Polyakov-loop potential from a massive extension of the background field gauge

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(Work in collaboration with J. Serreau, M. Tissier, N. Wschebor)

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SEWM, Lausanne, July 2014
Motivation
Motivation

Semi-analytical approaches to strongly interacting matter

Need for semi-analytical methods to investigate infrared properties of QCD or related theories.

These approaches (SD-eq, fRG, . . .) usually require gauge fixing:

Ex: Landau gauge action

\[ S = \int_x \left\{ \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \partial_\mu \bar{c}^a (D_\mu c)^a + i h^a \partial_\mu A_\mu^a \right\} \]

Valid only in the UV where the Gribov ambiguity is expected not to play a role.

\[ \Rightarrow \] Some additional input is needed in the IR.

Extended Landau gauge (eLG)

Alternative approach: find phenomenologically (and hopefully theoretically) motivated actions that could take into account the existence of Gribov copies.

A candidate for such an action is the extended Landau gauge (eLG) action

\[ S = \int_X \left\{ \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \partial_{\mu} \bar{c}^a (D_{\mu} c)^a + i h^a \partial_{\mu} A_{\mu}^a + \frac{1}{2} m^2 A_{\mu}^a A_{\mu}^a \right\} \]
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\]

- It is perturbatively renormalizable.
- A perturbative, calculation of the $T = 0$ propagators and vertices reproduces lattice data!

Tissier, Wschebor, Phys.Rev. D84 (2011);

- It could result from a gauge fixing procedure which averages over Gribov copies.
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\]

- It is perturbatively renormalizable. Extra parameter: for SU(3), \( m \approx 510 \text{ MeV} \).
- A perturbative, calculation of the \( T = 0 \) propagators and vertices reproduces lattice data!

- It could result from a gauge fixing procedure which averages over Gribov copies.
**Motivation**

**Tests at finite temperature**

One loop, finite $T$, eLG ghost and chromo-magnetic propagators agree well with lattice results:

\[ F(0, k) \]
\[ G_T(0, k) \]

UR, J. Serreau, M. Tissier and N. Wschebor, to appear in PRD.
Tests at finite temperature

One loop, finite $T$, eLG ghost and chromo-magnetic propagators agree well with lattice results:

The eLG fails in reproducing the lattice chromo-electric propagator in the vicinity of the confinement/deconfinement phase transition:

- could signal a failure of the eLG model
  → but similar limitations are observed in other approaches.

- could signal limitations of the use of the LG
  → explore “more appropriate” gauges and test whether the corresponding (IR) extended gauge models are capable to describe the phase transition.

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Motivation

Polyakov loop and center-symmetry breaking

Free-energy $F$ for having an isolated static quark located somewhere

$$e^{-\beta F} = \frac{1}{N} \left\langle \text{tr} \, P \, e^{ig \int_0^\beta d\tau A_0(\tau)} \right\rangle \equiv \langle L \rangle \quad \text{with} \quad A_0 = A_0^a t^a \quad (a = 1, \ldots, N)$$

The Yang-Mills action at finite $T$ is invariant under twisted or center (gauge) transformations

$$U(\beta, \bar{x}) = U(0, \bar{x}) \, V \quad \text{with} \quad V \in SU(N)_{\text{center}} = \left\{ e^{i2\pi k/\mathbb{N}} \mathbb{1} | k = 0, \ldots, N - 1 \right\}$$

Under a center transformation $\langle L \rangle \to \langle L \rangle \, e^{i2\pi k/\mathbb{N}}$:

- if center-symmetry is broken $\langle L \rangle \neq 0$ and $F < \infty$ (deconfined phase);
- if center-symmetry is restored $\langle L \rangle = 0$ and $F = \infty$ (confined phase);
Polyakov loop and center-symmetry breaking

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The lattice predicts a 2nd/1st order breaking of center-symmetry in the SU(2)/SU(3) case. Confirmed by the functional renormalization group:

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The lattice predicts a 2nd/1st order breaking of center-symmetry in the $SU(2)/SU(3)$ case. Confirmed by the functional renormalization group:

Can this physics be captured perturbatively?
Extended background field gauge
Choose a background $\bar{A}_\mu^a$. Fix the gauge according to $(\bar{D}_\mu (A_\mu - \bar{A}_\mu))^a = 0$.

In the limit $\xi \to 0$, one obtains the Landau-deWitt gauge:

$$S_{\bar{A}}[A] = \int_X \left\{ \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + (\bar{D}_\mu \bar{c})^a (D_\mu c)^a + i\hbar^a (\bar{D}_\mu (A_\mu - \bar{A}_\mu))^a \right\}$$

Why to consider such a gauge? From $S_{\bar{A}}[A]$, it is possible to construct $\tilde{\Gamma}[\bar{A}]$ such that

- the physics is obtained at the absolute minimum of $\tilde{\Gamma}[\bar{A}]$;
- center-symmetry is manifest because $\tilde{\Gamma}[\bar{A}^U] = \tilde{\Gamma}[\bar{A}]$. 
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We upgrade the background field gauge to the extended background field gauge (eBFG):

$$S_{\vec{A}}[A] = \int_x \left\{ \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + (\vec{D}_\mu \vec{c})^a (D_\mu c)^a + ih^a (\vec{D}_\mu (A_\mu - \vec{A}_\mu))^a + \frac{1}{2} m^2 (A_\mu^a - \vec{A}_\mu^a) (A_\mu^a - \vec{A}_\mu^a) \right\}$$

The mass term does not break center symmetry!

Feynman rules?
Feynman rules: simplifying remarks

We are interested in thermodynamical properties:

⇒ uniform background: $\bar{A}_\mu^a(\tau, \vec{x}) = \bar{A}_\mu^a$.

⇒ effective potential: $\gamma(\bar{A}) = \tilde{\Gamma}[\bar{A}] / (\beta V)$.

We are interested in the Polyakov loop:

⇒ temporal background $\bar{A}_\mu^a = \bar{A}_0^a \delta_{\mu 0}$.

One can always choose $\bar{A}_0$ in the Cartan sub-algebra:

⇒ SU(2): $\bar{A}_0 = \bar{A}_0^3 \sigma^3 / 2$

⇒ SU(3): $\bar{A}_0 = \bar{A}_0^3 \lambda^3 / 2 + \bar{A}_0^8 \lambda^8 / 2$

... 

The only role of the background is to lift the usual degeneracy between the three color directions.
Feynman rules: modes

**Ex.: SU(2) ghost propagator:**

\[
\begin{align*}
G^{11}(K) &= \frac{1}{K^2} \\
G^{22}(K) &= G^{11}(K) \\
G^{33}(K) &= G^{11}(K)
\end{align*}
\]

→ **eBFG:**

\[
\begin{align*}
G^0(K) &= \frac{1}{K^2} \\
G^{+(K)} &= \frac{1}{K^2} \\
G^{-}(K) &= \frac{1}{K^2}
\end{align*}
\]

\[K_\sigma = (\omega_n + \sigma gA_3, \vec{k})\]

**SU(2) gluon propagator:**

\[
\begin{align*}
G^{11}_{\mu\nu}(K) &= \frac{P^\perp_{\mu\nu}(K)}{K^2+m^2} + \frac{\xi P^\parallel_{\mu\nu}(K)}{K^2+\xi m^2} \\
G^{22}_{\mu\nu}(K) &= G^{11}_{\mu\nu}(K) \\
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G^0_{\mu\nu}(K) &= \frac{P^\perp_{\mu\nu}(K_+)}{K^2+m^2} + \frac{\xi P^\parallel_{\mu\nu}(K_+)}{K^2+\xi m^2} \\
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G^{-}_{\mu\nu}(K) &= \frac{P^\perp_{\mu\nu}(K_-)}{K^2+m^2} + \frac{\xi P^\parallel_{\mu\nu}(K_-)}{K^2+\xi m^2}
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\( \rightarrow \)

**EBFG:**

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G^0(K) &= \frac{1}{K^2} \\
G^+(K) &= \frac{1}{K^2_+} \\
G^-(K) &= \frac{1}{K^2_-}
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**SU(2) gluon propagator:**

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G^{\mu\nu}_{11}(K) &= \frac{P^\perp_{\mu\nu}(K)}{K^2 + m^2} + \frac{\xi P^\parallel_{\mu\nu}(K)}{K^2 + \xi m^2} \\
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G^-_{\mu\nu}(K) &= \frac{P^\perp_{\mu\nu}(K_-)}{K_-^2 + m^2} + \frac{\xi P^\parallel_{\mu\nu}(K_-)}{K_-^2 + \xi m^2}
\end{align*}
\]

For each charge eigenstate, we have:

3 massive transverse gluons, 1 massless longitudinal gluon (\(\xi \to 0\)), 2 massless ghosts.

\(K_\sigma = (\omega_n + \sigma g \bar{A}_3^3, \mathbf{k})\)
Polyakov-loop potential and center-symmetry breaking
To discuss center-symmetry breaking from $\gamma(\tilde{A})$, it is first necessary to identify $\tilde{A}$ as an order parameter for center-symmetry breaking.

At LO, the path ordering in $\langle L \rangle$ does not play a role

$$
\langle L \rangle = \frac{1}{N} \left( \text{tr} P e^{-ig \int_0^\beta d\tau (\tilde{A}_0 + a_0(\tau))} \right) = \frac{1}{N} \text{tr} e^{-i\beta g\tilde{A}_0} + \mathcal{O}(g^2)
$$

$$
\equiv \langle L \rangle_{lo}
$$

SU(2): $\tilde{A}_0 = \tilde{A}_0^3 \frac{\sigma^3}{2}$

$$
\langle L \rangle_{lo} = \cos \left( \frac{\beta g\tilde{A}_0^3}{2} \right) \quad \Rightarrow \quad \langle L \rangle_{lo} = 0 \quad \text{iff} \quad r_3 \equiv \beta g\tilde{A}_0 = \pi [2\pi]
$$

The background plays the role of an order parameter for center-symmetry breaking!
LO Polyakov-loop potential: SU(2) case

Only the charged modes contribute to the background dependence of the potential

\[
\gamma(\bar{A}_0^3) = 3 T \int_q \ln \left( 1 + e^{-2q \varepsilon_3} - 2 e^{-q \varepsilon_3} \cos(\beta g \bar{A}_0^3) \right) - T \int_q \ln \left( 1 + e^{-2q} - e^{-q} \cos(\beta g \bar{A}_0^3) \right)
\]

\( \equiv r_3 \)
Polyakov-loop potential and center-symmetry breaking

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Symmetries:

Center symmetry: $$\gamma(r_3) = \gamma(r_3 + 2\pi)$$

$$\Rightarrow$$ we can restrict to $$[0, 2\pi]$$

Center + C-symmetry:

$$\gamma(\pi + \delta r_3) = \gamma(-\pi - \delta r_3) = \gamma(\pi - \delta r_3)$$

$$\Rightarrow \begin{cases} \text{we can restrict to } r_3 \in [0, \pi] \\ 0 \text{ and } \pi \text{ are extrema} \end{cases}$$
Polyakov-loop potential and center-symmetry breaking

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\]

Thermal asymptotic behavior:

\[
\begin{align*}
T & \gg m, \quad 2T \int q \ln(1 + e^{-2\beta q} - e^{-\beta q} \cos(r_3)) \equiv \gamma_{\text{Weiss}}(r_3) \\
T & \ll m, \quad -T \int q \ln(1 + e^{-2\beta q} - e^{-\beta q} \cos(r_3)) = -\frac{1}{2} \gamma_{\text{Weiss}}(r_3)
\end{align*}
\]

\[
\gamma_{\text{Weiss}}(r_3) = \frac{(r_3 - \pi)^4}{24\pi^2} - \frac{(r_3 - \pi)^2}{12} + \frac{7\pi^2}{360}
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reverted Weiss potential!  
(ghost dominate at low \( T \);  
as in the fRG approach)
Polyakov-loop potential and center-symmetry breaking

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\[ \gamma(\bar{A}_0^3) = 3T \int_q \ln \left( 1 + e^{-2\beta q} - 2e^{-\beta q} \cos(\beta g A_0^3) \right) - T \int_q \ln \left( 1 + e^{-2\beta q} - e^{-\beta q} \cos(\beta g A_0^3) \right) \equiv r_3 \]

Thermal asymptotic behavior:

For large temperatures,

\[ T \gg m, \quad 2T \int_q \ln(1 + e^{-2\beta q} - e^{-\beta q} \cos(r_3)) \equiv \gamma_{\text{Weiss}}(r_3) \]

For small temperatures,

\[ T \ll m, \quad -T \int_q \ln(1 + e^{-2\beta q} - e^{-\beta q} \cos(r_3)) = -\frac{1}{2} \gamma_{\text{Weiss}}(r_3) \]

The Weiss function is given by

\[ \gamma_{\text{Weiss}}(r_3) = \frac{(r_3 - \pi)^4}{24\pi^2} - \frac{(r_3 - \pi)^2}{12} + \frac{7\pi^2}{360} \]
We obtain a mildly first order phase transition in agreement with lattice or fRG results.

We obtain $T_c/m \approx 0.363$ and since $m \approx 510$ MeV, we obtain $T_c \approx 185$ MeV.

Still far from the lattice ($T_c \approx 295$ MeV) or from fRG results ($T_c \approx 284$ MeV).
LO artifacts

The Polyakov loop reaches its limiting value at a finite temperature $T_a/T_c = 1.5$:

![Graph showing the Polyakov loop reaching its limiting value at $T_a/T_c = 1.5$.]

Similar conclusion for SU(3) again with $T_a/T_c = 1.38$:

![Graph showing the Polyakov loop reaching its limiting value at $T_a/T_c = 1.38$.]

Additional singularity in thermodynamical observables in the range $[T_c, 2T_c]$. 
Next-to-leading order results
Next-to-leading order results

NLO Polyakov-loop potential

\[ \gamma_{\text{nlo}}(\bar{A}) = - \]

(with background-dependent propagators and background-dependent derivative vertices)
→ At NLO, $\bar{A}$ plays the role of an order parameter. We find $\langle L \rangle_{\text{nlo}} = (1 + ag^2 \beta m) \times \langle L \rangle_{\text{lo}}$ with

$$a = \frac{3}{32\pi} + \sin^2\left(\frac{r_3}{2}\right) \int \frac{d^3 q}{(2\pi m)^3} \left[ \frac{1}{\cosh(\beta q) - \cos(r_3)} - \frac{q^2}{\varepsilon_q^2 \cosh(\beta \varepsilon_q) - \cos(r_3)} \right]$$

Since $a > 0$, it follows that $\langle L \rangle_{\text{nlo}} = 0$ iff $\langle L \rangle_{\text{lo}} = 0$ iff $r_3 = \pi[2\pi]$.

→ The NLO Polyakov loop potential is UV finite.

→ Our “predictions” concerning the orders of the SU(2)/SU(3) transitions remain the same.

→ We obtain improved values for $T_c$ in the SU(3) case:

<table>
<thead>
<tr>
<th></th>
<th>order</th>
<th>LO</th>
<th>NLO</th>
<th>FRG*</th>
<th>Lattice**</th>
</tr>
</thead>
<tbody>
<tr>
<td>SU(3)</td>
<td>1st</td>
<td>185 MeV</td>
<td>256 MeV (prelim.)</td>
<td>284 MeV</td>
<td>295 MeV</td>
</tr>
</tbody>
</table>


→ The LO artifact seems to be lifted or at least pushed to temperatures above $3T_c$:
we do not find additional thermodynamical singularities in the range $[T_c, 3T_c]$. 
Conclusions and Outlook

- A perturbative one-loop calculation of the Polyakov-loop potential within the extended BFG allows to capture the physics of center-symmetry breaking.

- Our approach allows for a systematic determination of higher order corrections.

- Two-loop corrections are important to reach a value of the transition temperature comparable to that obtained on the lattice or with an fRG approach and to get rid of certain artifacts of the one-loop calculation.

* * * * *

- eBFG propagators (in progress).
- Include quarks and chemical potential (in progress).
- ...
- Solid theoretical justification of extended massive gauges?

Thermodynamics: meaningful (monotonically increasing) pressure?
And please, visit the posters by:

- Marcela Peláez;
- Gergely Markó;
- Andréas Tresmontant.
eLG from the perspective of the eBFG

The loop expansion in the eLG looks like an expansion around an instable point.

\[ p = -\gamma(r_{\text{min}}) \]

eLG: \( r_3 = 0 \) (max)

eBFG: \( r_3 = \pi \) (min)

\[ p = T^4 \int_q \ln \left( 1 - e^{-q} \right) + T^4 \int_q \ln \left( 1 + e^{-2q} - 2e^{-q} \cos(r_3) \right) = 3 T^4 \int_q \ln \left( 1 - e^{-q} \right) < 0 \]

eBFG for \( T \ll m \):

\[ p = T^4 \int_q \ln \left( 1 - e^{-q} \right) + T^4 \int_q \ln \left( 1 + e^{-q} \right)^2 = -\frac{3}{4} T^4 \int_q \ln \left( 1 - e^{-q} \right) > 0 \]

Effective change of nature of the degrees of freedom in the presence of the background!
LO Polyakov-loop potential: SU(3) case

Each charged mode contributes as in the SU(2) case but with its own $Q_3$ and $Q_8$ charges:

\[
\gamma_{su(3)}(r_3, r_8) = \gamma_{su(2)}(r_3) + \gamma_{su(2)}\left(\frac{r_3 + r_8\sqrt{3}}{2}\right) + \gamma_{su(2)}\left(\frac{-r_3 + r_8\sqrt{3}}{2}\right)
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Center-symmetry
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Center-symmetry + Color invariance + C-symmetry
Each charged mode contributes as in the SU(2) case but with its own $Q_3$ and $Q_8$ charges:

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Next-to-leading order results

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Next-to-leading order results

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Each charged mode contributes as in the SU(2) case but with its own $Q_3$ and $Q_8$ charges:

$$
\gamma_{su(3)}(r_3, r_8) = \gamma_{su(2)}(r_3) + \gamma_{su(2)}\left(\frac{r_3 + r_8\sqrt{3}}{2}\right) + \gamma_{su(2)}\left(\frac{-r_3 + r_8\sqrt{3}}{2}\right)
$$

Center-symmetry + Color invariance + C-symmetry
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$(4\pi/3, 0)$ is always an extremum!
Each charged mode contributes as in the SU(2) case but with its own $Q_3$ and $Q_8$ charges:

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LO Polyakov-loop potential: SU(3) case

Each charged mode contributes as in the SU(2) case but with its own $Q_3$ and $Q_8$ charges:

$$\gamma_{su(3)} (r_3, r_8) = \gamma_{su(2)} (r_3) + \gamma_{su(2)} \left( \frac{r_3 + r_8 \sqrt{3}}{2} \right) + \gamma_{su(2)} \left( \frac{-r_3 + r_8 \sqrt{3}}{2} \right)$$

$(4\pi/3, 0)$ is always an extremum!
Each charged mode contributes as in the SU(2) case but with its own $Q_3$ and $Q_8$ charges:

\[
\gamma_{su(3)}(r_3, r_8) = \gamma_{su(2)}(r_3) + \gamma_{su(2)}(r_3 + r_8 \sqrt{3}) + \gamma_{su(2)}(-r_3 + r_8 \sqrt{3})
\]

$(4\pi/3, 0)$ is always an extremum!
Next-to-leading order results

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$$

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