Holographic thermalization at intermediate coupling

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D. Steineder, SS, A. Vuorinen, 1209.0291 (PRL), 1304.3404 (JHEP)
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FWF Der Wissenschaftsfonds.

Motivation, Goals & Strategy

Quark gluon plasma

- What are the dominant mechanisms behind the fast thermalization

Goals

- Gain insight into the fast thermalization process
- Which modes thermalise first: top down vs. bottom up
- Dependence on the coupling strength

Strategy

- SYM where weak and strong coupling regimes are accessible
- Relax infinite coupling limit
- Study quasinormal modes (near equilibrium)
- Retarded Green's functions far off equilibrium

Outline

- **Weak and strong coupling results**
- **Quasinormal modes**
- **Far off-equilibrium correlators**

Thermalization at weak coupling

Questions one wants to answer

- Parametric weak coupling estimate: How does the therm time depend on the coupling constant

$$t_{equ} \sim \frac{\alpha^n}{Q_s}$$

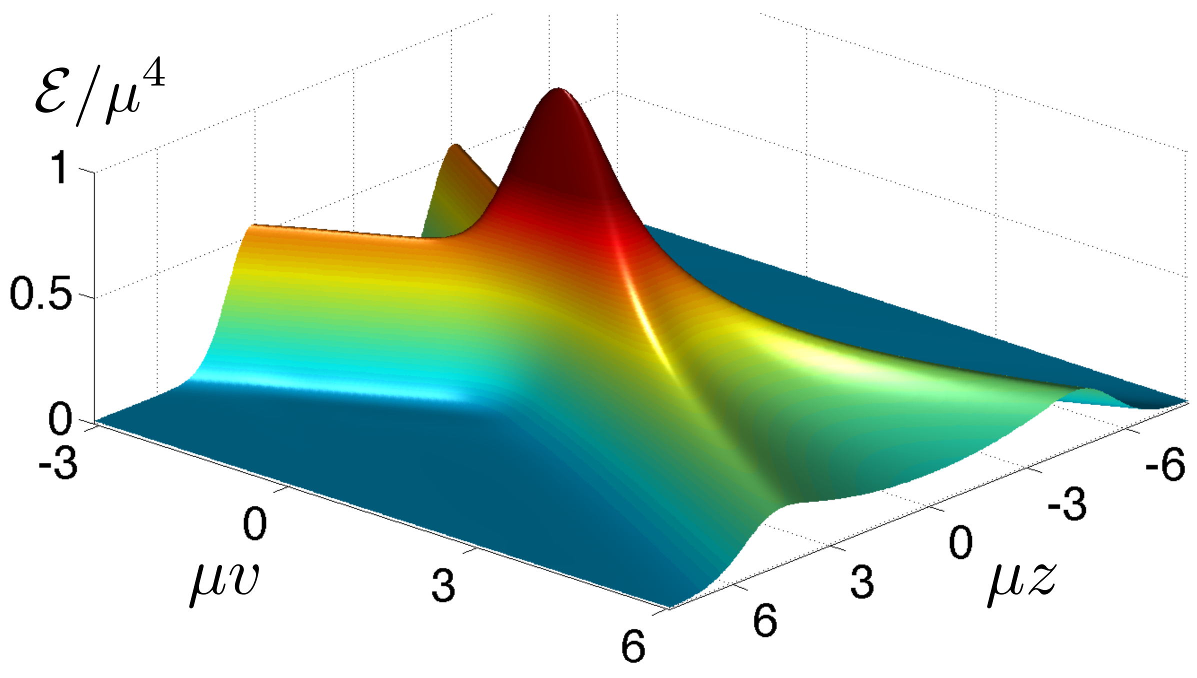
- what are the dominant processes?

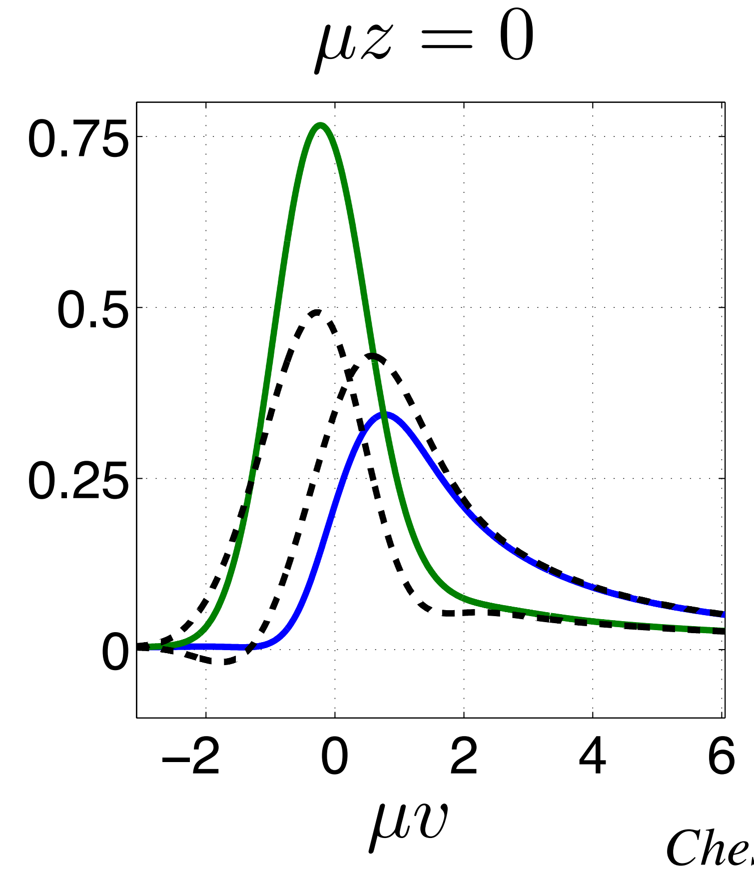
Bottom-up thermalization *(Baier et al (2001))*

- Scattering processes
  - In the early stages many soft gluons are emitted which then thermalize the system (*Baier et al (2001)*): $n_{BMSS} \sim -13/5$
- Driven by instabilities
  - Instabilities induce collinear radiation instead of scattering processes and make therm. faster (*Kurkela, Moore (2011)*): $n_{KM} \sim -5/2$

Thermalization at strong coupling

Thermalization process of strongly coupled N=4 SYM is mapped to black hole formation in asymptotically AdS space





Chesler & Yaffe

Lessons from gauge/gravity duality

- Thermalization time naturally short $t_{eq} \sim 1/T$
- Hydrodynamization ≠ thermalization, isotropization
- Thermalization always top down (causal argument)

Bridging the gap

Goal of this work: try to relax the infinite coupling limit and bring the two limiting cases closer together

Correlators for studying thermalization

Quasinormal modes

- characterize the response of the system to inf. perturbations
- Structure of retarded thermal Greens functions ⇒ Dispersion relation of field excitations

$$\omega_n(q) = M_n(q) - i\Gamma_n(q),$$

- Reveal striking difference between weakly and strongly coupled systems
  - At weak coupling long lived quasiparticles: $\mathrm{Im}(\omega_n) \ll \mathrm{Re}(\omega_n)$
  - At infinite coupling: infinite tower of modes $\omega_n|_{q=0} = n(\pm 1 - i)$
  - Magnitude of Γ_n related to thermalization pattern: At strong coupling highest energy modes decay fastest — top down thermalization

Time dependent off-equilibrium Greens functions probe how fast different energy (length) scales equilibrate

Two examples

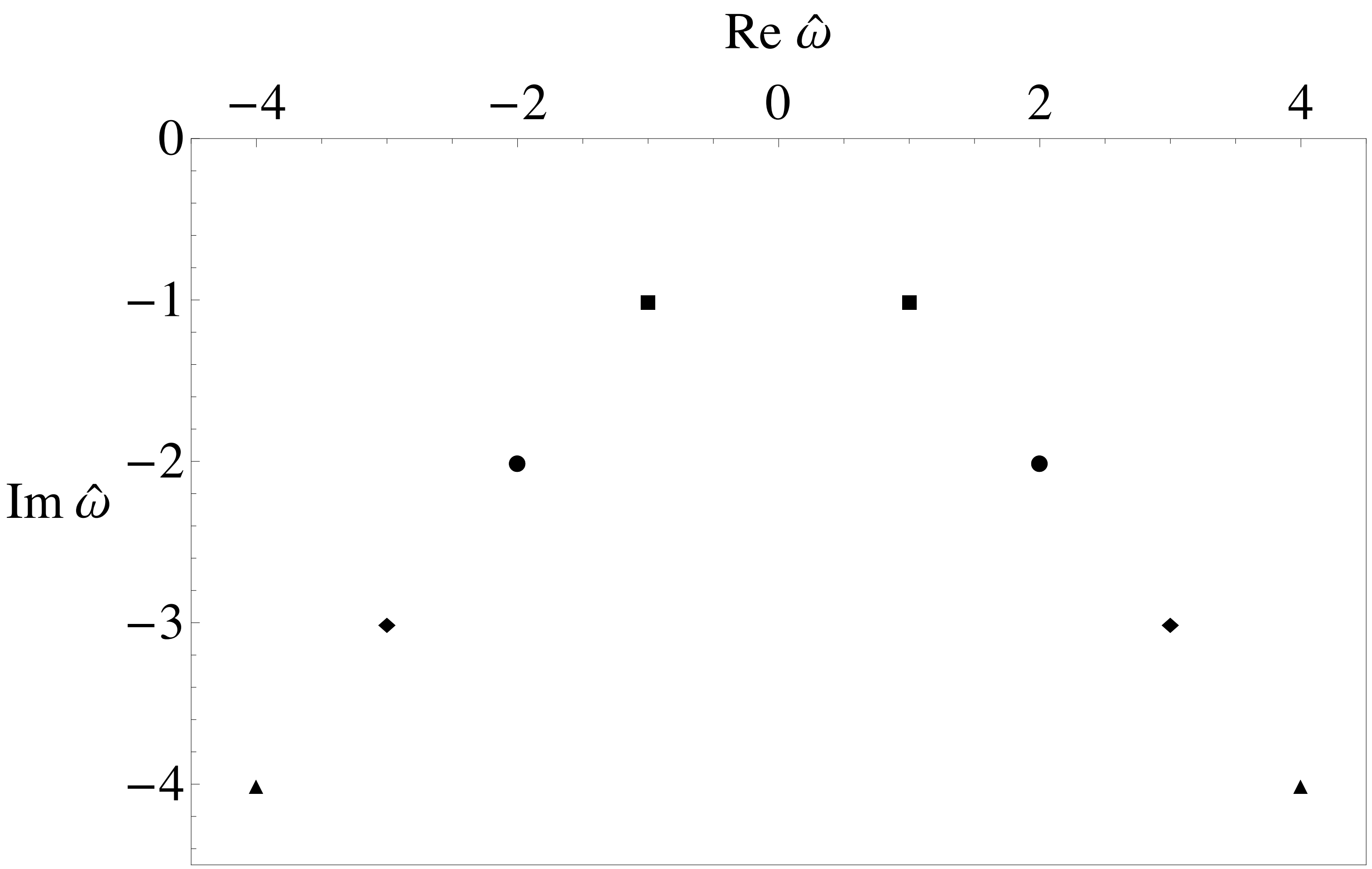
Energy momentum tensor correlators

- linearized perturbations of $g_{\mu\nu} \to g_{\mu\nu} + h_{\mu\nu}$
- construct gauge invariants from symmetry channels *(Kovtun, Starinets)*
  - scalar channel: h_{xy}
  - shear channel: $h_{tx},\ h_{zx}$
  - sound channel: $h_{tt},\ h_{tz},\ h_{zz},\ h$

EM current correlators — photon production

- Obtained by adding a U(1) vector field coupled to a conserved current corresponding to a subgroup of the $SU(4)_R$

QNM at infinite coupling: Photons



$$\omega_n|_{q=0} = n(\pm 1 - i)$$

(Kovtun, Starinets)

- Pole structure of EM current-current correlator displays usual quasinormal mode spectrum at infinite coupling
- How does the QNM spectrum get modified at finite coupling?

Finite coupling corrections

Key relation in AdS/CFT: $(L/l_s)^4 = L^4/\alpha'^2 = \lambda$

- Go beyond $\lambda = \infty$: add α' terms to SUGRA action, i.e. first non trivial terms in a small curvature expansion
- Leading order corrections: $\mathcal{O}(\alpha'^3) = \mathcal{O}(\lambda^{-3/2})$

Gubser et al; Pawelczyk, Theisen (1998)

Improved type IIB SUGRA action:

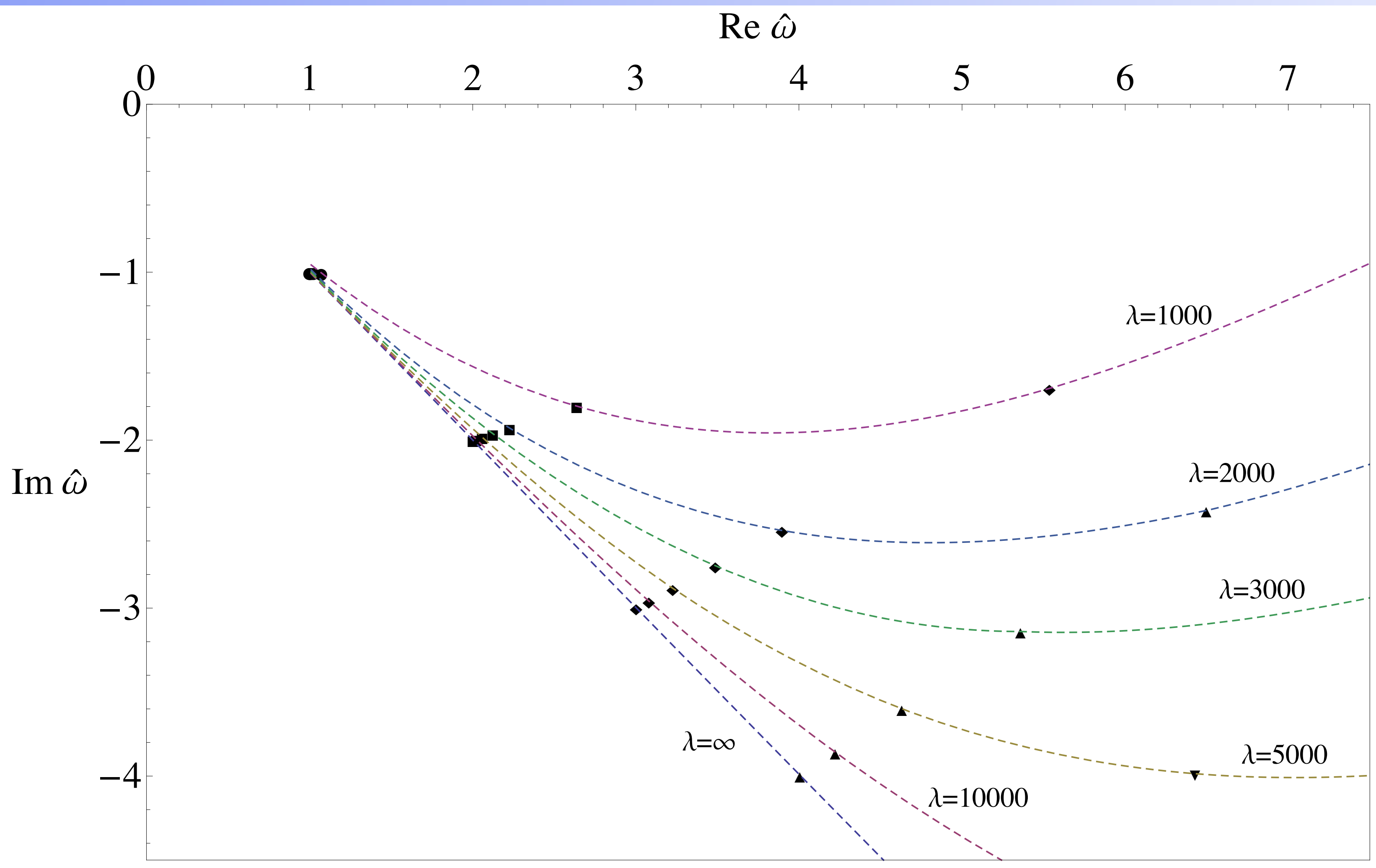
$$S^0_{IIB} = \frac{1}{2\kappa_{10}} \int d^{10}x\sqrt{-g}\left(R_{10} - \frac{1}{2}(\partial\phi)^2 - \frac{1}{4.5!}(F_5)^2 + \gamma e^{\frac{-3}{2}\phi}(C+\mathcal{T})^4\right)$$

$$\mathcal{T}_{abcdef} = i\nabla_a F^+_{bcdef} + \frac{1}{16}\left(F^+_{abcmn}F^{+\ mn}_{def} - 3F^+_{abfmn}F^{+\ mn}_{dec}\right), \quad \gamma \equiv \frac{1}{8}\zeta(3)\lambda^{-\frac{3}{2}}$$

Paulos (2008)

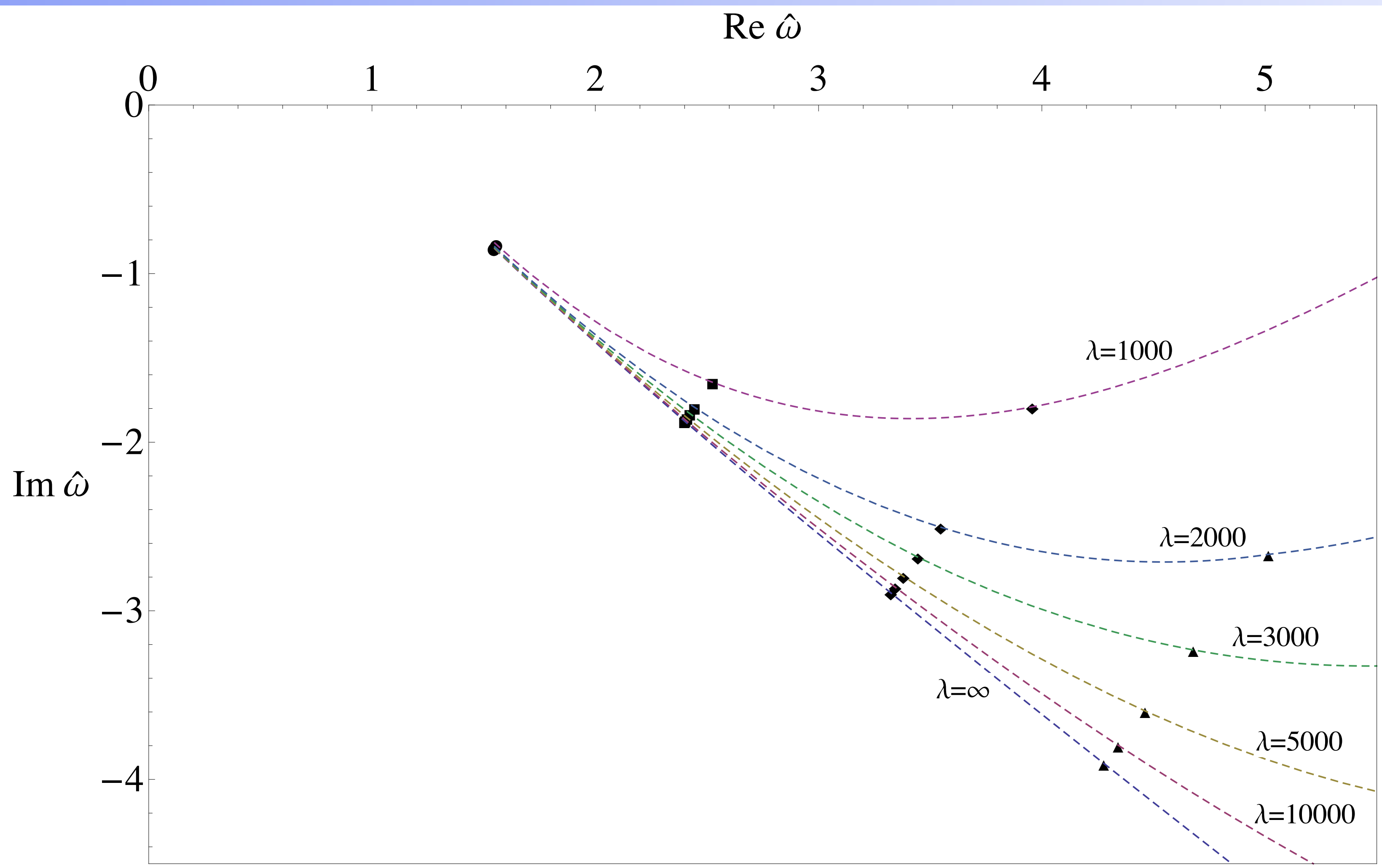
- Leads to γ-corrected metric
- EoM for different fields

QNM at finite coupling: photons



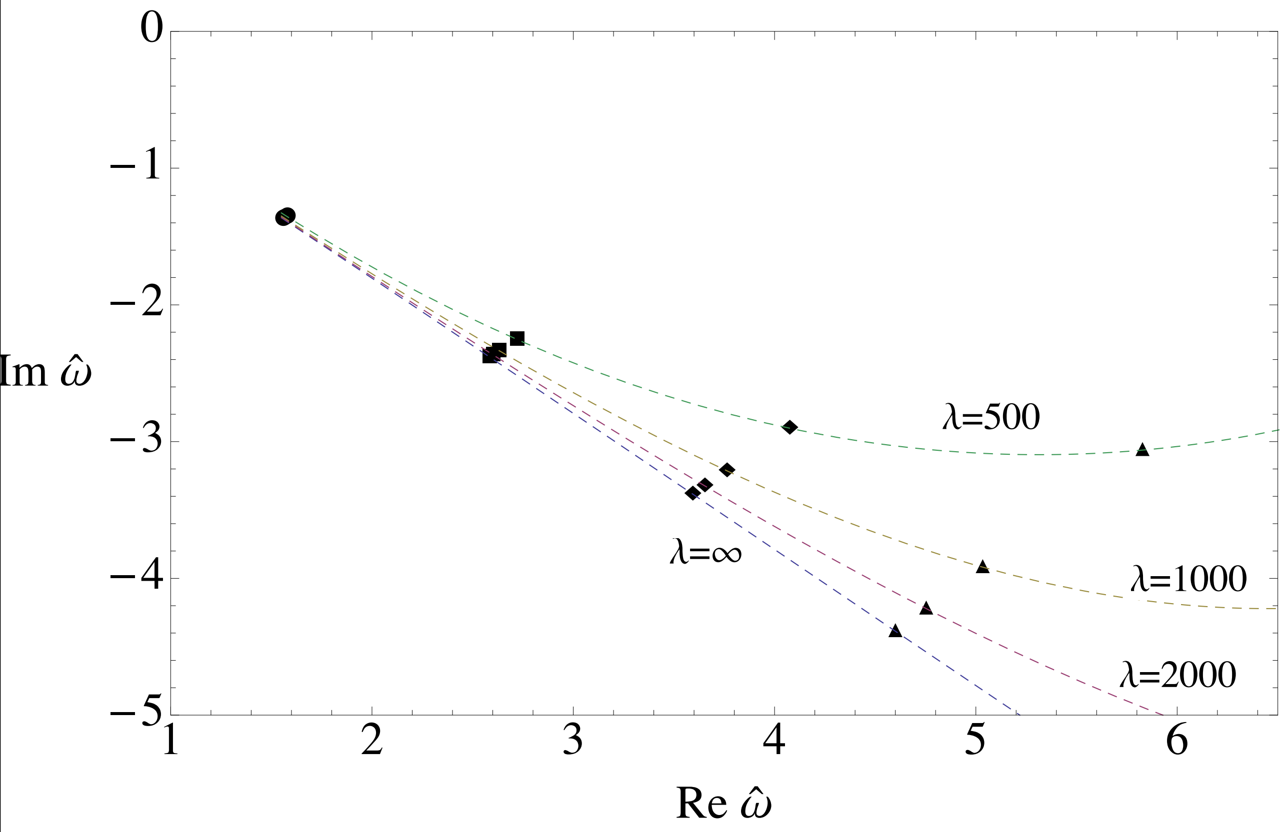
- Effect of decreasing coupling: Imaginary part increases, lowering the decay rate of the excitations $\Rightarrow$ modes become longer - lived
- Larger impact on higher energetic modes
- Convergence of strong coupling expansion not guaranteed when shift is of $\mathcal{O}(1)$

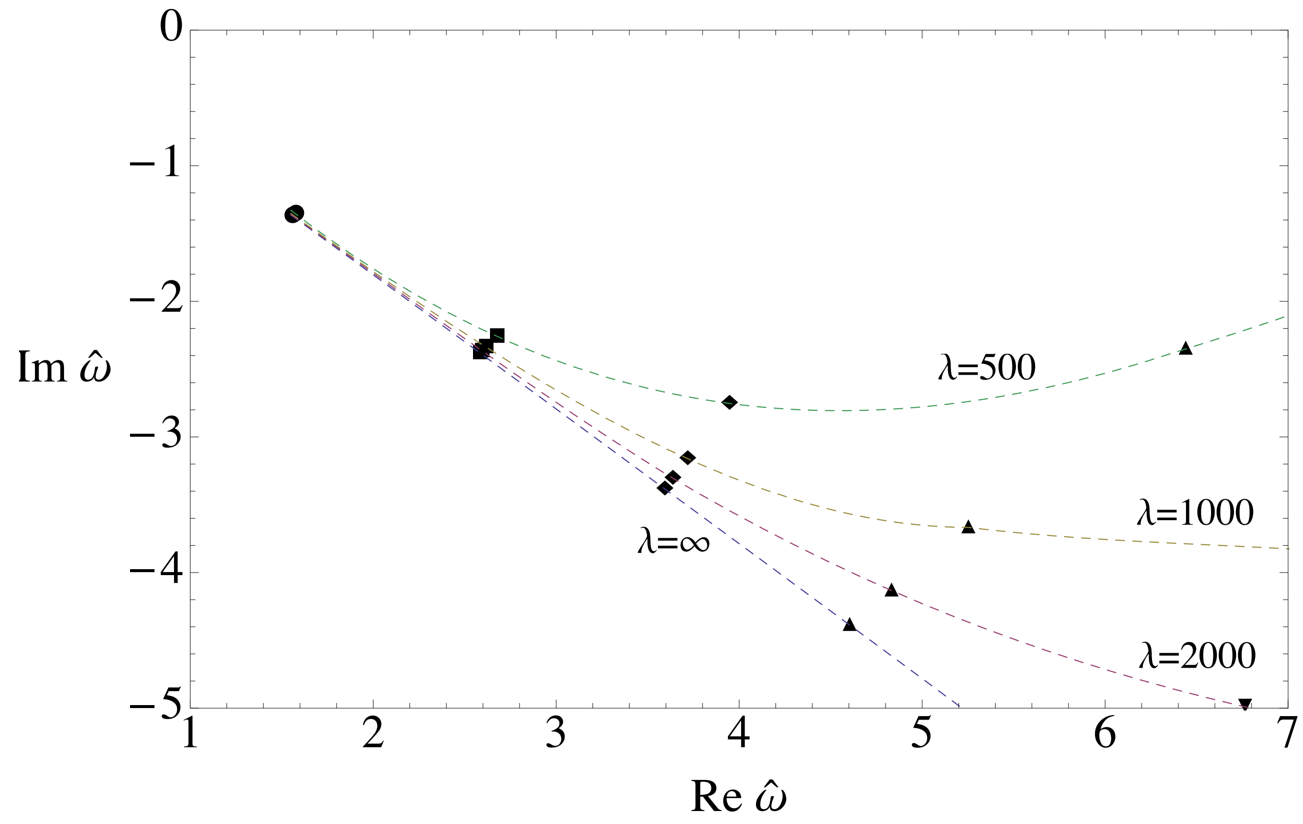
QNM at finite coupling: Photons



- similar shift at nonzero three momentum: q=2πT

QNM at finite coupling: $T_{\mu\nu}$ correlators

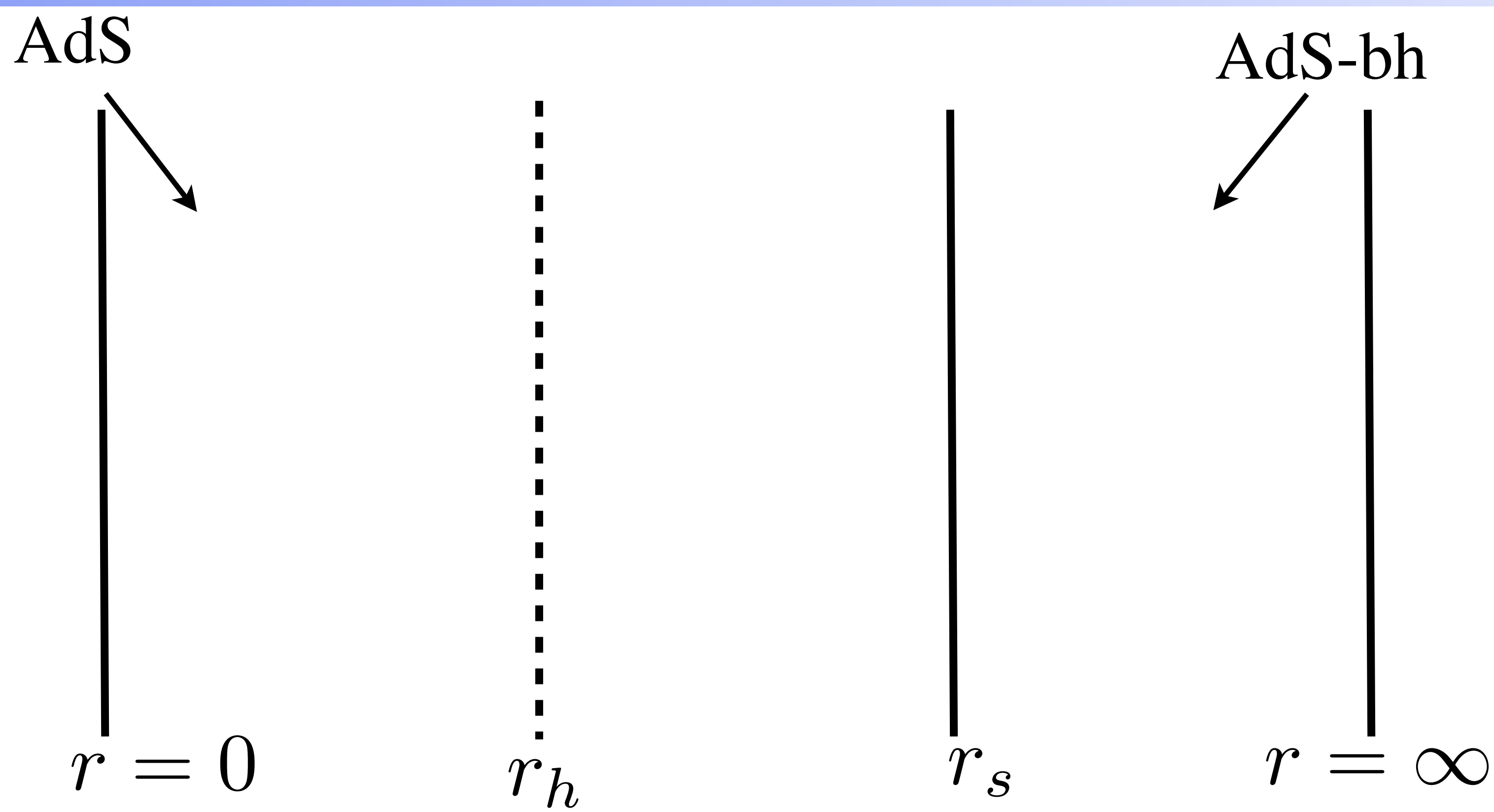




Same effect for the shear (left) and sound (right) channel (here q=0)

- Outside the infinite coupling, the response of a strongly coupled plasma appears to change, with the QNM mode spectrum moving towards a quasiparticle one
- What happens if we the take the system further away from equilibrium by using the collapsing shell model?

The falling shell setup



Danielsson, Keski-Vakkuri, Kruczenski (1999)

Lin & Shuryak (2008)

Outside and inside spacetime

- metric: $$ds^2 = \frac{(\pi T L)^2}{u}\left(f(u)dt^2 + dx^2 + dy^2 + dz^2\right) + \frac{L^2}{4u^2 f(u)}du^2 \qquad u = \frac{r_h^2}{r^2}$$

$$f(u) = \begin{cases} f_+(u) = 1 - u^2\,, & \text{for } u > 1 \\ f_-(u) = 1\,, & \text{for } u < 1 \end{cases},$$

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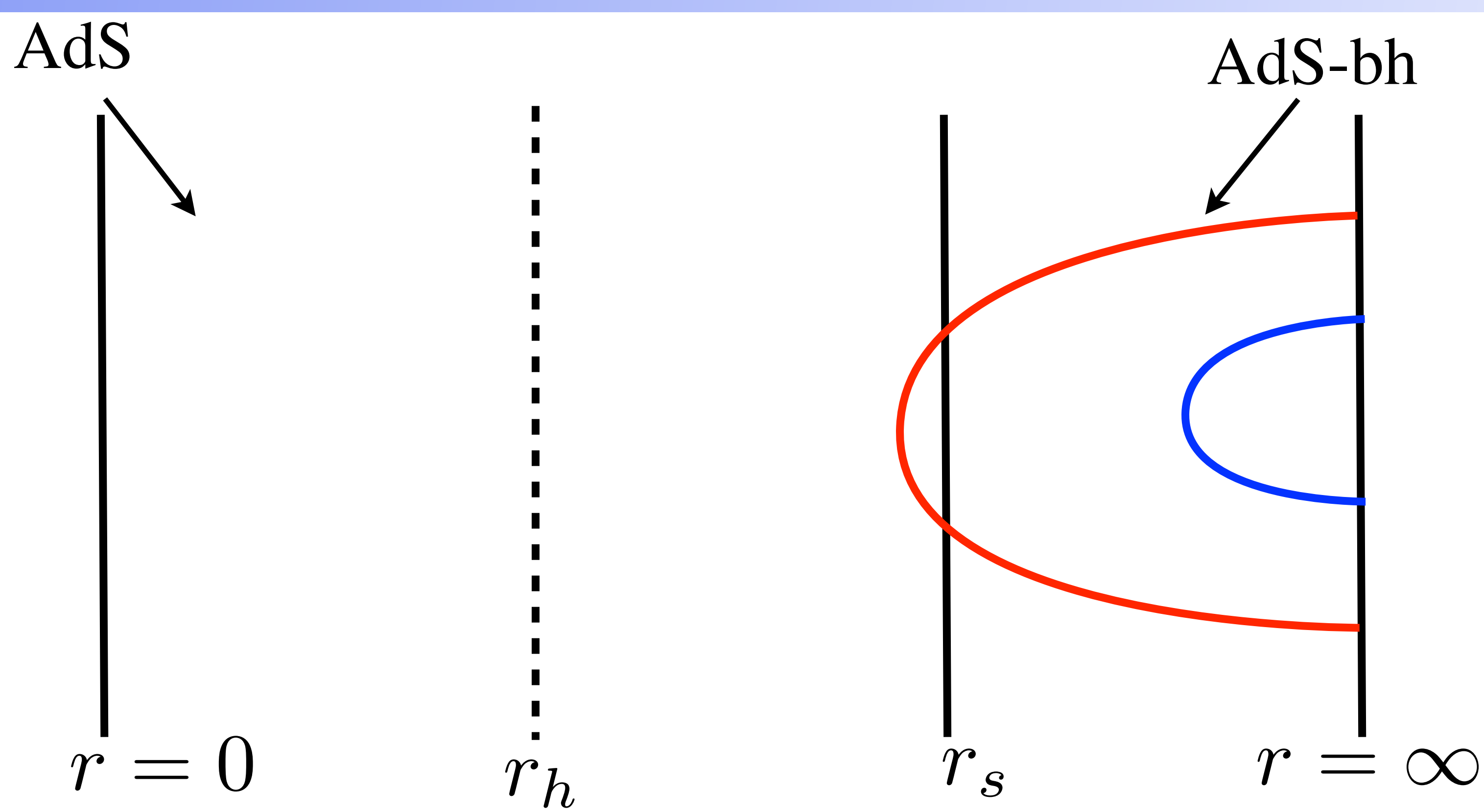

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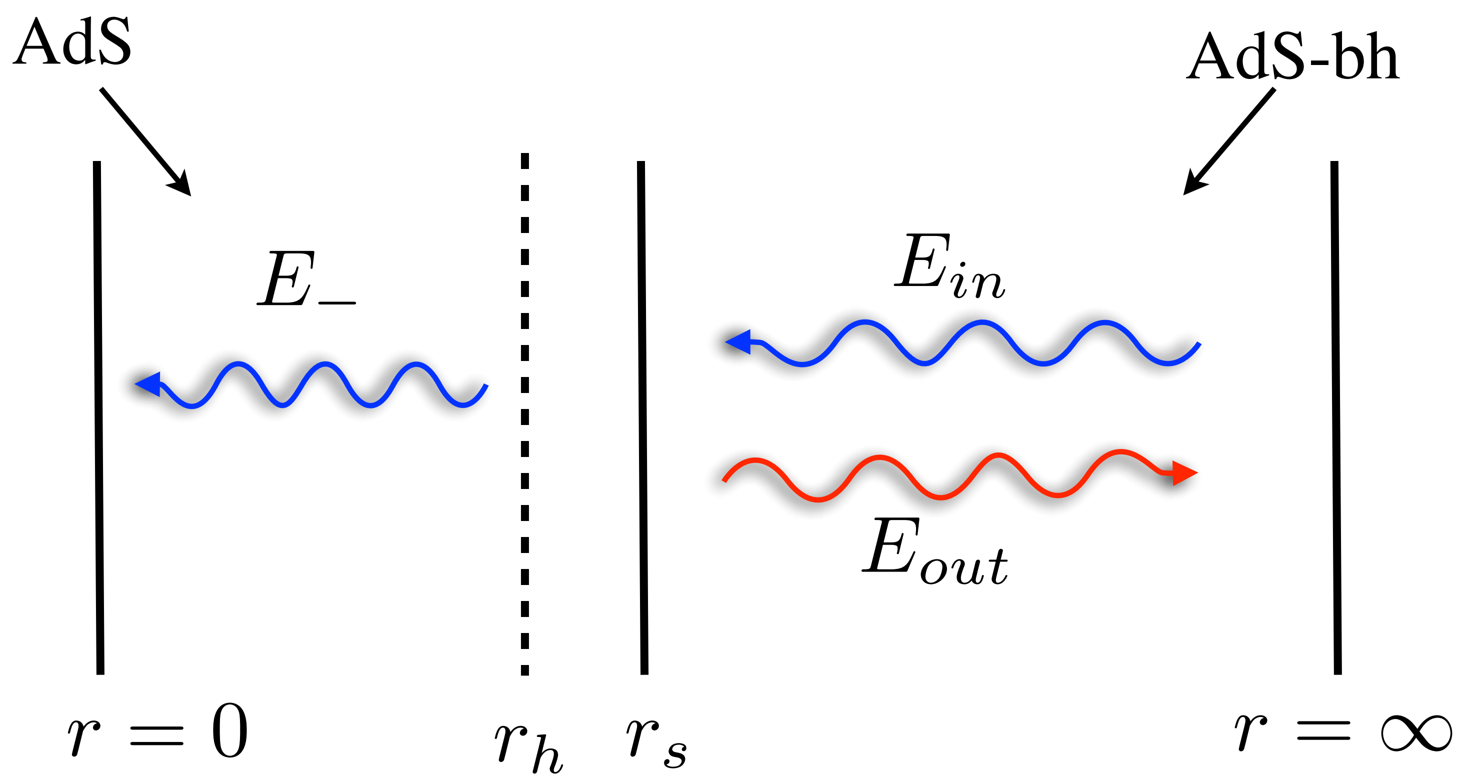
Thermalization from geometric probes:

- Entanglement entropy and Wilson loop: always top down thermalization

The falling shell setup



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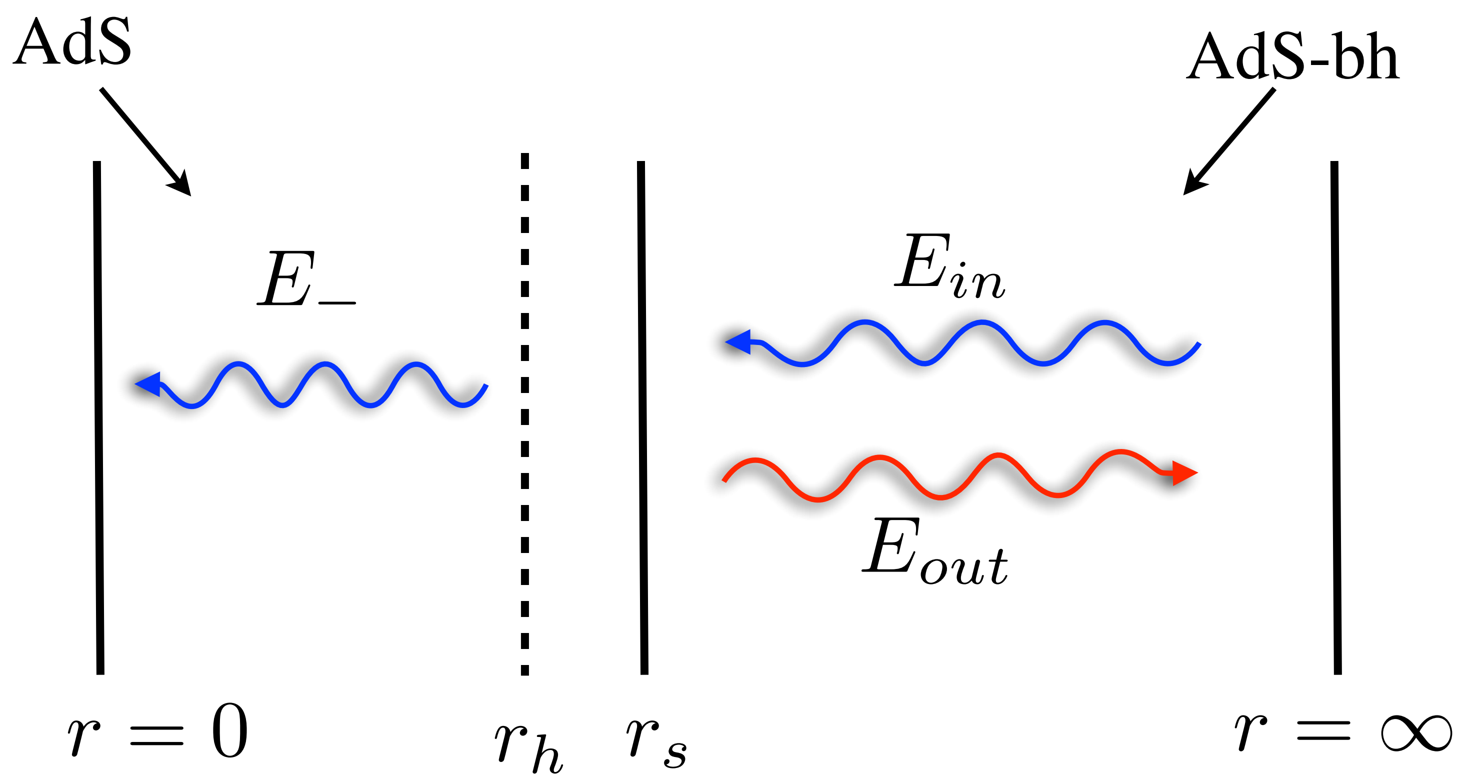
Outside solution

$$E_+ = c_+ E_{in} + c_- E_{out}$$

The falling shell setup



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Quasistatic approximation:

- Energy scale of interest >> characteristic time scale of shell's motion

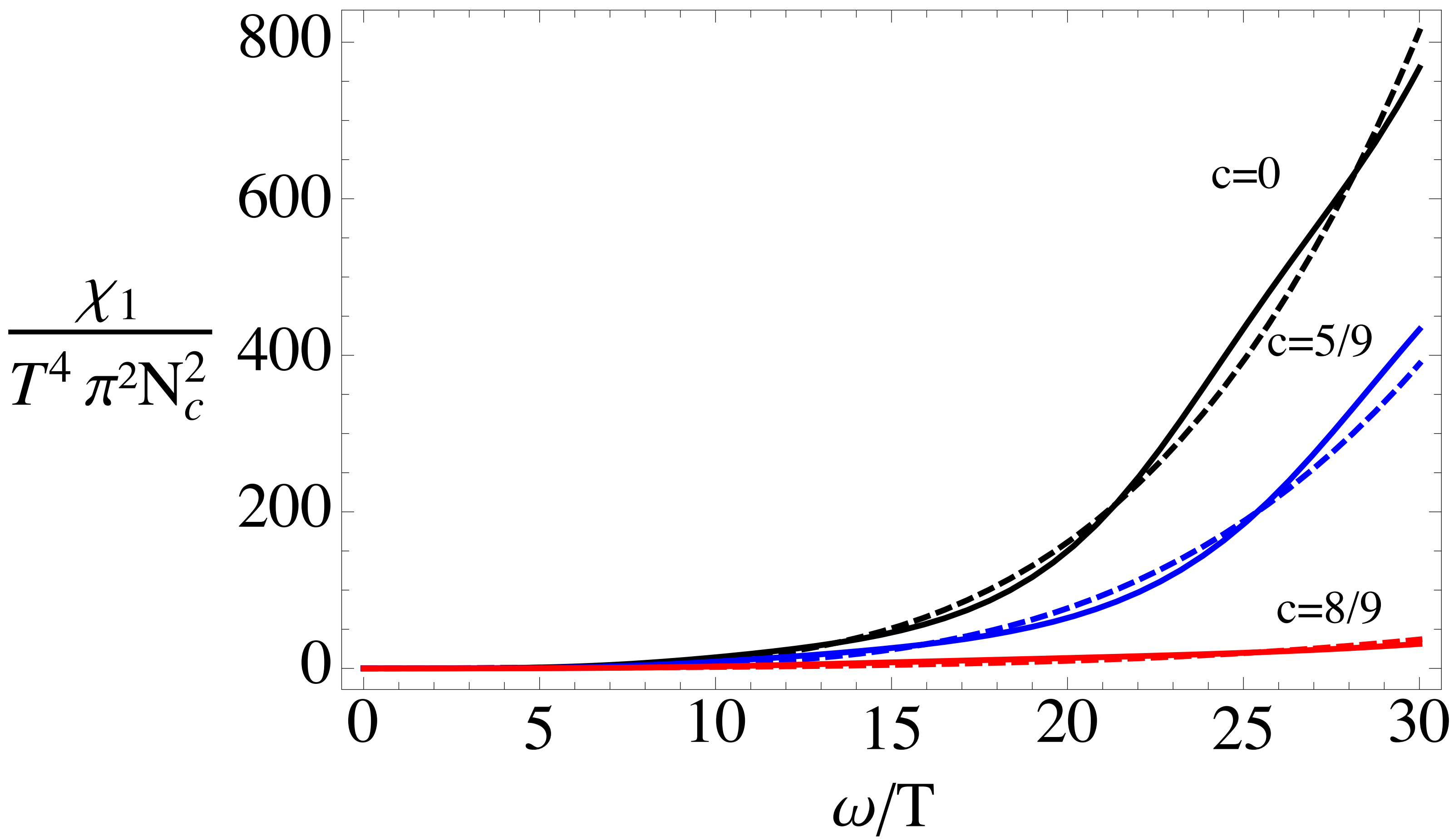
Spectral density: scalar channel

natural quantity to study: spectral density: $\chi^{\mu}_{\mu} = -2\mathrm{Im}(\Pi^{\mathrm{ret}})^{\mu}_{\mu}(k_0)$

- virtuality

$$v = \frac{\hat{\omega}^2 - \hat{q}^2}{\hat{\omega}^2}$$

- parametrize $q = c\,\hat{\omega}$



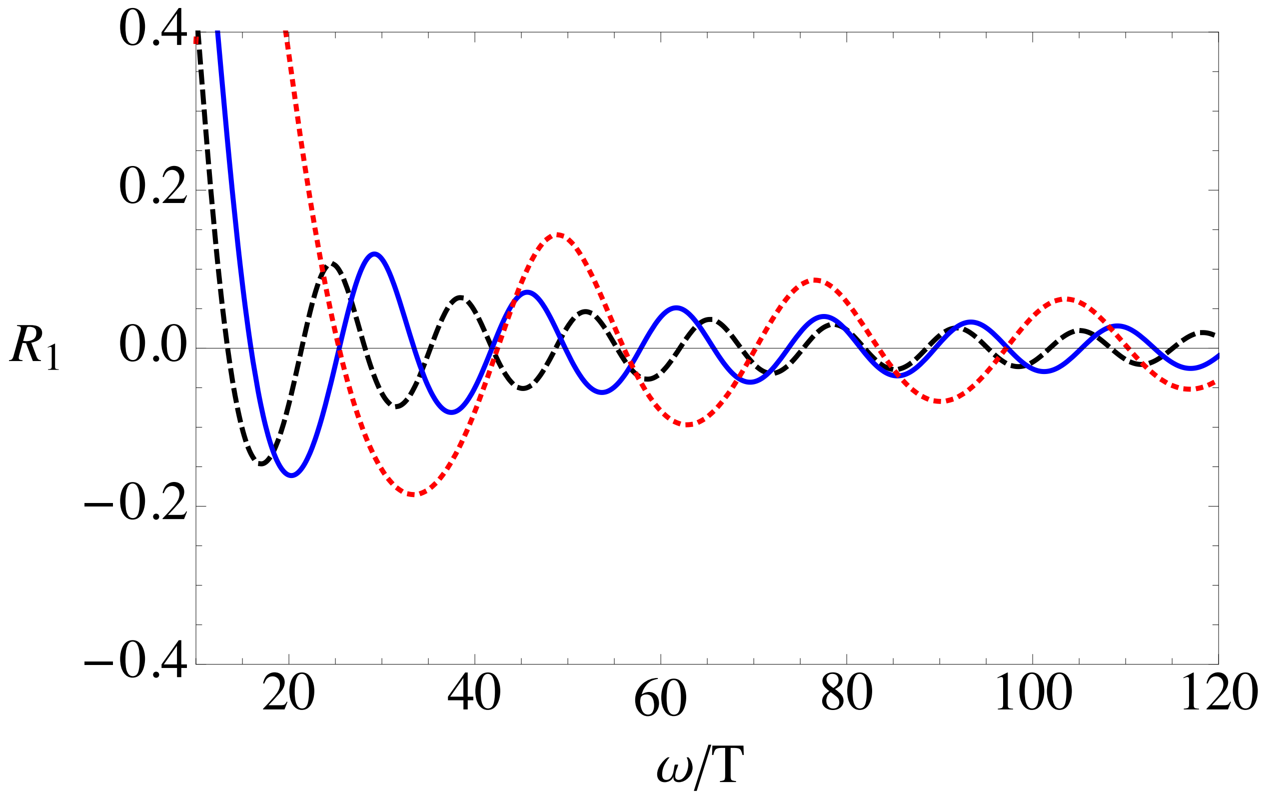
spectral density for u_s =0.5 for different virtualities

- Out of equilibrium effect: oscillations around thermal value

Relative deviation of spectral density: scalar channel

- Relative deviation from thermal equilibrium

$$R(\hat{\omega}) = \frac{\chi(\hat{\omega}) - \chi_{th}(\hat{\omega})}{\chi_{th}(\hat{\omega})}$$

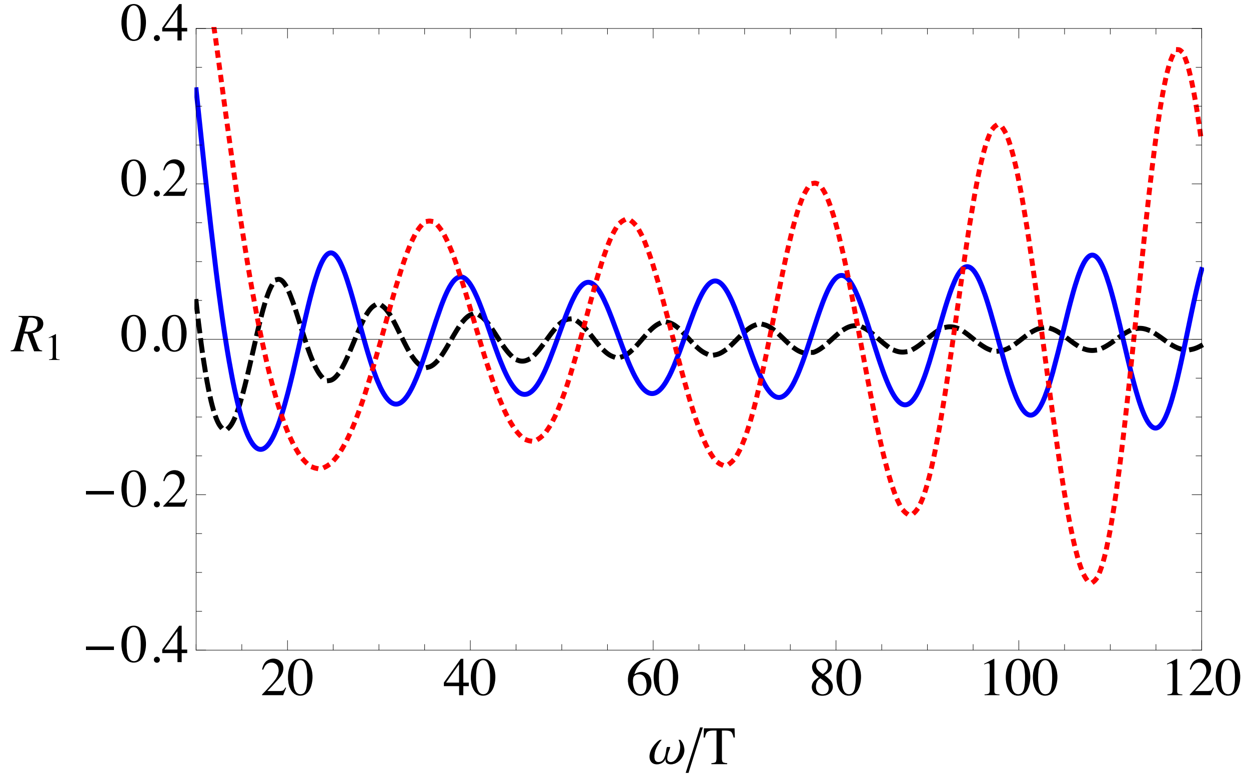


relative deviation R for u_s=0.5 and c=8/9 (red), 5/9 (blue), 0 (black)

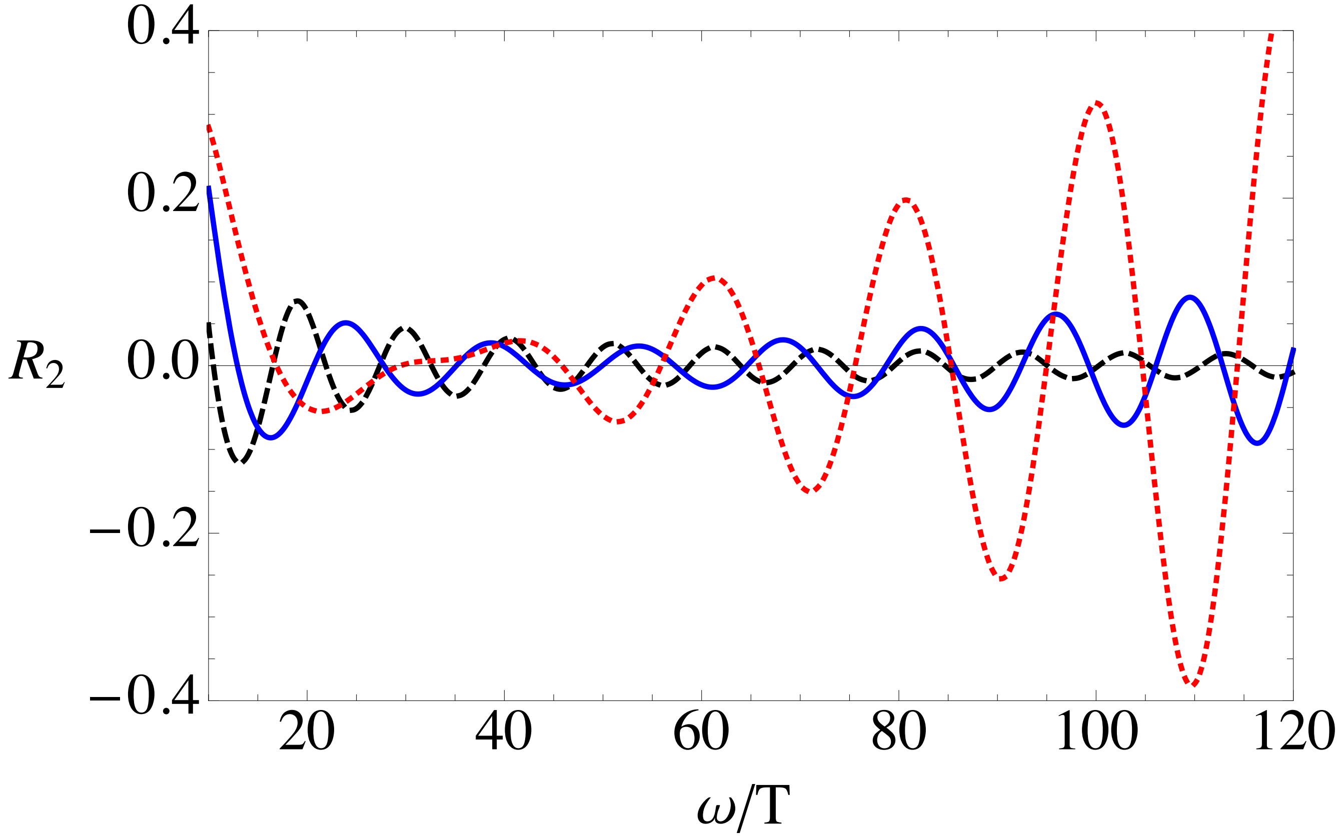
- Top down thermalization: highly energetic modes are closer to equ. value
- Dependence on c: smaller c $\rightarrow$ R closer to equilibrium
- As the shell approaches the horizon spectral density approaches equilibrium value

Relative deviation at finite coupling

Scalar channel



Shear channel



Relative deviation for the scalar/shear channel for u_s=0.5, c=0, 6/9, 8/9 and $\lambda = 100$

- For c=0: R approaches a constant for large frequencies
- As c increases: fluctuation amplitude starts to grow at some critical ω_{crit}
- Indication of weakening the top-down thermalization pattern
- Decreasing the coupling: change happens at lower frequency
- Same behaviour for all three channels

Relative deviation: photons

Infinite coupling:

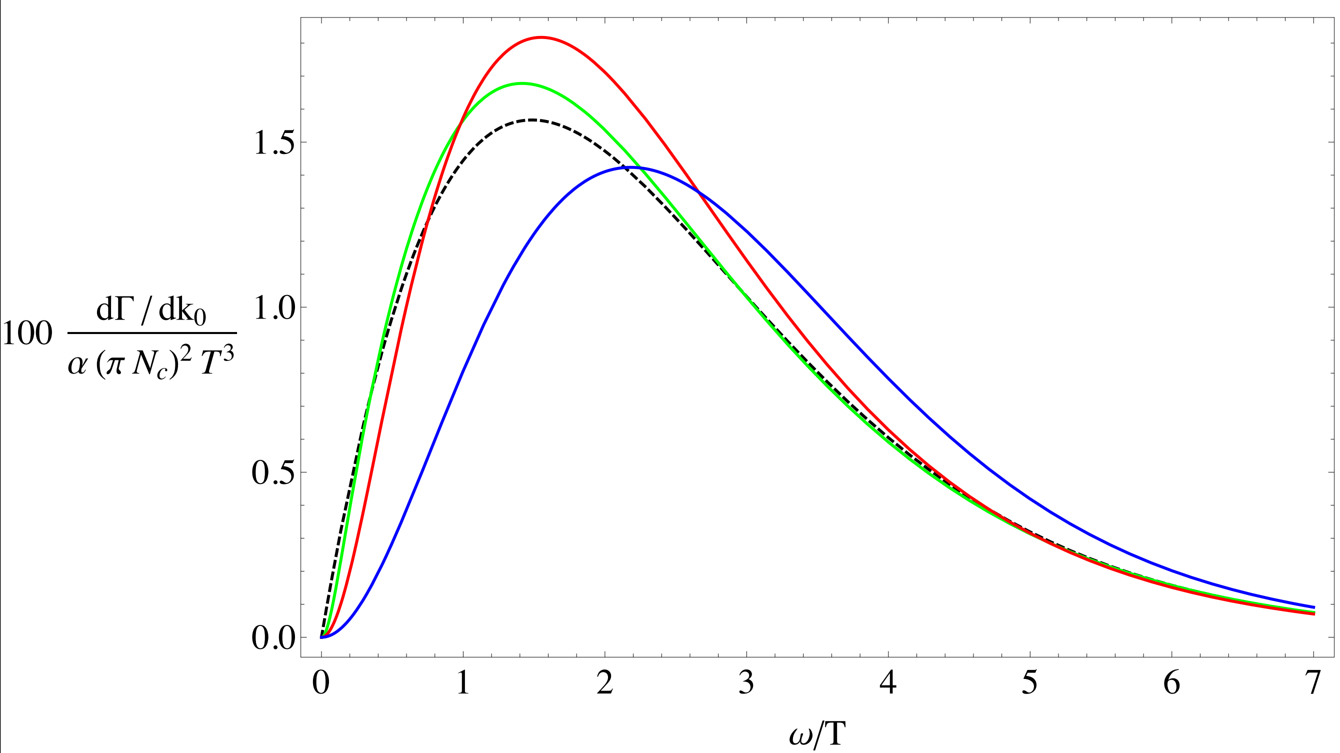
- Highly virtual photons thermalise first
- Top down pattern $$\chi(\hat{\omega}) \approx \hat{\omega}^{\frac{2}{3}} \left(1 + \frac{f_1(u_s)}{\hat{\omega}}\right), \qquad R \approx \frac{1}{\hat{\omega}}$$

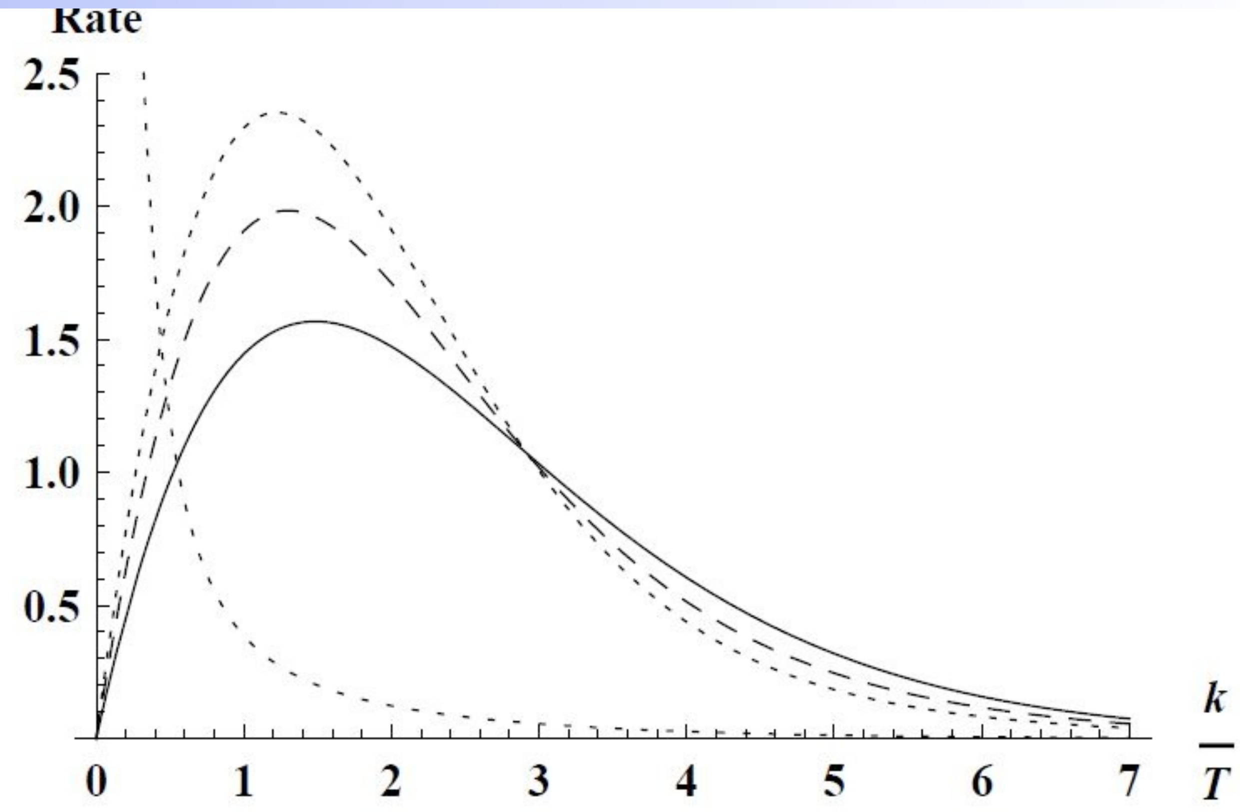
Finite coupling:

- For maximally virtual photons (c=0) R approaches a constant as $\omega \to \infty$
- For on-shell photons (c=1): amplitude of R rises linearly with ω

$$\chi(\hat{\omega}) \approx \hat{\omega}^{\frac{2}{3}} \left(1 + \frac{3\zeta(3)}{8\lambda^{\frac{3}{2}}} + \frac{f_1(u_s)}{\hat{\omega}} + \frac{f_2(u_s)\hat{\omega}}{\lambda^{\frac{3}{2}}}\right)$$

Photon production rate at infinite coupling

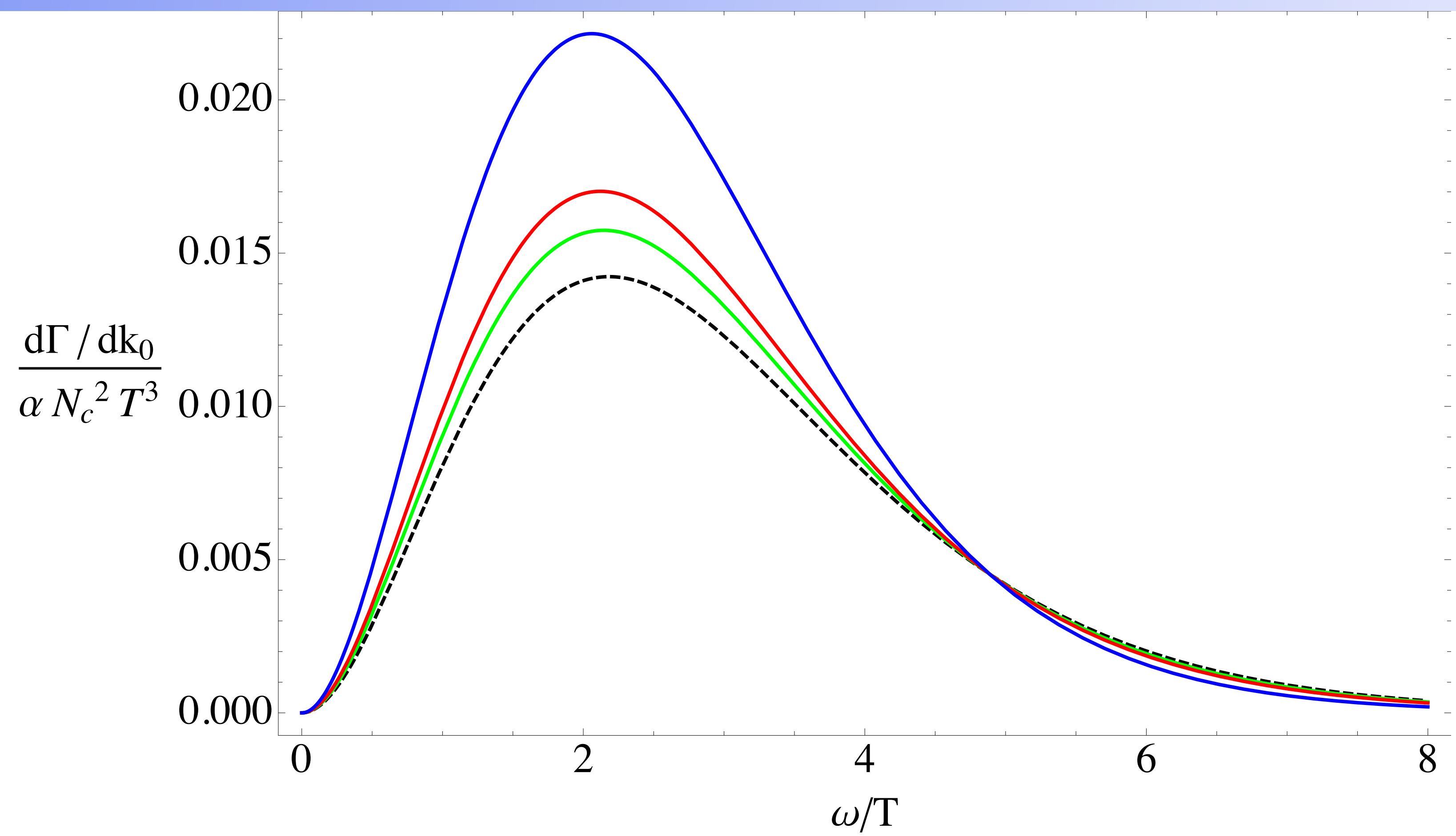




photon production rate for r_s/r_h=1.1, 1.01, 1.001

- Enhancement of production rate
- Hydro peak broadens and moves right
- Apparently no dramatic observable signature in off-equilibrium photon production
- Combining the two allows to study thermalization at finite coupling!

Photon production rate at intermediate coupling



emission rate for photons r_s/r_h=1.01 and $\lambda = \infty, 120, 80, 40$

- Behaviour qualitatively similar to equilibrium case: in particular the result is much less sensitive to finite coupling corrections than QNM spectrum

Implications for holography

- For a given (equilibrium) quantity

$$X(\lambda) = X(\lambda = \infty) \times \left(1 + X_1/\lambda^{3/2} + \mathcal{O}(1/\lambda^3)\right)$$

- Define critical coupling λ_c such that $|X_1/\lambda_c^{3/2}| = 1$. Then:

Quantity	$\boldsymbol{\lambda_c}$
Pressure	0.9
Transport/hydro coeffs. (η/s, τ_H, κ)	7 ± 1
Quasinormal mode n for photons / $T_{\mu\nu}$	$\lambda_c(n=1) = 200$, $\lambda_c(n=2) = 500$ $\lambda_c(n=3) = 1000$,...
Spectral densities in equilibrium	$\lambda_c(\omega = 0) = 40$, $\lambda_c(\omega \to \infty) = 0.8$, ...

- Lesson: What is weak/strong coupling depends strongly on the quantity. Thermalization properties appear to be sensitive to strong coupling corrections

Conclusions

- Holographic (thermalization) calculations at finite coupling are possible and potentially a very fruitful exercise

- Indications that a holographic systems obtains weakly coupled characteristic within the realm of a strong coupling expansion
  - QNM modes: flow towards quasiparticle picture, independent of the thermalization model
  - Top-down thermalization pattern weakens and moves towards bottom-up

- Naive conclusion: to describe the physical heavy ion system using holography $(\lambda \sim 20)$ accounting for finite coupling corrections mandatory

- As always: more work needed
  - in particular go beyond the quasistatic approximation and study full dynamical problem