Infrared dynamics of quantum scalar fields in de Sitter space

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MOTIVATIONS

Early Universe cosmology

Success of inflationary paradigm

- radiative corrections to inflationary observables
- coherent (quantum) theory of inflation

QFT in curved spaces

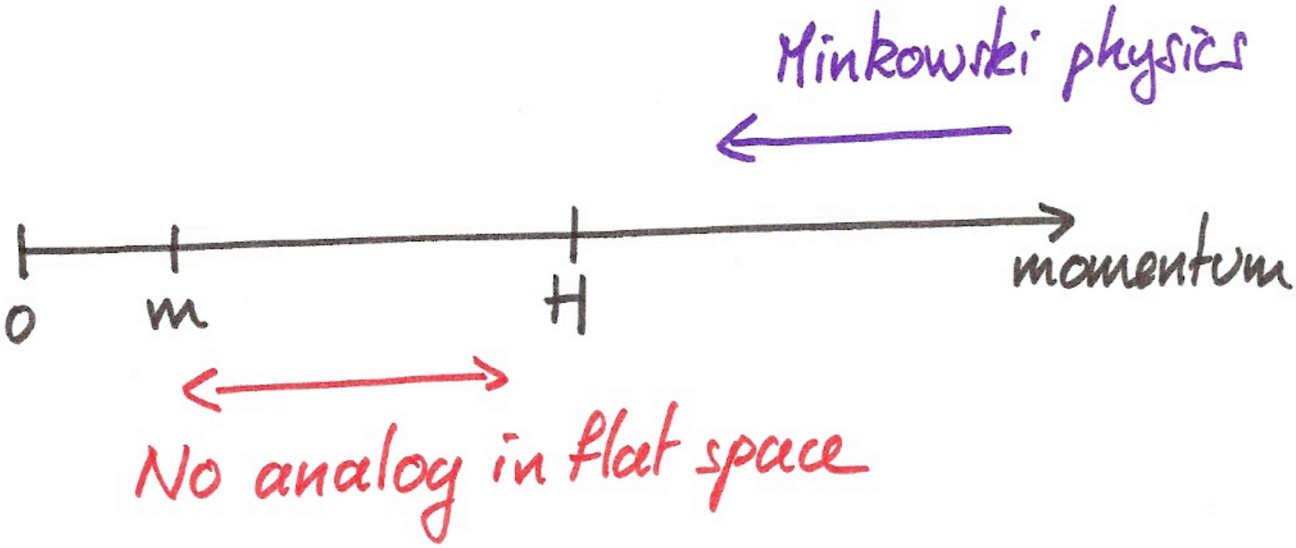
Fundamental issues due, e.g., to gravitational redshift

e.g. trans-Planckian / decoupling problem

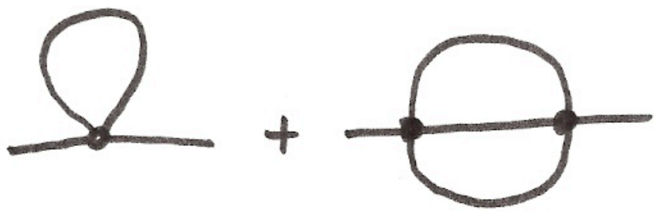
steps towards a deeper understanding of fundamental laws of Nature and, maybe, towards the discovery of new laws

Quantum fields in de Sitter space

The case of light fields : $m \ll H$



Loop corrections in perturbation theory :



$+ \cdots$

- Infrared (IR) divergences $\sim \frac{H}{m}$
- Large (IR) logarithms $\sim \ln(P/H)$ (secular terms)

NEED FOR RESUMMATION

IR / secular issues in QFT

e.g.
- critical point
- bosonic fields at high T
- nonequilibrium systems

Resummation techniques / Nonperturbative approaches

- Large-N
- Renormalization group
- Schwinger-Dyson equations
- Two-particle-irreducible (2PI) techniques

⋮

Resummation techniques in dS

Exploit full dS invariance

$$G(x,x') = \langle T\,\varphi(x)\varphi(x')\rangle = G(z)$$

dS invariant

$$(\Box + m^2)\,G(x,x') = \delta^{(D)}(x,x') + \underbrace{\int d^D x''\, \Sigma(x,x'')\, G(x'',x')}$$

Difficult to formulate in terms of z only

[see, however, Youssef, Kreimer ('13)]

Exploit momentum space representation

$$ds^2 = a^2(\eta)\left[-d\eta^2 + d\vec{X}.d\vec{X}\right] \quad , \; D = d+1$$

$$\phi(x) = a(\eta)^{\frac{D-2}{2}}\varphi(x)$$

$$G_\phi(x,x') = G_\phi(\eta,\eta',|\vec{X}-\vec{X}'|) = \int \frac{d^d K}{(2\pi)^d}\, e^{i\vec{K}\cdot(\vec{X}-\vec{X}')}\, G_\phi(\eta,\eta',K)$$

$$\Rightarrow \left[\partial_\eta^2 + K^2 + m^2 a^2 - \frac{a''}{a}\right] G_\phi(\eta,\eta',K) = \delta(\eta-\eta') + \int d\eta''\, \Sigma(\eta,\eta'',K)\, G(\eta'',\eta',K)$$

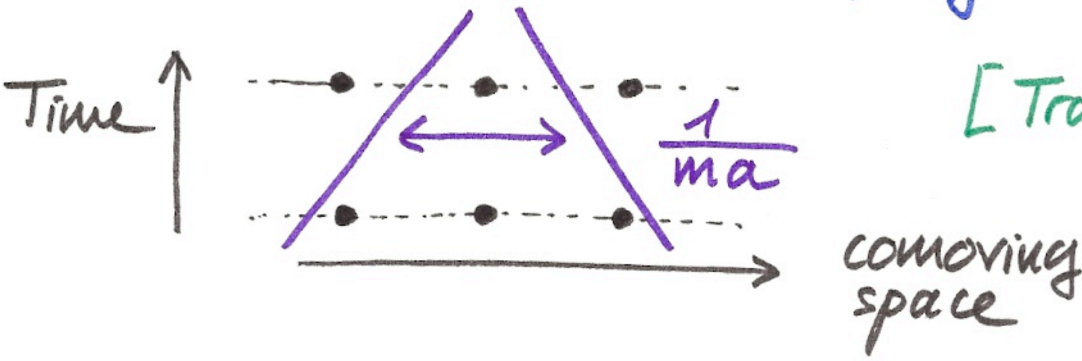
A typical nonequilibrium problem : Numerical solution ?

The trans-Planckian issue.

Numerical solution of Schwinger-Dyson eqs.?

Comoving grid

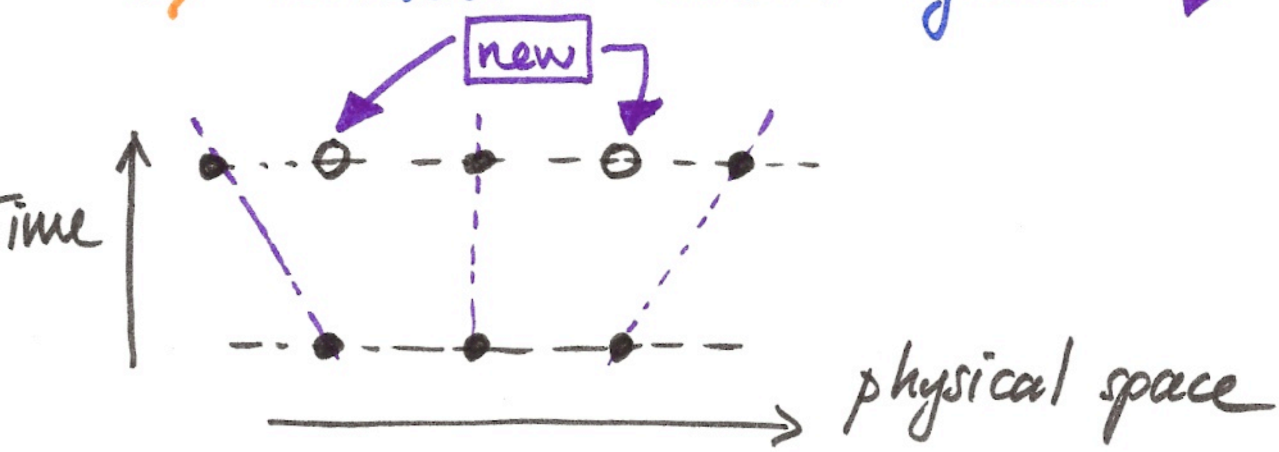
- fixed number of d.o.f ✓
- "mass" term $m a(\eta)$ grows ↯



[Tranberg ('08)]

Physical grid

- physical mass scale $m = \text{const.}$ ✓
- number of d.o.f grows ↯



[Weiss ('85); Jacobson ('99)]

The p-representation

[Parentani, Serreau ('13) ; Adamek, Busch, Parentani ('13)]

$$ds^2 = a^2(\eta)\left[-d\eta^2 + d\vec{x}.d\vec{x}\right]$$ ⇐ homogeneous non stationary

$$ds^2 = -(1-x^2)dt^2 - 2\vec{x}.d\vec{x}\,dt + d\vec{x}.d\vec{x}$$

⇐ inhomogeneous and stationary

$$G(\eta,\eta',K) = \frac{1}{K}\hat{G}(p,p')$$

$p = -K\eta$ and $p' = -K\eta'$: physical momenta

$$\Sigma(\eta,\eta',K) = K^3\hat{\Sigma}(p,p')$$

$$\left[\partial_p^2 + 1 - \frac{\nu^2 - 1/4}{p^2}\right]\hat{G}(p,p') = \delta(p-p') + \int dp''\,\hat{\Sigma}(p,p'')\,\hat{G}(p'',p')$$

$$\nu^2 = \frac{d^2}{4} - \frac{m^2}{H^2}$$

- Grav. redshift accounted for
- Reduces to a 0+1 dimensional problem

Schwinger-Dyson equations in the p-representation.

Standard closed-time-path (CTP; "in-in") formalism $\rightsquigarrow$ closed-momentum-path

$$\hat{G}(p,p') = \hat{F}(p,p') - \frac{i}{2}\,\mathrm{sign}_c(p-p')\,\hat{\rho}(p,p')$$

$$\hat{\Sigma}(p,p') = \hat{\Sigma}_F(p,p') - \frac{i}{2}\,\mathrm{sign}_c(p-p')\,\hat{\Sigma}_\rho(p,p')$$

$$\left[\partial_p^2 + 1 - \frac{\nu^2 - 1/4}{p^2}\right]\hat{F}(p,p') = -\int_p^\infty dp''\,\hat{\Sigma}_\rho(p,p'')\,\hat{F}(p'',p') + \int_{p'}^\infty dp''\,\hat{\Sigma}_F(p,p'')\,\hat{\rho}(p'',p')$$

$$\left[\partial_p^2 + 1 - \frac{\nu^2 - 1/4}{p^2}\right]\hat{\rho}(p,p') = \int_{p'}^{p} dp''\,\hat{\Sigma}_\rho(p,p'')\,\hat{\rho}(p'',p')$$

$\Rightarrow$ Momentum-flow equations

Approximation schemes in the p-rep.

- Standard perturbation theory (diagrammatic techniques) ✓

- Large-N techniques ✓

- Schwinger-Dyson equations ✓

- 2PI techniques (self-consistent approximations) ✓

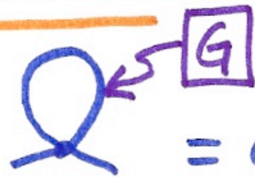
⋮

Application : $O(N)$ theory at large-N

$$S = \int d^Dx \sqrt{-g}\left(\frac{1}{2} g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{m^2}{2}\varphi^2 - \frac{\lambda}{4!N}(\varphi^2)^2\right)$$

Two-point function : mass resummation

$$\boxed{G^{-1} = G_0^{-1} - \Sigma \quad ; \quad \Sigma = \quad = \text{const}}$$



mass correction

$$\left(\partial_p^2 + 1 - \frac{\nu^2 - 1/4}{p^2}\right)\hat{F}(p,p') = 0$$

$$\hookrightarrow \hat{F}(p,p') \propto \mathrm{Re}\left(H_\nu(p) H_\nu^*(p')\right)$$

$$\boxed{\nu^2 = \frac{d^2}{4} - \frac{M^2}{H^2}}$$

$$\boxed{M^2(\phi^2) = m_{cl}^2(\phi^2) + \frac{\lambda}{6}\int \frac{d^dp}{(2\pi)^d}\frac{\hat{F}(p,p)}{p}}$$

$$m_{cl}^2(\phi^2) = m^2 + \frac{\lambda}{6}\phi^2$$

$$\boxed{|H_\nu(p)|^2 \underset{p\ll 1}{\sim} p^{-2\nu}}$$

$$\Rightarrow \boxed{M^2(\phi^2) \simeq m_{cl}^2(\phi^2) + \frac{c\lambda H^4}{M^2(\phi^2)}}$$

Effective potential : symmetry restoration

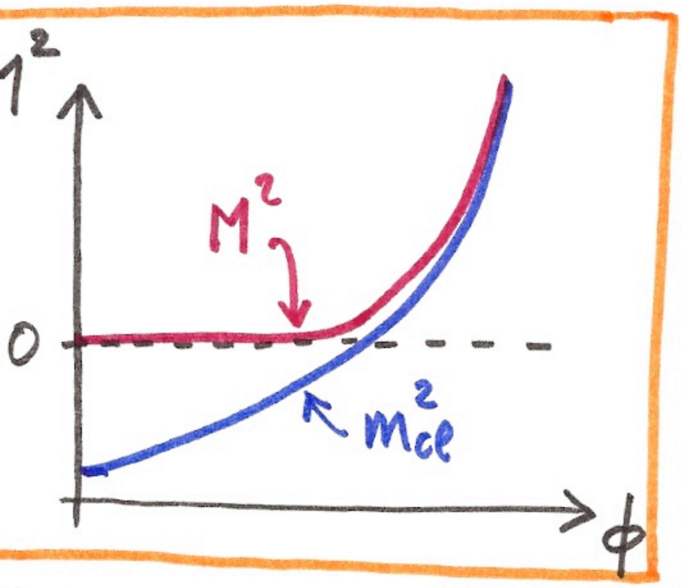
[Serreau ('11)]

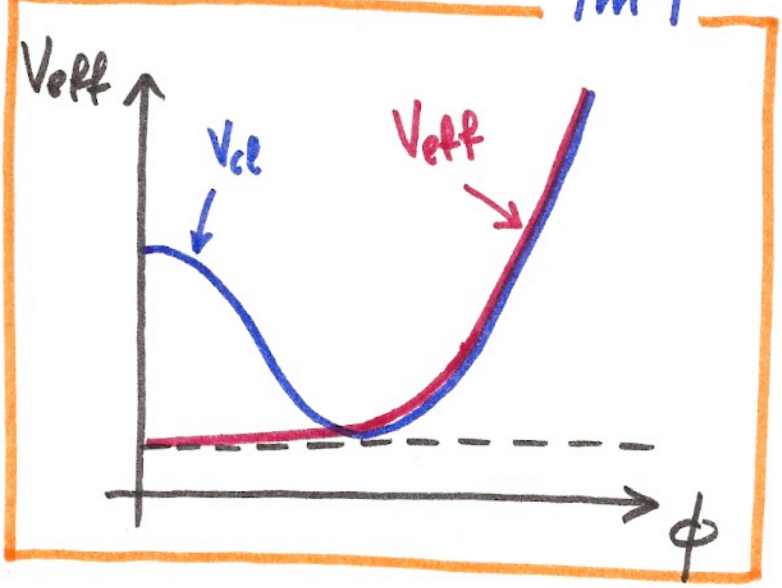
$$M^2(\phi^2) = \frac{m_{cl}^2(\phi^2)}{2} + \sqrt{\left(\frac{m_{cl}^2(\phi^2)}{2}\right)^2 + c\lambda H^4}$$

$$\Rightarrow \boxed{V_{eff}(\phi) = \int_0^{\phi^2} du\, M^2(u)}$$

1. $m_{cl}^2(\phi^2) > 0 \rightsquigarrow M^2(\phi^2) \approx m_{cl}^2(\phi^2)$
2. $m_{cl}^2(\phi^2) = 0 \rightsquigarrow M^2(\phi^2) \approx \sqrt{c\lambda}\, H^2$

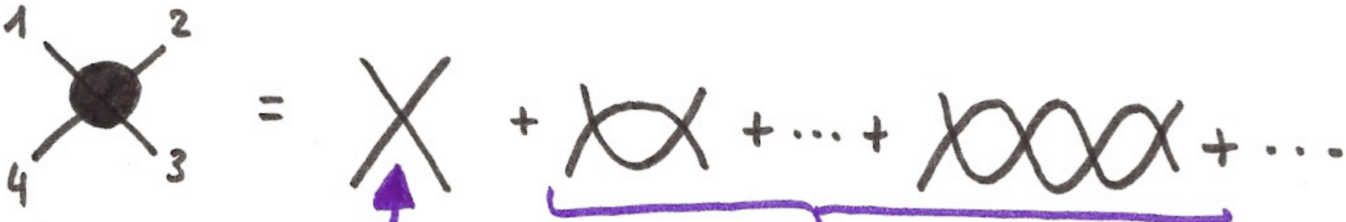
   [Starobinsky, Yokoyama ('94)]
3. $m_{cl}^2(\phi^2) < 0 \rightsquigarrow M^2(\phi^2) \approx \frac{c\lambda H^4}{|m^2|}$





Four-point vertex : infrared logs.

[Serreau, Parentani ('13)]



$$\Gamma^{(4)}_{1234} = -\lambda\, \delta_{12}\delta_{13}\delta_{14} + i\, \delta_{12} \boxed{\mathbb{I}_{13}} \delta_{34} + \text{perm.}$$

$$\mathbb{I}(x,x') = \Pi(x,x') + i \int d^D z\, \Pi(x,z)\, \mathbb{I}(z,x')$$

with $\Pi(x,x') = -\frac{\lambda}{6} G^2(x,x') =$ 

p-representation :

$$\hat{\Pi}(p,p') = -\frac{\lambda}{6} \int \frac{d^d q}{(2\pi)^d} \frac{\hat{G}(qp,qp')}{q} \frac{\hat{G}(rp,rp')}{r}$$

$r = |\vec{q}+\vec{e}|$, $\vec{e}$ unit vector

$$\hat{\mathbb{I}}(p,p') = \hat{\Pi}(p,p') - i \int_C ds\, \hat{\Pi}(p,s)\, \hat{\mathbb{I}}(s,p')$$

- Each bubble brings IR logs $\sim \ln(P/H)$
- The integral (SD) equation resums the infinite series of bubble diagrams

The IR dynamics is one-dimensional

IR modes $q p \lesssim H$ dominate the loop integral for IR external momenta $p, p' \lesssim H$

$$\nu = \sqrt{\frac{d^2}{4} - \frac{M^2}{H^2}} \equiv \frac{d}{2} - \varepsilon \quad ; \quad \varepsilon \ll 1$$

$$\boxed{\hat{\Pi}_F(p,p') = \frac{\Pi_F}{(pp')^{\kappa+1/2}} \quad ; \quad \hat{\Pi}_\rho(p,p') = \frac{\Pi_\rho}{\sqrt{pp'}} \mathcal{P}_\nu^\varepsilon(\ln p/p')}$$

$$\kappa = \nu - \varepsilon \quad ; \quad \mathcal{P}_\nu^\varepsilon(x) = \frac{\operatorname{sh} \nu x}{\nu} e^{-\varepsilon|x|}$$

The bubble summation equation rewrites

$$\hat{I}_F(p,p') = \frac{\Pi_F H^{2\kappa}}{\sqrt{pp'}} \bar{A}\left(\ln \frac{p}{H}\right) \bar{A}\left(\ln \frac{p'}{H}\right)$$

$$\hat{I}_\rho(p,p') = \frac{\Pi_\rho}{\sqrt{pp'}} \bar{I}(\ln p/p')$$

$$\bar{I}(x) = \mathcal{P}_\nu^\varepsilon(x) + \Pi_\rho \int_x^0 dy\, \mathcal{P}_\nu^\varepsilon(x-y) \bar{I}(y)$$

$$\bar{A}(x) = e^{-\kappa x} + \Pi_\rho \int_x^0 dy\, \bar{I}(x-y) e^{-\kappa y}$$

One-dimensional integral equation

Exact Solution

$$P_\nu^\varepsilon(x) = \frac{\mathrm{sh}(\nu x)}{\nu} e^{-\varepsilon|x|}$$

$$\bar{I}(x) = P_\nu^\varepsilon(x) + \pi_\rho \int_x^0 dy\, P_\nu^\varepsilon(x-y)\, \bar{I}(y)$$

$$\boxed{\bar{I}(x) = P_{\bar{\nu}}^\varepsilon(x) \quad \text{with} \quad \bar{\nu} = \sqrt{\nu^2 + \pi_\rho}}$$

$$\hat{\mathbb{I}}_F(p,p') = \frac{\pi_F}{(pp')^{\bar{\kappa}+1/2}} \; ; \; \bar{\kappa} = \bar{\nu} - \varepsilon$$

$$\hat{\mathbb{I}}_\rho(p,p') = \frac{\pi_\rho}{\sqrt{pp'}} P_{\bar{\nu}}^\varepsilon(\ln p/p')$$

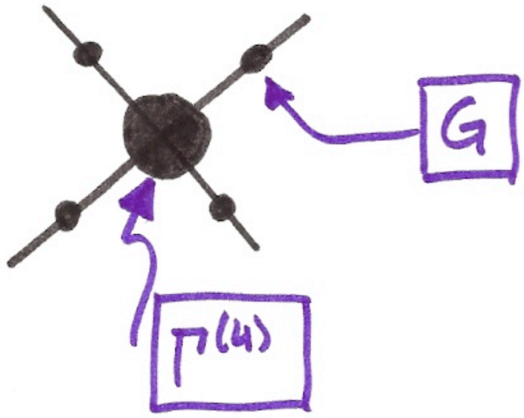
<u>The dangerous logs resum to a modified power law</u>

- Akin to anomalous dimension in critical phenomena
- Akin to mass correction in dS

Four-point correlator : non Gaussianities

[Serreau ('13)]

$$\langle \varphi_1 \varphi_2 \varphi_3 \varphi_4 \rangle =$$



$$= \frac{\lambda}{3N} \frac{F_\nu^3}{2\nu} \frac{(-\eta)^{2-4\nu}}{(K_1 \ldots K_4)^{2\nu}} \delta_{ab}\delta_{cd} \boxed{g\left(\ln\frac{\eta}{\eta_0}, \vec{K}_i\right)}$$

$$+ \text{perm.}$$

ex : massless case ($m_{dS}^2 = 0$) in the deep IR

$$\boxed{g(x, \vec{K}_i) = -\frac{1}{4\varepsilon}\left(K_1^{2\nu} + \ldots + K_4^{2\nu}\right) + \frac{1}{8\varepsilon} \frac{(K_1^{2\nu} + K_2^{2\nu})(K_3^{2\nu} + K_4^{2\nu})}{|\vec{K}_1 + \vec{K}_2|^{2\nu}}}$$

→ Non perturbative enhancement of loop contributions due to IR effects

→ Loop contrib's are of the same order as tree level ones.

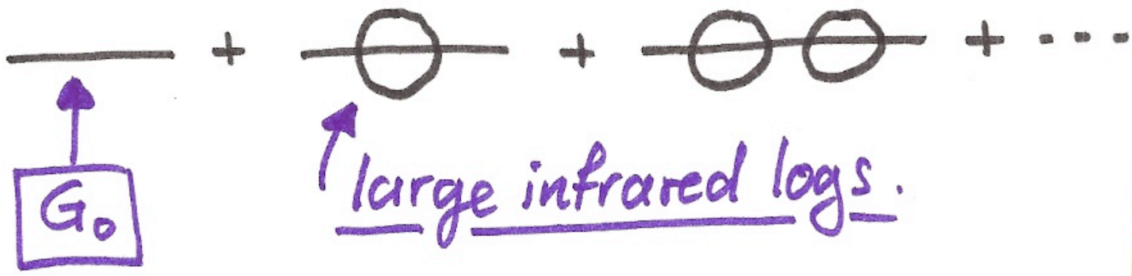
Application 2 : Solving the Schwinger-Dyson equation in de Sitter space

[Gautier, Serreau ('13)]

Beyond local mass resummation :

$$\Sigma^{2loop} =$$

$$= - \frac{\lambda^2 (N+2)}{18 N^2} G^3$$

$G =$


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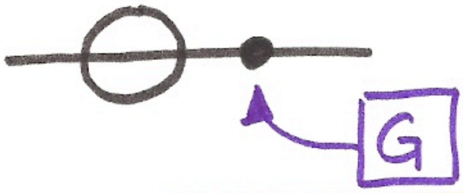

The Schwinger-Dyson equation resums the infinite series of (non local) self-energy insertions

⇛ resums IR logarithms

SUMMARY

- The IR dynamics of light fields in dS is nontrivial

- The p-representation provides a powerful tool for resummation and/or nonperturbative techniques

- IR/secular divergences can be exactly resummed (in some cases) into finite, well-behaved expressions

PERSPECTIVES

- Nonperturbative renormalization group techniques [Kaya ('13), Serreau ('13)]
  - Phase structure of O(N) theories in dS space [Guilleux, Serreau ...]
- Applications to inflationnary cosmology
  - Decoherence and the quantum to classical transition
  - Slow-roll corrections for quasi-deSitter space [Herranen, Markkanen, Tranberg ('13)]

ON FRIDAY

SOME RELATED WORK (non exhaustive)

- **Symmetry restoration**

  [Ratra ('85) ; Prokopec ('12) ; Arai ('12) ; Boyanovsky ('12) ; Lazzari, Prokopec ('13)]

- **Nonperturbative IR effects**

  - Stochastic approach [Starobinsky, Yokoyama ('94)] ...
  - Schwinger-Dyson [Youssef, Kreimer ('13)]

    [see also : Akhmedov et al. ('12) ('13) ; Jaktar, Leblond, Rajaraman ('12) } massive fields]
  - 2PI [Riotto, Sloth ('08) ; Garbrecht, Rigopoulos ('11)]
  - Wigner-Weisskopf [Boyanovsky, Holman ('11) ; Boyanovsky ('12)]
  - Renormalization group

    [Burgess, Leblond, Holman, Shandera ('10) ; Kaya ('13)]