The Silver Blaze Problem in the Presence of an External Magnetic Field.

Prifysgol Abertawe
Swansea University

Chrisanthi Praki, Gert Aarts

Department of Physics, College of Science, Swansea University, United Kingdom.

Introduction

A complex scalar field with ϕ^4 interactions and a chemical potential, μ, is studied in the presence of an external magnetic field. Complex Langevin Dynamics is utilised in order to circumvent the sign problem which arises due to the inclusion of a non-zero chemical potential [1]. Standard numerical approaches cannot be successfully applied to such a system, as the inclusion of the chemical potential makes the action complex, meaning that importance sampling is no longer applicable.

The thermodynamic quantities of the system as a function of μ will be studied in order to determine the nature of its phase structure. At zero temperature, physical observable are independent of the chemical potential up to a critical value of the chemical potential, μ_c. The equations describing the observables contains terms proportional to μ, but there is an exact cancellation of the μ dependence in the region $\mu < \mu_c$ (known as the Silver Blaze region). This exact cancellation is referred to as the Silver Blaze phenomenon [1][2].

The System

The continuum action for a self-interacting complex scalar field with chemical potential μ in the presence of an external magnetic field is given by

$$S = \int d^4x[(\partial_\nu - iqA_\nu)\phi^*(\partial^\nu + iqA^\nu)\phi + m^2\phi^*\phi - \mu^2\phi^*\phi + \lambda(\phi^*\phi)^2 + \mu\phi^*\partial_4\phi - \mu\phi\partial_4\phi^*] \tag{1}$$

where q is the charge associated to the field ϕ, μ is the chemical potential and the magnetic vector potential, $\mathbf{A}$, is defined as $\mathbf{A} = \frac{B}{2}(-y, x, 0)$.
The complex field may be written in terms of two real fields as $\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$. The action is discretised in order to allow numerical analysis. The magnetic vector potential may be incorporated into the action as complex phases, $u_\nu(n) \in U(1)$, in the spatial directions orthogonal to the direction of the magnetic field [3].
If $n_x = N_x - 1$

$$u_x(n) = e^{-iq|B|N_x n_y} \tag{2}$$

If $n_x \neq N_x - 1$

$$u_x(n) = 1 \tag{3}$$

In the $\hat{y}$ direction

$$u_y(n) = e^{iq|B|n_x} \tag{4}$$

where N_ν is the total number of lattice sites in the ν direction and n_ν are the individual lattice sites in the ν direction and may take integer values from 0 to $N_\nu - 1$.
The external magnetic field takes discrete values and is described by [3]

$$q|B| = \frac{2\pi N_b}{N_x N_y} \tag{5}$$

where N_b is a positive integer.

$$0 \le N_b < \frac{N_x N_y}{4}, \quad 0 \le q|B| < \frac{\pi}{2} \tag{6}$$

due to the periodicity of the magnetic field in N_b.

Langevin Dynamics

The Langevin equations, in terms of the Langevin time, θ, for the fields ϕ_a $(a = 1, 2)$ are given by [1]

$$\frac{\partial}{\partial\theta}\phi_{a,x}(\theta) = -\frac{\delta S[\phi]}{\delta\phi_{a,x}(\theta)} + \eta_{a,x}(\theta) \tag{7}$$

The Gaussian distributed noise, η, is normalised as follows

$$\begin{aligned} \langle\eta_{a,x}(\theta)\rangle &= 0 \\ \langle\eta_{a,x}(\theta)\eta_{b,x'}(\theta')\rangle &= 2\delta_{a,b}\delta_{x,x'}\delta(\theta - \theta') \end{aligned} \tag{8}$$

The field is complexified as $\phi_a \to \phi_a^R + i\phi_a^I$. The complex Langevin equations, with the noise chosen to be real, may then be written as

$$\begin{aligned} \frac{\partial}{\partial\theta}\phi^R_{a,x}(\theta) &= K^R_{a,x}(\theta) + \eta_{a,x}(\theta) \\ \frac{\partial}{\partial\theta}\phi^I_{a,x}(\theta) &= K^I_{a,x}(\theta) \end{aligned} \tag{9}$$

where the real and imaginary drift terms, respectively, are defined to be

$$\begin{aligned} K^R_{a,x} &= -\mathrm{Re}\frac{\delta S}{\delta\phi_{a,x}}|_{\phi_a \to \phi_a^R + i\phi_a^I} \\ K^I_{a,x} &= -\mathrm{Im}\frac{\delta S}{\delta\phi_{a,x}}|_{\phi_a \to \phi_a^R + i\phi_a^I} \end{aligned} \tag{10}$$

The relevant physical observables may be written in terms of derivatives of the logarithm of the partition function, $\mathcal{Z}$.

Results

The following results were obtained for a 10^4 lattice with several different values for the external magnetic field with q|B| taking discrete values between 0 and $\frac{\pi}{2}$. The value of the magnetic field for a 10^4 lattice is given by $q|B| = \frac{\pi N_b}{50}$.

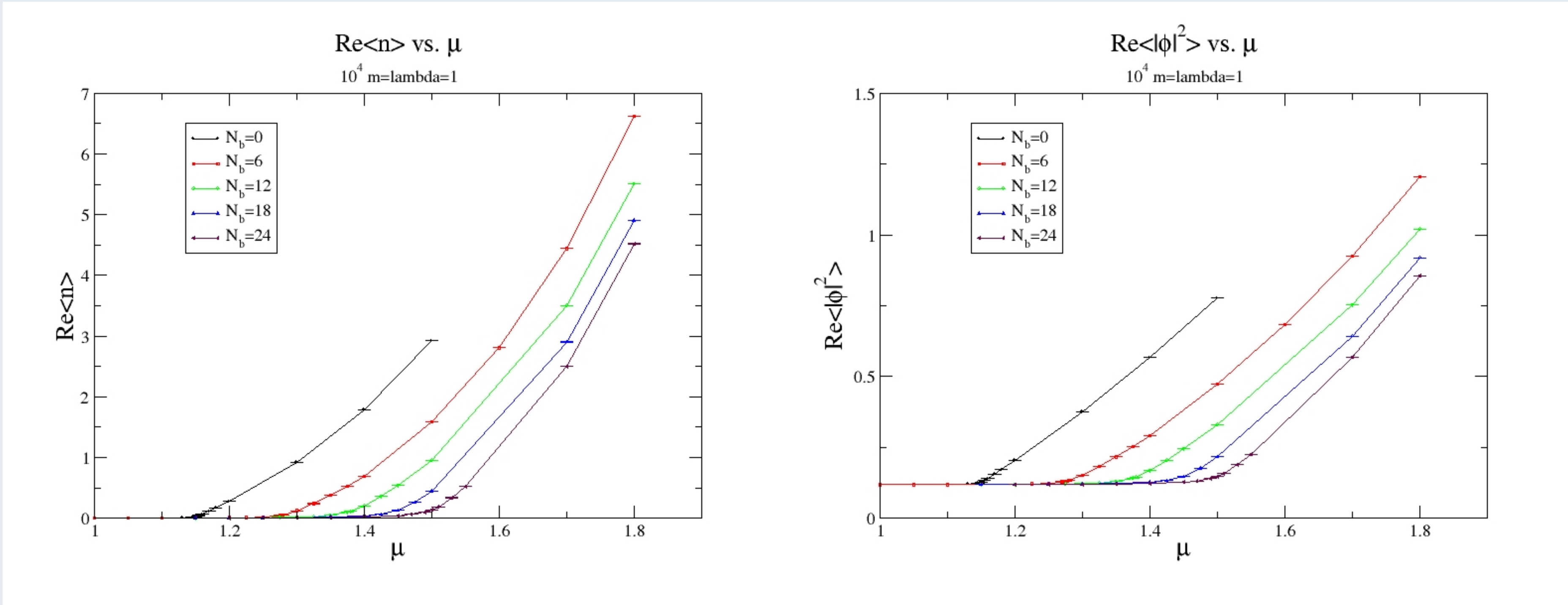

Figure 1: The real part of the density, $\langle n\rangle$, (left) and of the modulus of the field squared, $\langle|\phi|^2\rangle$, (right) against μ for different values of the external magnetic field. For increasing B the onset occurs at larger values of μ and the Silver Blaze region is extended.

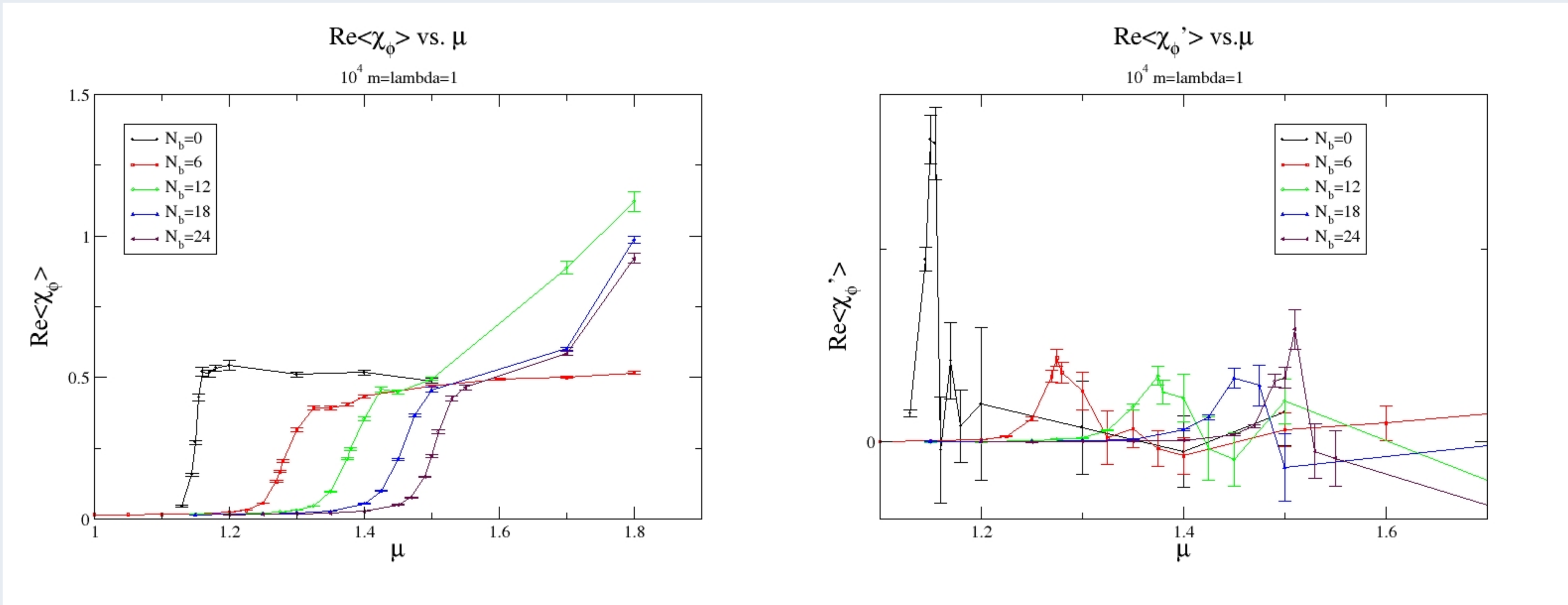

Figure 2: The real part of the $\langle|\phi|^2\rangle$ susceptibility, χ_{ϕ^2}, (left) of the derivative of χ_{ϕ^2} (right) against μ for different values of the external magnetic field.

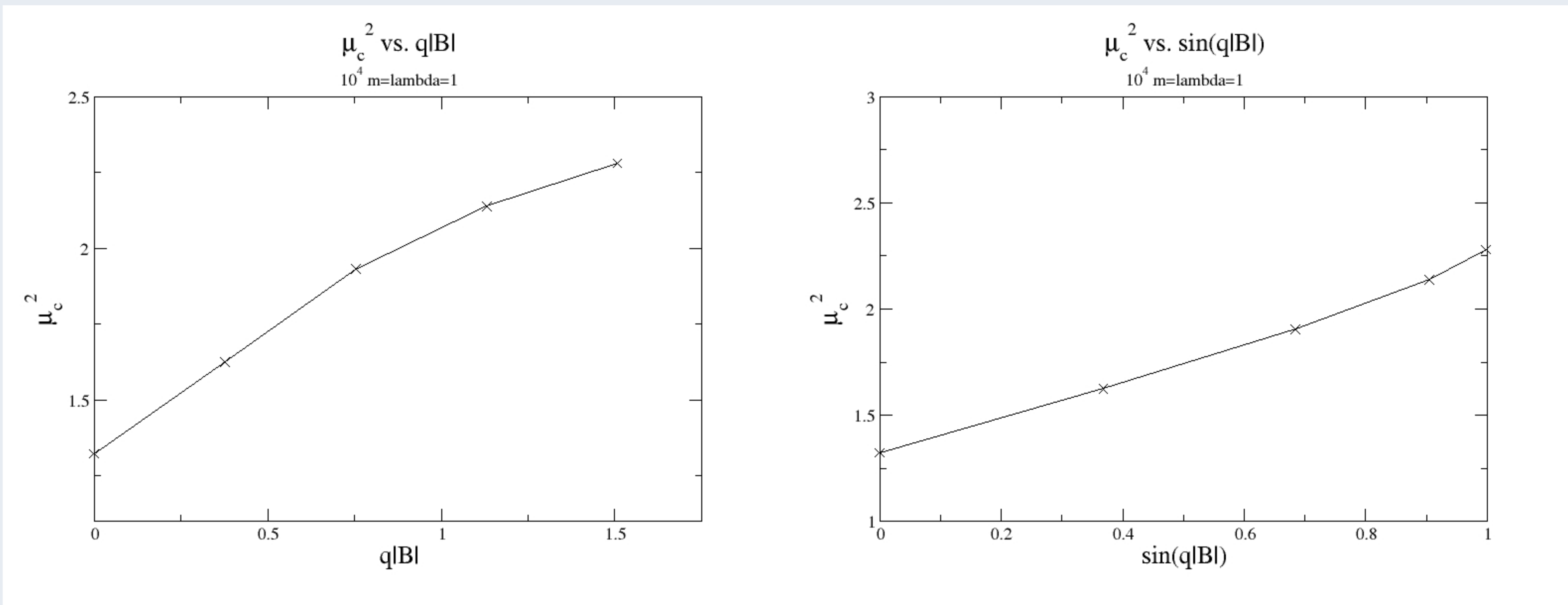

Figure 3: The critical chemical potential squared, μ_c^2, against the magnetic field strength (left) and against sin(q|B|) (right).

The results in figure 3 were determined by estimating the positions of the peaks in χ_{ϕ^2}. From a lowest Landau level approximation it can be shown that $\mu_c^2 = \mathrm{m}^2 + \mathrm{q}|\mathrm{B}|$, [4] [5] which would imply that μ_c^2 should be proportional to q|B|. However, q|B| enters into the lattice formulation as sin(q|B|) and so in this case it would be expected that μ_c^2 should be proportional to sin(q|B|), rather than q|B|.

Conclusion

The numerical results indicate that for increasing external magnetic field strength the value of μ_c increases. This is due to the external magnetic field acting to increase the effective mass, meaning that a larger value of μ is required in order to counteract this. The increase in μ_c is expected to be proportional to $\sqrt{\mathrm{sin(q|B|)}}$ and the results obtained from the 10^4 lattice seem to be in accordance with this. The Silver Blaze region is extended as B is increased. An open question is the phase structure in the T, μ, B phase diagram.

References

[1] G. Aarts and I. -O. Stamatescu, PoS LATTICE **2008** (2008) 176 [arXiv:0809.5227 [hep-lat]].
[2] T. D .Cohen, Phys. Rev. Lett. **91** (2003) 222001 [hep-ph/0307089].
[3] G. S. Bali, F. Bruckmann, G. Endrodi, Z. Fodor, S. D. Katz, S. Krieg, A. Schafer and K. K. Szabo, JHEP **1202** (2012) 044 [arXiv:1111.4956 [hep-lat]].
[4] G. EndrÃűdi, JHEP **1304** (2013) 023 [arXiv:1301.1307 [hep-ph]].
[5] G. Endrodi, arXiv:1407.1216 [hep-lat].