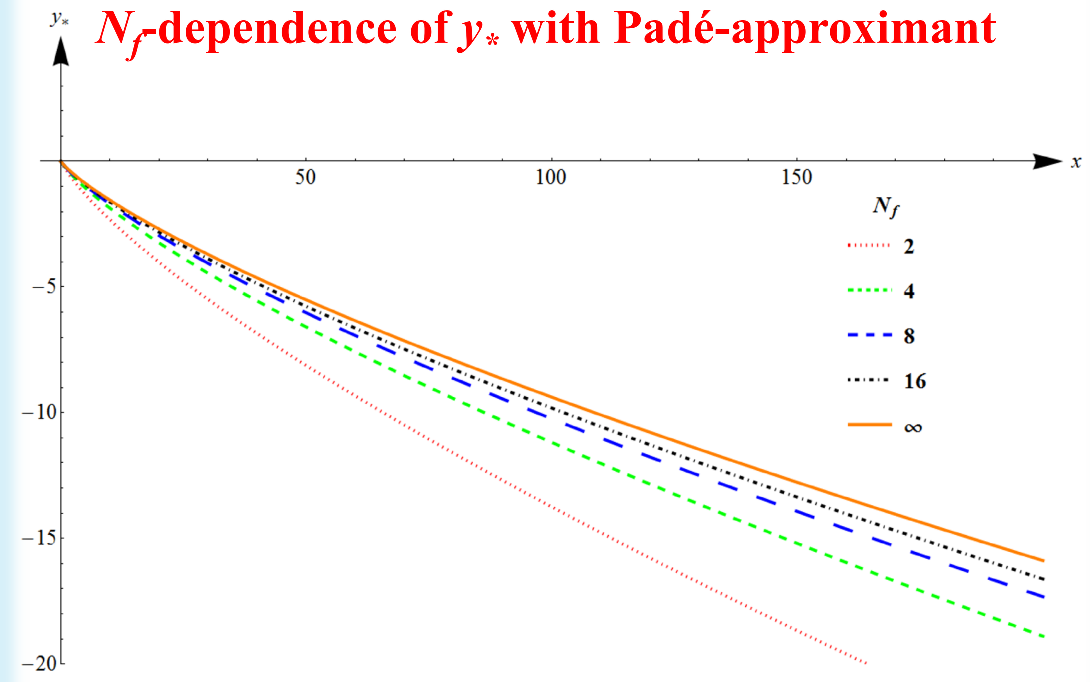
$$y_*(x)=(1+x^2)^{d/(4(d-1))}\lim_{N\to\infty}\mathrm{Padé}_N^N\left[\frac{\sum_{n=1}^{2N}l_{n*}x^n}{(1+x^2)^{d/(4(d-1))}}\right]$$

THE NON-GAUSSIAN FIXED POINT POTENTIAL

$N_f=2$, dependence on n_{max} of y_*



N_f-dependence of y_* with Padé-approximant



Scaling exponents

Lower triangle structure of the linearized RGE:

$$k\partial_k\delta l_n=-\left[d-2n\left(1+\frac{(n-1)(d-2)}{2N_f-1}\right)\right]\delta l_n+\sum_{j=1}^{n-1}\frac{\partial F(l_1,...,l_{n-1}}{\partial l_j}\Big|_{l_*}\delta l_j$$

$$\Theta^{GN}=d-2n\left(1+\frac{(n-1)(d-2)}{2N_f-1}\right)$$

d=3: single UV-unstable (relevant) operator*: n=1*

Stability of the results
under scale dependent wavefunction renormalisation

NLO of the gradient expansion of RGE)

$$\partial_kZ_k\frac{\delta(0)}{(2\pi)^d}=\frac{1}{N_f}\frac{d}{dq^2}\left\{-iq_m\gamma_m^{\alpha_1\alpha_2}\frac{\delta}{\delta\bar\psi_l^{\alpha_2}(-q)}\partial_k\Gamma_k\frac{\delta}{\delta\psi(q)_l^{\alpha_1}}\right\}_{|q=0}$$

$$\partial_tZ_k=0,\qquad \text{all}\quad N_f.$$

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