Jet quenching and EQCD

Marco Panero, Kari Rummukainen and Andreas Schäfer

UNIVERSITY OF HELSINKI

HELSINKI INSTITUTE OF PHYSICS

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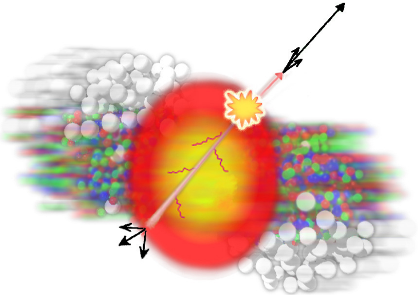
Jet quenching

Jet quenching is evidence for strongly coupled quark-gluon plasma (QGP)



A fast parton

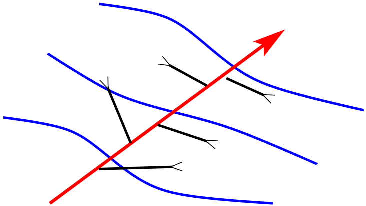
- is generated in a hard collision (large Q^2)
- Interacts with the expanding QGP
- hadronization into a jet

Parton-plasma cross-section $\sigma(q_\perp, Q^2)$

We study σ on the lattice using electrostatic QCD, EQCD

Hard parton propagation in QGP

- Multiple soft-scattering, eikonal approximation ($v = 1$)



- *Transverse* momentum broadening described by jet quenching parameter: [Baier et al.]

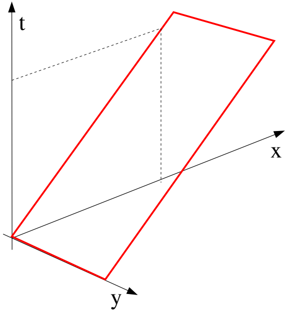
$$\hat{q} = \frac{\langle p_\perp^2 \rangle}{L}$$

- Can be evaluated in terms of a *collision kernel* $C(p_\perp)$

$$\hat{q} = \int^\Lambda \frac{\mathrm{d}^2 p_\perp}{(2\pi)^2} p_\perp^2 C(p_\perp)$$

Light-like Wilson loop

- $C(p_\perp)$ is the Fourier transformed "potential" of the light-like Wilson loop $W(r,T) = \exp(-V(r)T)$



- The collision kernel $C(p_\perp)$ is known to leading order (g^2) [Arnold,Xiao] and next-to-leading order (g^4) [Caron-Huot]:

$$C(p_\perp) = g^2 T C_F \left(\frac{1}{p_\perp^2} - \frac{1}{p_\perp^2 + m_D^2} \right) + g^4 + g^6 + \dots$$

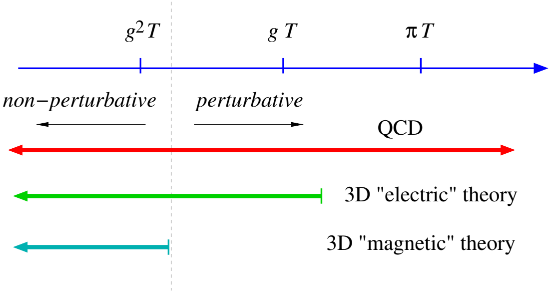
- At order g^6 IR divergences (soft physics) make the result non-perturbative $\rightarrow$ lattice?

Scale hierarchies and effective theories

- At high T (weak coupling), QCD has 3 distinct scales:

$$g^2 T \text{ (ultrasoft)} \ll gT \text{ (soft)} \ll \pi T \text{ (hard)}$$

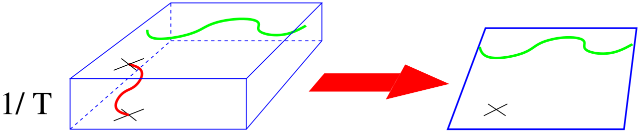
- Hierarchy of effective theories (for static quantities) by successive "integration" over hard modes:
  - Scales $p \lesssim gT$: Electrostatic QCD, EQCD
  - scales $p \lesssim g^2 T$: Magnetostatic QCD, MQCD



EQCD

- Starting from Euclidean QCD with N_f (massless) quarks, integrate modes $p \gtrsim T$: *fermions, non-zero Matsubara frequencies*

$\rightarrow$ 3d effective theory (dimensional reduction)



EQCD action:

$$\mathcal{L}_{\rm EQCD} = \frac{1}{4} F_{ij}^a F_{ij}^a + \mathrm{Tr}\left((D_i A_0)^2\right) + m_{\rm E}^2 \,\mathrm{Tr}\left(A_0^2\right) + \lambda_3 \left(\mathrm{Tr}\left(A_0^2\right)\right)^2$$

[Braaten and Nieto; Kajantie, Laine, K.R., Shaposhnikov]

- $g_{\rm E}^2 = g^2 T + \ldots$
  $m_{\rm E}^2 = (1 + \frac{1}{6} N_f) g^2 T^2 + \ldots$
  $\lambda_3 = \frac{9 - N_f}{24\pi^2} g^4 T + \ldots$
- We use $N_f = 2$ massless quarks
- Theory superrenormalizable: lattice counterterms (1 and 2-loop) known

EQCD

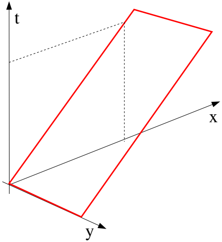
- Lattice simulations of EQCD used successfully in computations of *static* quantities:
  - QCD pressure [Kajantie et al.]
  - screening lengths [Kajantie et al,; Laine, Philipsen; Hart et al.]
  - quark number susceptibilities [Hietanen, K.R]
- Works reasonably well down to $T \approx$ few $\times$ T_c, depending on the observable

Same methodology applied to Electroweak theory [Kajantie et al; D'Onofrio et al; ...]

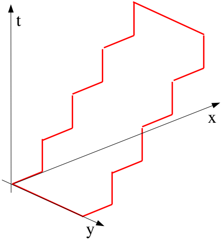
- Works even better because of the small coupling

Back to $\hat{q}$: how to compute $C(p_\perp)$ on the lattice?

- Minkowski space (real-time) object $\rightarrow$ Minkowski lattice? Not possible!



$\Rightarrow$

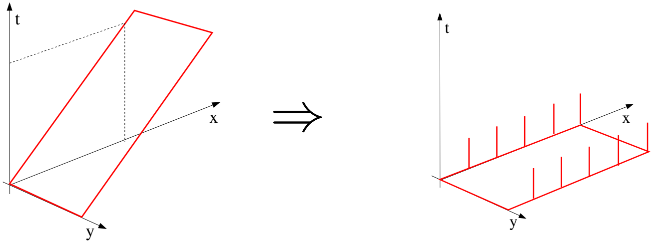


- Euclidean finite-T lattice? [Majumder] Tricky (and costly) analytical continuation required [Laine; Laine and Rothkopf]
- Classical field theory simulation? [Laine and Rothkopf]
  - Minkowski
  - Captures (static) g^2T physics correctly
  - Treats hard modes incorrectly
- Calculation using EQCD?

Evaluating $C(p_\perp)$ with EQCD

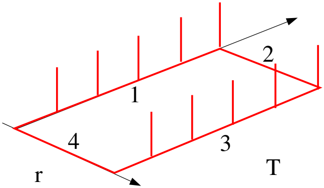
- Intuitively: **soft physics is slow physics**
- Overdamped evolution
- Soft fields along the light cone $\sim$ soft fields along $t = \mathrm{const}$ plane

$\Rightarrow$ Can evaluate soft contribution to $C(p_\perp)$ using static EQCD [Caron-Huot; Aurenche,Gelis,Zaraket]



- The decorations on x-direction lines are insertions of temporal "parallel transporters", constucted from Euclidean$\rightarrow$Minkowski rotated A_0's.
- Shown rigorously by [Caron-Huot; Ghiglieri et al.]
- Well-defined renormalization on the lattice [D'Onofrio et al.]

Evaluating $\hat{q}$ with EQCD



More precisely: construct "potential" $V(r)$ from generalised Wilson loop

$$
\begin{aligned}
\exp(-V(r)T) &= \mathcal{W}(r,T) = \mathrm{Tr}\, L_1 L_2 L_3^{-1} L_4^{-1} \\
L_1 &= U_x(0,0)\, H(a,0)\, U_x(a,0)\, H(2a,0) \ldots U_x(T-a,0)\, H(T,0) \\
L_2 &= U_y(T,0) U_y(T,a) \ldots U_y(T,r) \\
L_3 &= U_x(0,r)\, H(a,r) \ldots U_x(T-a,r)\, H(T,r) \\
L_4 &= U_y(0,0) \ldots U_y(0,r)
\end{aligned}
$$

where $U_x \in \mathrm{SU}(3)$ is the standard lattice x-direction link matrix and

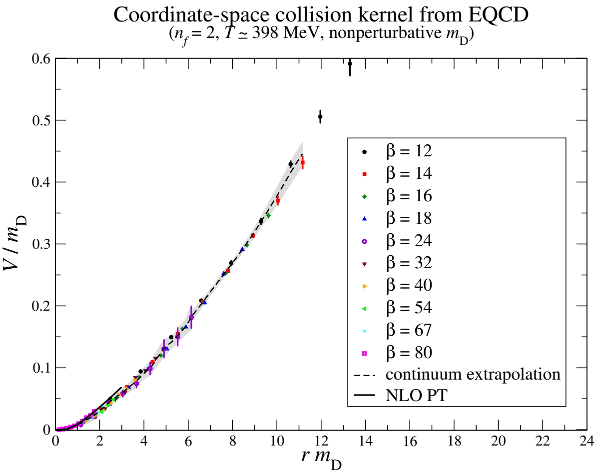
$$H(x) = \exp(a g_{\mathrm{E}} A_0)$$

is a *Hermitian* Wick-rotated "parallel transporter"

Measurements

- Lattice spacings used: $ag_{\mathrm{E}}^2 = 0.5 \ldots 0.075$ ($\beta_G = 12 \ldots 80$)
- Volumes up to $120^2 \times 168$
- Loops are measured using a modified version of the multi-level algorithm [Lüscher and Weisz] $\rightarrow$ very large loops possible, accurate results.
- Two temperatures: $T \approx 398$ MeV and 2 GeV
- For comparison, we also measure std. Wilson loop in MQCD (3D pure gauge theory)

Measured $V(r)$ at $T \approx 398\,\text{GeV}$



- $T \approx 2\,\text{GeV}$ similar
- No sign of the "Coulomb" term in the potential $V(r_\perp)$
- NLO PT: perturbative result w. non-perturbatively measured m_D [Laine and Philipsen]

Extracting $\hat{q}$ from $V(r)$

- $C(p_\perp)$ is 2d Fourier transform of $-V(r_\perp)$
- $\hat{q}$ can now be in principle obtained from

$$\hat{q} = \int \frac{\mathrm{d}^2 p_\perp}{(2\pi)^2} p_\perp^2 C(p_\perp) = \int d^2 r_\perp \nabla^2 V(r_\perp)$$

($+$ suitable cut-offs needed [Laine])

- Good fits to $V(r_\perp)$ are obtained with the perturbatively motivated ansatz

$$V(r_\perp)/g_\mathrm{E}^2 = A r_\perp + B r_\perp^2 + C r_\perp^2 \ln(g_\mathrm{E}^2 r)$$

in the range $0.3 \leq g_\mathrm{E}^2 r_\perp \leq 3$.

- Subtract the perturbative LO and NLO contributions

Results

- The soft EQCD contribution is large:

$$\delta\hat{q}_{\rm EQCD} \simeq \begin{cases} 0.55(5)g_{\rm E}^6 & \text{for } T \simeq 398 \text{ MeV} \\ 0.45(5)g_{\rm E}^6 & \text{for } T \simeq 2 \text{ GeV} \end{cases}$$

  (note: all beyond-NLO contributions included in the $g_{\rm E}^6$-term)
- *Approximate* estimate: $\hat{q} \sim 6$ GeV^2/fm at RHIC temperatures
- Alternatively, using perturbative NLO result with non-perturbatively determined m_D [Laine and Philipsen]

$$\hat{q}_{\rm NLO} = g^4 T^2 m_D C_F C_A \frac{3\pi^2 + 10 - 4\ln 2}{32\pi^2}$$

  gives again $\hat{q} \sim 6\,\text{GeV}^2/\text{fm}$.
- in MQCD (3d pure gauge) Wilson loop gives only the ultrasoft (static) contribution to $\hat{q}$:

$$\hat{q}_{\rm MQCD} \approx 0.08 g_{\rm E}^6 \qquad [\textit{Laine}]$$

$\Rightarrow$ Electric sector important!

Conclusions

- EQCD is a natural extension to perturbative analysis – can be applied to (some) real-time problems
- Large soft contribution to $\hat{q}$
- Comparable with holographic models [Liu et al; Armesto et al; Gürsoy et al.] and phenomenological models [Dainese et al; Eskola et al; Bass et al]
- Screening masses of conserved vector currents can be related to precisely the same "potential" $V(r)$ calculated here: $V_{\mathrm{EQCD}}(r)$ improves the match to 4d lattice [Brandt et al.]
- Apply to other real time problmes, e.g. photon production rate [Ghiglieri et al]?