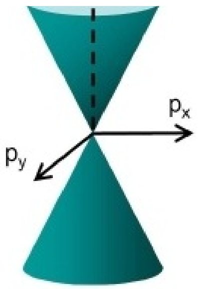


# The Relativistic World of Graphene


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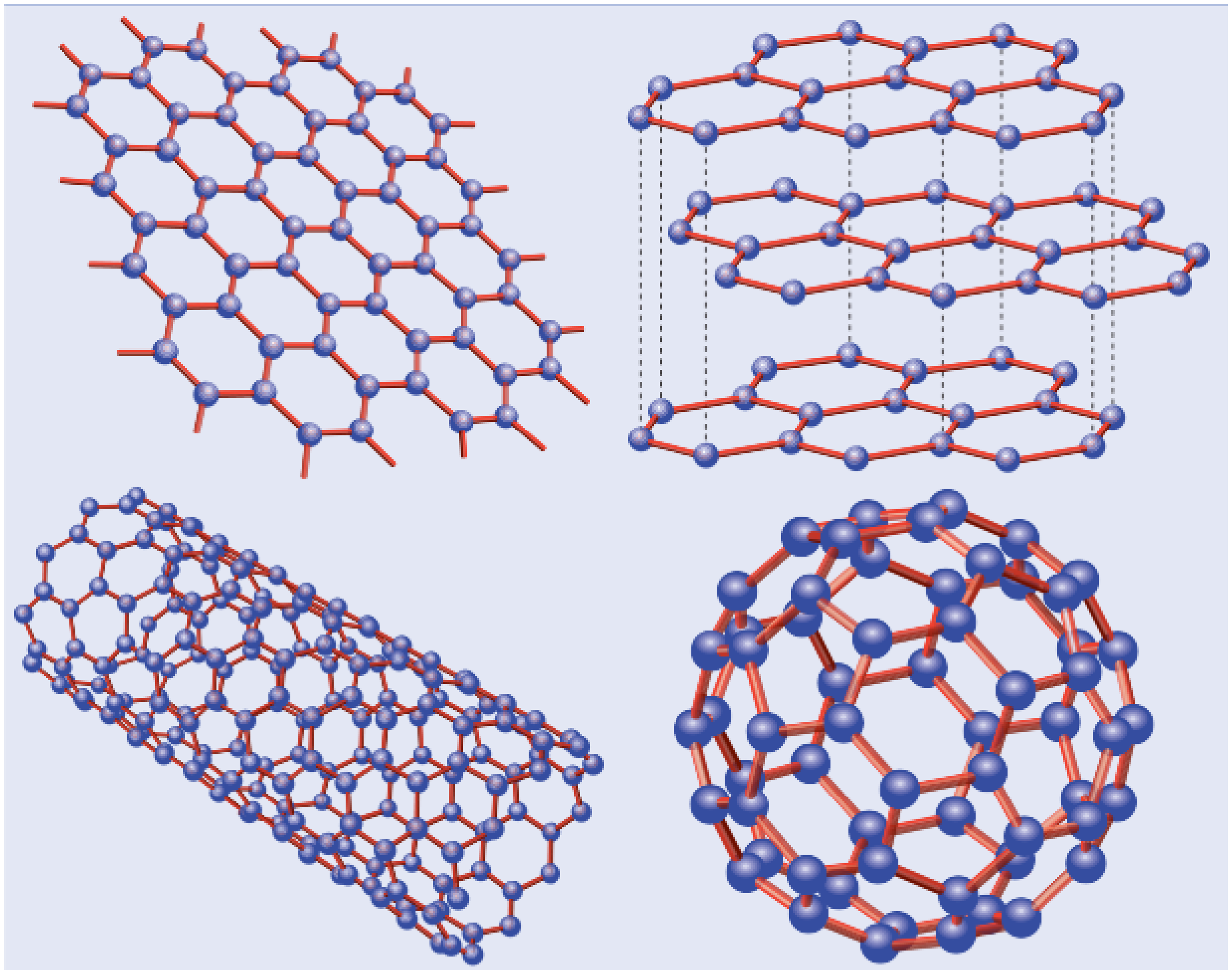
University of British Columbia


Strong and Electroweak Matter
Lausanne, Switzerland
July 18, 2014

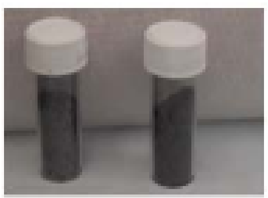
## Graphene is a 2-dimensional array of carbon atoms with a hexagonal lattice structure



## Graphene was produced and identified in the laboratory in 2004

- Micromechanical cleavage of bulk graphite up to 100 micrometer in size via adhesive tapes !

Novoselov et al, Science **306**, 666 (2004)



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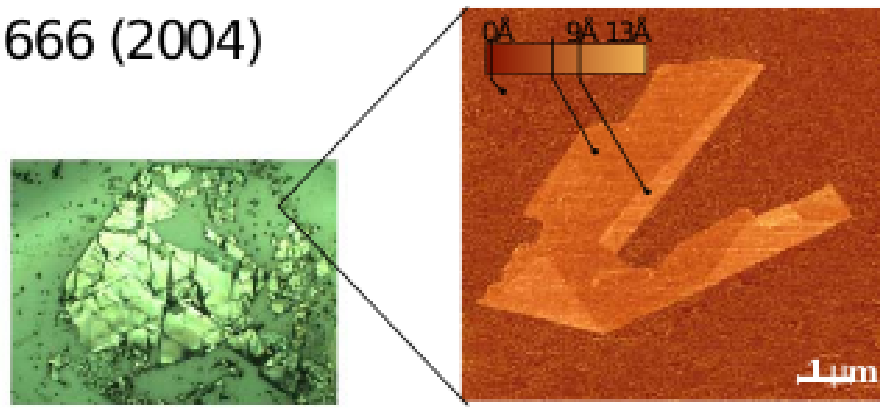
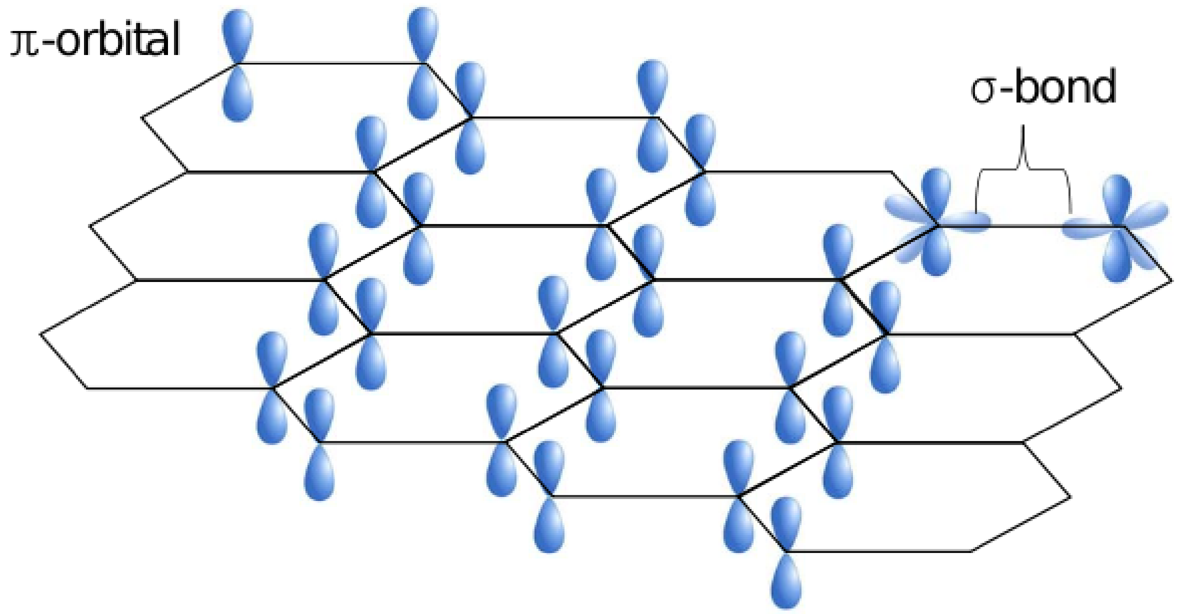


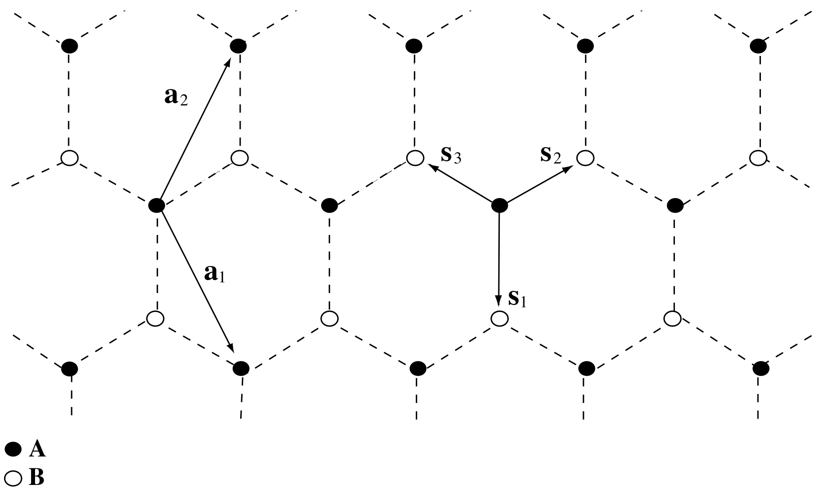

Kostya Novoselov

Andre Geim




A carbon atom has four valence electrons. Three of these electrons form strong covalent $\sigma$-bonds with neighboring atoms. The fourth, $\pi$-orbital is un-paired.

## Tight-binding model




hexagonal lattice = two triangular sub-lattices $\vec{A}$ and $\vec{B}$ connected by vectors $\vec{s}_1, \vec{s}_2, \vec{s}_3$.

$$H = \sum_{\vec{A},i} \left( t\ b^{\dagger}_{\vec{A}+\vec{s}_i} a_{\vec{A}} + t^*\ a^{\dagger}_{\vec{A}} b_{\vec{A}+\vec{s}_i} \right) \quad , \quad t \sim 2.7ev \quad |\vec{s}_i| \sim 1.4\mathring{A}$$

**P. R. Wallace,** *Phys. Rev. 71, 622 (1947)*

**J. C. Slonczewsi and P. R. Weiss,** *Phys. Rev. 109, 272 (1958).*

**G. W. S.,** *Phys. Rev. Lett. 53, 2449 (1984)*

## Tight-binding model

$$H=\sum_{\vec{A},i}\left(t\; b^{\dagger}_{\vec{A}+\vec{s}_i}a_{\vec{A}}+t^*\; a^{\dagger}_{\vec{A}}b_{\vec{A}+\vec{s}_i}\right)$$

$$i\hbar\frac{da_{\vec{A}}}{dt}=t\sum_i b_{\vec{A}+\vec{s}_i}\quad,\quad i\hbar\frac{db_{\vec{B}}}{dt}=t^*\sum_i a_{\vec{B}-\vec{s}_i}$$

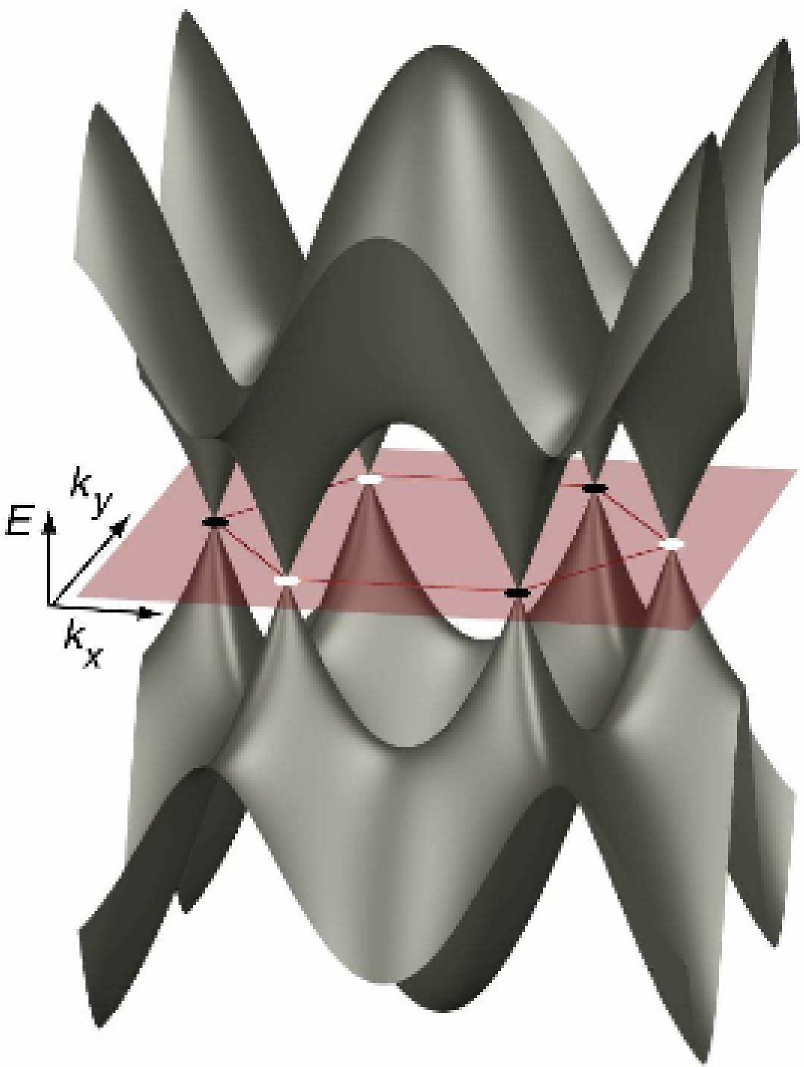
$$a_{\vec{A}}=e^{i\frac{E}{\hbar}t+i\vec{k}\cdot\vec{A}}a_0\quad,\quad b_{\vec{B}}=e^{-i\frac{E}{\hbar}t+i\vec{k}\cdot\vec{B}}b_0$$

$$E\begin{bmatrix}a_0\\ b_0\end{bmatrix}=\begin{bmatrix}0 & t\sum_i e^{i\vec{k}\cdot\vec{s}_i}\\ t^*\sum_i e^{-i\vec{k}\cdot\vec{s}_i} & 0\end{bmatrix}\begin{bmatrix}a_0\\ b_0\end{bmatrix}$$

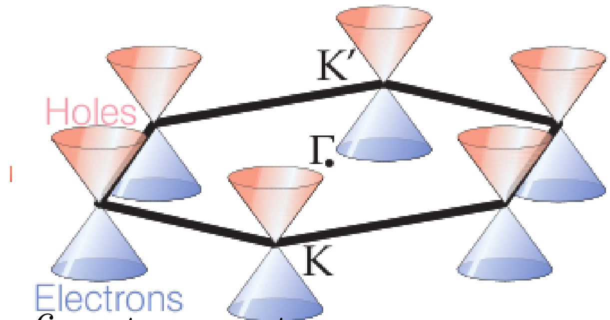
**Two energy bands:**

$$E(k)=\pm|t|\sqrt{(1+2\cos(\frac{3k_y}{2})\cos(\frac{\sqrt{3}k_x}{2}))^2+\sin^2(\frac{3k_y}{2})}$$

# Band structure of graphene



## Linearize spectrum near degeneracy points




$$E(k) = \hbar v_F |\vec{k}|$$

$v_F \sim 10^6 m/s \sim c/300$, good up to $\sim 1ev$

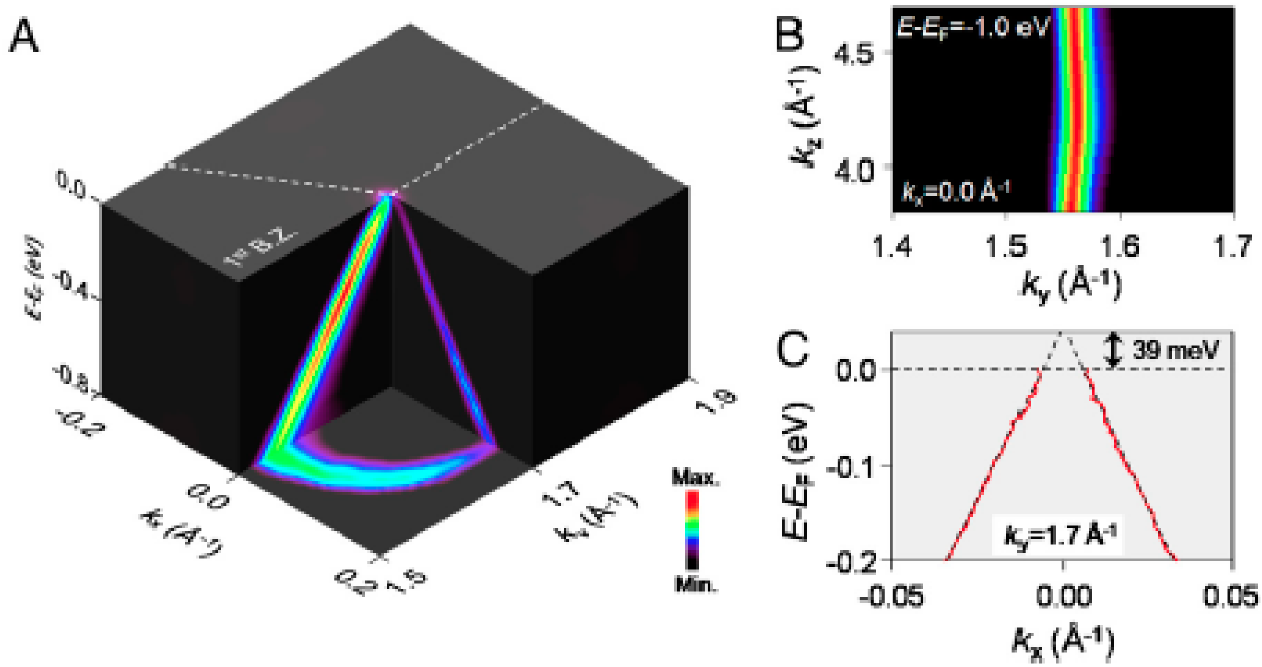
2 valleys $\times$ 2 spin states = 4 2-component spinors $\psi$

$$H\psi = \hbar v_F \begin{bmatrix} 0 & k_x - ik_y & 0 & 0 \\ k_x + ik_y & 0 & 0 & 0 \\ 0 & 0 & 0 & k_x - ik_y \\ 0 & 0 & k_x + ik_y & 0 \end{bmatrix} \begin{bmatrix} \psi_A(k-K) \\ \psi_B(k-K) \\ \psi_B(k-K') \\ \psi_A(k-K') \end{bmatrix}$$

$$S = \int d^3x \sum_{\sigma=1}^{4} \bar{\psi}^\sigma \; i\gamma^\mu \partial_\mu \; \psi^\sigma + \text{interactions}$$

# Electron dispersion relation with ARPES

## D.A. Siegel et. al. PNAS,1100242108



## The Dirac equation in condensed matter

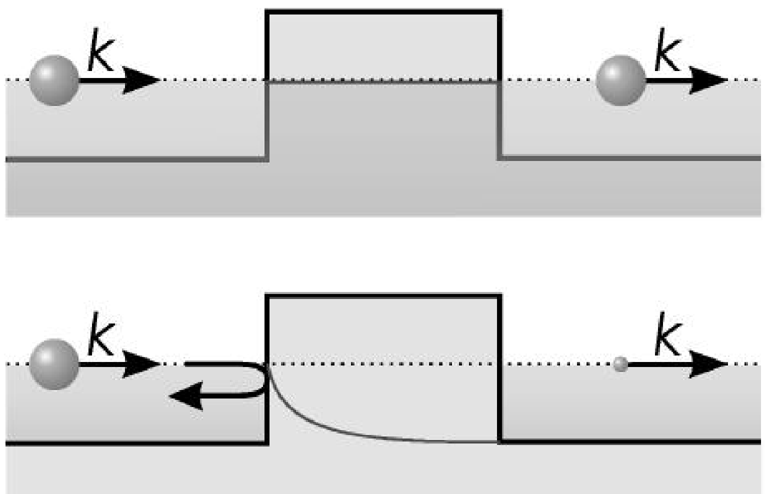
- unusual electronic properties:
  redo semiconductor physics with
  Schrödinger $\rightarrow$ Dirac
- electronics using graphene
- nanotechnology using graphene
- explore issues in relativistic quantum mechanics which are otherwise inaccessible to experiment
  Zitterbewegung
  Klein effect
  supercritical atoms
- explore dynamical issues in graphene as an analog of those in quantum field theory, e.g. symmetry breaking, phase transitions, quantum critical behavior

## Klein Effect

**O. Klein,** *Z. Phys. 33, 157 (1929)*

**M. Katsnelson, K. S. Novoselov and A. Geim,** *Nature Physics 2, 620 (2006)*

Unsuppressed tunneling through a potential barrier



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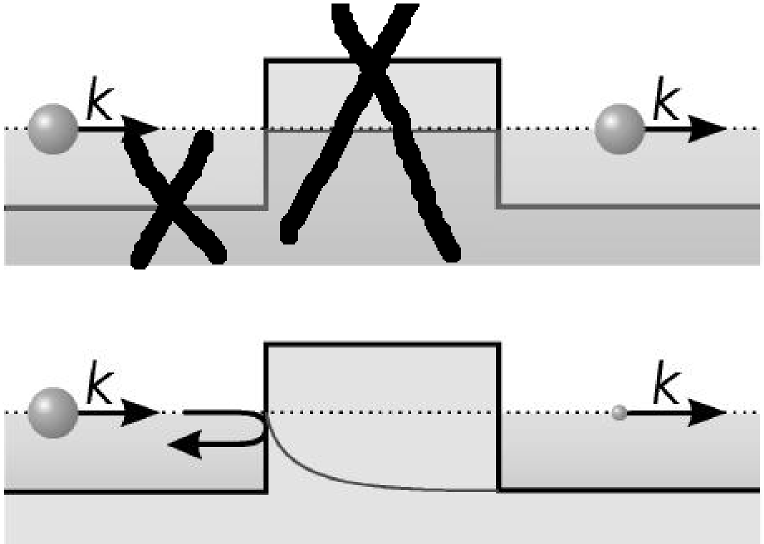


(attempts to observe in QED in collisions of large $Z$ nuclei)

## Graphene with Coulomb interaction $V(r) = \frac{e^2}{4\pi r}$

$$S = \int dtdxdy \sum_{k=1}^{4} \bar{\psi}_k \left[ \gamma^t (i\partial_t - A_t) + v_F \vec{\gamma} \cdot (i\vec{\nabla} - \vec{A}) \right] \psi_k$$

$$+ \frac{1}{4e^2} \int dtdxdydz \left[ \frac{1}{c} F_{0i} F_{0i} - c F_{ij} F_{ij} \right]$$

- Scale invariant but the Kinetic terms have different speeds of light. ( $v_F \sim c/300$).
- The graphene fine structure constant is larger than one,

$$\alpha_{\text{graphene}} = \frac{\frac{e^2}{4\pi\lambda}}{\hbar v_F/\lambda} = \frac{e^2}{4\pi\hbar v_F} = \frac{e^2}{4\pi\hbar c}\frac{c}{v_F} \approx \frac{300}{137}$$

- radiative corrections $v_F(\omega) = v_F \left( 1 + \pi \frac{e^2}{4\pi\hbar v_F} \ln \frac{\Lambda}{\omega} + \ldots \right)$

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$$+ \frac{1}{4e^2} \int dtd^2x \left[ F_{0i} \frac{1}{2\sqrt{\partial_t^2 - c^2\nabla^2}} F_{0i} - F_{ij} \frac{c^2}{2\sqrt{\partial_t^2 - c^2\nabla^2}} F_{ij} \right]$$

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## AC Conductivity of Neutral Graphene $\omega >> k_B T$

Two-loop correction

$$\sigma(\omega) = \frac{e^2}{4\hbar}\left[1 + \frac{11-3\pi}{6}\cdot 4\pi \cdot \frac{e^2}{4\pi\hbar v_F} + \ldots\right]$$

**V. Juricic et.al. Phys. Rev. B 82, 235402 (2010)**

Experiments $\sigma(\omega) \simeq \frac{e^2}{4\hbar}$, $\omega$-independent

**R. Nair et.al., Science 320, 1308 2008.**

## Large N approximation

$$S = \int dtd^2x \sum_{k=1}^{N} \bar{\psi}_k \left[\gamma^t(i\partial_t - A_t) + v_F\vec{\gamma}\cdot(i\vec{\nabla} - \vec{A})\right]\psi_k$$

$$+ \frac{1}{4e^2}\int dtd^2x \left[F_{0i}\frac{1}{2\sqrt{\partial_t^2 - c^2\nabla^2}}F_{0i} - F_{ij}\frac{c^2}{2\sqrt{\partial_t^2 - c^2\nabla^2}}F_{ij}\right]$$

In this large N limit, we integrate out fermions to get effective action

$$S = \frac{N}{32}\int dtd^2x \left[\ F_{0i}\frac{1}{2\sqrt{\partial_t^2 - v_F^2\nabla^2}}F_{0i} - v_F^2\ F_{ij}\frac{1}{2\sqrt{\partial_t^2 - v_F^2\nabla^2}}F_{ij}\right] + \ldots$$

$$+ \frac{1}{4e^2}\int dtd^2x \left[F_{0i}\frac{1}{2\sqrt{\partial_t^2 - c^2\nabla^2}}F_{0i} - F_{ij}\frac{c^2}{2\sqrt{\partial_t^2 - c^2\nabla^2}}F_{ij}\right]$$

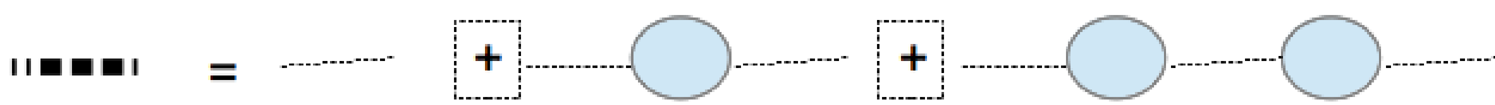
## AC Conductivity of Neutral Graphene

Perturbation theory in coupling constant:

$$\sigma(\omega) = \frac{e^2}{4\hbar}\left[1 + \frac{11-3\pi}{6}\cdot 4\pi \cdot \frac{e^2}{4\pi\hbar v_F} + \ldots\right]$$

Large N approximation:

$$\sigma(\omega) = \frac{e^2}{4\hbar}\frac{N}{4}\left[1 + \frac{4}{N}\left(\frac{92}{9\pi^2} - 1\right) + \ldots\right]$$

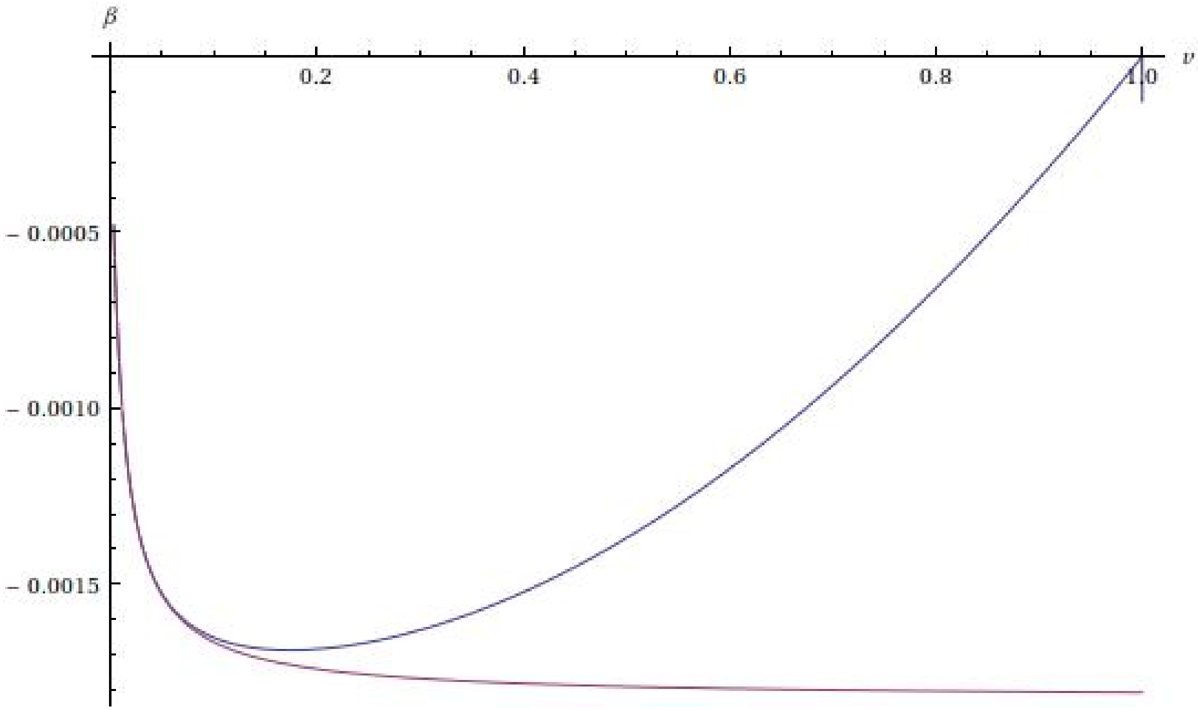


$$\frac{92}{9\pi^2} - 1 = 0.045$$

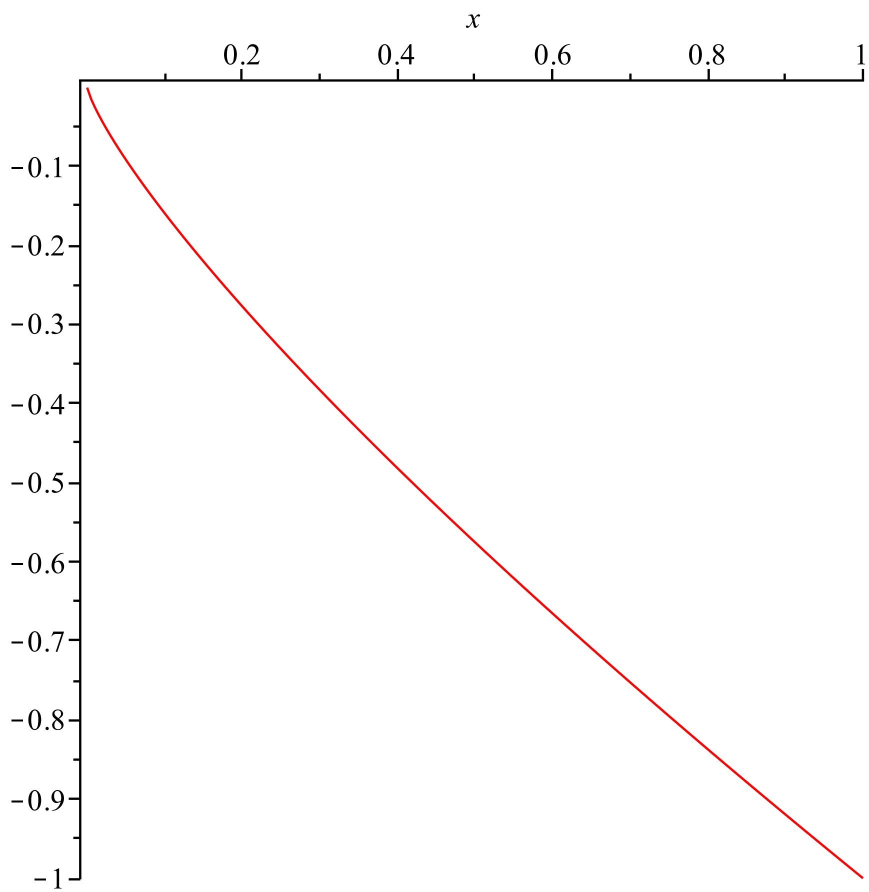
## Beta function at Large N

## D.T.Son cond-mat/0701501

$$\beta = \Lambda \frac{d}{d\Lambda} v_F$$

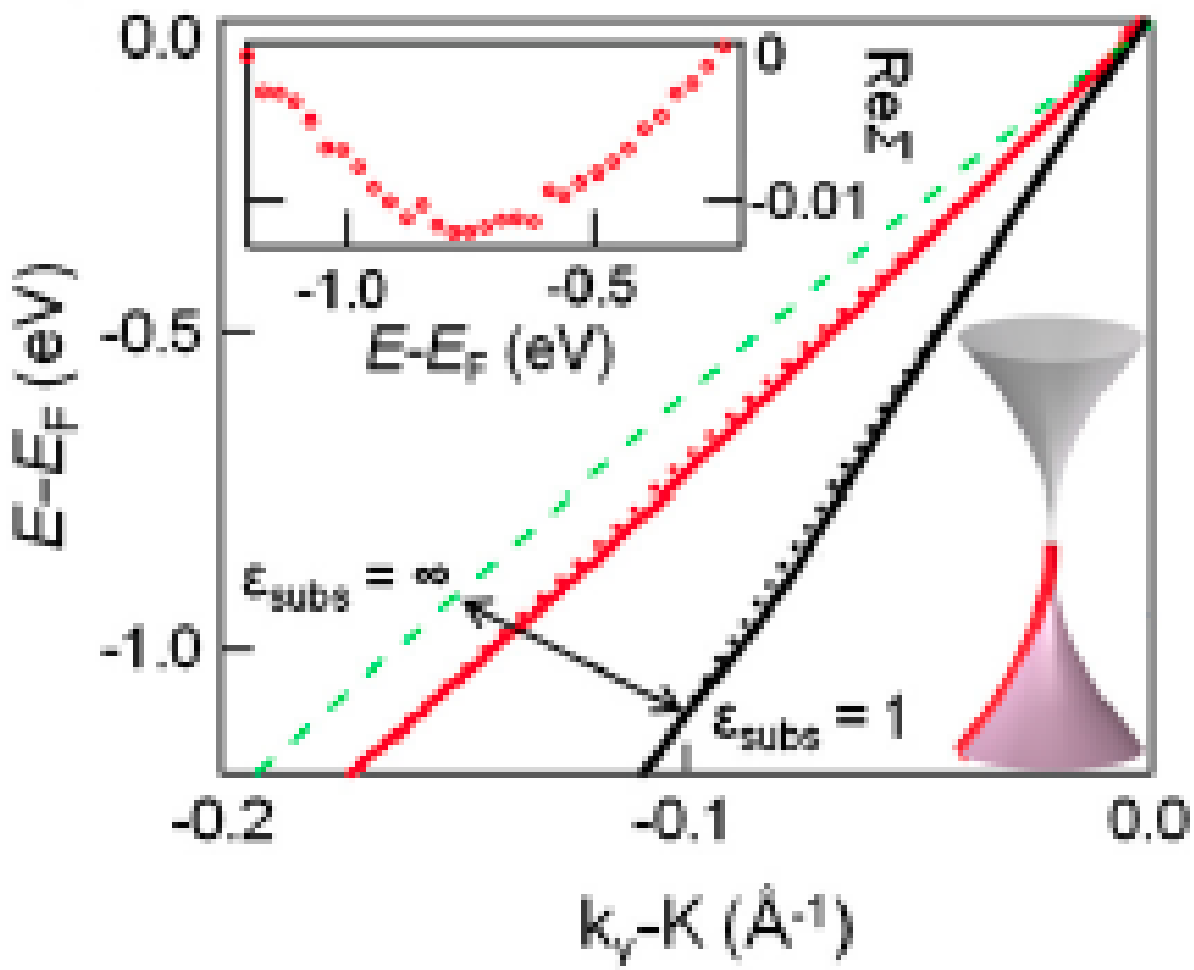



$$v_F \approx \left(\frac{p}{\Lambda}\right)^{-\frac{8}{\pi^2 N}}$$


x
0.2
0.4
0.6
0.8
1
−0.1
−0.2
−0.3
−0.4
−0.5
−0.6
−0.7
−0.8
−0.9
−1

## Does $v_F$ really run?

D.A. Siegel et. al. PNAS,1100242108

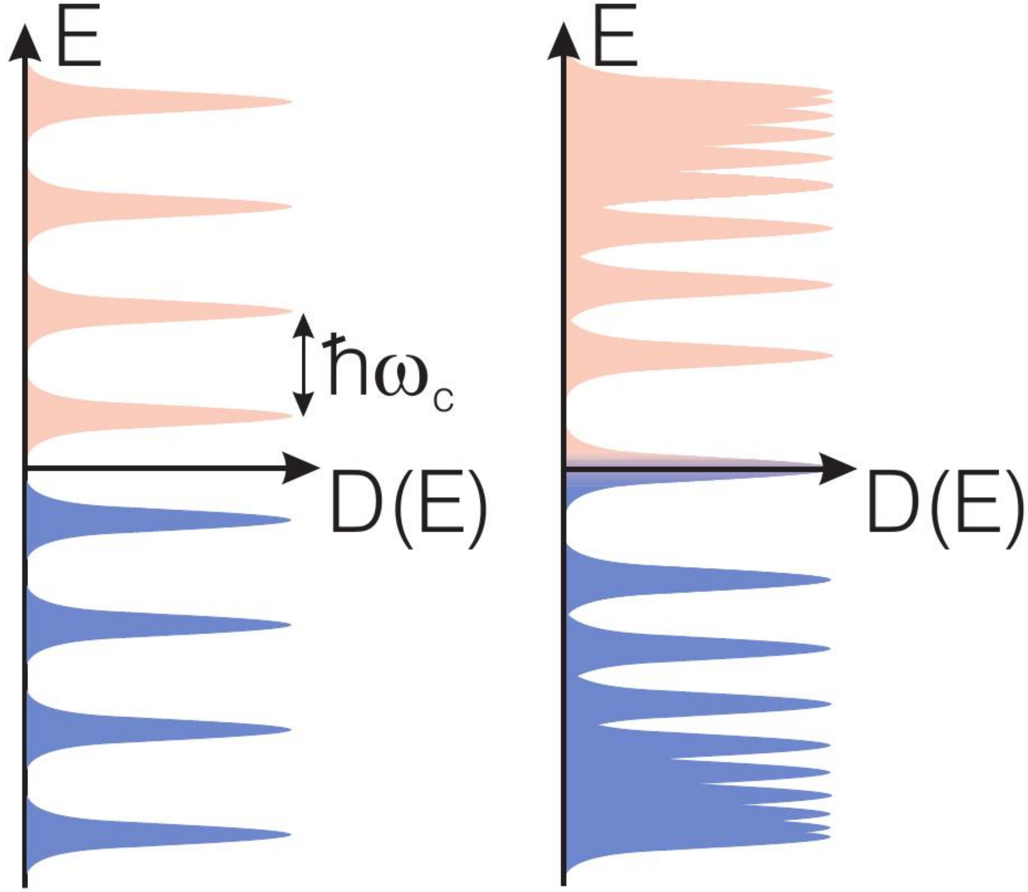


## Does $v_F$ really run?

**"Dirac cones reshaped by interaction effects in suspended graphene"**
D.C.Elias, R.V.Gorbachev, A.S.Mayorov, S.V.Morozov, A.A.Zhukov, P.Blake, L.A.Ponomarenko, I.V.Grigorieva, K.S.Novoselov, F.Guinea, A.K.Geim
Nature Physics 7, 701704 (2011) doi:10.1038/nphys2049
Received 01 April 2011 Accepted 17 June 2011 Published online 24 July 2011 Corrected online 21 December 2011 Corrigendum (February, 2012)

**"Renormalization of the Graphene Dispersion Velocity Determined from Scanning Tunneling Spectroscopy"**,
J.Chae, S.Jung, A.F.Young, C.R.Dean, L.Wang, Y.Gao, K.Watanabe, T.Taniguchi, J.Hone, K.L.Shepard, P.Kim, N.B.Zhitenev, J.A.Stroscio Phys. Rev. Lett. 109, 116802 (2012).

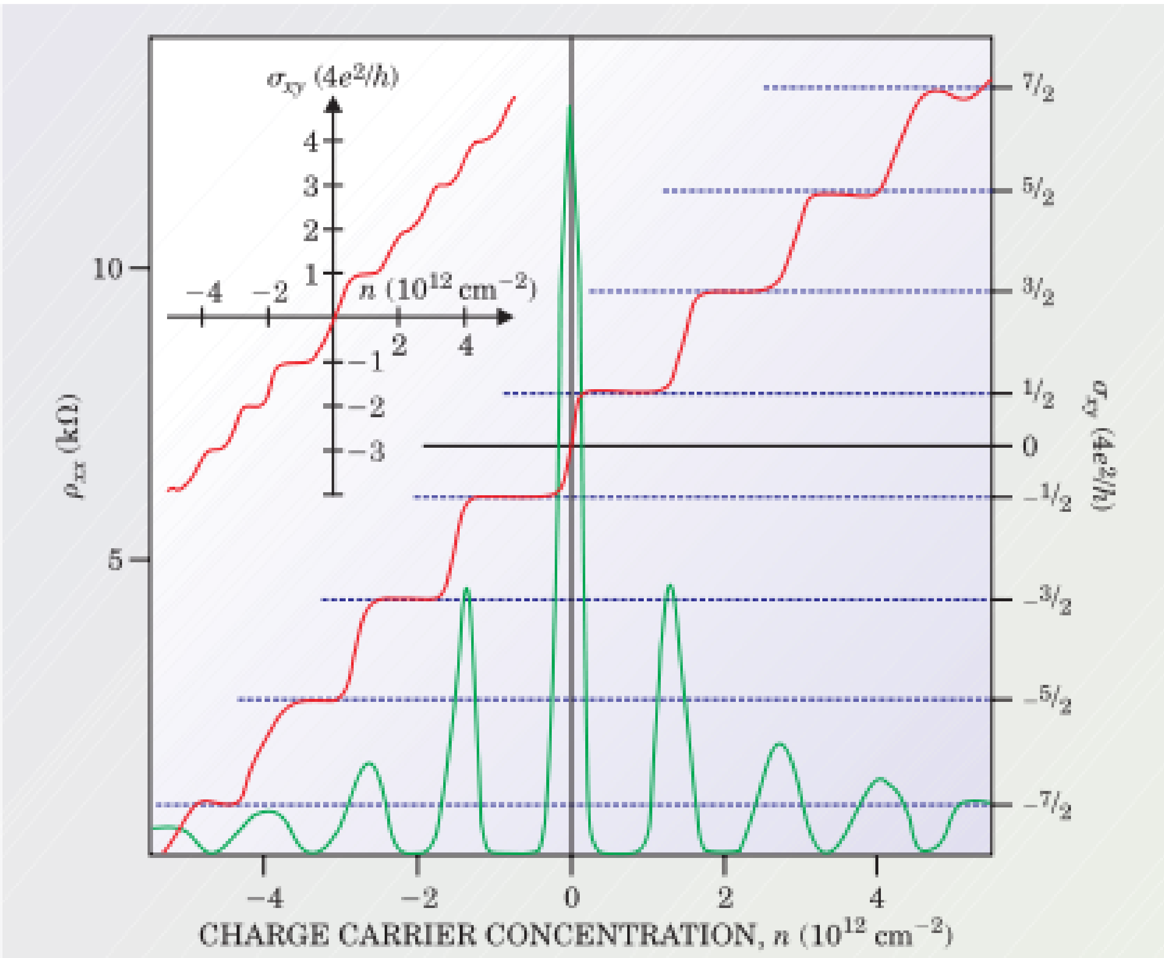
## Graphene Landau Levels




Relativistic $E = \pm\hbar v_F \sqrt{|B|n}$ , $n = 0, 1, 2, ...$ degeneracy $= 4 \cdot \frac{e|B|}{2\pi}$

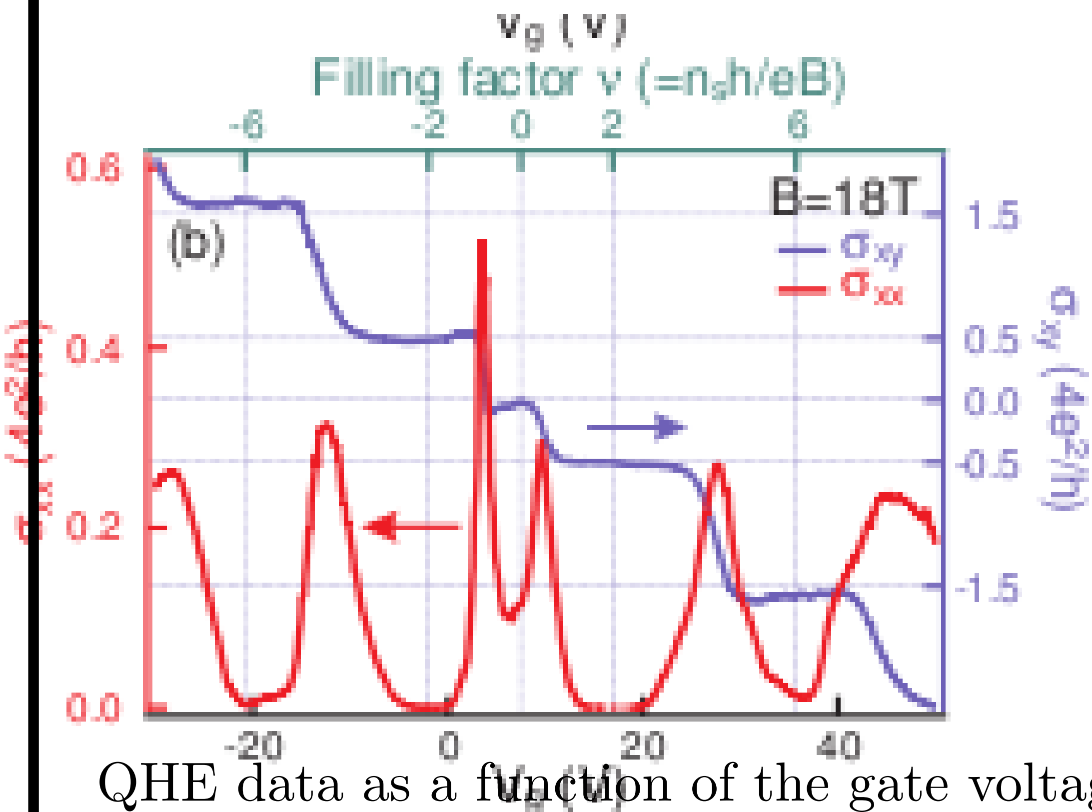
**K. Novoselov et. al.** *Nature 438, 197 (2005)*

**Y. Zhang et. al.** *Nature 438, 201 (2005)*



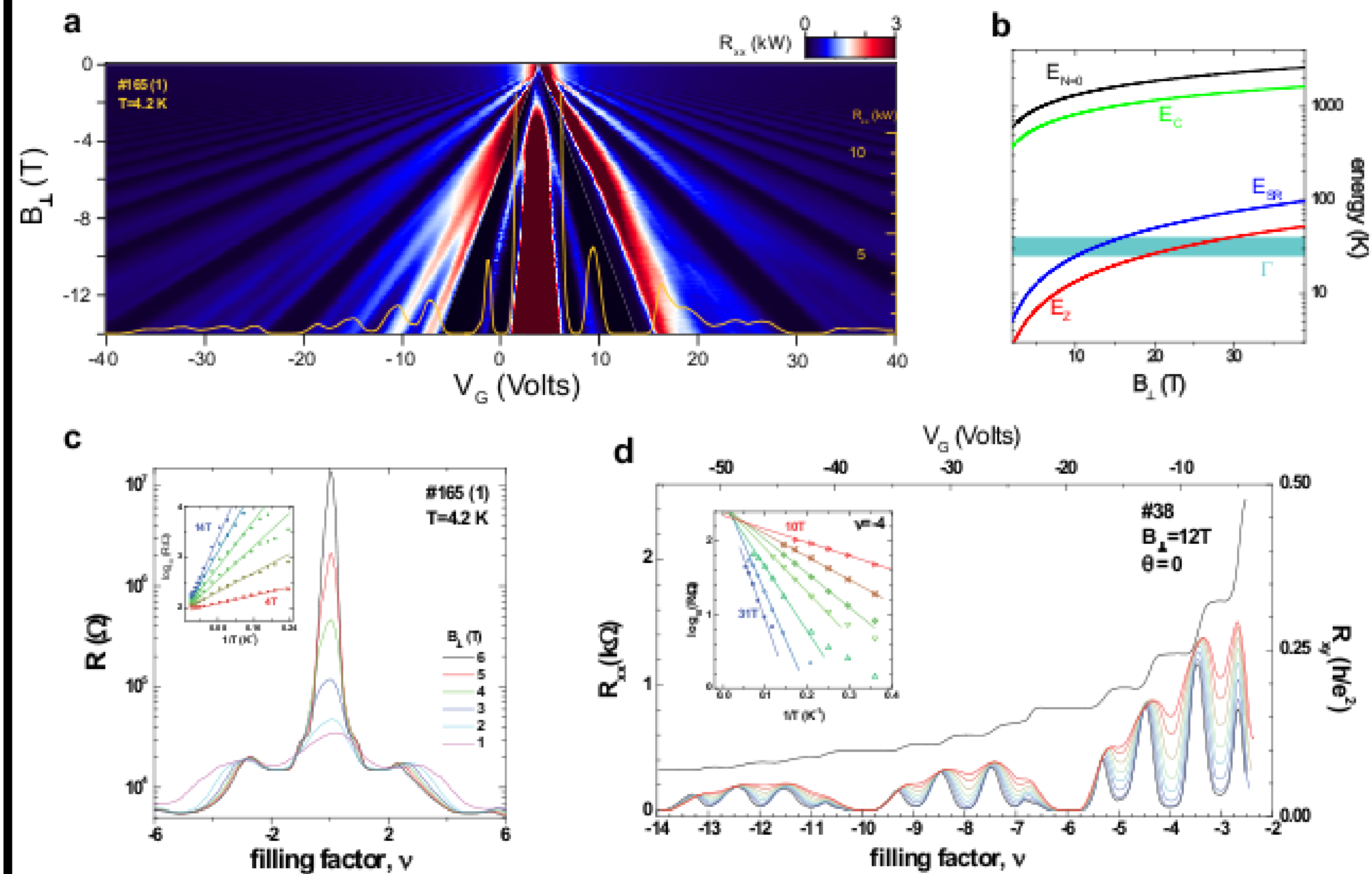

$$\sigma_{xy} = 4\frac{e^2}{h}\left(n + \frac{1}{2}\right)$$

**Splitting of $\nu = 0$ Landau level** **Zhang et.al. arXiv:1003.2738**

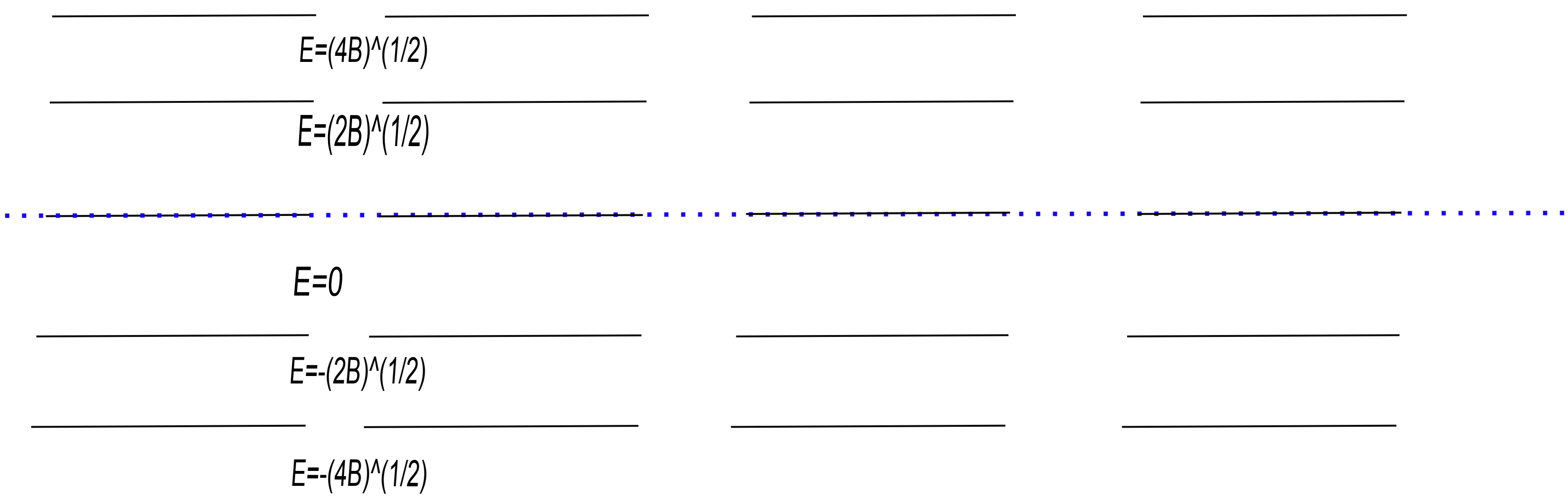



QHE data as a function of the gate voltage $V_g$, for B = 18 T at T = 0.25 K

## Splitting of $\nu = 0$ Landau level A.F.Young et.al., Nat. Phys. 2012



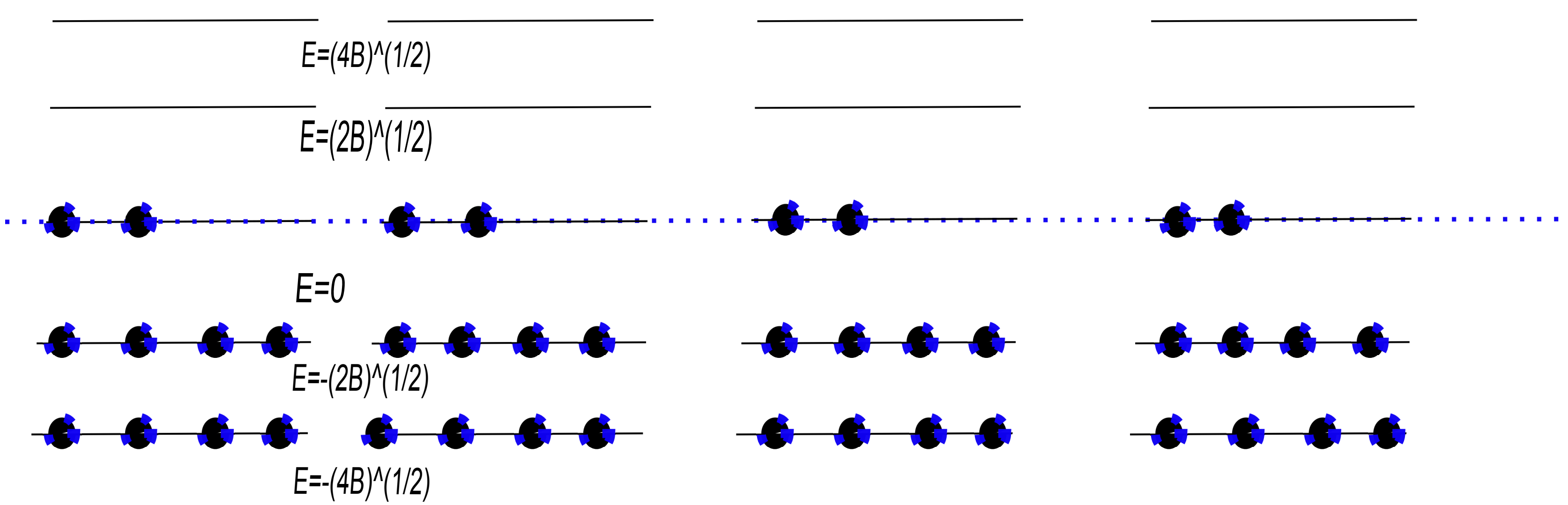
# Four flavors of massless fermions in a magnetic field: Landau levels



**Four flavors of massless fermions in a magnetic field: Landau levels**

Ground state has negative energy levels filled

**The zero energy states should be half-filled**



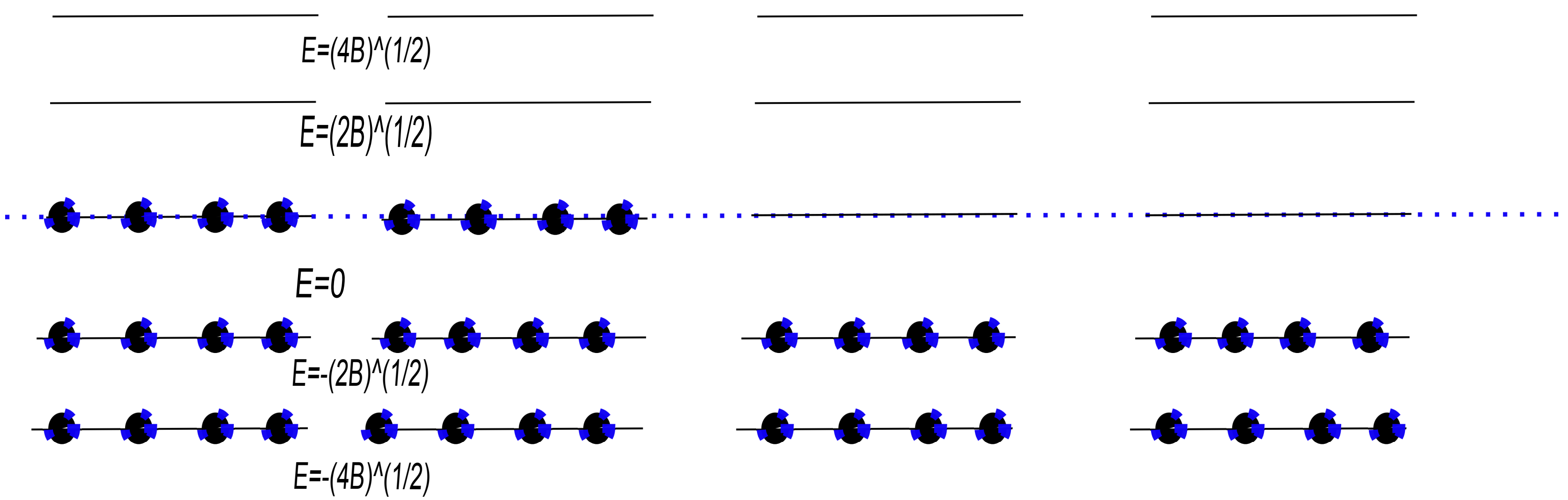
## Quantum Hall Ferromagnet/Magnetic Catalysis:

Spontaneous breaking $U(4) \to U(2) \times U(2)$

**Weak** Coulomb interaction

$$H_{\rm Coulomb} = \frac{e^2}{8\pi\epsilon}\int \psi^\dagger(r)\psi(r)\frac{1}{|\vec{r}-\vec{r'}|}\psi^\dagger(r')\psi(r')$$

$$\rho = \langle\psi^\dagger\psi\rangle = \frac{B}{4\pi}(1,1,-1,-1) \quad , \quad \langle\bar{\psi}\psi\rangle = \frac{B}{4\pi}(1,1,-1,-1)[1+\ldots]$$



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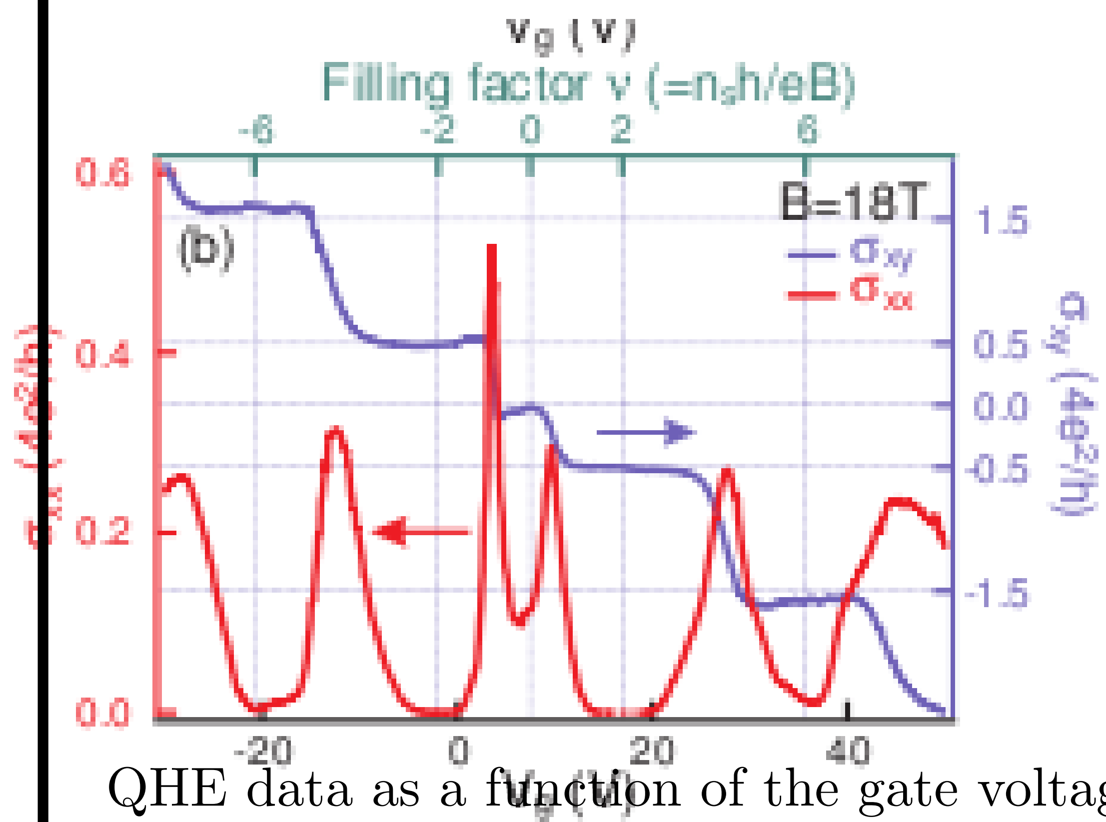



QHE data as a function of the gate voltage $V_g$, for B = 18 T at T = 0.25 K

## Holographic quantum Hall ferromagnet

Quantum Hall ferromagnetic states in strong coupling limit:

D3-probe-D5 branes:

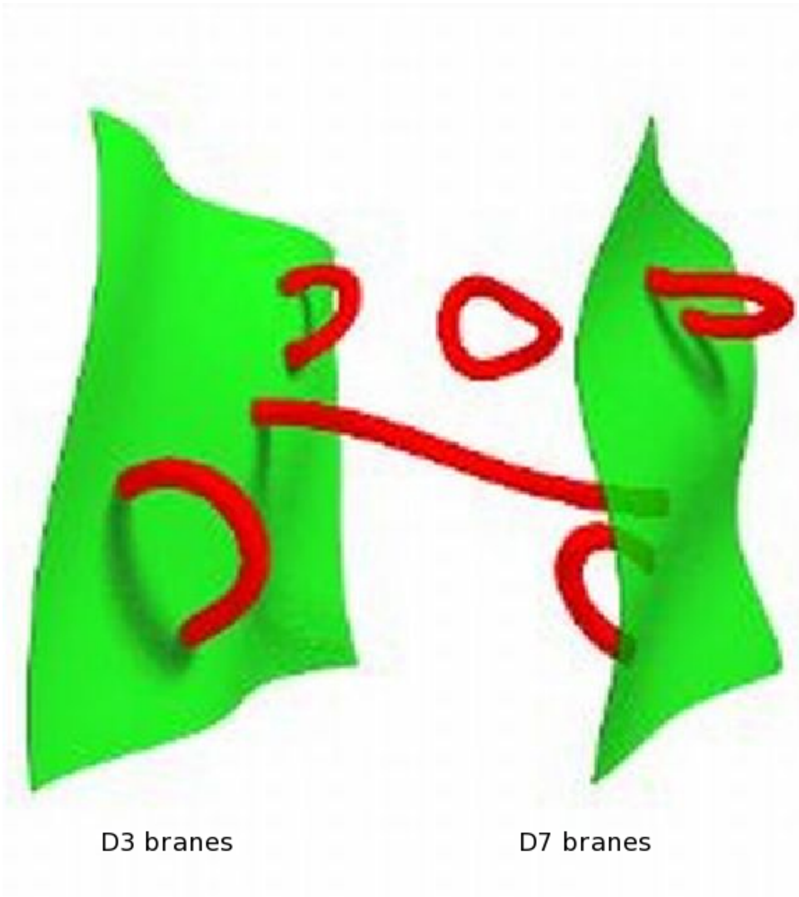
**C. Kristjansen and G. W. Semenoff, Giant D5 Brane Holographic Hall State, JHEP 1306, 048 (2013) [arXiv:1212.5609 [hep-th]].**

**C. Kristjansen, R. Pourhasan and G. W. Semenoff, A Holographic Quantum Hall Ferromagnet, arXiv:1311.6999 [hep-th].**

D3-probe-D7 branes:

**appearing soon**

## Holographic graphene



## D3-D7 system

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| $D3$ | $X$ | $X$ | $X$ | $X$ | $O$ | $O$ | $O$ | $O$ | $O$ | $O$ |
| $D7$ | $X$ | $X$ | $X$ | $O$ | $X$ | $X$ | $X$ | $X$ | $X$ | $O$ |

branes extend in directions $X$

$\#ND = 6$ system – no supersymmetry – no tachyon – only zero modes of 3-7 strings are in R-sector and are 2-component fermions ($N_7$ flavors and $N_3$ colors).

$$S = \frac{1}{g_{\rm YM}^2}\int d^4x {\rm Tr}\left[-\frac{1}{2}F_{\mu\nu}^2 + (D_\mu\Phi^I)^2 + \ldots\right]$$

$$+\int d^3x \sum_{\sigma=1}^{N_7}\sum_{\alpha=1}^{N_3}\bar\psi_\alpha^\sigma[i\gamma^\mu\partial_\mu + \gamma^\mu A_\mu - g\Phi^9]\psi_\alpha^\sigma$$

$N_3 \to \infty$, $\lambda = g_{\rm YM}^2 N_3$ fixed $\to$ replace D3's by $AdS_5 \times S^5$, large $\lambda$

## D3-D7 system

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| $D3$ | $X$ | $X$ | $X$ | $X$ | $O$ | $O$ | $O$ | $O$ | $O$ | $O$ |
| $D7$ | $X$ | $X$ | $X$ | $O$ | $X$ | $X$ | $X$ | $X$ | $X$ | $O$ |

D7 brane worldvolume $AdS_4 \subset AdS_5 \ \times \ S^4 \subset S^5$

**S. J. Rey, Talk at Strings 2007;**

**Prog. Theor. Phys. Suppl. 177, 128 (2009) arXiv:0911.5295**

**D. Kutasov, J. Lin, A.Parnachev, arXiv:1107.2324**

This embedding is unstable: Fluctuation of $x^9$ violate BF bound for $\text{AdS}_4$

CFT when $\lambda < \lambda^*$, Chiral symmetry broken when $\lambda > \lambda^*$ with $< \bar{\psi}\psi > \sim \Lambda e^{-1/\sqrt{\lambda - \lambda^*}}$

Stabilize with internal flux

Embed in black D3-brane background (resembles cutoff)

## Some Results:

### AC conductivity

$$\sigma(\omega) \simeq \frac{2e^2}{\pi^2 \hbar} \qquad \left( \sigma(\omega) = \frac{e^2}{4\hbar} \right)$$

### Debye screening length

$$L_D \simeq \frac{e}{\mu} \simeq \frac{5}{\mu} \qquad \left( L_D \simeq \frac{1.6}{\mu} \right)$$

### Diamagnetism

$$M \simeq -(0.24) e \sqrt{B} \simeq -1.25 \sqrt{B} \qquad \left( M \simeq -0.28 \sqrt{B} \right)$$

## Conclusions

- Graphene contains emergent massless relativistic electrons
- Graphene is a promising material for electonic technology.
- Coulomb interaction is strong.
- Is graphene in a nontrivial 3-dimensional conformal field theory?
- D3-D7 brane model
- three computations of the AC conductivity
- Magnetic catalysis of chiral symmetry breaking with D7 branes.