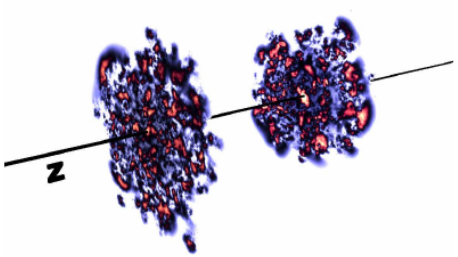
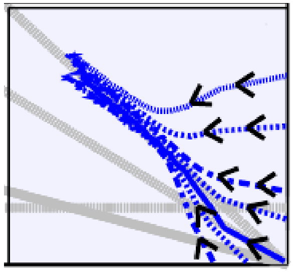
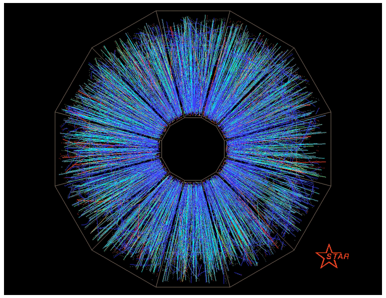
Thermalization process in weakly coupled field theories far from equilibrium







Soeren Schlichting

Strong & ElectroWeak Matter 2014, Lausanne, 07/15/2014

BROOKHAVEN NATIONAL LABORATORY

How is thermal equilibrium reached?

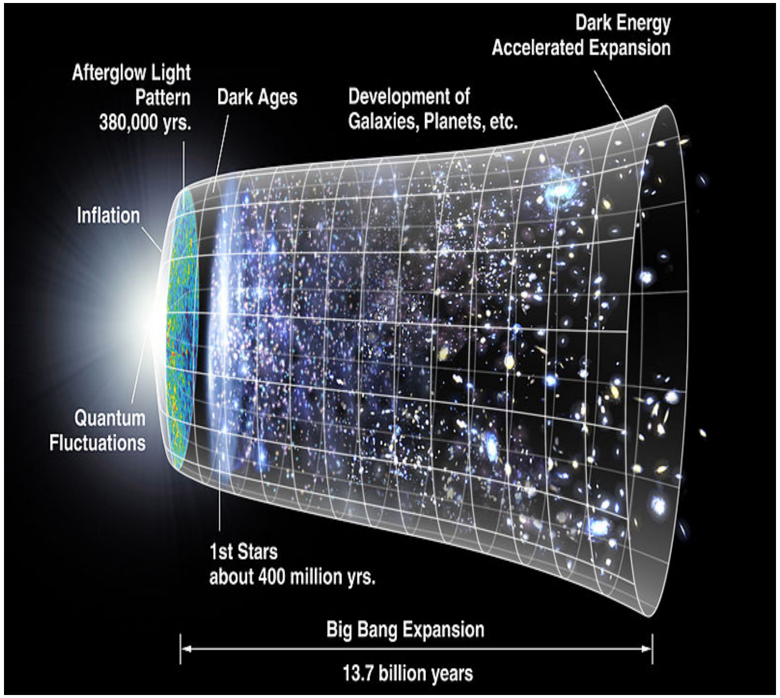
Initial state:
Far from equilibrium

Non-equilibrium dynamics

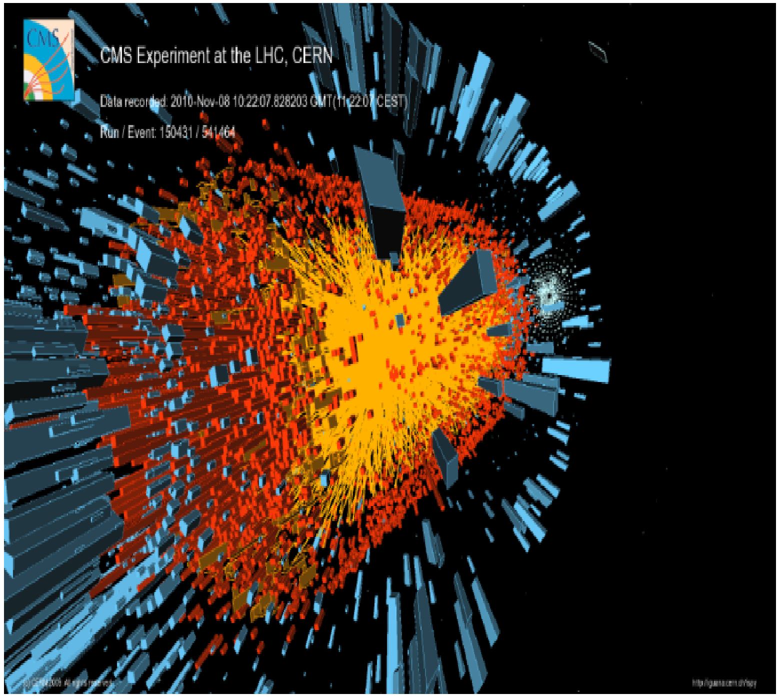
Final state:
Thermal equilibrium

Examples of far-from equilibrium systems

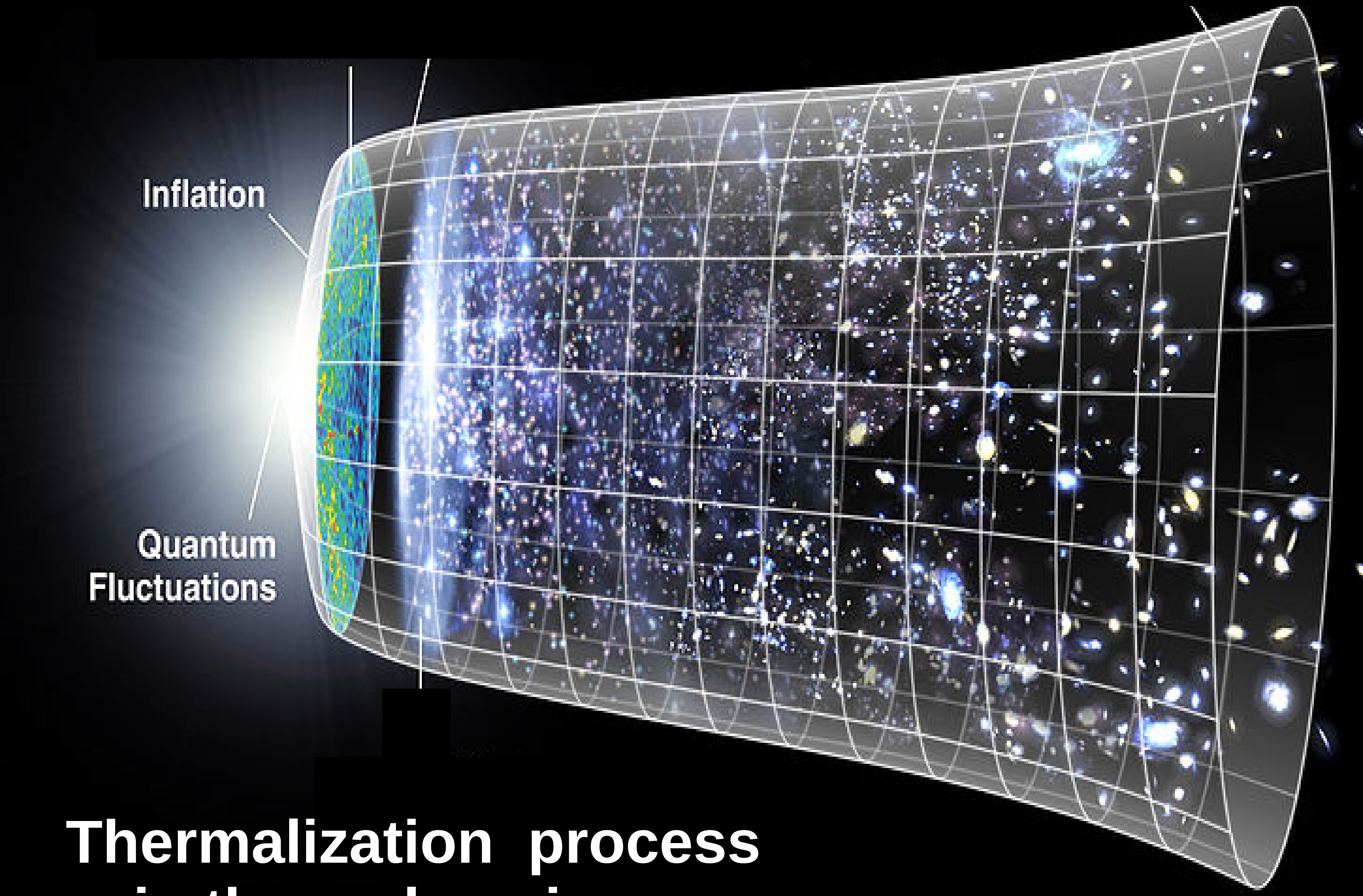
***Early universe –
Scenarios for reheating
after inflations***



***Heavy-Ion collision in the weak
coupling picture***



Non-equilibrium phenomena may be shared by different systems



Thermalization process in the early universe

Cosmology – Reheating after Inflation

Many models for thermalization of the early universe have been studied

Will consider here a simple massless scalar field theory ($\lambda\Phi^4$), which using a conformal transformation can be mapped onto a non-expanding non-expanding scalar field theory:

$$S[\phi] = \int d^4x \left(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{\lambda}{24}\phi^4 \right)$$

(c.f. Micha, Tkachev PRD 70 (2004) 043538)

Coupling constant is typically very small

$$\lambda \sim 10^{-8}$$

The initial field amplitude of the inflation field is large

$$\phi \sim 1/\sqrt{\lambda}$$

→ ***Weakly coupled but strongly interacting***

Non-equilibrium dynamics at weak coupling

Classical-statistical field theory

Whenever the occupancy/field amplitudes are large (f >> 1) a description in terms of classical field equations of motion is applicable

$$\Box\phi - \lambda\phi^3 = 0$$

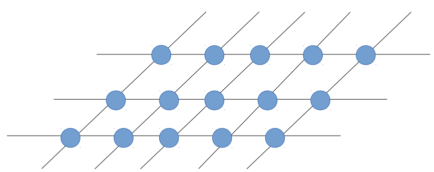
→ Can be solved numerically for a discretized space-time using lattice techniques (first principles description)

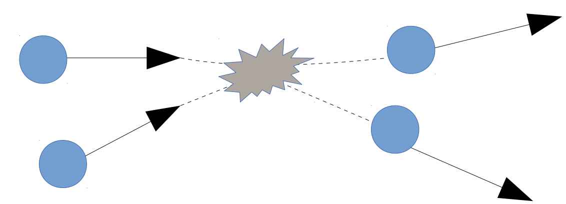
Kinetic theory

Whenever the occupancy becomes less than (f <1/λ) a description in terms of quasi particle excitations should also be applicable

$$\partial_t f(t,p) = C[f](t,p)$$

→ Can study the effect of individual processes (e.g. 2 ↔2 or 2↔3 scattering)





$f(t,p) \sim 1/\lambda$

('strongly interacting system')

$1/\lambda > f(t,p) > 1$

('classical particles')

$f(t,p) \sim 1$

('quantum')

→ Overlap in the range of applicability

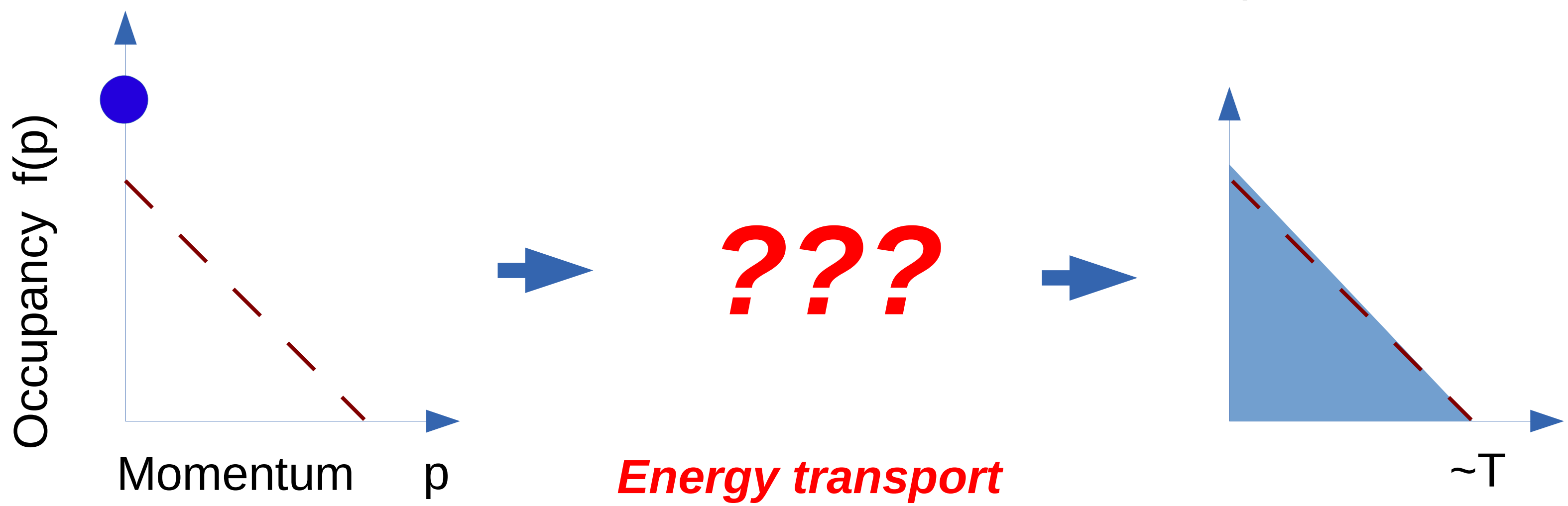
What happens during thermalization?

Initially most energy is stored in the inflaton field

In thermal equilibrium energy is mostly carried by momentum modes ~T

Initial state –
Far from equilibrium

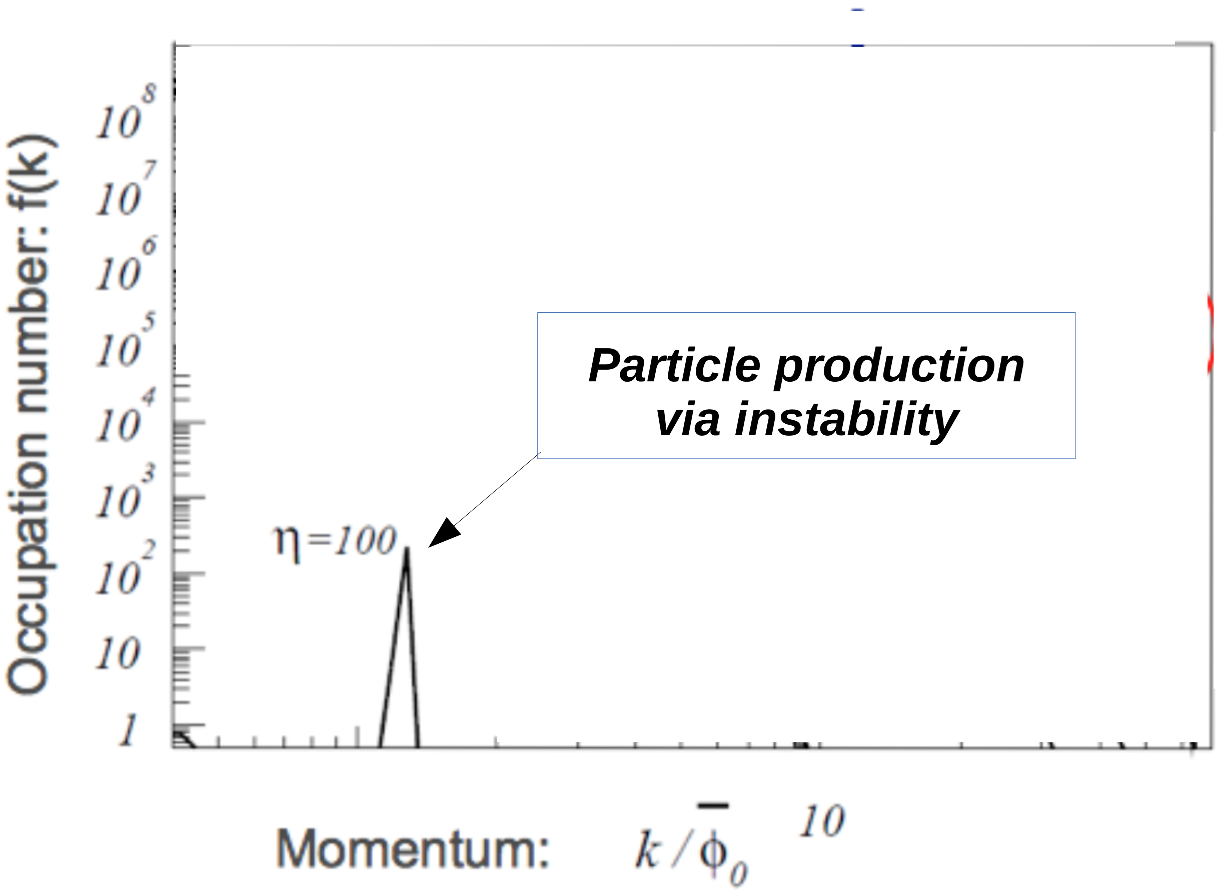
Thermal
equilibrium



Energy transport towards Ultra-Violet

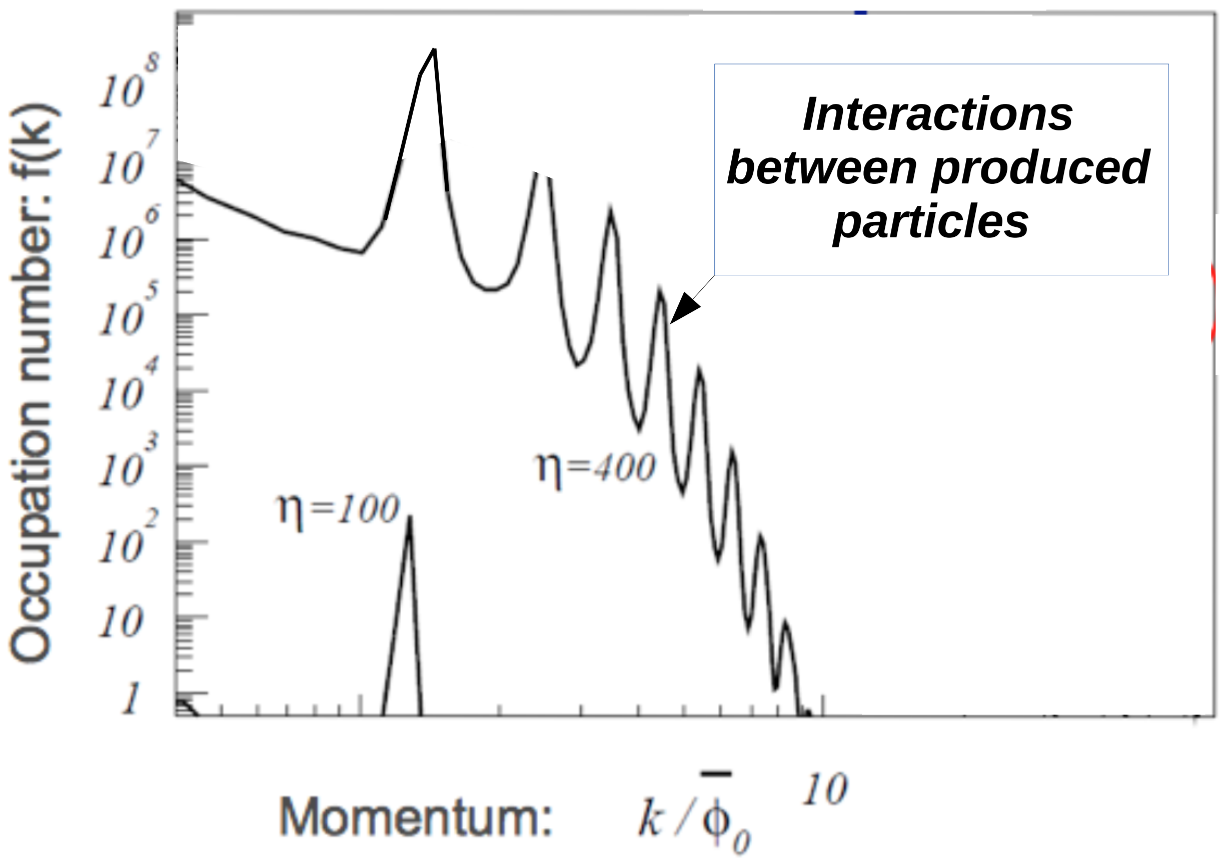
Early time dynamics (Preheating)

Inflaton field oscillates and begins to decay via a "parametric resonance" instability
(Kofman, Linde,Starobinsky, Gacia-Bellido, Boyanovsky, Khlebnikov, Tkachev,...)



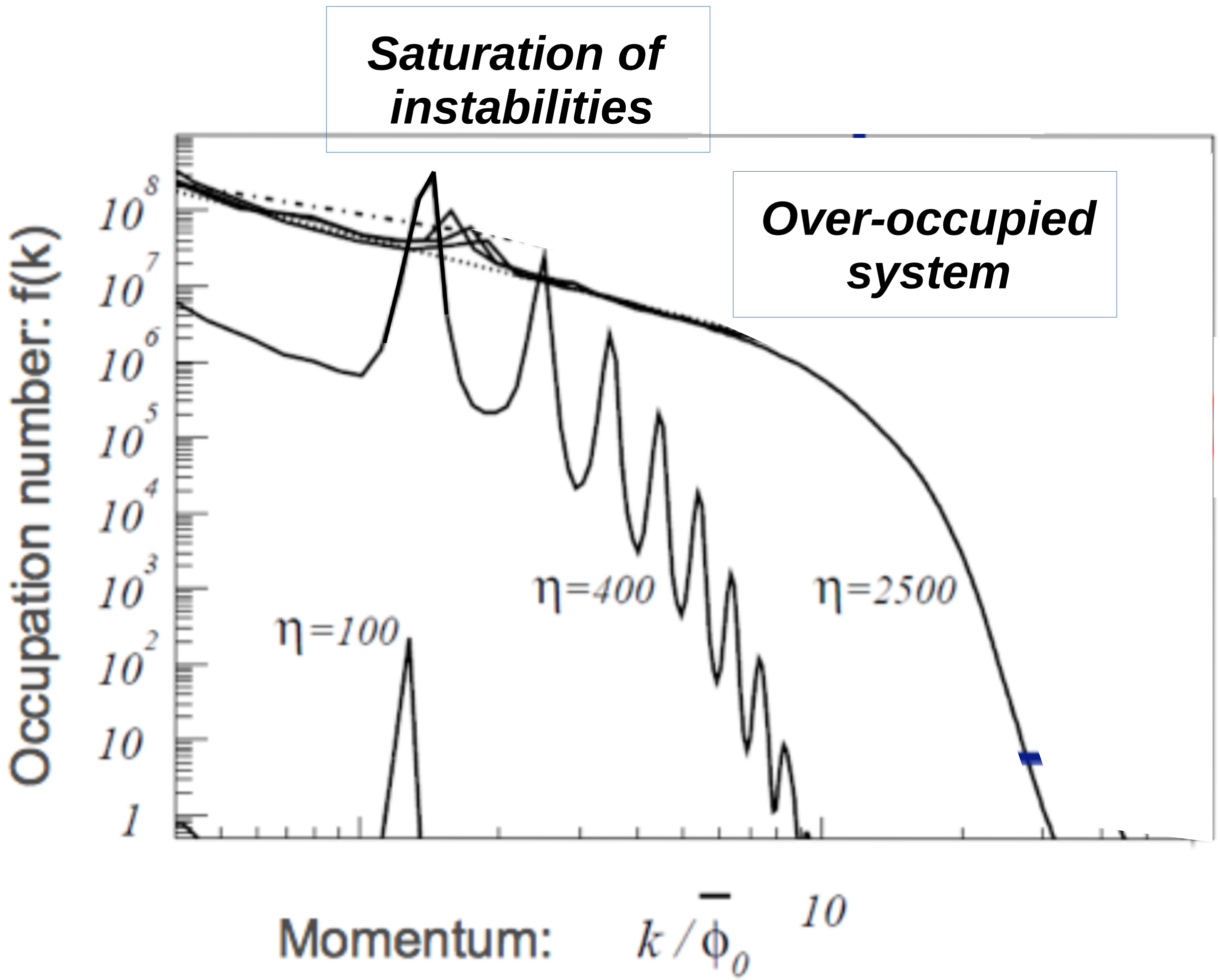
(Micha, Tkachev PRD 70 (2004) 043538)

Early time dynamics (Preheating)



(Micha, Tkachev PRD 70 (2004) 043538)

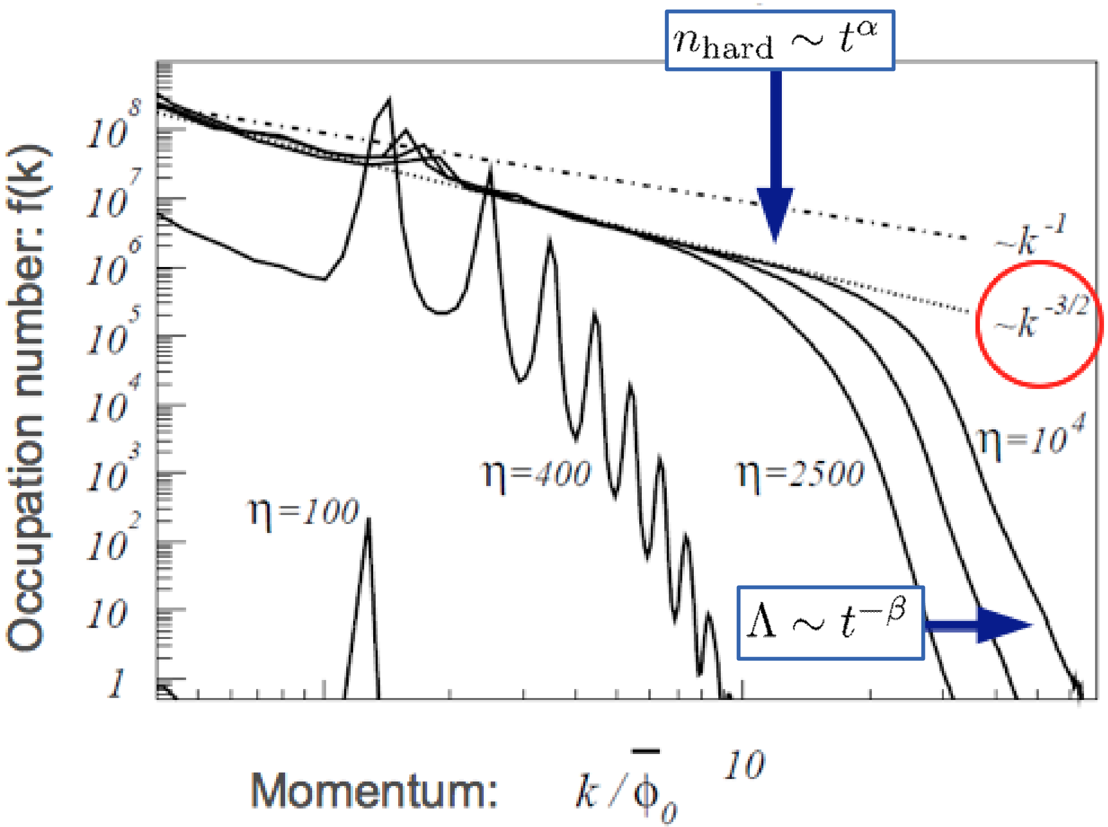
Early time dynamics (Preheating)



System is still far from equilibrium!

(Micha, Tkachev PRD 70 (2004) 043538)

Thermalization process (Reheating)



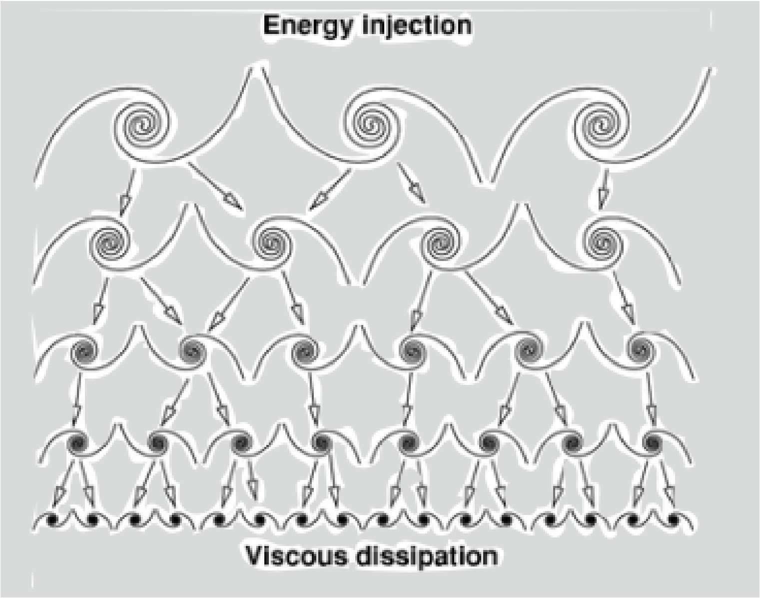
- The evolution becomes ***self-similar*** $f(p,t) = t^{\alpha}\ f_S(t^{\beta}p)$

- The thermalization process is described by a ***quasi-stationary evolution*** with ***scaling exponents*** Dynamic: α=-4/5 β=-1/5 Spectral: κ=-3/2

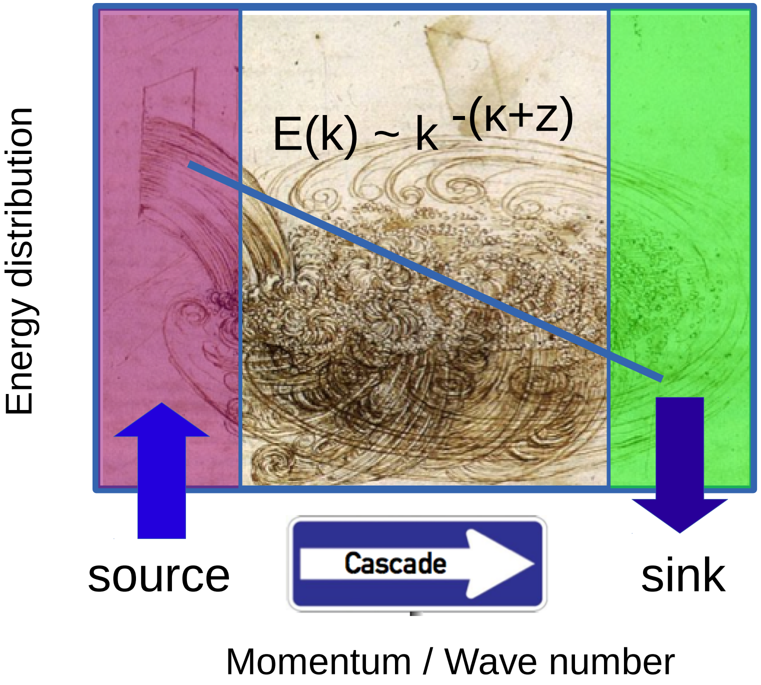
(Micha, Tkachev PRD 70 (2004) 043538)

Turbulent thermalization – Classical picture of wave turbulence

Richardson cascade



Kolmogorov spectra



Uriel Frisch, *"Turbulence. The Legacy of A. N. Kolmogorov."*

Zakharov, V. E.; L'vov, V. S.; Falkovich, G, *"Kolmogorov spectra of turbulence 1. Wave turbulence."*

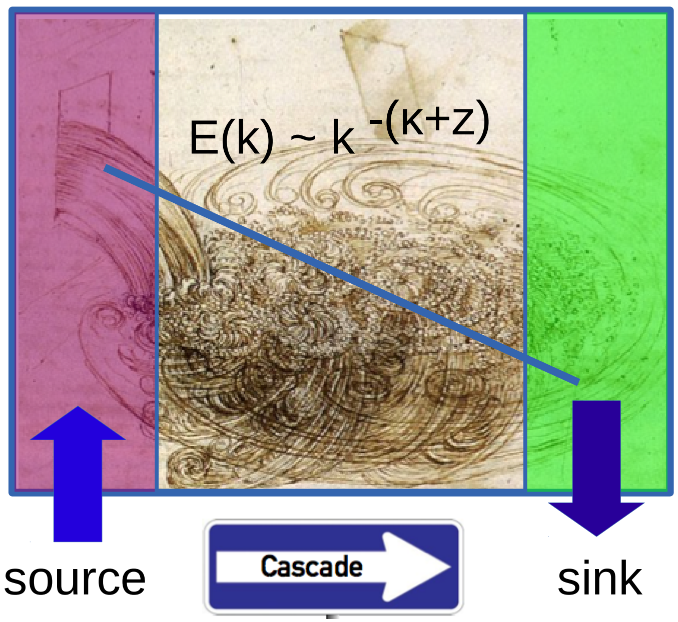
- ***Stationary scaling solution*** associated to scale invariant energy flux

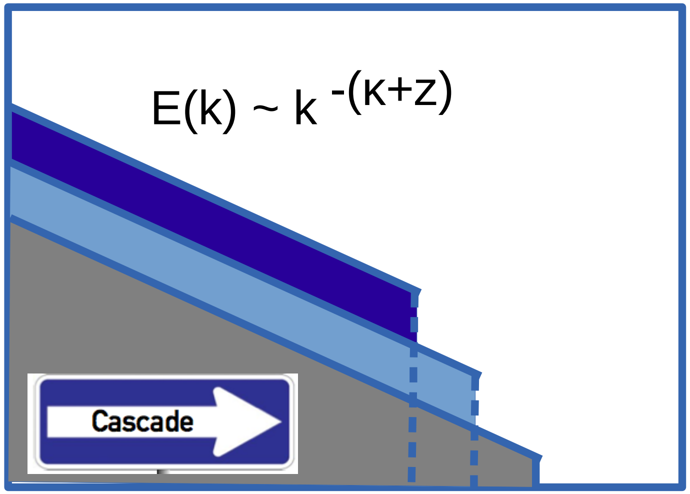
Turbulent thermalization – Wave turbulence in closed systems

"Driven" Turbulence – Kolmogorov wave turbulence

vs.

"Free" Turbulence – Turbulent Thermalization





closed system

- ***Stationary scaling solution*** associated to scale invariant energy flux

- ***Quasi-stationary scaling solution***

- ***Self-similar time evolution*** associated to energy transport towards the ultra-violet

Kinetic interpretation

- Search for ***self-similar scaling solutions*** of the Boltzmann equation

$$\partial_t f(p,t) = C[f](p,t) \xrightarrow[(f \gg 1)]{\textit{scale invariance}} C[f](p,t) = t^{\mu} C[f_S](t^{\beta} p)$$

- The dynamic ***scaling exponents*** are uniquely determined by

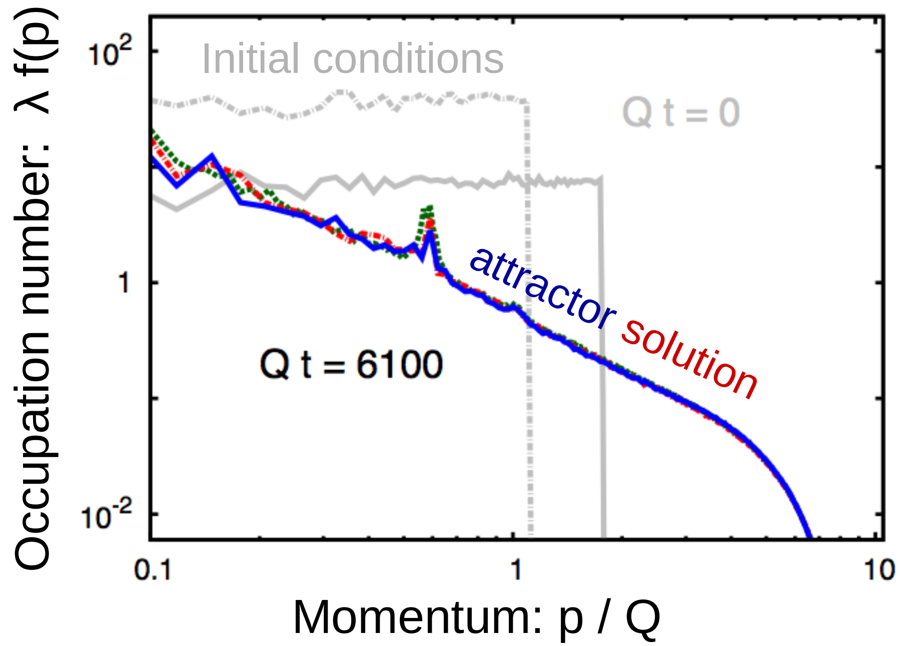
Canonical scaling of the collision integral + *Conservation laws* $\longrightarrow$ ***Universality far from equilibrium***

- Classification scheme for relativistic field theories *(Micha, Tkachev)*

Interaction	*Hard scale evolution (Exponent α)*	*Occupancy evolution (Exponent β)*	*Spectral Shape (Exponent κ)*
2<->1+soft	***-1/5***	***-4/5***	***3/2***
2<->2	***-1/7***	***-4/7***	***4/3 5/3***

"free" cascade (Hard scale and Occupancy evolution for 2<->2) — ***stationary cascade*** (Spectral Shape for 2<->2)

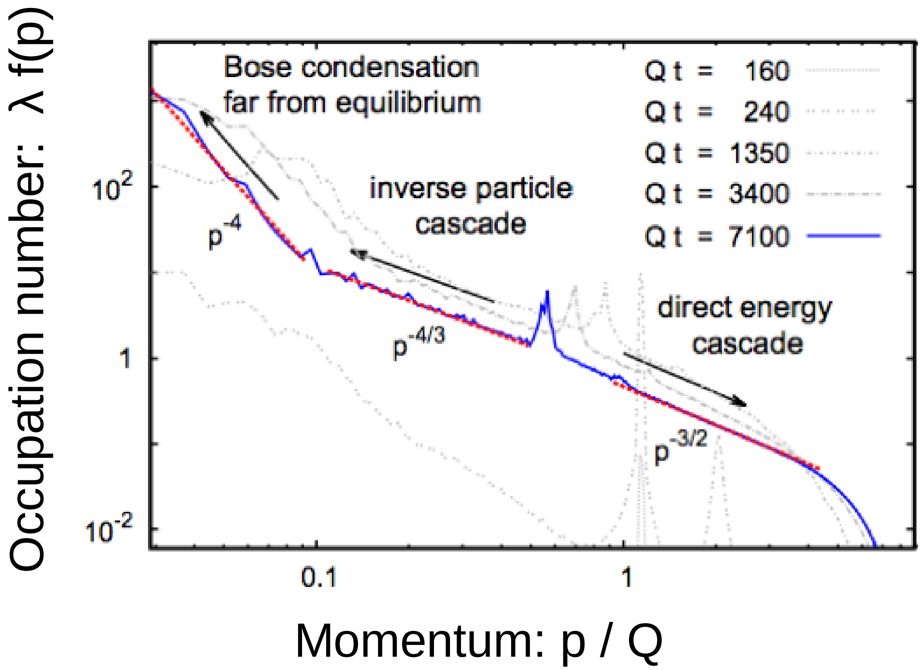
Independence of Initial conditions

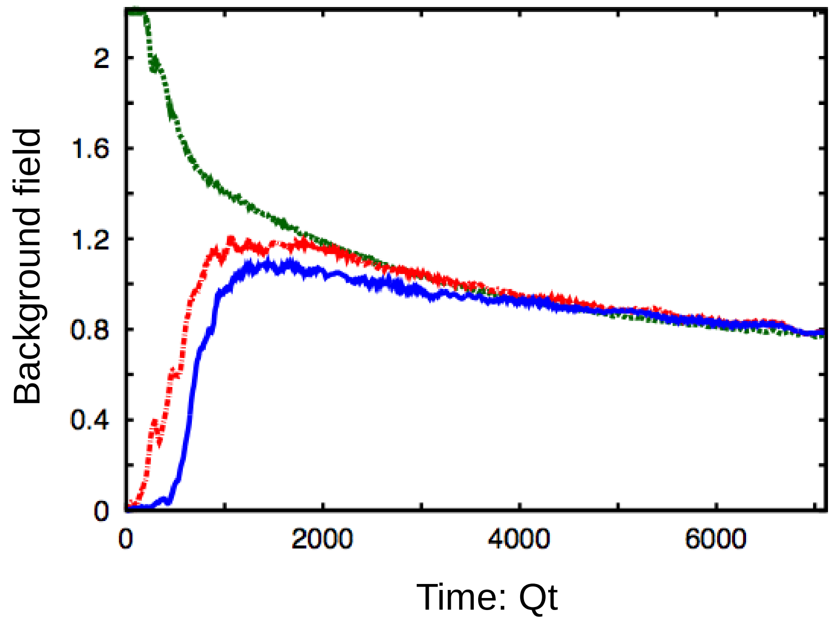


- The turbulent scaling behavior is a property of the thermalization process – ***independent of the underlying initial conditions***
- An ***effective memory loss*** occurs ***already at the early stages*** of the thermalization process

(Berges,Boguslavski,SS,Venugopalan arXiv:1312.5216)

Bose-Condensation far from equilibrium



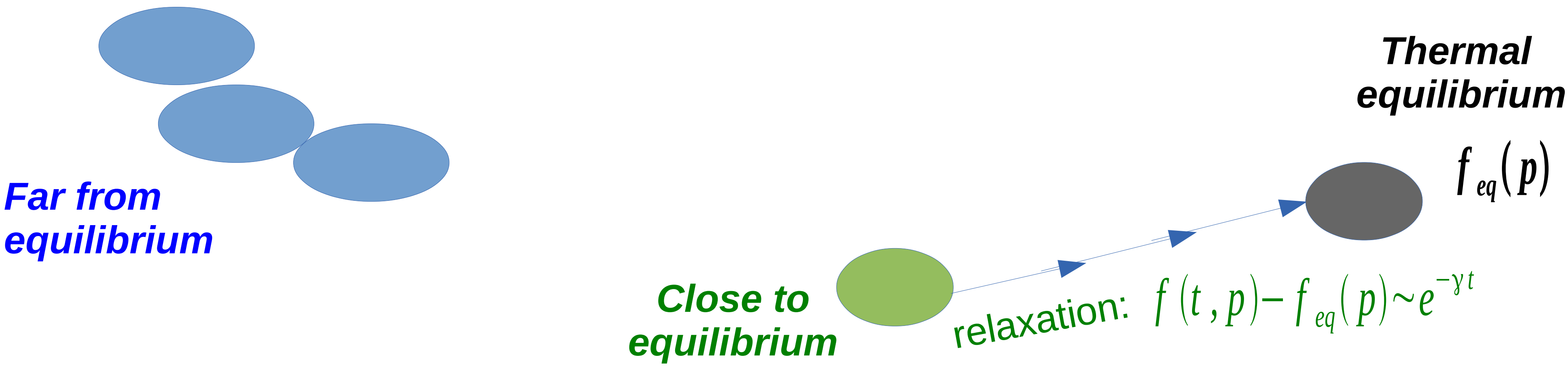


- Dynamical formation of zero mode (Bose condensation) even the though the system is in the symmetric phase

(Berges, Sexty PRL 108 (2012) 161601 Berges,Boguslavski,SS,Venugopalan arXiv:1312.5216)

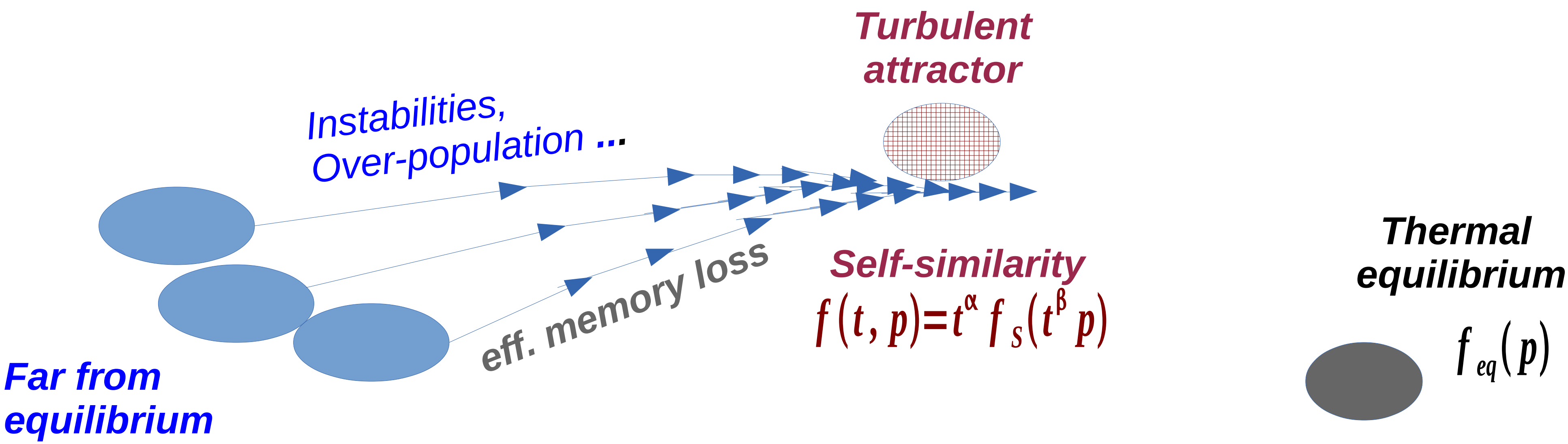
Turbulent thermalization

While ***close to thermal equilibrium***, one may expect thermalization to occur as a ***relaxation procedure*** ...



Turbulent thermalization

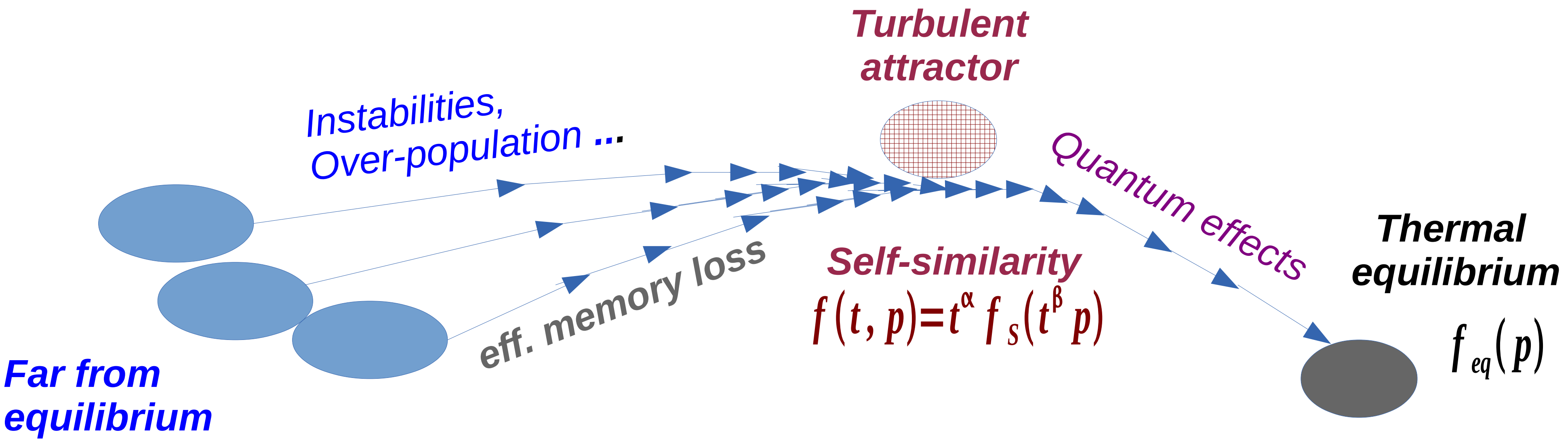
… The thermalization process for a system far from equilibrium proceeds as a ***self-similar evolution*** associated with the presence of a (classical) ***turbulent attractor*** ...



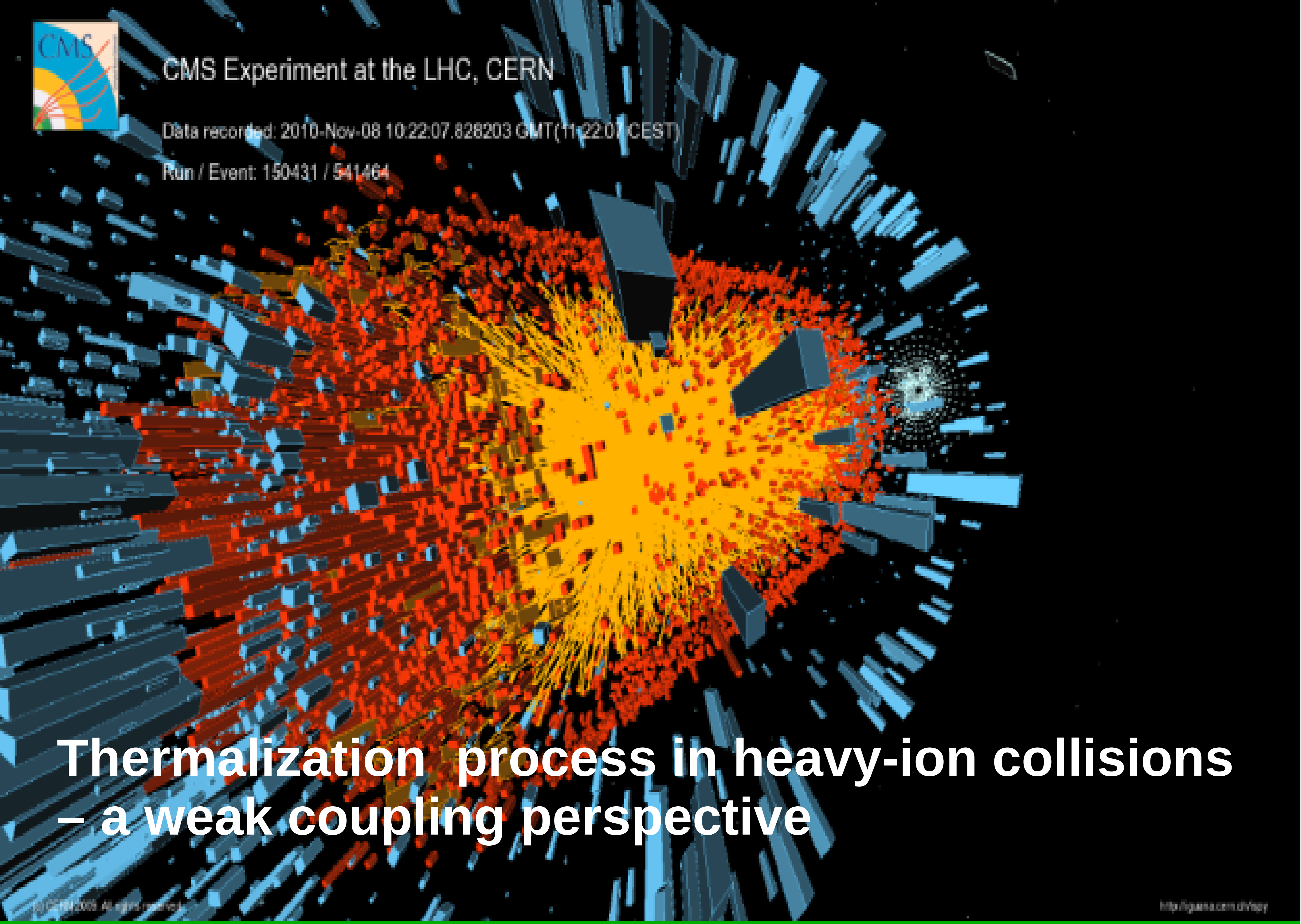
...until the occupancies become small $(f(p) \sim 1)$ and quantum effects can no longer be neglected.

Turbulent thermalization

The classical turbulent attractor is then no longer stable. The energy transfer to the ultra-violet has (parametrically) been accomplished at that time, system should be not too far from equilibrium.



→ Quantum effects will drive the system to thermal equilibrium

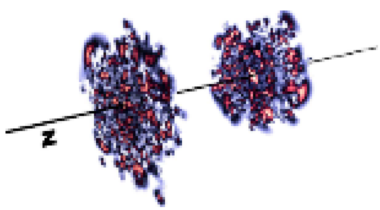


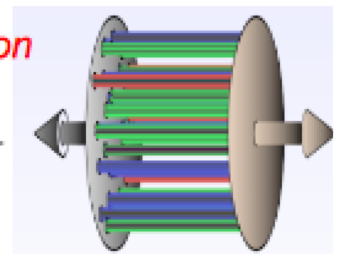
Thermalization process in heavy-ion collisions – a weak coupling perspective

General picture at weak coupling

Thermalization process

Collision





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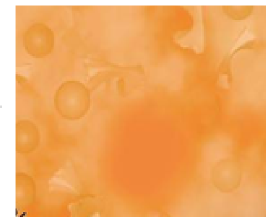


Time

Colliding nuclei

Gluons produced in highly anisotropic state *'Glasma'*

Thermalized QGP

Coupling constant assumed to be small

$$\alpha_s(Q) \ll 1$$

Many gluons prodcued in the collision

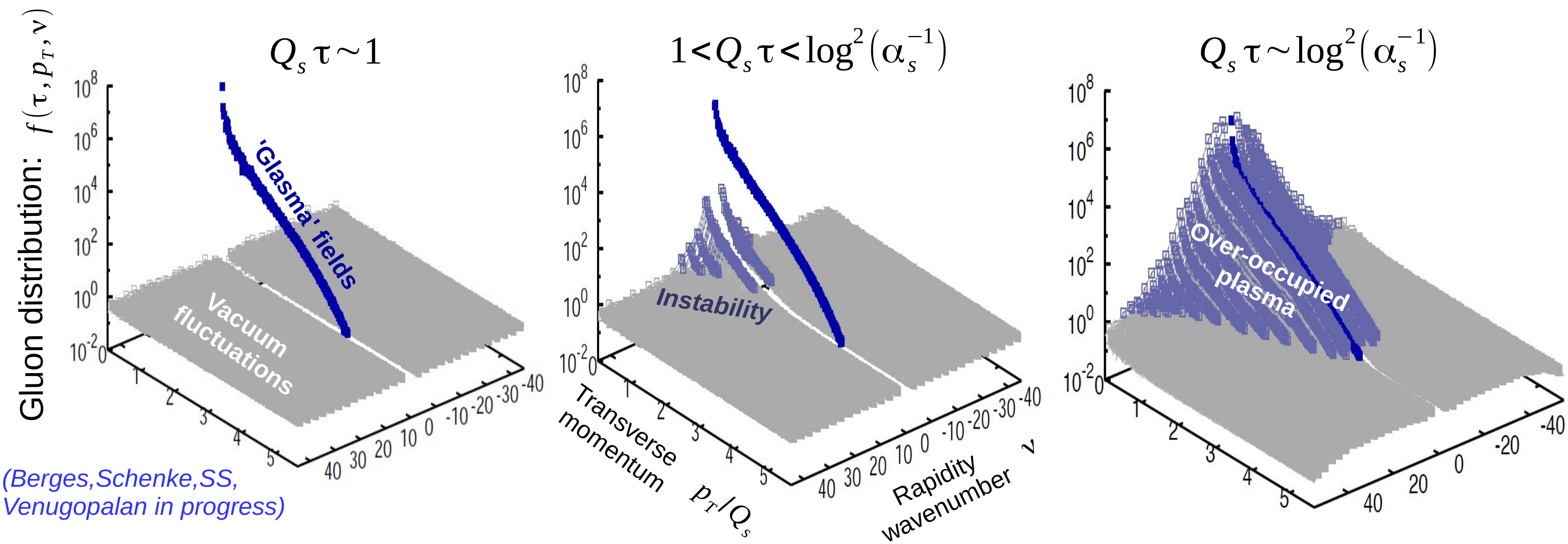
$$f(p) \sim 1/\alpha_s$$

→ ***Weakly coupled but strongly interacting***

Early time dynamics

Initial state is highly anisotropic → Plasma instabilities lead to exponential growth of low momentum modes *(c.f. Mrowczynski, Romatschke, Strickland, Rebhan, Attems , Venugopalan, Epelbaum, Gelis, Fukushima, Berges, Sexty ...)*

→ ***Over-occupied plasma*** $f(p \lesssim Q_s) \sim 1/\alpha_s$ ***formed on a time scale*** $\tau \sim Q_s^{-1} \log^2(\alpha_s^{-1})$

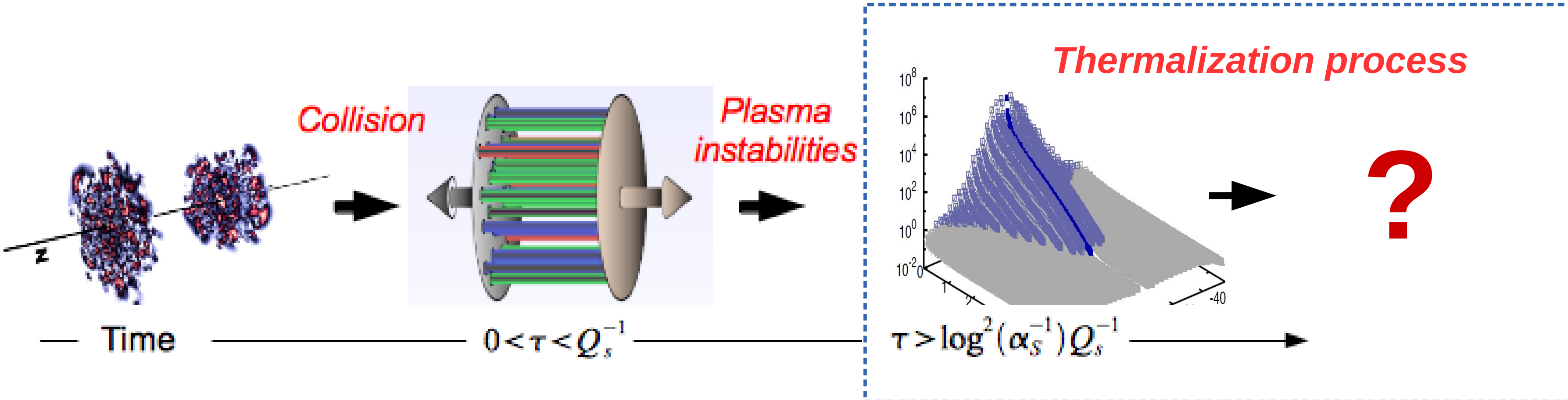


(Berges,Schenke,SS, Venugopalan in progress)

Classical-statistical simulation with CGC initial conditions with the NLO spectrum of vacuum fluctuations (c.f. Epelbaum & Gelis) at weak coupling $\alpha_s \sim 10^{-6}$

Thermalization process at weak coupling

System is still far from equilibrium at times $\tau_0 = 1/Q_s \ln^2(1/\alpha_s)$



Different thermalization scenarios proposed in kinetic theory:

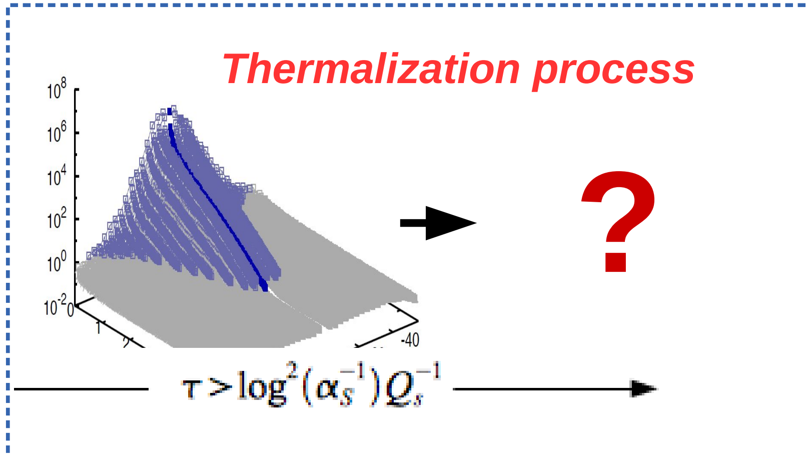
Baier et al. (BMSS),
PLB 502 (2001) 51-58

Kurkela, Moore (KM),
JHEP 1111 (2011) 120

Blaizot et al. (BGLMV),
Nucl. Phys. A 873 (2012) 68-80

Differences due the treatment of soft (non-perturbative) physics below m_D

Thermalization of the over-occupied QGP



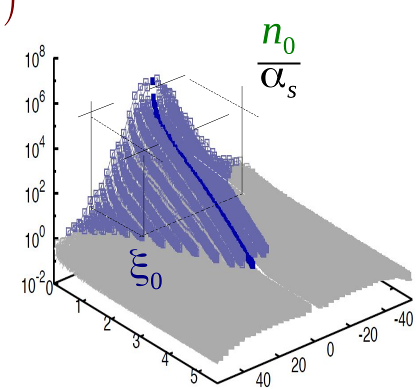
Classical regime can be studied non-perturbatively within classical-statistical lattice simulations

Study thermalization process for a variety of different initial conditions which describe the the over-occupied plasma at initial time $\tau_0 = 1/Q_s \ln^2(1/\alpha_s)$

Over-occupation

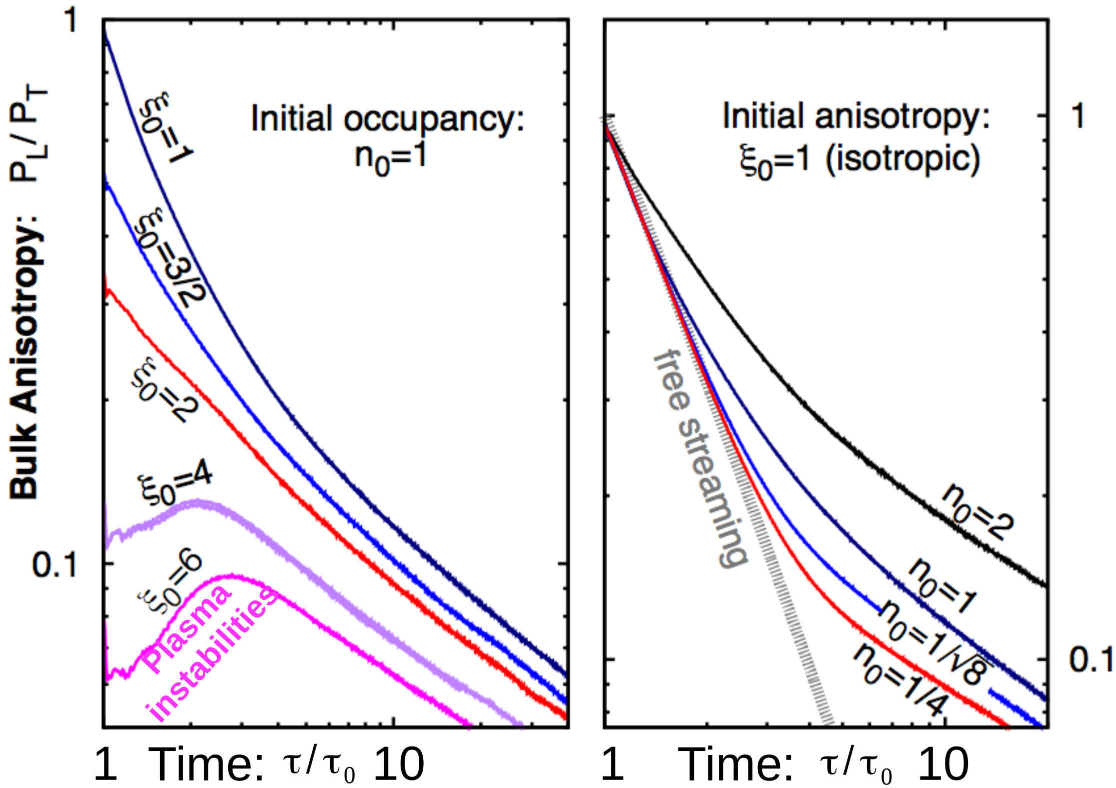
Momentum space anisotropy

$$f(p_T, p_Z, \tau_0) = \frac{n_0}{\alpha_s} \theta\left(Q_s - \sqrt{p_T^2 + \xi_0^2 p_Z^2}\right)$$



(Berges, Boguslavski, SS, Venugopalan PRD 89 074011 & arXiv:1311.3005)

Bulk anisotropy



- Competition between interactions and longitudinal expansion leads to an ***increase of the anisotropy.***
- Nevertheless the system remains ***significantly interacting*** throughout the entire evolution.
- The evolution becomes ***insensitive to the initial conditions*** and exhibits a ***universal scaling behavio***r at late times.

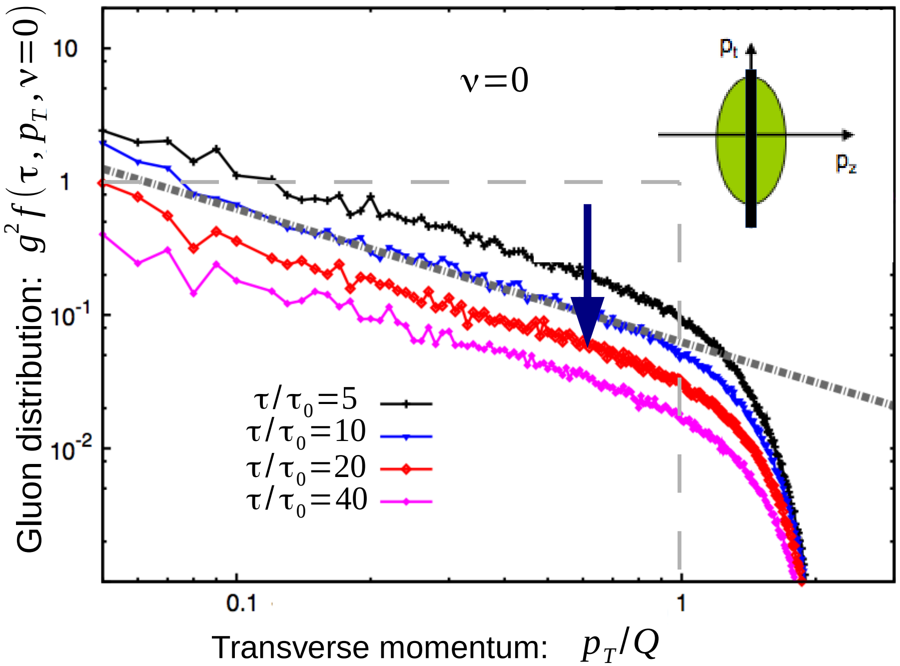
n_0 controls initial over-occupancy

ξ_0 controls initial anisotropy

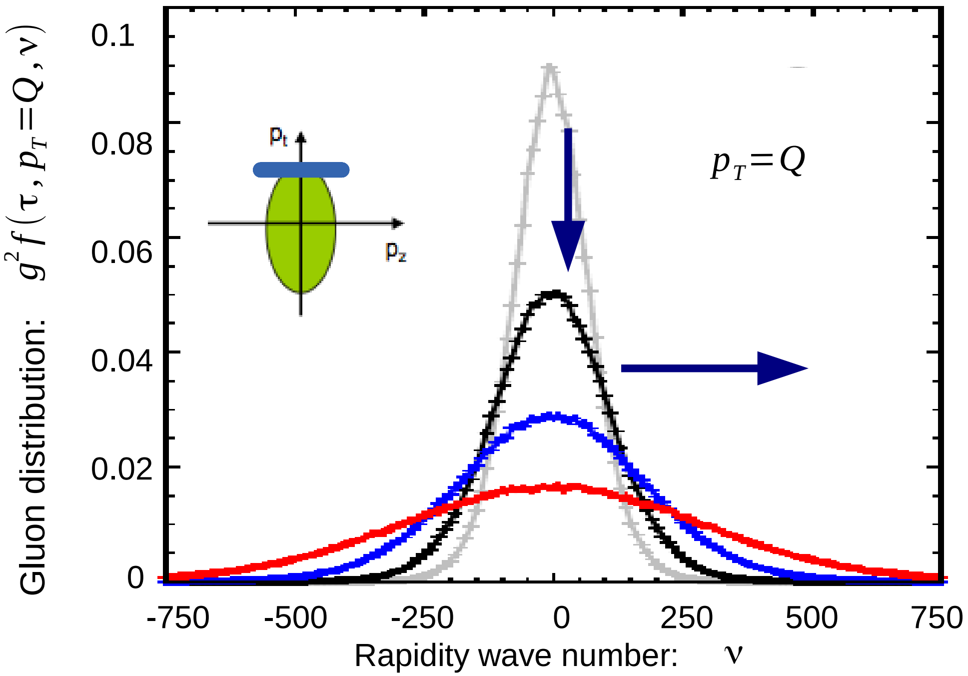
(Berges, Boguslavski, SS, Venugopalan PRD 89 074011 & arXiv:1311.3005)

Single particle gluon spectra

Transverse spectrum



Longitudinal spectrum



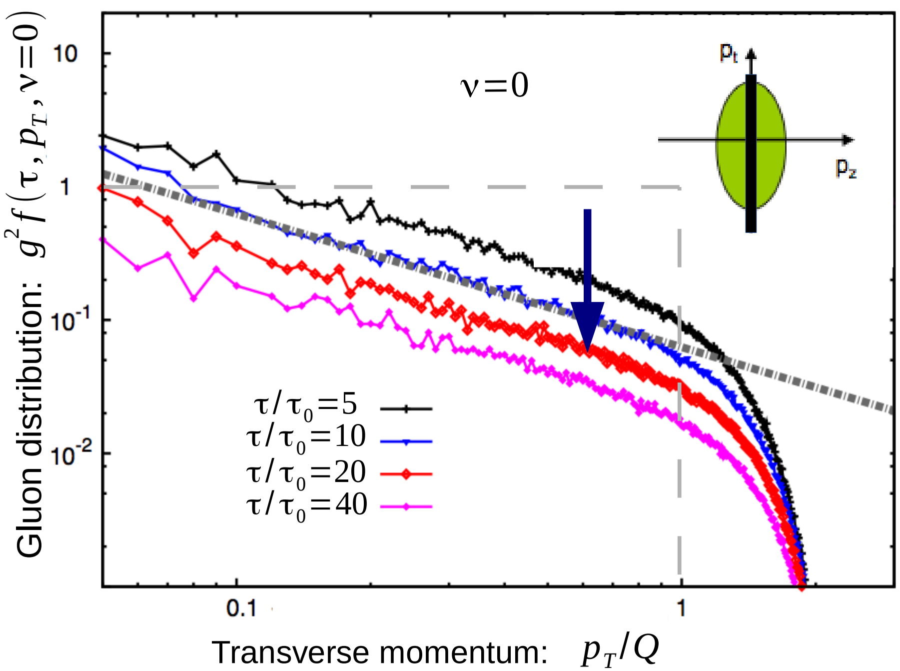
Transverse spectrum quickly approaches 'thermal' like T/p_T shape, with decreasing amplitude

Significant momentum broadening in the longitudinal direction observed.

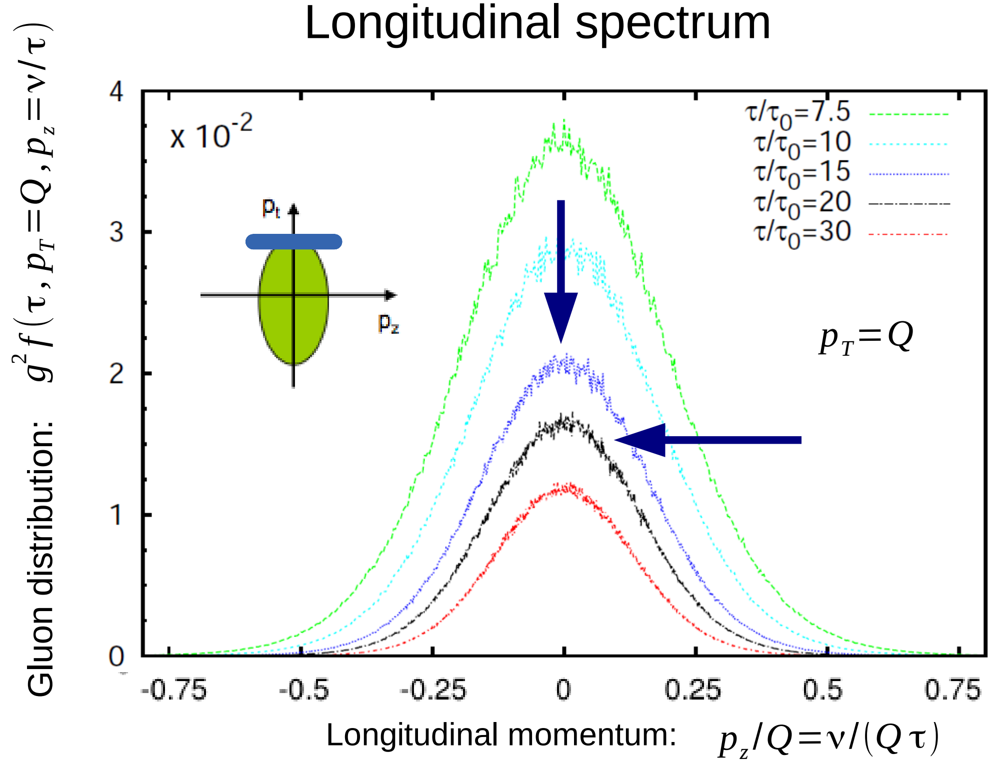
(Berges, Boguslavski, SS, Venugopalan PRD 89 074011 & arXiv:1311.3005)

Single particle gluon spectra

Transverse spectrum



Longitudinal spectrum

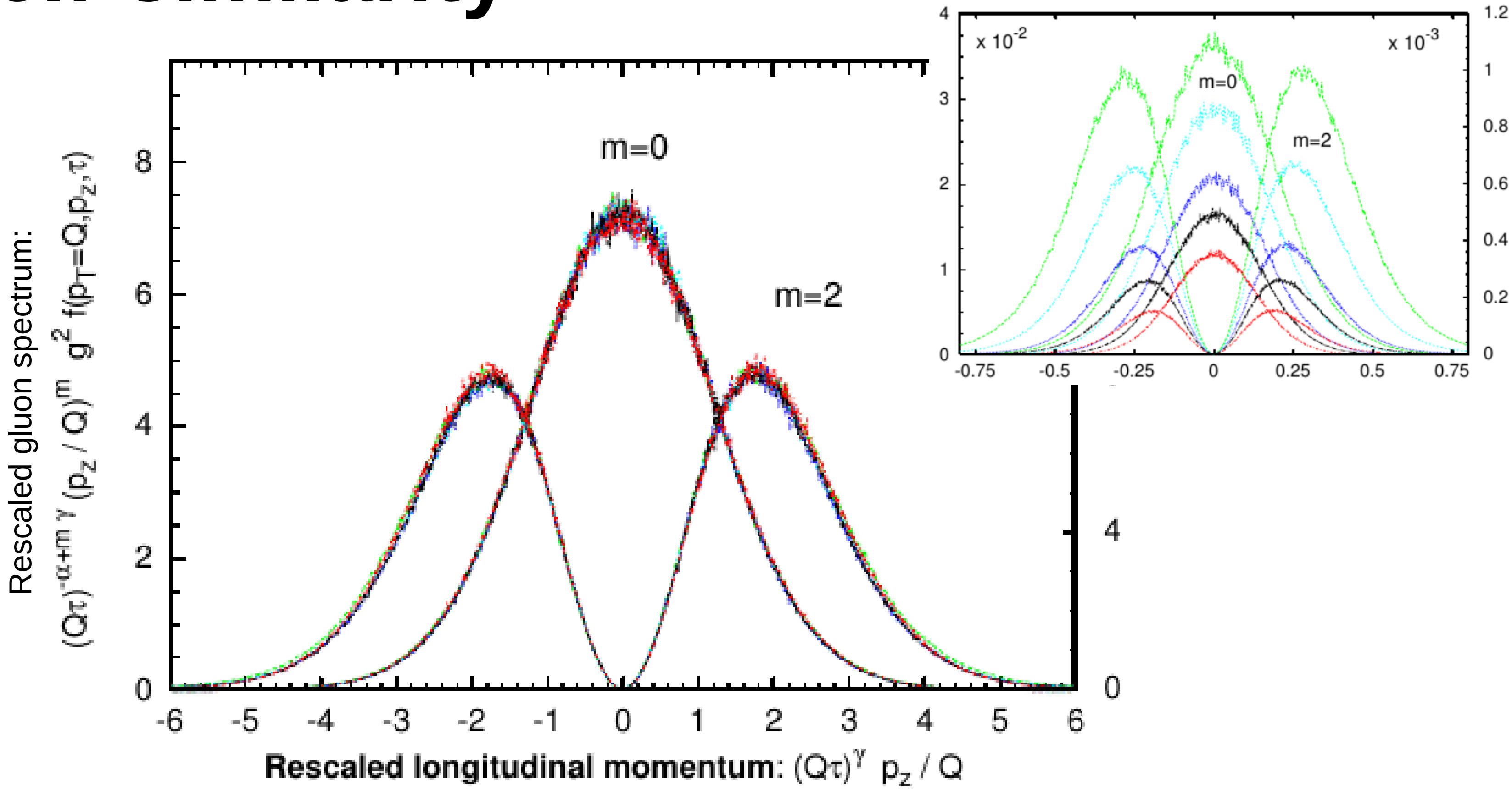


Transverse spectrum quickly approaches 'thermal' like T/p_T shape, with decreasing amplitude

However not strong enough to compensate completely for the red shift due to the longitudinal expansion.

(Berges, Boguslavski, SS, Venugopalan PRD 89 074011 & arXiv:1311.3005)

Self-similarity



The system reaches a non-thermal fixed point, where the space-time evolution

becomes self-similar, i.e. $f(p_T, p_Z, \tau) = (Q\tau)^{\alpha} f_S((Q\tau)^{\beta} p_T, (Q\tau)^{\gamma} p_Z)$

with a ***stationary distribution*** f_S and scaling exponents $\alpha = -2/3, \beta = 0, \gamma = 1/3$

(Berges, Boguslavski, SS, Venugopalan PRD 89 074011 & arXiv:1311.3005)

Kinetic interpretation

Consider the Boltzmann equation

$$\left[\partial_\tau - \frac{p_Z}{\tau}\partial_{p_Z}\right] f(p_T, p_Z, \tau) = C[f](p_T, p_Z, \tau)$$

with a self-similar evolution

$$f(p_T, p_Z, \tau) = (Q\tau)^{\alpha} f_S\left((Q\tau)^{\beta} p_T, (Q\tau)^{\gamma} p_Z\right)$$

→ ***Non-thermal fixed point solution*** $(f \gg 1)$

$$\left[\alpha + \beta p_T \partial_{p_T} + (\gamma - 1) p_Z \partial_{p_Z}\right] f_S(p_T, p_Z) = Q^{-1} C[f_S](p_T, p_Z)$$

→ ***Scaling exponents determined by scaling relations for***

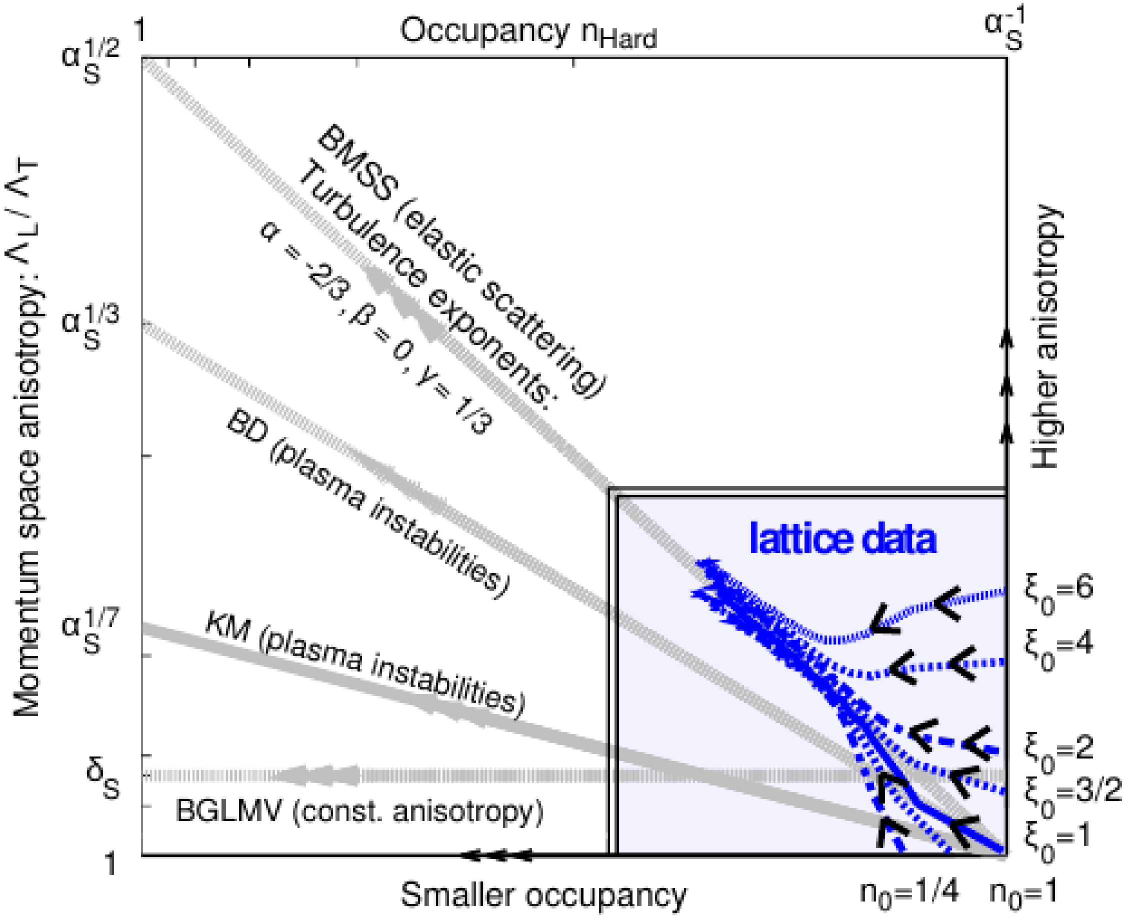
- Small angle elastic scattering $(2\alpha - 2\beta + \gamma = -1)$
- Energy conservation $(\alpha - 3\beta - \gamma = -1)$
- Particle number conservation $(\alpha - 2\beta - \gamma = -1)$

→ $\alpha = -2/3, \beta = 0, \gamma = 1/3$ ***in excellent agreement with lattice data!***

Confirms "bottum-up" thermalization scenario (Baier et al. PLB 502 (2001) 51-58***)***

(Berges, Boguslavski, SS, Venugopalan PRD 89 074011 & arXiv:1311.3005)

Comparison with weak-coupling thermalization scenarios

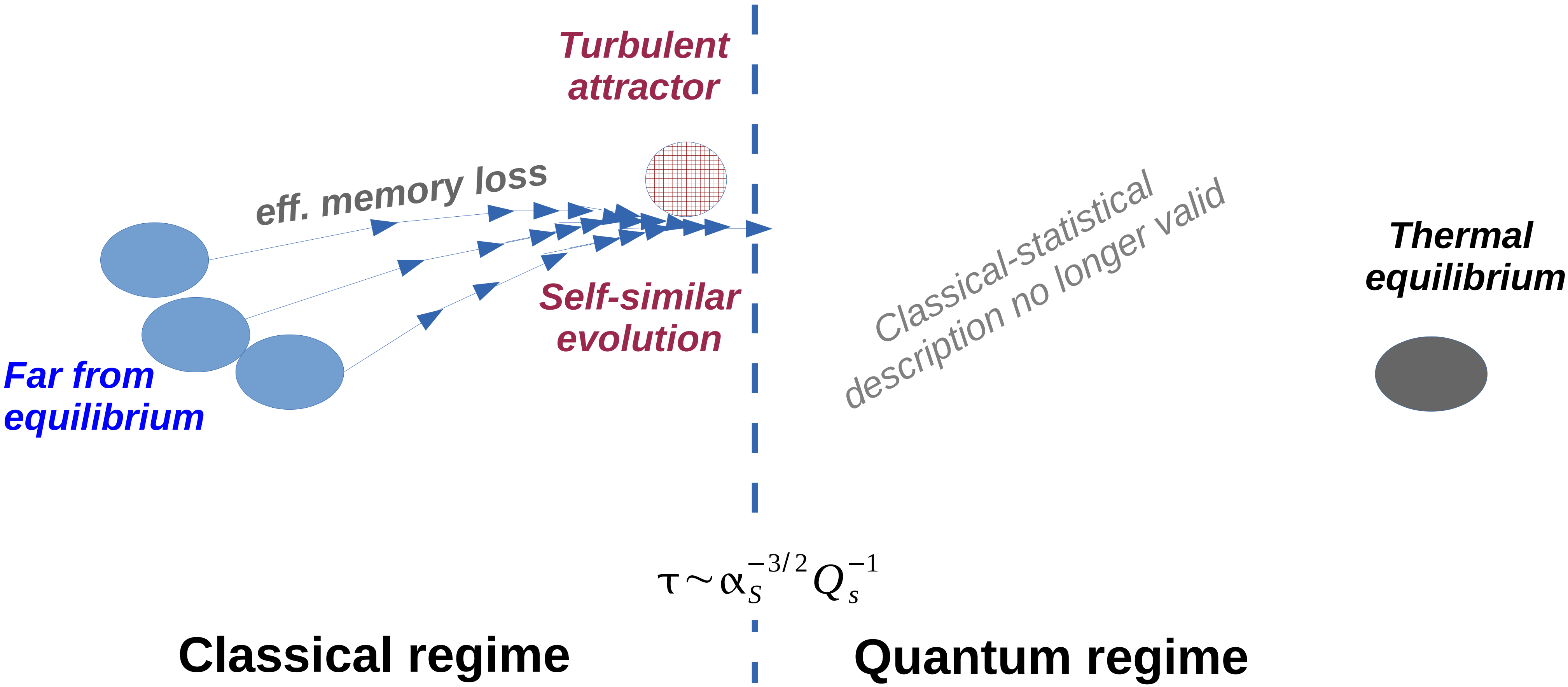


'Bottum up' scenario* emerges as a turbulent attractor of the evolution!

**Baier,Mueller,Schiff and Son, Phys. Lett. B 502, 51 (2001)*

Thermalization process at weak coupling

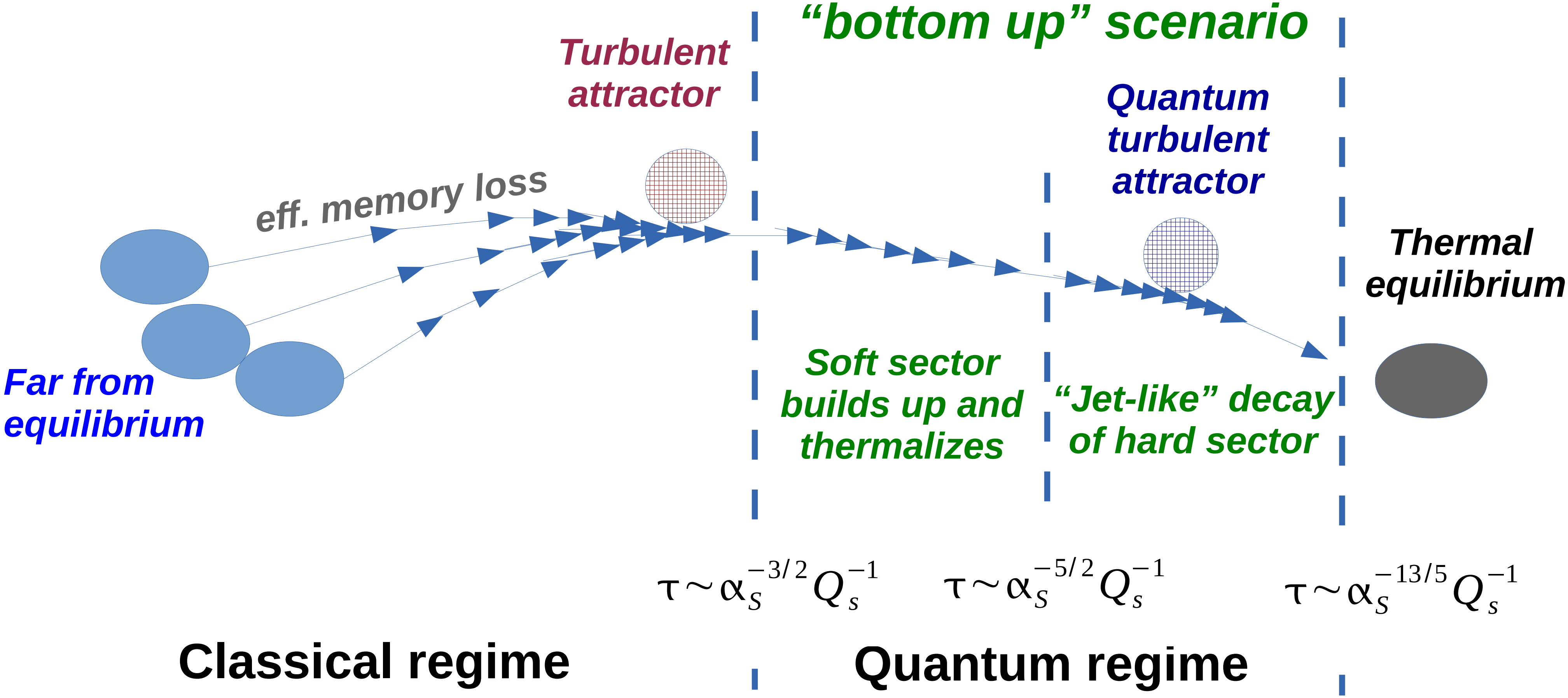
The thermalization process again reaches a ***turbulent attractor***, However at the end of the classical regime the system is ***still far from equilibrium.***



Thermalization of the expanding plasma

Classical statistical simulations no longer applicable in the quantum regime. However kinetic theory predictions provide route to thermal equilibrium

Kinetic theory: "bottom up" scenario



c.f. talks by Kurkela & Blaizot

Summary & Conclusions

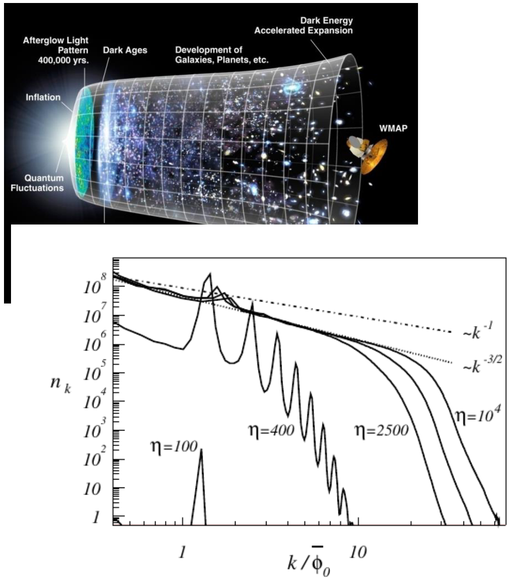
Thermalization process of far from equilibrium systems is controlled by the ***transport of conserved quantities*** (energy and particle number) over a large separation of scales

→ ***Wave turbulence***

Similar manifestations across far from equilibrium systems at all energy scales

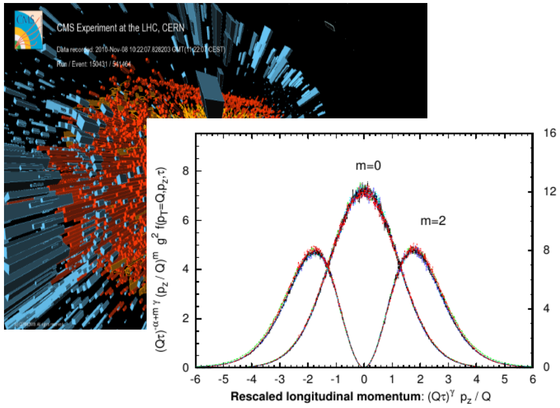
Cosmology

WMAP Science Team



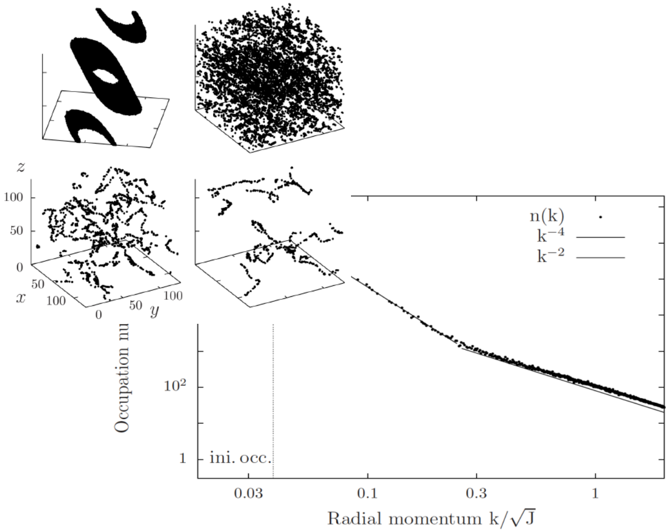
Micha, Tkachev PRD 70 (2004) 043538

Heavy-ion collisions



→ ***Universality far from equilibrium***

Cold quantum gases



Nowak, Sexty, Gasenzer, PRB 84 (2011) 020506(R)

Summary & Conclusions

Many open questions concerning the thermalization in high-energy heavy-ion collisions:

– How exactly does the thermalization process at weak coupling continue beyond the classical regime? *(c.f. talks by Kurkela & Blaizot)*

– Can weak coupling methods describe the bulk physics at RHIC/LHC where $\alpha_s \sim 0.3$? *(c.f. work by Epelbaum & Gelis)*

– Can one connect the weak coupling picture compare to strong coupling (AdS/CFT) picture?

While generally the thermalization process is much better understood in scalar models, there also remain some interesting questions e.g.

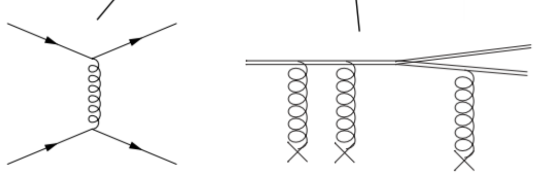
– Detailed dynamic of infrared sector and onset of Bose-Einstein condensation?

Matching classical Yang-Mills evolution to kinetic theory

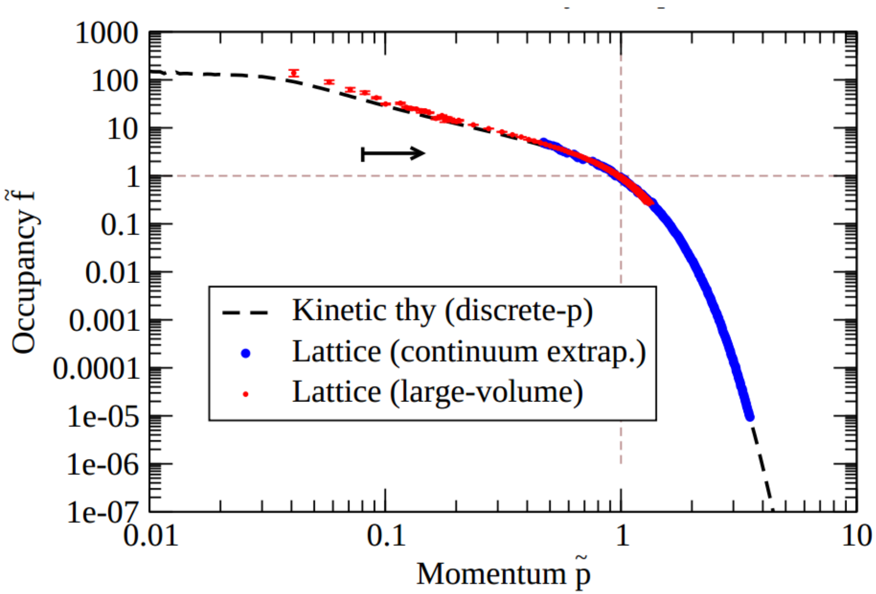
Comparison between classical-statistical lattice and kinetic theory for non-expanding gauge theory

Effective kinetic theory

$$\frac{df}{dt} = -C_{2\leftrightarrow 2}[f] - C_{1\leftrightarrow 2}[f]$$



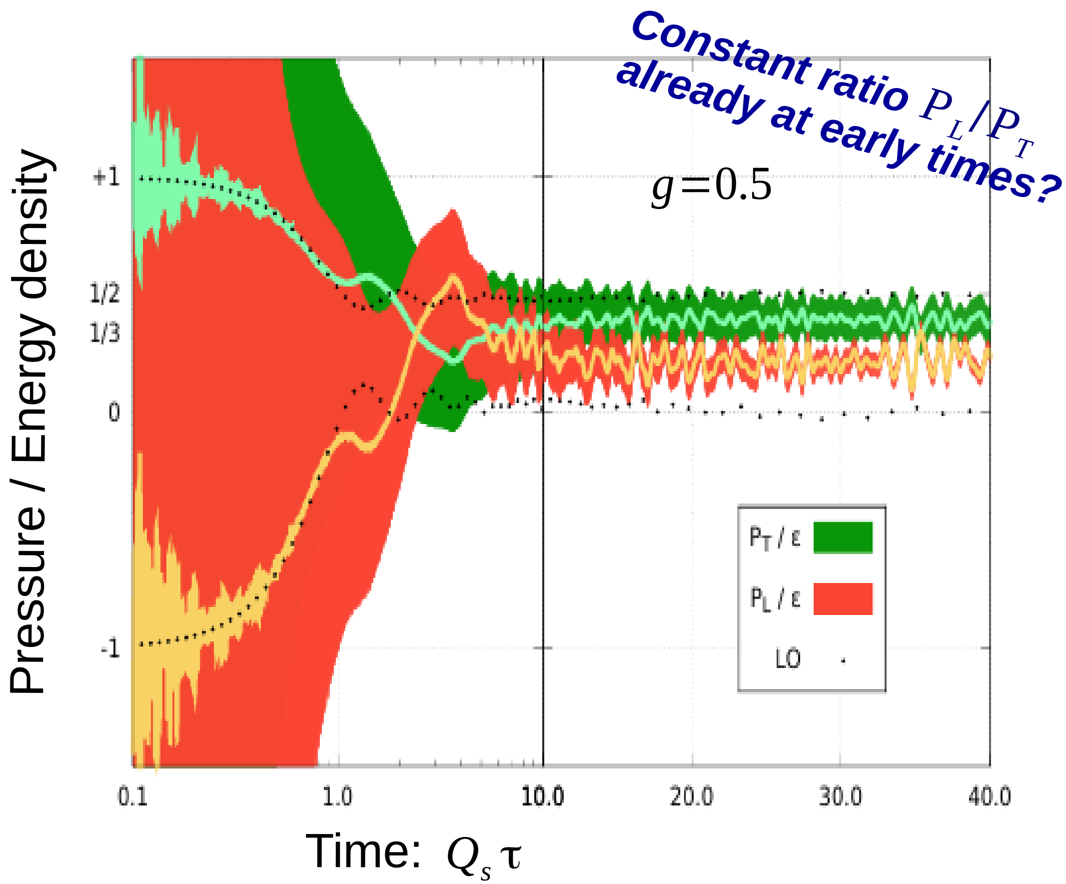
Arnold et al. hep-ph/0209353



Abraao York, AK, Lu, Moore arXiv:1401.3751

→ Kinetic theory can be extended straightforward to the quantum regime *(c.f. talk by Kurkela)*

Weak coupling methods at stronger couplings



Epelbaum & Gelis PRL 111 (2013) 232301

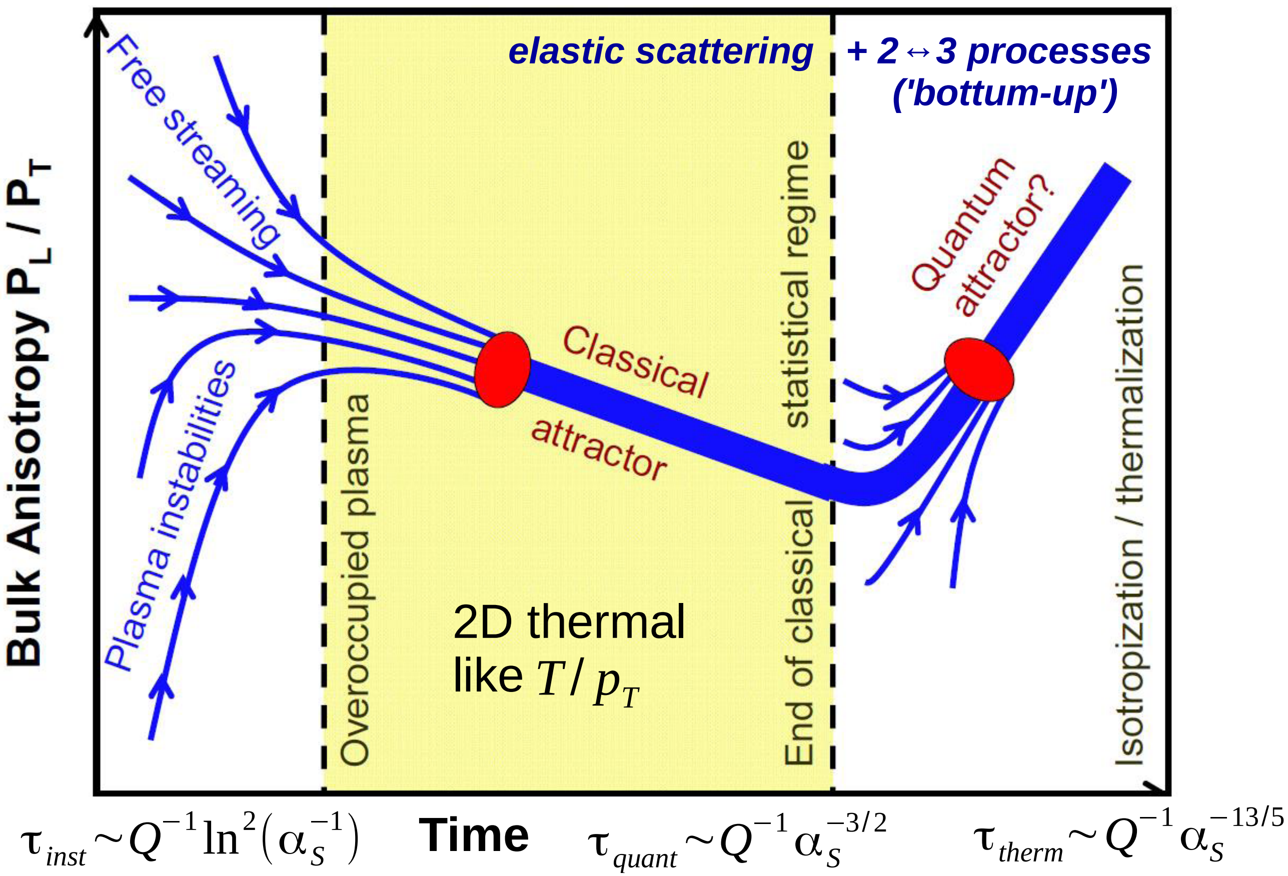
+ Simulations based on CGC initial conditions with NLO spectrum of vacuum fluctuations

- Conceptual problems related to classical-statistical method:

Non-renormalizability of vacuum fluctuations?

Within the range of validity?

Thermalization process at weak coupling



- Different initial conditions all lead to the same attractor solution.
- The anisotropy of the system increases until the occupancies become of order unity. Beyond this point classical-statistical dynamics is no longer applicable.
- In the quantum regime kinetic theory predicts the system to restore isotropy (c.f. Baier et al.) with the possiblity of a second 'quantum attractor' (c.f. Blaizot, Iancu, Mehtar-Tani)

Extrapolation to realistic coupling $\alpha_S \sim 0.3$ for $Q \sim 2\,GeV$

$\tau_{inst} \sim 0.1\, fm/c$ $\tau_{quant} \sim 0.6\, fm/c$ $\tau_{therm} \sim 2\, fm/c$