Jet quenching and EQCD

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Jet quenching

Jet quenching is evidence for strongly coupled quark-gluon plasma (QGP)

A fast parton
- is generated in a hard collision (large $Q^2$)
- Interacts with the expanding QGP
- hadronization into a jet

Parton-plasma cross-section $\sigma(q_\perp, Q^2)$
We study $\sigma$ on the lattice using electrostatic QCD, EQCD
Hard parton propagation in QGP

- Multiple soft-scattering, eikonal approximation ($\nu = 1$)

- Transverse momentum broadening described by jet quenching parameter: [Baier et al.]

\[ \hat{q} = \frac{\langle p_{\perp}^2 \rangle}{L} \]

- Can be evaluated in terms of a collision kernel $C(p_{\perp})$

\[ \hat{q} = \int_{\Lambda} \frac{d^2 p_{\perp}}{(2\pi)^2} p_{\perp}^2 C(p_{\perp}) \]
Light-like Wilson loop

- $C(p_{\perp})$ is the Fourier transformed “potential” of the light-like Wilson loop $W(r, T) = \exp(-V(r)T)$

The collision kernel $C(p_{\perp})$ is known to leading order ($g^2$) [Arnold,Xiao] and next-to-leading order ($g^4$) [Caron-Huot]:

$$C(p_{\perp}) = g^2 TC_F \left( \frac{1}{p^2_{\perp}} - \frac{1}{p^2_{\perp} + m^2_D} \right) + g^4 + g^6 + \ldots$$

- At order $g^6$ IR divergences (soft physics) make the result non-perturbative $ightarrow$ lattice?
At high $T$ (weak coupling), QCD has 3 distinct scales:

$$g^2 T \text{ (ultrasoft)} \ll gT \text{ (soft)} \ll \pi T \text{ (hard)}$$

Hierarchy of effective theories (for static quantities) by successive “integration” over hard modes:

- Scales $p \lesssim gT$: Electrostatic QCD, EQCD
- Scales $p \lesssim g^2 T$: Magnetostatic QCD, MQCD
Starting from Euclidean QCD with $N_f$ (massless) quarks, integrate modes $p \gtrsim T$: fermions, non-zero Matsubara frequencies

$\rightarrow$ 3d effective theory (dimensional reduction)

**EQCD action:**

$$\mathcal{L}_{\text{EQCD}} = \frac{1}{4} F_{ij}^a F_{ij}^a + \text{Tr} \left( (D_i A_0)^2 \right) + m_E^2 \text{Tr} \left( A_0^2 \right) + \lambda_3 \left( \text{Tr} \left( A_0^2 \right) \right)^2$$

[Braaten and Nieto; Kajantie, Laine, K.R., Shaposhnikov]

- $g_E^2 = g^2 T + \ldots$
- $m_E^2 = (1 + \frac{1}{6} N_f) g^2 T^2 + \ldots$
- $\lambda_3 = \frac{9 - N_f}{24\pi^2} g^4 T + \ldots$
- We use $N_f = 2$ massless quarks
- Theory superrenormalizable: lattice counterterms (1 and 2-loop) known
Lattice simulations of EQCD used successfully in computations of static quantities:

- QCD pressure [Kajantie et al.]
- screening lengths [Kajantie et al.; Laine, Philipsen; Hart et al.]
- quark number susceptibilities [Hietanen, K.R]

Works reasonably well down to $T \approx \text{few} \times T_c$, depending on the observable.

Same methodology applied to Electroweak theory [Kajantie et al; D’Onofrio et al; . . .]

Works even better because of the small coupling.
Back to $\hat{q}$: how to compute $C(p_\perp)$ on the lattice?

- Minkowski space (real-time) object $\rightarrow$ Minkowski lattice? Not possible!

- Euclidean finite- $T$ lattice? [Majumder] Tricky (and costly) analytical continuation required [Laine; Laine and Rothkopf]

- Classical field theory simulation? [Laine and Rothkopf]
  - Minkowski
  - Captures (static) $g^2 T$ physics correctly
  - Treats hard modes incorrectly

- Calculation using EQCD?
Intuitively: *soft physics is slow physics*

Overdamped evolution

Soft fields along the light cone \( \sim \) soft fields along \( t = \text{const} \) plane

\[ \Rightarrow \text{Can evaluate soft contribution to } C(p_\perp) \text{ using static EQCD} \]

[Caron-Huot; Aurenche, Gelis, Zaraket]

The decorations on \( x \)-direction lines are insertions of temporal “parallel transporters”, constucted from Euclidean \( \rightarrow \) Minkowski rotated \( A_0 \)'s.

Shown rigorously by [Caron-Huot; Ghiglieri et al.]

Well-defined renormalization on the lattice [D’Onofrio et al.]
More precisely: construct “potential” $V(r)$ from generalised Wilson loop

$$
\exp(-V(r)T) = W(r, T) = \text{Tr} L_1 L_2 L_3^{-1} L_4^{-1}
$$

$$
L_1 = U_x(0, 0) H(a, 0) U_x(a, 0) H(2a, 0) \ldots U_x(T - a, 0) H(T, 0)
$$

$$
L_2 = U_y(T, 0) U_y(T, a) \ldots U_y(T, r)
$$

$$
L_3 = U_x(0, r) H(a, r) \ldots U_x(T - a, r) H(T, r)
$$

$$
L_4 = U_y(0, 0) \ldots U_y(0, r)
$$

where $U_x \in \text{SU}(3)$ is the standard lattice $x$-direction link matrix and

$$
H(x) = \exp(agaE A_0)
$$

is a Hermitian Wick-rotated “parallel transporter”
Measurements

- Lattice spacings used: $a g_E^2 = 0.5 \ldots 0.075 \ (\beta_G = 12 \ldots 80)$
- Volumes up to $120^2 \times 168$
- Loops are measured using a modified version of the multi-level algorithm [Lüscher and Weisz] → very large loops possible, accurate results.
- Two temperatures: $T \approx 398 \ \text{MeV}$ and $2 \ \text{GeV}$
- For comparison, we also measure std. Wilson loop in MQCD (3D pure gauge theory)
Measured $V(r)$ at $T \approx 398$ GeV

$T \approx 2$ GeV similar
No sign of the “Coulomb” term in the potential $V(r_\perp)$
NLO PT: perturbative result w. non-perturbatively measured $m_D$ [Laine and Philipsen]
Extracting $\hat{q}$ from $V(r)$

- $C(p_\perp)$ is 2d Fourier transform of $-V(r_\perp)$
- $\hat{q}$ can now be in principle obtained from

$$\hat{q} = \int \frac{d^2 p_\perp}{(2\pi)^2} p_\perp^2 C(p_\perp) = \int d^2 r_\perp \nabla^2 V(r_\perp)$$

(+ suitable cut-offs needed [Laine])

- Good fits to $V(r_\perp)$ are obtained with the perturbatively motivated ansatz

$$V(r_\perp)/g_\perp^2 = A r_\perp + B r_\perp^2 + C r_\perp^2 \ln(g_\perp^2 r_\perp)$$

in the range $0.3 \leq g_\perp^2 r_\perp \leq 3$.

- Subtract the perturbative LO and NLO contributions
The soft EQCD contribution is large:

\[ \delta \hat{q}_{\text{EQCD}} \simeq \begin{cases} 0.55(5)g_E^6 & \text{for } T \simeq 398 \text{ MeV} \\ 0.45(5)g_E^6 & \text{for } T \simeq 2 \text{ GeV} \end{cases} \]

(note: all beyond-NLO contributions included in the \( g_E^6 \)-term)

- **Approximate** estimate: \( \hat{q} \sim 6 \text{ GeV}^2/\text{fm} \) at RHIC temperatures
- Alternatively, using perturbative NLO result with non-perturbatively determined \( m_D \) [Laine and Philipsen]

\[ \hat{q}_{\text{NLO}} = g^4 T^2 m_D C_F C_A \frac{3\pi^2 + 10 - 4 \ln 2}{32\pi^2} \]

gives again \( \hat{q} \sim 6 \text{ GeV}^2/\text{fm} \).

- in MQCD (3d pure gauge) Wilson loop gives only the ultrasoft (static) contribution to \( \hat{q} \):

\[ \hat{q}_{\text{MQCD}} \simeq 0.08g_E^6 \] [Laine]

⇒ Electric sector important!
Conclusions

- EQCD is a natural extension to perturbative analysis – can be applied to (some) real-time problems
- Large soft contribution to $\hat{q}$
- Comparable with holographic models [Liu et al; Armesto et al; Gürsoy et al.] and phenomenological models [Dainese et al; Eskola et al; Bass et al]
- Screening masses of conserved vector currents can be related to precisely the same “potential” $V(r)$ calculated here: $V_{\text{EQCD}}(r)$ improves the match to 4d lattice [Brandt et al.]
- Apply to other real time problems, e.g. photon production rate [Ghiglieri et al]?