Higgs-Dilaton Cosmology
Universality vs. Criticality

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ABSTRACT

The Higgs-Dilaton model is able to produce an early inflationary expansion followed by a dark energy dominated era responsible for the late time acceleration of the Universe. At tree-level, the model predicts a small tensor-to-scalar ratio ($0.0021 \lesssim r \lesssim 0.0034$), a tiny negative running of the spectral tilt ($-0.00057 \lesssim dns/d\ln k \lesssim -0.00034$) and a non-trivial consistency relation between the spectral tilt of scalar perturbations and the dark energy equation of state, which turns out to be close to a cosmological constant ($0 \lesssim 1 + w \lesssim 0.014$). We reconsider the validity of these predictions in the vicinity of the critical value of the Higgs self-coupling giving rise to an inflection point in the inflationary potential. The value of the inflationary observables in this case strongly depends on the parameters of the model. The tensor-to-scalar ratio can be large ($r \sim O(0.1)$) and notably exceed its tree-level value. If that happens, the running of the scalar tilt becomes positive and rather big ($dns/d\ln k \sim O(0.01)$) and the equation-of-state parameter of dark energy can significantly differ from a cosmological constant ($1 + w \sim O(0.1)$).

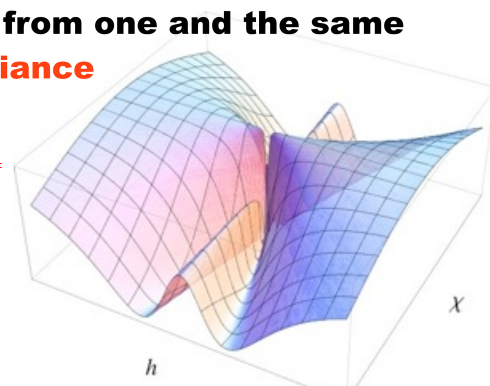
THE NO-SCALE SCENARIO

In the SM, some scales, such as the Newton's constant or the vev of the Higgs field, are a priori completely unrelated to the Higgs mechanism. In the Higgs-Dilaton model

$$\frac{\mathcal{L}_{SI}}{\sqrt{-g}} = \frac{1}{2}\left(\xi_\chi \chi^2 + \xi_h h^2\right)R - \frac{1}{2}(\partial h)^2 - \frac{1}{2}(\partial \chi)^2 - U(h,\chi)$$

all these scales are generated from one and the same source: the SSB of scale invariance

$$U(h,\chi) = \frac{\lambda}{4}\left(h^2 - \frac{\vartheta}{\lambda}\chi^2\right)^2 + \beta\chi^4$$



Infinitely degenerate vacuum

A singlet under the SM group

No couplings with SM particles/No problems with EWC

"Who gives mass to whom?"

Irrelevant question The dilaton is massless

EINSTEIN FRAME ANALYSIS

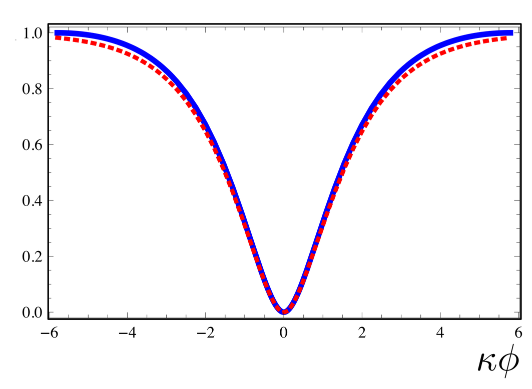
Performing a conformal transformation to the so-called Einstein frame, together with a field redefinition, the lagrangian density can be written in a very simple form

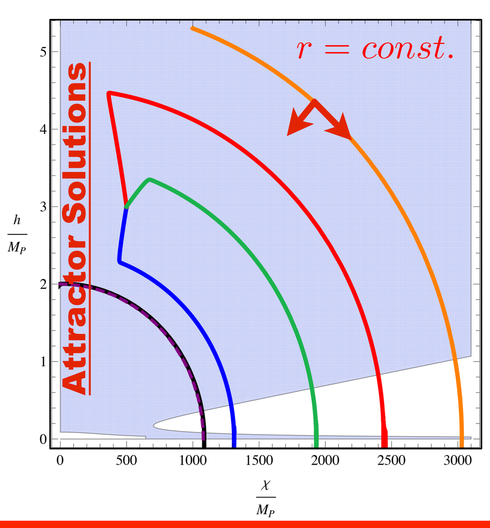
$$\frac{\tilde{\mathcal{L}}}{\sqrt{-\tilde g}} = \frac{M_P^2}{2}R - \frac{e^{2b(\phi)}}{2}(\partial\rho)^2 - \frac{1}{2}(\partial\phi)^2 - \tilde U(\phi)$$

$$e^{2b(\phi)} \equiv \sigma\cosh^2\left[\alpha\kappa\left(\phi_0 - |\phi|\right)\right]$$

The potential closely resembles that of Higgs Inflation and depends only on one of the two scalar fields

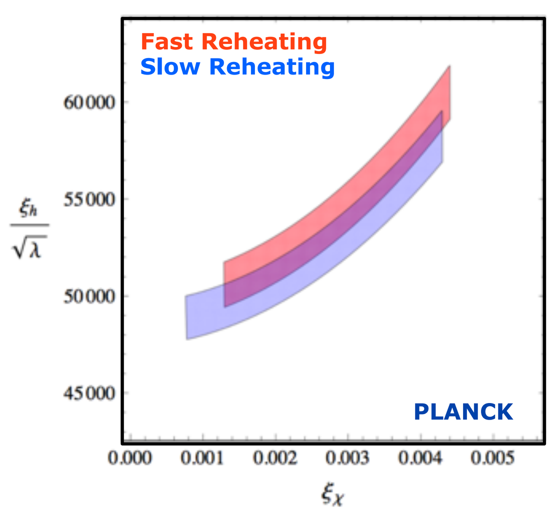
$$\tilde U(\phi) = U_0\left(1 - e^{2b(\phi)}\right)^2$$





INFLATIONARY OBSERVABLES

Accurate predictions for the non-minimal couplings yielding the different inflationary observables in the current observational range can be easily found. The scalar tilt and the tensor-to-scalar ratio at tree-level only depends on ξ_χ. The observational value of the spectral tilt forces the tensor-to-scalar ratio to be small



Scalar spectral tilt	Amplitude
$n_s(k^*) \simeq 1 - 8\xi_\chi \coth(4\xi_\chi N^*)$	$\Delta_\zeta^2(k^*) \simeq \frac{\lambda\sinh^2(4\xi_\chi N^*)}{1152\pi^2\xi_\chi^2\xi_h^2}$
Running of the tilt	**Tensor-to-scalar ratio**
$\alpha_\zeta(k^*) \simeq -32\xi_\chi^2\sinh^{-2}(4\xi_\chi N^*)$	$r(k^*) \simeq 192\xi_\chi^2\sinh^{-2}(4\xi_\chi N^*)$

SCALE INVARIANCE + UNIMODULAR GRAVITY

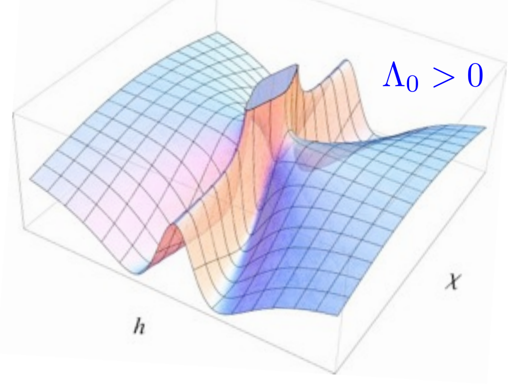
A spontaneously broken scale invariance symmetry forbids the appearance of a cosmological term. One possible way out is to consider Unimodular Gravity

General Relativity	Unimodular Gravity
CC at the level of the action	No CC at the level of the action
$S = \int d^4x \left(\mathcal{L}(g_{\mu\nu}, \partial g_{\mu\nu}, \Phi, \partial\Phi) + \Lambda_0\right)$	$S = \int d^4x \mathcal{L}(g_{\mu\nu}, \partial g_{\mu\nu}, \Phi, \partial\Phi)$
Unrestricted metric determinant	Restricted metric determinant
$\|g\|$	$\|g\| = 1$
Variation of the action	Variation of the action
	$\partial_\mu\lambda(x) = 0 \leftrightarrow \lambda(x) = \Lambda_0$

Same equations of motion
Different interpretation

Rather than a cosmological constant, the Λ_0-term becomes the strength of a quintessence potential, being its value related only to initial conditions.

$$V(\phi) + \Lambda_0\left(\frac{1 + 6\xi_\chi}{\xi_\chi}\right)^2 \sigma^2\cosh^4\left[\bar\alpha\kappa\left(\phi_0 - |\phi|\right)\right]e^{-4\gamma\kappa\rho}$$



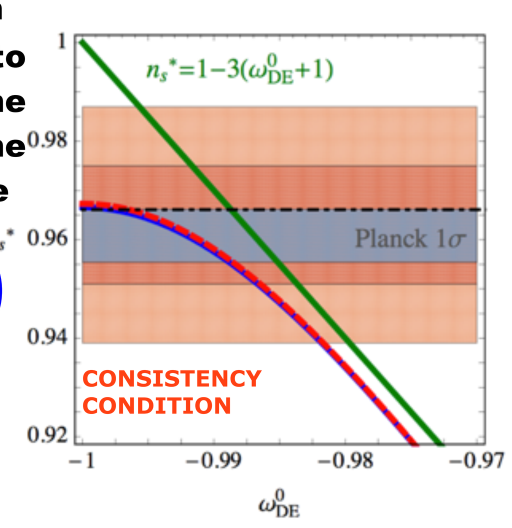
The new term lifts the valleys and breaks the degeneracy of the classical ground state. After inflation, we are left with just one dynamical degree of freedom, ρ, with the dynamics of a "thawing quintessence field".

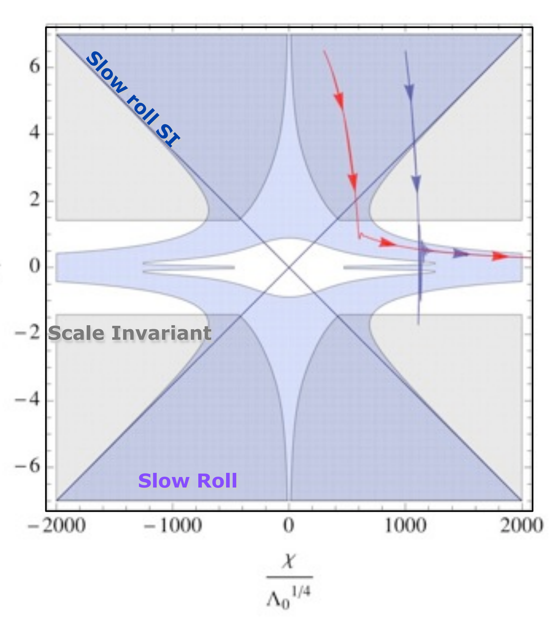
CONSISTENCY CONDITIONS

The tree-level parameters of the theory are determined by the inflationary observables. Any subsequent period, as the mentioned DE dominated stage, should be consistent with that choice of parameters. This gives rise to a full hierarchy of non-trivial consistency relations, which allows us to use the measurable observables from CMB anisotropies to make firm testable predictions in the widely unknown DE sector. In particular, it is possible to derive a relation between the scalar spectral index and the dark energy equation of state

$$n_s^* - 1 \simeq -\frac{12\eta_{DE}^0}{4 - 9\eta_{DE}^0}\coth\left(\frac{6N^*\eta_{DE}^0}{4 - 9\eta_{DE}^0}\right)$$

$$\eta_{DE}^0 = 1 + w_{DE}^0$$





On the other hand, the DE domination period can be used to further constraint the initial inflationary conditions. One can easily conclude that any viable trajectory should originate from an inflationary region in which the effect of the Λ_0-term was negligible.

QUANTUM CORRECTIONS

- Assume the SM valid up to the Planck scale
- Regularize keeping scale invariance intact

Dimensional regularization	$\mu \to F(\chi, h)$
$\overline{MS}$ subtraction scheme	

- No (new) higher dimensional operators beyond those required by the consistency of the theory

EW theory in the inflationary region

CHIRAL $m_{A,f} \propto h$, $\Omega \propto h$ $\Rightarrow$ $m_{A,f} \to \frac{m_{A,f}}{\Omega} = \text{const.}$

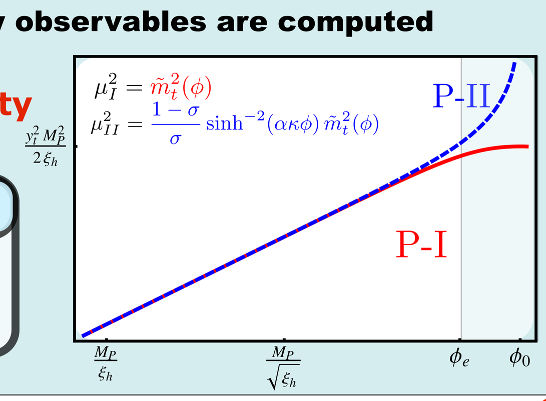
1. 2-loop running SM RGE until the chiral SM region
2. Obtained values as input of chiral phase, 1-loop RGE are run until a given scale
3. RGE effective potential at inflation is computed

$$\tilde U_{RGE}(\phi) = \frac{\lambda(\mu(\phi))}{4}\frac{M_P^4}{\xi_h^2(\mu(\phi))(1-\sigma)^2}\left(1 - \sigma\cosh^2\frac{\alpha\phi}{M_P}\right)^2$$

4. Higgs non-minimal coupling fixed with COBE normalization. Inflationary observables are computed

An intrinsic uncertainty

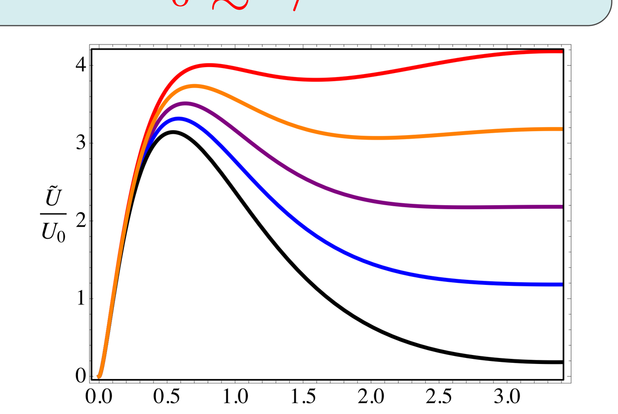
Jordan frame def.	
P-I	$\xi_\chi\chi^2 + \xi_h h^2$
P-II	$\xi_\chi\chi^2$



The effective self-coupling at the inflationary scale can be well approximated by $\lambda(\mu(\phi)) = \lambda_0 + b\log^2\left(\frac{\sqrt{1 - e^{2b(\phi)}}}{q_{\rm eff}\sqrt{1-\sigma}}\right)$. Here $b \simeq 2.3 \times 10^{-5}$ and λ_0 is some function of the top quark pole mass, the Higgs mass and the strong coupling constant at the inflationary scale, whose precise form will not be relevant for our discussion.

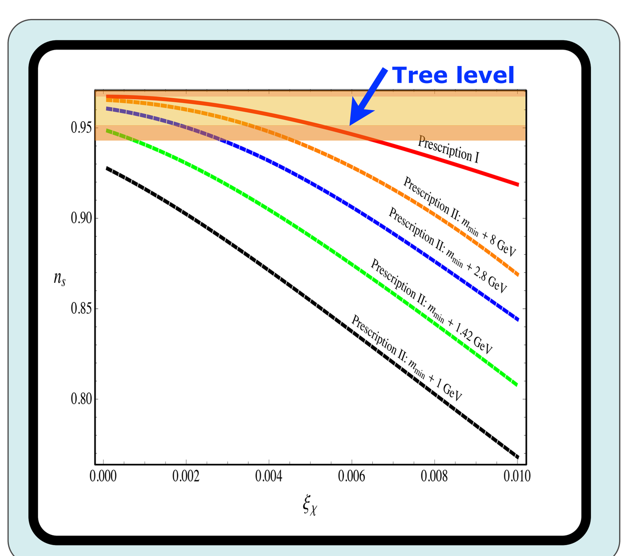
FORBIDDEN REGION $\lambda_0 \lesssim b/16$

The potential develops a wiggle and inflation is not longer possible. Smaller values of λ_0 make the electroweak vacuum unstable

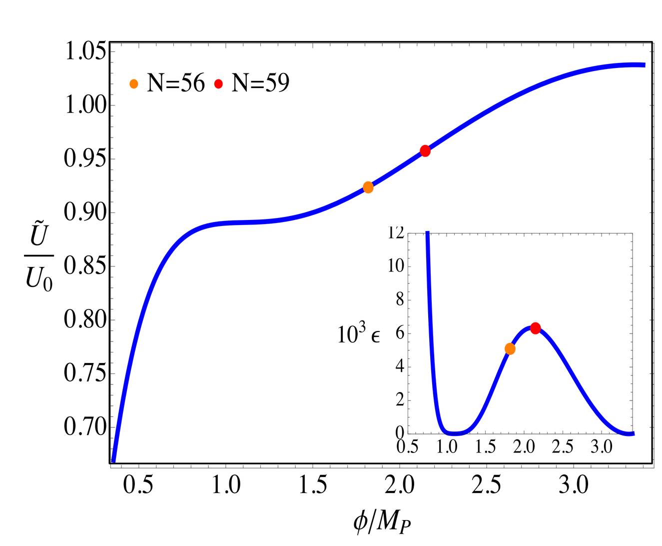


UNIVERSALITY $\lambda_0 \gg b/16$

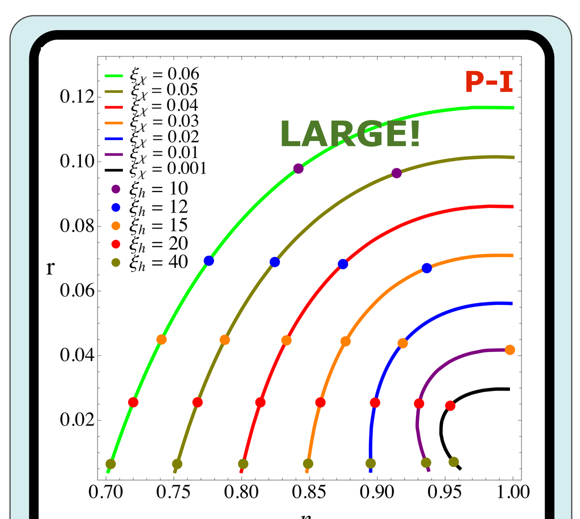
The form of the potential becomes almost independent of the values of the parameters ξ_χ, ξ_h and $q_{\rm eff}$ appearing within the logarithmic correction. As in the tree-level case, the potential effectively depends on two parameters which can be completely fixed by observations. The tensor-scalar ratio remains smaller.



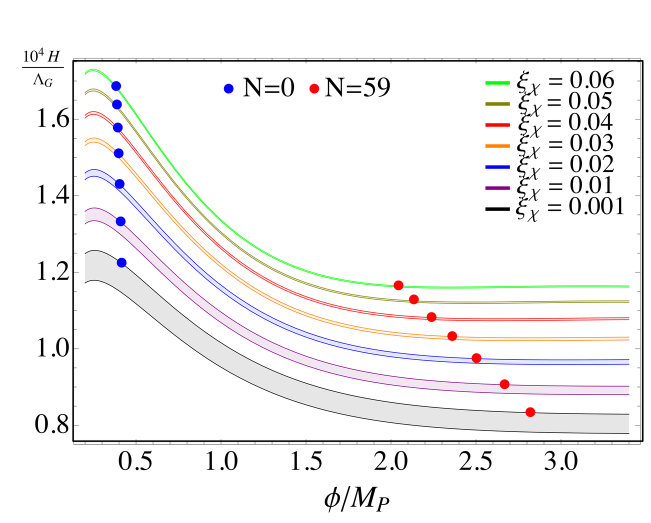
CRITICALITY $\lambda_0 \simeq b/16$

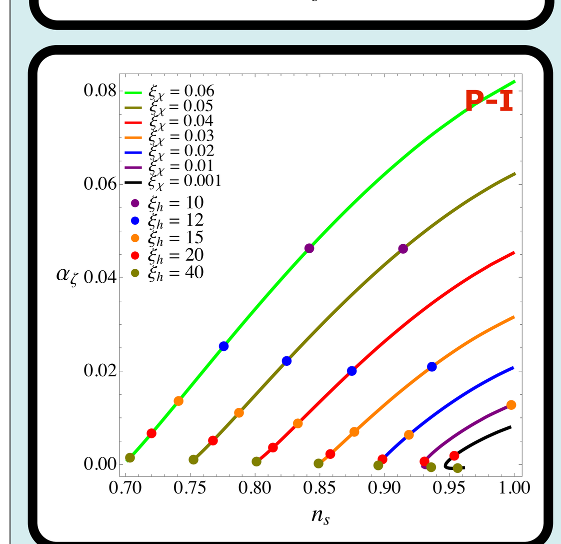


In this case, the first and second derivatives of the potential are equal to zero at some field value along the inflationary evolution. The effective potential strongly differs from the tree-level case. The slow-roll parameter becomes non-monotonic, opening the possibility of getting a sizable tensor-to-scalar ratio.



The self-consistency of the model is also guaranteed when perturbations are computed around inflationary Higgs field values. The typical energy of the scalar perturbations produced during inflation $H \sim O(\lambda M_P/\xi_h)$ is well below the cutoff of the theory at those energies, $\Lambda_G \sim O(M_P/\xi_h)$





Based on [1] J. Rubio, M. Shaposhnikov arXiv:1406.5182
[2] F.Bezrukov, G. Karananas, J. Rubio, M. Shaposhnikov,Phys.Rev. D87 (2013) 096001