Magnetic Catalysis in Nuclear Matter

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Motivation

Cold dense matter in strong magnetic fields is found in ...

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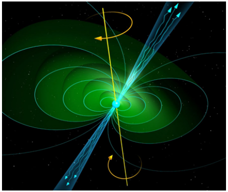
- magnetars

on the surface $B \sim 10^{15}\,\mathrm{G}$

R.C. Duncan and C. Thompson, Astrophys.J. 392, L9 (1992)

in the core possibly up to $B \sim 10^{18}\,\mathrm{G}$

D. Lai and S. Shapiro, Astrophys.J. 383, 745 (1991)



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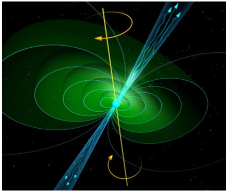

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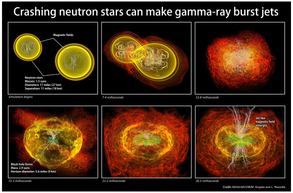
- binary neutron star mergers



Gravitational wave signature sensitive to equation of state

J.S. Read et al., Phys.Rev. D88, 044042 (2013)

The magneto-rotational instability may increase the initial B-field by one order of magnitude

D.M. Siegel et al., Phys.Rev. D87, 121302 (2013)

Extended Linear Sigma Model (eLSM) part I

Mesons:

D. Paraganlija et al., Phys.Rev. D82, 054024 (2010) & Phys.Rev. D87, 014011 (2013) & Phys.Rev. D87, 014011 (2013)

The eLSM is based on

- degrees of freedom of the QCD vacuum: hadrons
- symmetries of QCD: chiral and dilatation invariance

Relevant ingredients ($N_f = 2$):

- chiral partner of pions σ: a finite VEV breaks chiral symmetry fluctuations correspond to the state $f_0(1370)$
- the lightest scalar resonance $f_0(500)$ corresponds to an $SU(2)_L \times SU(2)_R$ singlet state χ; tetraquark or pion-pion resonance
- the $SU(2)_L \times SU(2)_R$ singlet vector meson ω_μ corresponding to $\omega(782)$

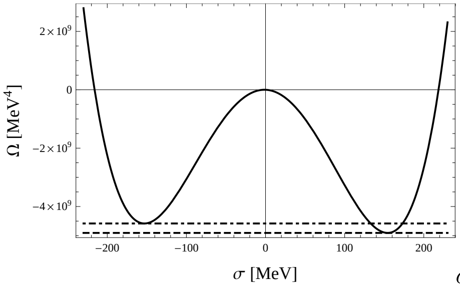
eLSM part I

Tree level free energy:

$$\Omega_{\rm mes}^{\rm tree} = -\frac{1}{2} m\sigma^2 + \frac{\lambda}{4}\sigma^4 - \epsilon\sigma - \frac{1}{2} m_\omega^2 \omega_0^2 + \frac{1}{2} m_\chi^2 \chi^2 - g\chi\sigma^2,$$

with gap equation for χ and ω_0

$$\omega_0 = 0, \quad \chi = g\sigma^2 / m_\chi^2$$



$\sigma_{\rm min} = Zf_\pi \simeq 154\ {\rm MeV}$

eLSM part II

Including nucleons in the mirror assignment:

C. DeTar and T. Kunihiro, Phys.Rev. D39, 2805 (1989)
S. Gallas, F. Giacosa, and D.Rischke, Phys.Rev. D82, 014004 (2010)

Isospin-douplet spinor ψ_1 and its parity partner ψ_2 obeying

$$\psi_{1,R/L} \to U_{R/L}\psi_{1,R/L}, \quad \psi_{2,R/L} \to U_{L/R}\psi_{2,R/L},$$

allow for a chirally symmetric mass term

$$m_0(\overline{\psi}_2\gamma_5\psi_1 - \overline{\psi}_1\gamma_5\psi_2).$$

dilatation invariance $\to$ m_0 generated dynamically, e.g. $m_0 \propto \chi$.
Interaction Lagrangian:

$$\mathcal{L}^{\rm int} = -\sum_i \overline{\psi}_i(\frac{g_i\sigma}{2} + g_\omega\gamma^0\omega_0)\psi_i - a\chi(\overline{\psi}_2\gamma_5\psi_1 - \overline{\psi}_1\gamma_5\psi_2)$$

eLSM part II

Effective fermion mass and chemical potential:

$$
\begin{aligned}
M_{N,N^*} &= \pm\frac{g_1 - g_2}{2}\sigma + \sqrt{\left(\frac{g_1 + g_2}{4}\sigma\right)^2 + (a\chi)^2} \\
\mu^* &= \mu - g_\omega \omega_0
\end{aligned}
$$

Free Energy in "no sea approximation"

N.K. Glendenning, Phys.Lett. B208, 335 (1988) & Nucl.Phys. A493, 521 (1989)

$$
\begin{aligned}
\Omega &= \Omega_{\mathrm{mes}}^{\mathrm{tree}} + \sum_i \int \frac{\mathrm{d}^3 k}{2\pi^3} \left\{ -E_i(k) + \left[E_i(k) - \mu^*\right] \Theta\left[\mu^* - E_i(k)\right] \right\} \\
E_{N,N^*}(k) &= \sqrt{\vec{k}^{\,2} + M_{N,N^*}^2}
\end{aligned}
$$

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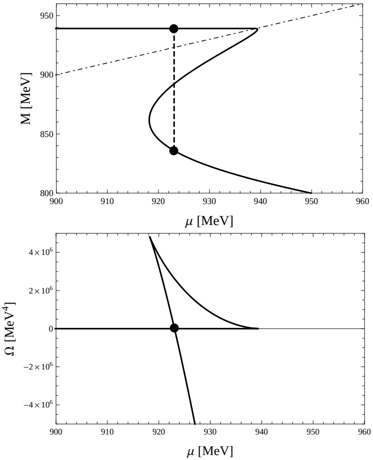
eLSM part II

Solving the gap equation - gas/liquid phase transition:



$$M_N^{\mathrm{vac}} = 939.12\,\mathrm{MeV}$$
$$\mu_c = 923.06\,\mathrm{MeV}$$
$$\Rightarrow E_B = 16.06\,\mathrm{MeV}$$
$$\rho_0 = 0.15\,\mathrm{fm}^{-3}$$

$M_{N^*}^{\mathrm{vac}} = 1535.55\ \mathrm{MeV}$ is identified with $N(1535)$

Magnetic catalysis

Fermions in a magnetic field

the dispersion relations for fermions with charge q in a magnetic field are altered: Landau level quantization (anomalous magnetic moment ignored)

$$E_\ell(k_z) = \sqrt{k_z^2 + M^2 + 2\,|q|\,B\ell}$$

Free energy for one charged fermion species ψ at $\mu = T = 0$

$$\Omega_\mathrm{f} \quad = \quad \frac{B^2}{2} - \frac{|q|\,B}{4\pi^2}\sum_\ell (2 - \delta_{\ell,0}) \int \mathrm{d}\,k_z\, E_\ell(k_z)$$

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&= -2\int \frac{\mathrm{d}^3\,k}{(2\pi)^3}\sqrt{\vec{k}^{\,2} + M^2} + \frac{B_r^2}{2} - \frac{(|q_r|\,B_r)^2}{24\pi^2}\ln\frac{2\,|q_r|\,B_r}{Q^2 A^{12}} \\
&\quad - \frac{(|q_r|\,B_r)^2}{2\pi^2}\left[\frac{x^2}{4}(3 - 2\ln x) + \frac{x}{2}\left(\ln\frac{x}{2\pi} - 1\right) + \psi^{(-2)}(x)\right],
\end{aligned}$$

with $x = M^2/(2\,|q_r|\,B_r)$, A being the Glaisher constant and Q is the renormalization scale

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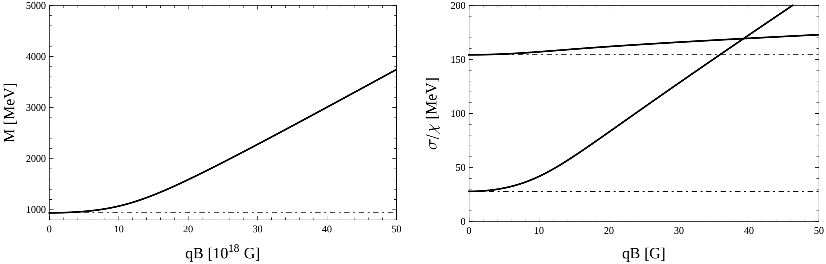
Magnetic catalysis

Magnetic catalysis in the eLSM

This contribution gives the well known magnetic catalysis effect

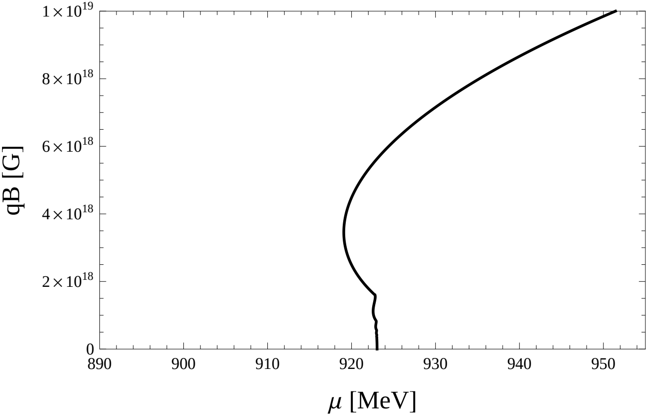
V.P. Gusynin, V.A. Miransky, and I.A. Shovkovy, Phys.Lett., B349:477–483 (1995)



$$\frac{M_N(B)}{M_N^{\mathrm{vac}}} \simeq 1 + \left(\frac{|q|\, B}{3.06 \times 10^{19}\,\mathrm{G}} \right)^2$$

Magnetic catalysis in the onset of nuclear matter

Phase diagram:



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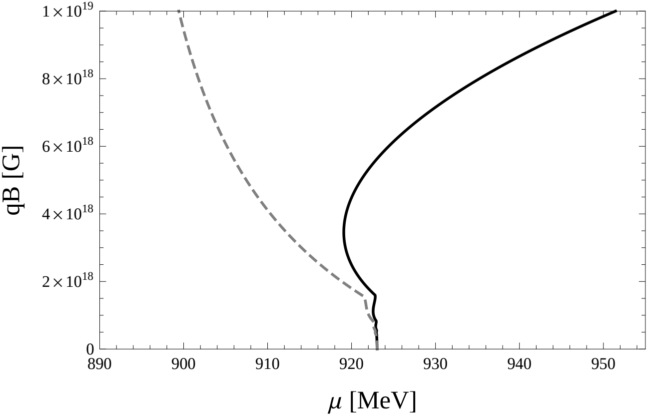
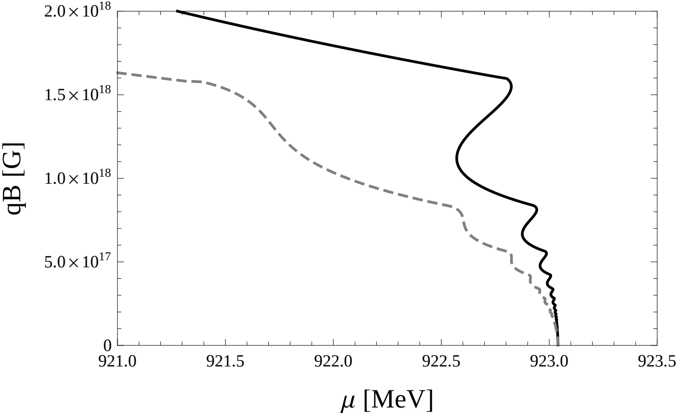


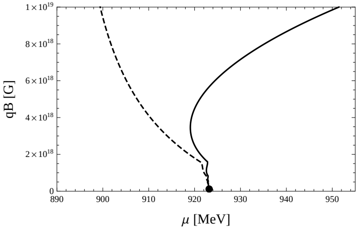
Magnetic catalysis in the onset of nuclear matter

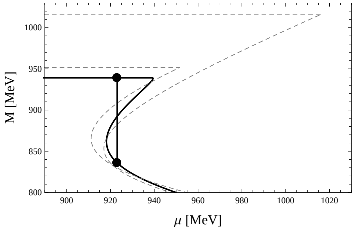
Phase diagram: (zoom-in)



Magnetic catalysis in the onset of nuclear matter

Phase diagram and mass gap:





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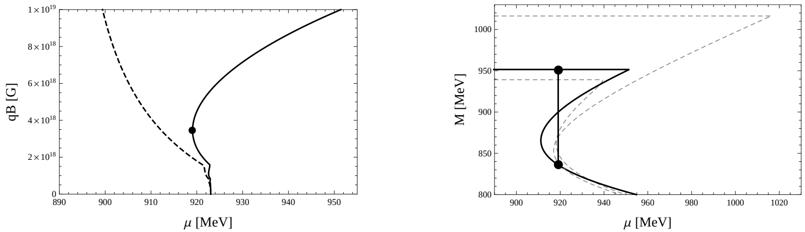

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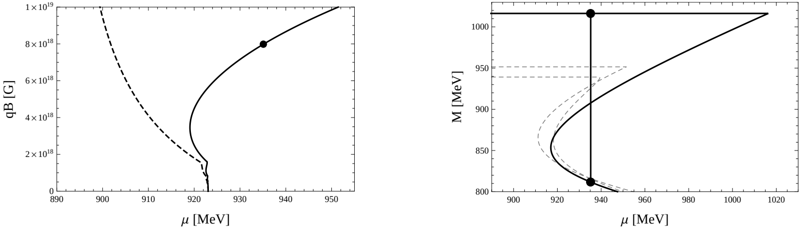
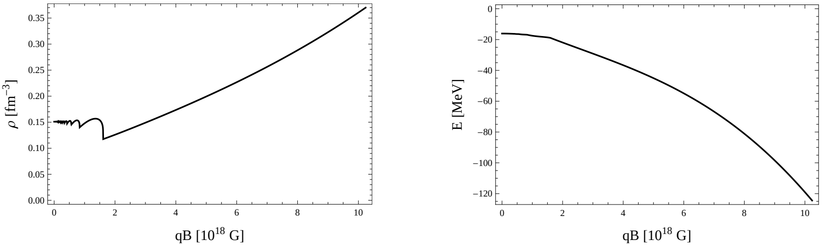


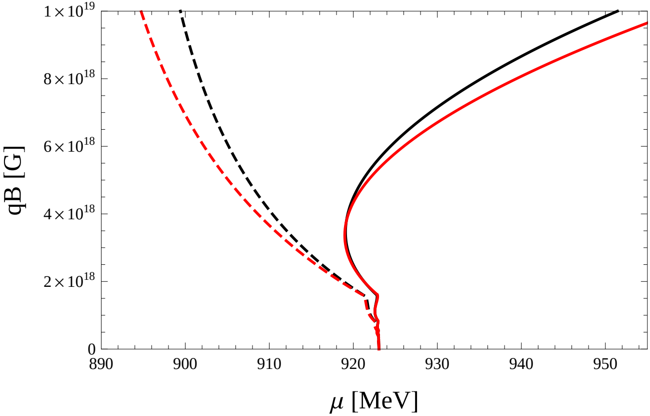
Magnetic catalysis in the onset of nuclear matter

density and binding energy along the critical line:



Magnetic catalysis in the onset of nuclear matter

Comparison with results from the Walecka model:



Conclusion

So far the effect of the B-dependent vacuum terms has always been neglected:

A. Broderick, M.Prakash, and J.M. Lattimer, Astrophys. J. 537, 351 (2000) & Phys.Lett. B531, 167 (2002),
M. Sinha, B. Mukhopadhyay, and A. Sedrakian, Nucl.Phys. A898, 43 (2013),
A. Rabhi, P. Panda, and C. Providencia, Phys.Rev. C84, 035803 (2011),
FP, A. Rebhan, and A. Schmitt, J.Phys. G39, 054006 (2012),
J. Dong, W. Zuo, and J. Gu, Phys.Rev. D87, 103010 (2013),
R.C.R. de Lima, S.Avancini, and C. Providencia, Phys.Rev. C89, 035804 (2013),
R. Casali, L.Castro, and D. Menezes, Phys.Rev. C89, 015805 (2014),

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R. Casali, L.Castro, and D. Menezes, Phys.Rev. C89, 015805 (2014),

incorporation of magnetic catalysis in models of nuclear matter is the first important step

Outlook

Possible further research:

- include the anomalous magnetic moment
- allow for anisotropic condensates: (magnetic) chiral spiral
  A. Rebhan, S. Stricker, and A. Schmitt, JHEP 0905, 084 (2009),
  I. Frolov, V.C. Zhukovsky, and K. Klimenko, Phys.Rev. D82, 076002 (2010),
  A. Rebhan, S. Stricker, and A. Schmitt, JHEP 0905, 084 (2009),
  FP, A. Rebhan, and A. Schmitt, JHEP 1103, 033 (2011),
  FP, A. Rebhan, and A. Schmitt, J.Phys. G39, 054006 (2012)
- impose neutrality and β-equilibrium conditions
- analyze the chiral phase transition in the eLSM