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Sound modes and the two-stream instability in relativistic superfluids

M.G. Alford, S.K. Mallavarapu, A. Schmitt, S. Stetina, PRD 87, 065001 (2013)

M.G. Alford, S.K. Mallavarapu, A. Schmitt, S. Stetina, PRD 89, 085005 (2014)

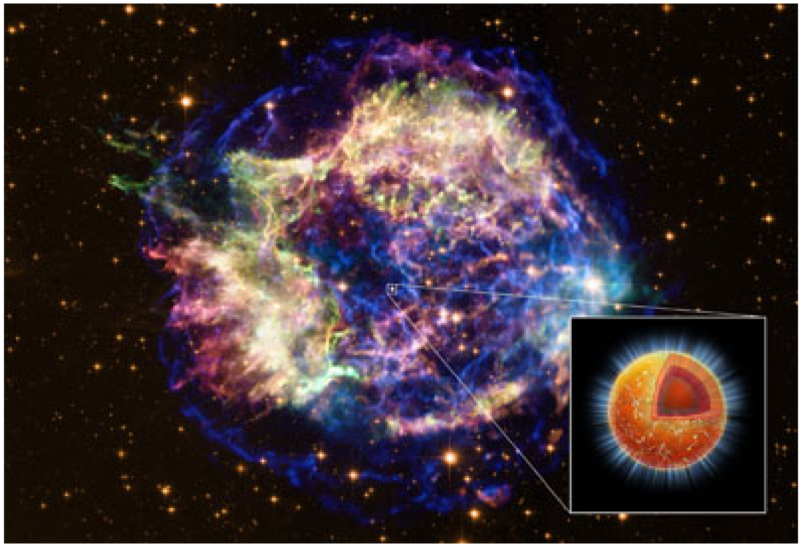
A. Schmitt, PRD 89, 065024 (2014)

A. Schmitt, arXiv:1404.1284, to appear in Lect. Notes Phys.

- two-fluid picture of a superfluid
- role reversal in first and second sound
- two-stream instability

- **Superfluid hydrodynamics: relevance for compact stars**

  - r-mode instability
  - pulsar glitches
  - precession
  - asteroseismology
  - superfluid turbulence (?)



- **Superfluidity in dense matter**

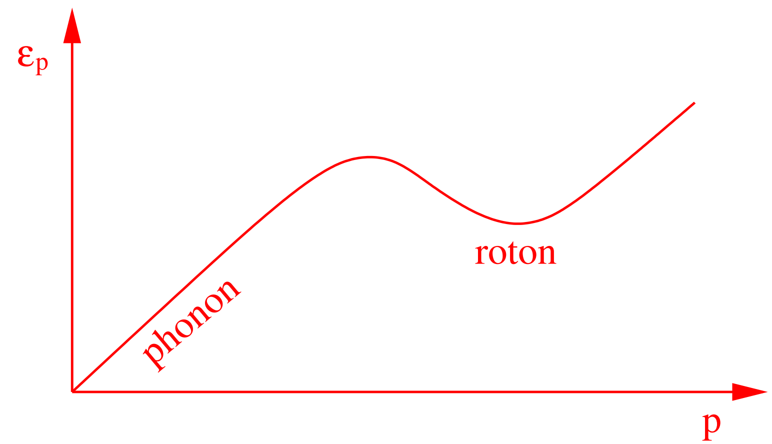
Nuclear matter	Quark matter
neutrons ($T_c \lesssim 10\,\mathrm{keV}$)	color-flavor locked phase ($T_c \sim 10\,\mathrm{MeV}$)
hyperons	color-spin locked phase ($T_c \sim 10\,\mathrm{keV}$)

• Two-fluid picture of a superfluid (liquid helium)

London, Tisza (1938); Landau (1941)

relativistic: Khalatnikov, Lebedev (1982); Carter (1989)

- "superfluid": condensate, carries no entropy
- "normal fluid": excitations (Goldstone mode), carries entropy



$\boxed{\text{Hydrodynamic eqs.} \Rightarrow \textbf{two sound modes}}$

1st sound	**2nd sound**
in-phase oscillation (primarily) density wave	out-of-phase oscillation (primarily) entropy wave

• Goals

$\boxed{\text{How does the (covariant) two-fluid picture arise from a microscopic field theory?}}$

M.G. Alford, S.K. Mallavarapu, A. Schmitt, S. Stetina, PRD 87, 065001 (2013)

this talk:

$\boxed{\text{Compute sound modes in a relativistic superfluid (and in the presence of a superflow)}}$

M.G. Alford, S.K. Mallavarapu, A. Schmitt, S. Stetina, PRD 89, 085005 (2014)

A. Schmitt, PRD 89, 065024 (2014)

• Microscopic calculation (page 1/2)

- starting point:
  complex scalar field φ

$$\boxed{\mathcal{L} = \partial_\mu \varphi^* \partial^\mu \varphi - m^2 |\varphi|^2 - \lambda |\varphi|^4}$$

- Bose condensate $\langle \varphi \rangle = \rho\, e^{i\psi} \to$ superfluid velocity $v^\mu = \dfrac{\partial^\mu \psi}{\mu}$

- effective action density in the 2PI formalism (CJT)

$$\boxed{\Gamma[\rho, S] = -U(\rho) - \frac{1}{2}\mathrm{Tr}\ln S^{-1} - \frac{1}{2}\mathrm{Tr}[S_0^{-1}(\rho)S - 1] - V_2[\rho, S]}$$

- $V_2[\rho, S]$: two-loop two-particle irreducible (2PI) diagrams

• Microscopic calculation (page 2/2)

- minimize w.r.t. condensate ρ and solve Dyson-Schwinger equation (Hartree approximation; impose Goldstone theorem by hand)

- restrict to weak coupling
  $\rightarrow$ no dependence on renormalization scale

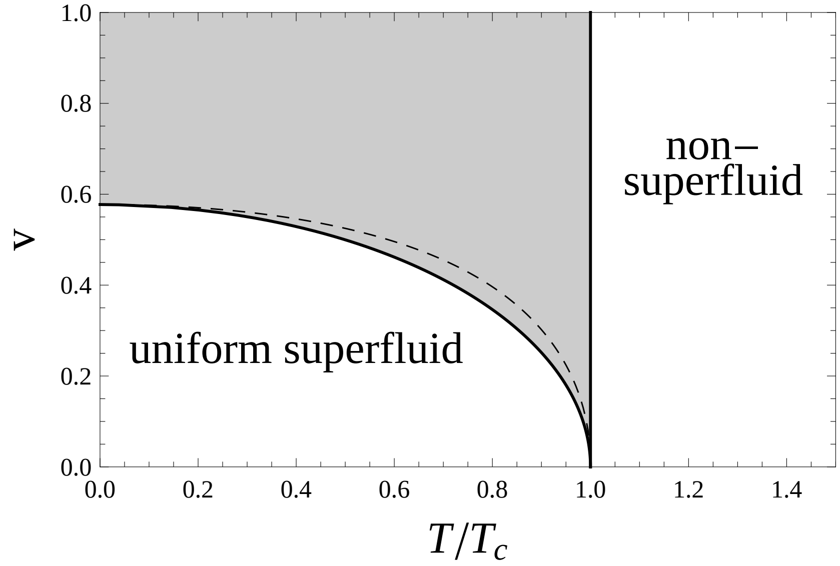
- consider uniform superflow $\mathbf{v}$

- neglect dissipation, compute sound modes in linear regime
  $\rightarrow$ thermodynamics with $(\mu, T, \mathbf{v})$

• Results I: critical velocity

- instability at $v = v_c(T)$:
  negative energies in Goldstone dispersion $\epsilon_{\mathbf{k}}(\mathbf{v}) < 0$

- generalization of Landau's original argument $\epsilon_{\mathbf{k}} - \mathbf{k} \cdot \mathbf{v} < 0$

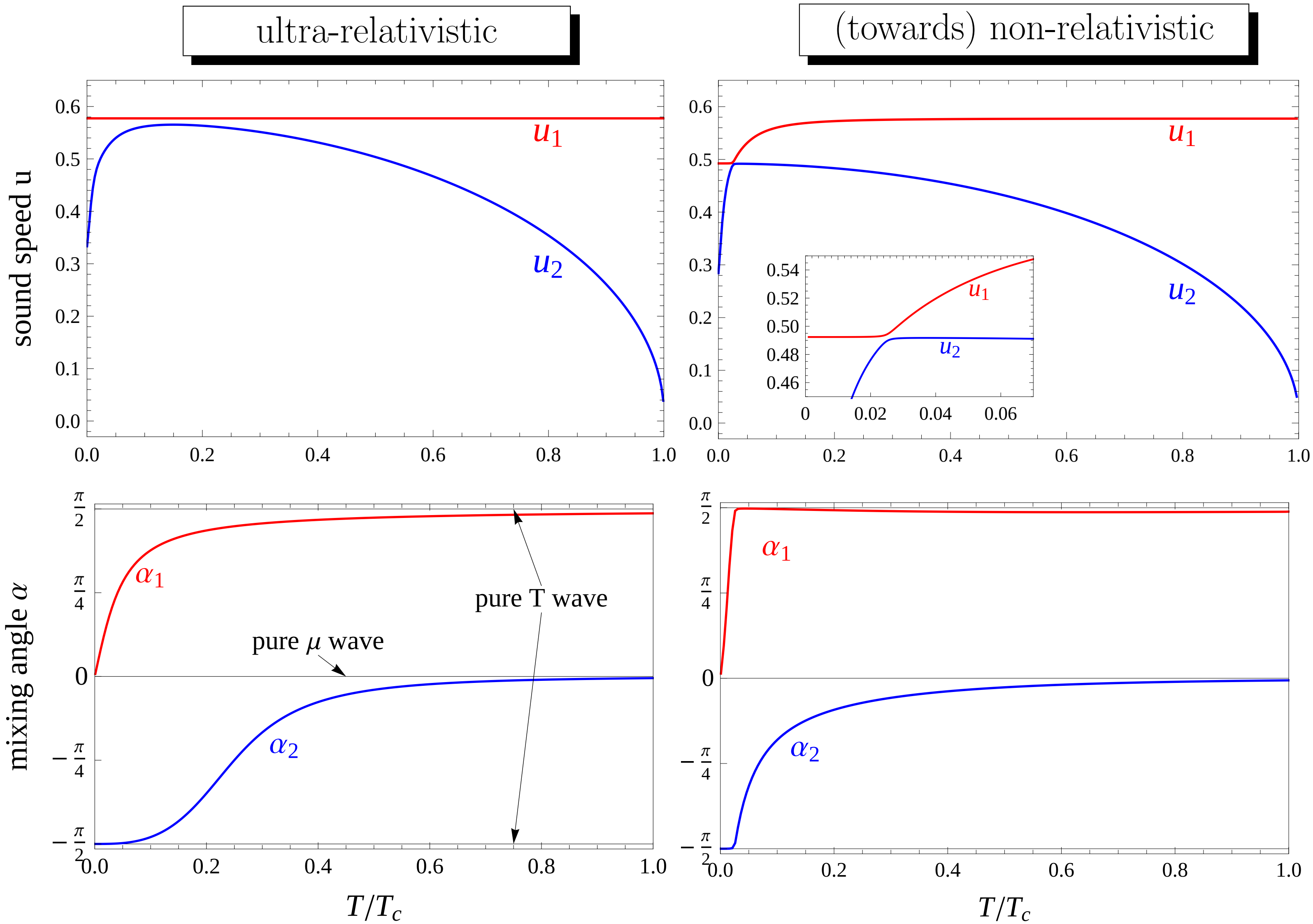


- dashed line: without backreaction of condensate

- shaded region: dissipation, turbulence?

- similar phase diagram for holographic superfluid I. Amado, D. Arean, A. Jimenez-Alba, K. Landsteiner, L. Melgar and I. S. Landea, JHEP 1402, 063 (2014)

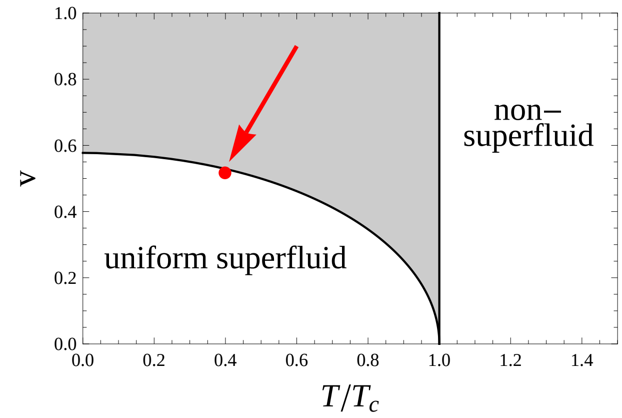
• Results II: sound speeds and mixing angle



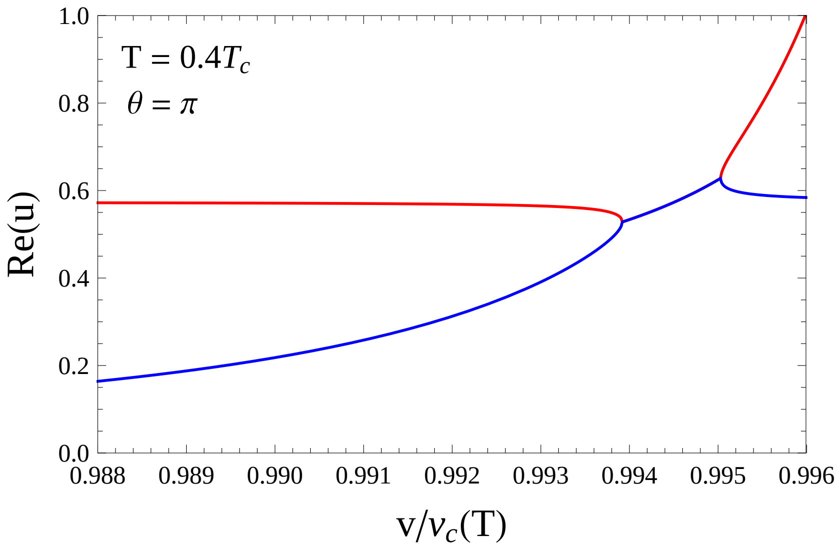
$$\alpha = \arctan \frac{\delta T}{\delta \mu}$$

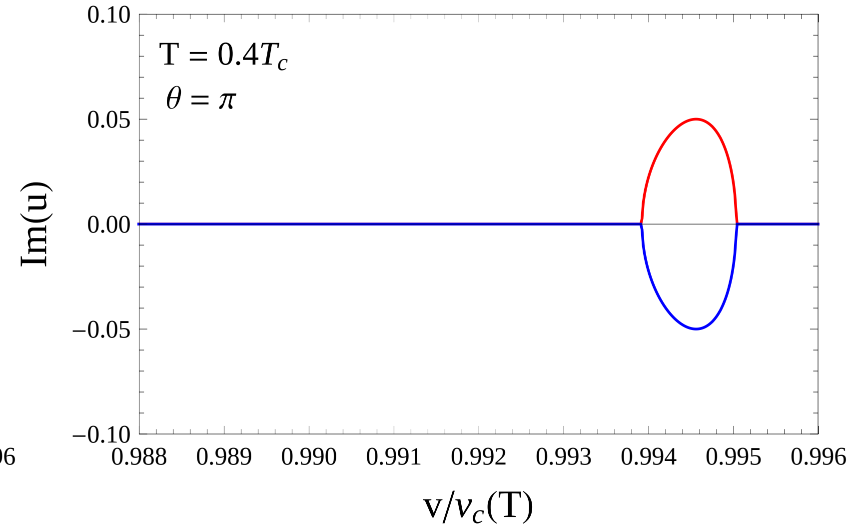
role reversal in first and second sound!

• Results III: two-stream instability



- compute sound speed close to Landau's critical velocity





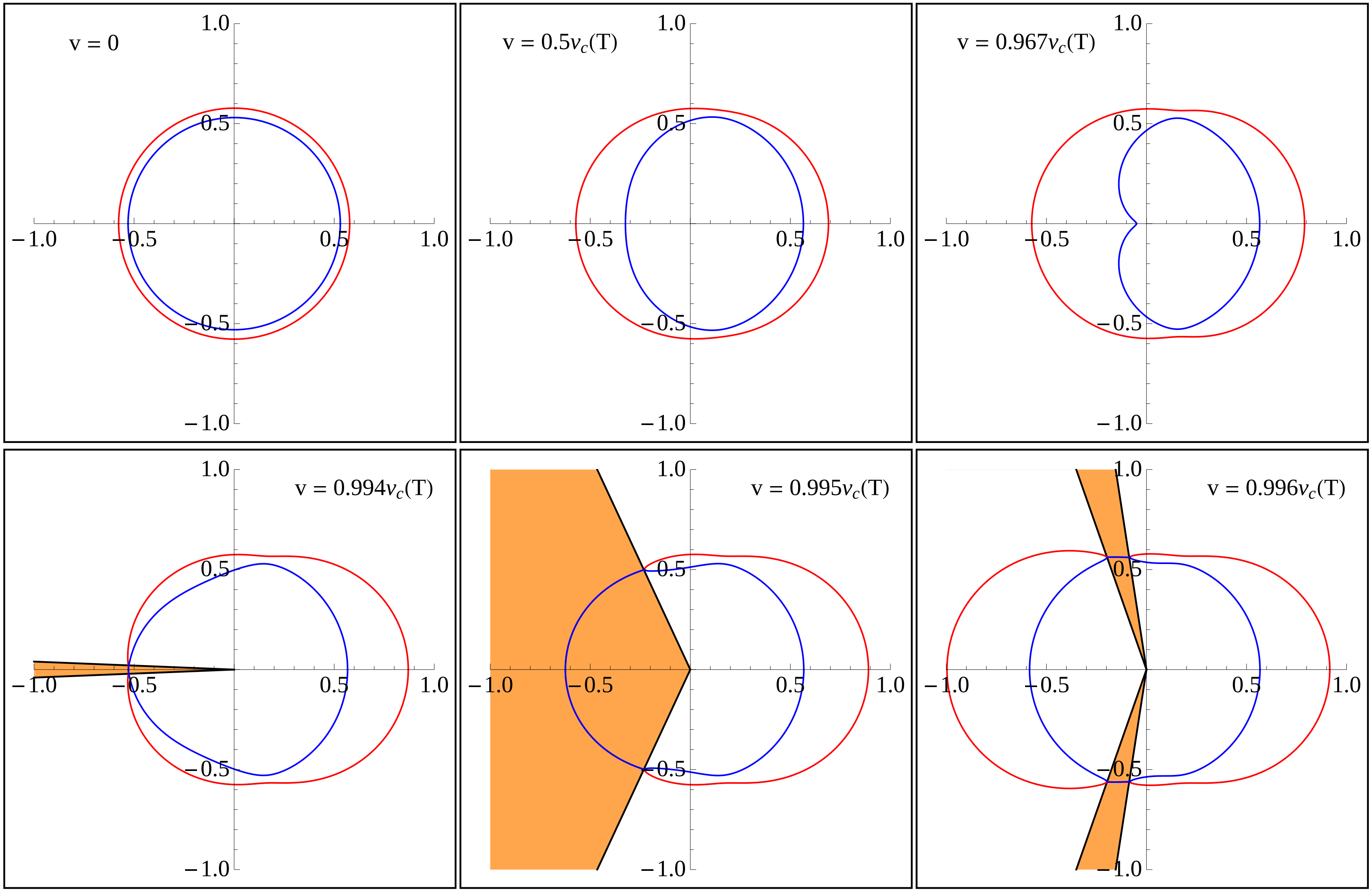
- complex sound speeds → one mode damped, one mode explodes

plasma physics: O. Buneman, Phys.Rev. 115, 503 (1959); D.T. Farley, PRL 10, 279 (1963)

general two-fluid system: L. Samuelsson, C. S. Lopez-Monsalvo, N. Andersson, G. L. Comer, Gen. Rel. Grav. 42, 413 (2010)

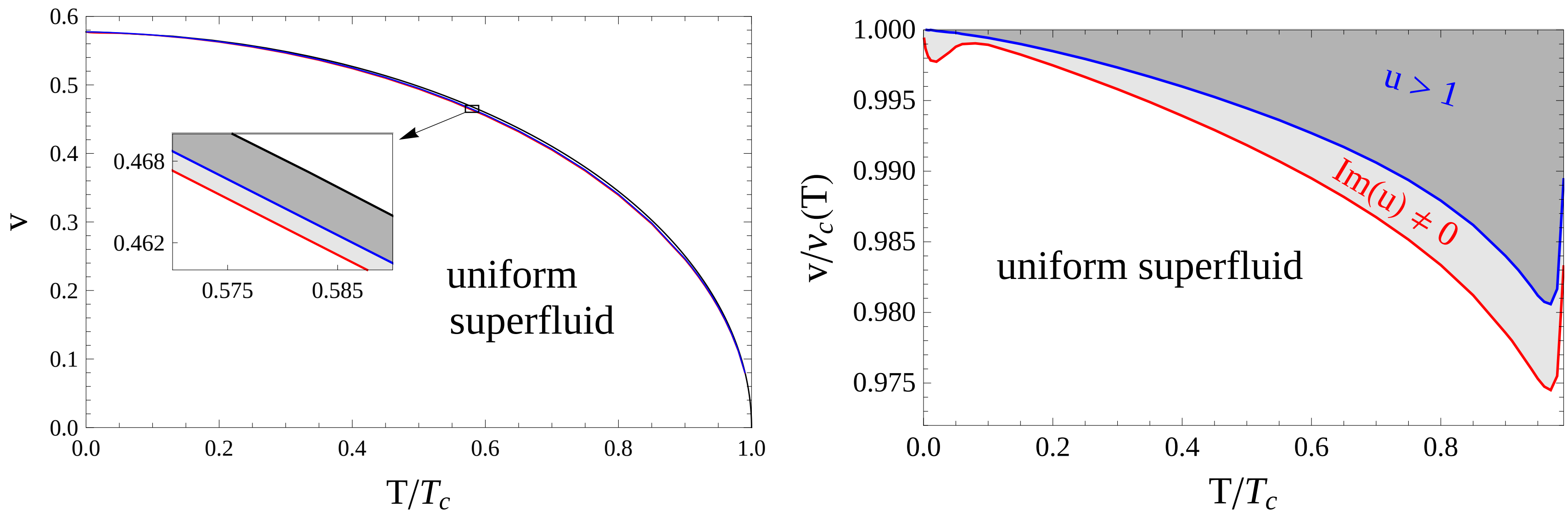
relevance for superfluids: N. Andersson, G. L. Comer, R. Prix, MNRAS 354, 101 (2004)

- ## All directions



(superflow pointing to the right)

• Instability window in phase diagram



- tiny window for weak coupling $\lambda = 0.05$
  (varying λ shows that the window grows with λ)
- region with $u > 1$: problem in the formalism?
  (Hartree? enforced Goldstone theorem?)
- very small T: qualitatively different angular structure of instability

• Outlook

- role reversal of sound modes:
  - relevance for compact stars?
  - analogy to superfluid r-modes

    M. E. Gusakov, A. I. Chugunov and E. M. Kantor, PRL 112, 151101 (2014)

- two-stream instability:
  - instability more prominent at strong coupling?

    holographic approach: C.P.Herzog and A.Yarom, PRD 80, 106002 (2009); I.Amado, D.Arean, A.Jimenez-Alba, K.Landsteiner, L.Melgar, I.S.Landea, JHEP 1402, 063 (2014)
  - time evolution of instability

    I. Hawke, G. L. Comer and N. Andersson, Class. Quant. Grav. 30, 145007 (2013)
  - relevance for compact stars, e.g., pulsar glitches

    N. Andersson, G. L. Comer, R. Prix, MNRAS 354, 101 (2004)

- start from fermionic theory

  D. Müller, A. Schmitt, work in progress

- behavior beyond critical velocity

Thank you for your attention!