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Towards understanding chiral vortical effect

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- Consider a hydrodynamic theory with

  u^μ, T and μ

- The motion of the medium is governed by

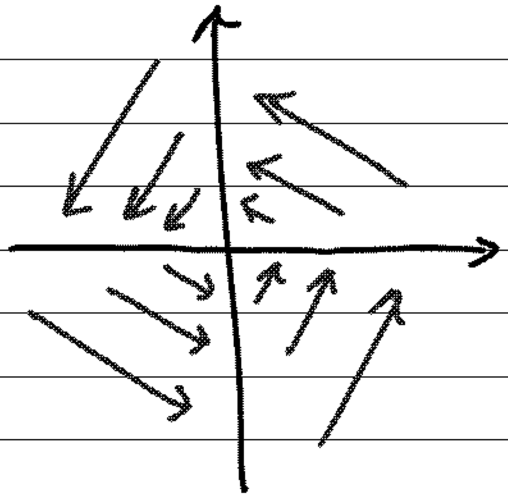
  $$\partial_\mu T^{\mu\nu} = 0 , \qquad \partial_\mu j^\mu = 0$$

- Specifically: $j^\mu = n u^\mu - \sigma T (\eta^{\mu\nu} + u^\mu u^\nu) \partial_\nu \left(\frac{\mu}{T}\right) + \xi \omega^\mu$

  Anomaly

- Here vorticity is defined by

$$\omega^\mu = \frac{1}{2} \epsilon^{\mu\nu\lambda\rho} u_\nu \partial_\lambda u_\rho$$



ξ : D. Son and P. Surówka (2009)

- But vorticity in flat space is problematic

for relativistic theories.

- we know this problem from angular momentum ..
- Lorentz group is origin dependent.

- Example: Consider fluids motion confined to a plane 2D; $\omega_z = \frac{1}{r}\frac{\partial}{\partial r}(r u_\phi)$ $\quad u_\phi = a r^n$

$$\Rightarrow \omega_z = a(n+1) r^{n-1} \quad n > 1$$

- Vorticity becomes larger as we go far from the origin $\Rightarrow$ fluid's speed $> c$

- Solution: Curved space

- We already know that curved-space configurations allow vorticity.

$$T^{\mu\nu} = T^{\mu\nu}_{ideal} + \cdots + \lambda_1 \sigma^{\langle\mu}{}_{\rho}\sigma^{\rho\nu\rangle} + \lambda_2 \sigma^{\langle\mu}{}_{\rho}\Omega^{\rho\nu\rangle}$$

$$\Omega^{\mu\nu} := \nabla^{\mu}u^{\nu} - \nabla^{\nu}u^{\mu} \qquad + \lambda_3 \Omega^{\langle\mu}{}_{\rho}\Omega^{\rho\nu\rangle}$$

- Consider small perturbations around flat space:

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}(x)$$

with $h_{0x}(y) \neq 0$ $\quad (t, x, y, z)$

rest frame: $u_\mu = (-1, h_{0x}, 0, 0)$

- You get vorticity vector $\omega^z = -\frac{1}{2}\partial_y h_{0x}$

- Our goal is to understand the chiral vortical effect microscopically.

- Let's study fermions on curved space with a chemical potential as a warm-up.

- Local Lorentz frames: $e_{\alpha\mu} e_{\beta}^{\ \mu} = \eta_{\alpha\beta} \quad e_{\alpha\mu} e^{\alpha}_{\ \nu} = g_{\mu\nu}$

- Modifications: $\gamma^{\mu} = \gamma_{\alpha} e^{\alpha\mu} \leftarrow \{\gamma_{\alpha}, \gamma_{\beta}\} = 2\eta_{\alpha\beta}$

$$\{\gamma^{\mu}, \gamma^{\nu}\} = 2g^{\mu\nu}$$

- Lagrangian:

$$\mathcal{L} = i g^{\frac{1}{2}} \bar{\Psi} \gamma^{\mu} \left(\partial_{\mu} + \frac{1}{2} G_{[\alpha\beta]} \omega_{\mu}^{\alpha\beta} \right) \qquad G_{[\alpha\beta]} \sim [\gamma_{\alpha}, \gamma_{\beta}]$$

$$\omega_{\mu}^{\alpha\beta} = e^{\alpha\sigma} e^{\beta\nu} \Gamma_{\sigma\nu\mu} - e^{\alpha}_{\nu,\mu} e^{\beta\nu}$$

- We can calculate the Hamiltonian: $\mathcal{H} = \Pi\dot{\Psi} - \mathcal{L}$

or even better $\hat{H}^2$:

$$\hat{H}^2 = \Big(\underbrace{-i\gamma^i \frac{\partial}{\partial x^i}}_{\gamma^i p_i} - i\frac{\gamma^\mu}{2} G_{[\alpha\beta]}\,\omega^{\alpha\beta}_{\ \ \mu} + i\mu\gamma^0\Big)^2$$

$$H^2 = \bar{\Psi}\left[\vec{p}^{\,2} + \frac{1}{2}\mu\,\sigma_z\,\omega^z\right]\Psi$$

- We can solve the equation of motion:

$$\omega \chi + \begin{pmatrix} \partial_y - \mu & -\sigma^x\left(-\frac{\omega^z}{2} + \partial_x\right) - \sigma^z \partial_z \\ \sigma^x\left(-\frac{\omega^z}{2} - \partial_x\right) - \sigma^z \partial_z & -\partial_y - \mu \end{pmatrix} \chi = 0$$

$$\chi^+ = \begin{bmatrix} \dfrac{-\partial_y + \sqrt{\partial_\perp^2 + \left(\partial_z + i\frac{\omega z}{2}\right)^2}}{i\partial_z - \frac{\omega z}{2} - \partial_x} \\[2ex] i\,\dfrac{-\partial_y + \sqrt{\partial_\perp^2 + \left(\partial_z + i\frac{\omega z}{2}\right)^2}}{i\partial_z - \frac{\omega z}{2} - \partial_x} \\[2ex] i \\ 1 \end{bmatrix}, \quad \chi^- = \begin{bmatrix} \dfrac{-\partial_y + \sqrt{\partial_\perp^2 + \left(\partial_z + i\frac{\omega z}{2}\right)^2}}{i\partial_z - \frac{\omega z}{2} - \partial_x} \\[2ex] -i\,\dfrac{\partial_y + \sqrt{\partial_\perp^2 + \left(\partial_z + i\frac{\omega z}{2}\right)^2}}{i\partial_z - \frac{\omega z}{2} - \partial_x} \\[2ex] i \\ 1 \end{bmatrix}$$

- $$\begin{cases} \omega^{+} - \mu = -\sqrt{\partial_\perp^2 + \left(\partial_z + \frac{i\omega z}{2}\right)^2} \\ \omega^{-} - \mu = +\sqrt{\partial_\perp^2 + \left(\partial_z + \frac{i\omega z}{2}\right)^2} \end{cases}$$

- Poincaré generators:

$$M^{\alpha\beta} = \int_\Sigma d\Sigma_\rho \left(x^\alpha T^{\rho\beta} - x^\beta T^{\rho\alpha}\right)$$

$$[P^\mu, M^{\rho\sigma}] = i(\eta^{\mu\sigma} P^\rho - \rho \leftrightarrow \sigma)$$

$$[M^{0i}, H] = i\hbar P^i$$

with

$$T^{\mu\nu} = \frac{i\sqrt{g}}{8}\left(\nabla^\mu \bar{\Psi}\gamma^\nu \Psi + \nabla^\nu \bar{\Psi}\gamma^\mu \Psi - \bar{\Psi}\gamma^\mu \nabla^\nu \Psi - \bar{\Psi}\gamma^\nu \nabla^\mu \Psi\right)$$

Conclusion:

- It's possible to study a theory microscopically with vorticity using the curved space methods.

- we can apply it to study chiral vortical effect by computing axial current.