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# QUANTUM CORRECTIONS DURING INFLATION

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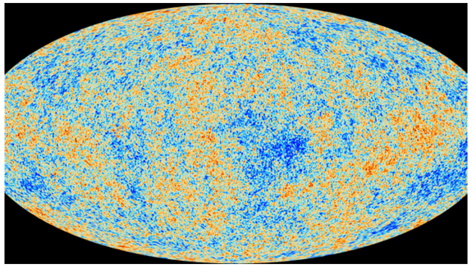
# Inflation

- The interpretation of modern Cosmological observables points to a stage of accelerating expansion in the very early Universe. Planck (Talks: Lesgourgues, Enqvist, Hindmarsh, … )
- Standard dynamics:
  - Inflation from classically slow-rolling homogeneous field: inflaton.
  - CMB from free, light scalar field modes in deSitter space vacuum, freezing in semi-instantaneously at horizon crossing.
- New observables:
  - Non-gaussianity (bi-spectrum, tri-spectrum, spikes, …).
  - Scale dependence beyond power law (spectral index, running, running of running…).
  - Efolds with precision +/- 10.
- But: Inflaton is an interacting quantum field.



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# Classical slow-roll inflation

- Homogeneous field in FRW background.
- Friedmann equations.

- If H is roughly constant
- If kinetic energy is much smaller than potential energy

→ "Slow roll" inflation.

Realized for certain V with certain initial conditions for the field.

Slow-roll works if
V = V = V.

$$\ddot{\bar{\phi}} + 3H\dot{\bar{\phi}} + V_{,\bar{\phi}} = 0, \qquad H = \dot{a}/a$$

$$3M_{\rm pl}^2 H^2 = \frac{1}{2}\dot{\bar{\phi}}^2 + V[\bar{\phi}]$$

$$3M_{\rm pl}^2\left(\dot{H} + H^2\right) = -\dot{\bar{\phi}}^2 + V[\bar{\phi}]$$

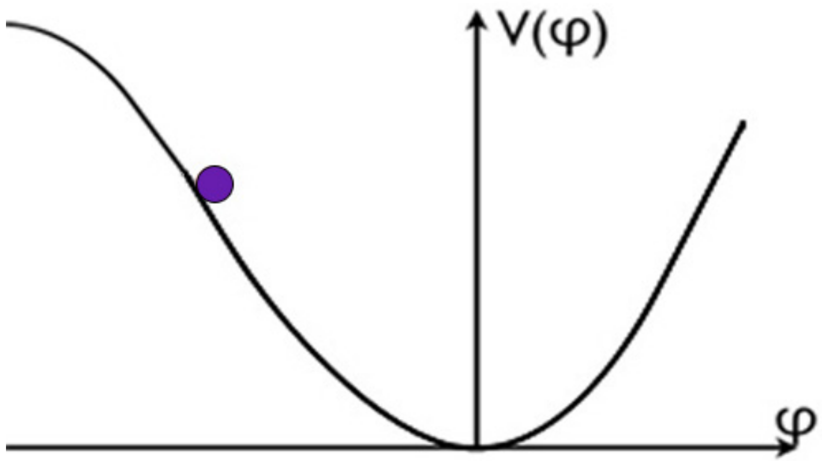
$$ds^2 = dt^2 - a^2(t)d\mathbf{x}^2$$

$$-\frac{\dot{H}}{H^2} = \frac{\dot{\bar{\phi}}^2}{2M_{\rm pl}^2} \equiv \epsilon$$

$$3H\dot{\bar{\phi}}\left(1 + \frac{\ddot{\bar{\phi}}}{3H\dot{\bar{\phi}}}\right) + V_{,\bar{\phi}} = 0, \qquad \frac{\ddot{\bar{\phi}}}{3H\dot{\bar{\phi}}} \simeq \delta/3$$

# What we all know, but rarely state.

- The ”inflaton” is really the mean-field (1-point function) of a quantum degree of freedom (fundamental scalar field, composite order parameter, …).
- The ”potential” V is really the quantum effective potential, computed to some order in some expansion.
- Degree of freedom displaced from potential minimum $\longrightarrow$ inflation.




$$\bar{\phi}(t) \equiv \langle \hat{\phi}(\mathbf{x}, t) \rangle$$

$$V[\bar{\phi}] \equiv V_{\rm eff}[\bar{\phi}]$$

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# Semi-classical approximation

$$3M_{\rm pl}^2 H^2 = \langle T^{00}\rangle = \frac{1}{2}\dot{\bar\phi}^2 + V^{\rm eff}[\bar\phi]$$

$$3M_{\rm pl}^2\left(\dot H + H^2\right) = \frac{1}{2}\langle T^{00} + 3T^{ii}\rangle = -\dot{\bar\phi}^2 + V^{\rm eff}[\bar\phi]$$

$$\ddot{\bar\phi} + 3H\dot{\bar\phi} + V^{\rm eff}_{,\bar\phi} = 0$$

- Can't quantize gravity: treat as classical FRW.
- Can quantize scalar in that background: treat quantum.
- Solve for the vacuum…
- …compute the renormalized energy-momentum tensor.

Vacuum? (At least) three options:

- Minkowski space
- Expansion around H = 0 (adiabatic, WKB).
  - Parker, Toms (70', 80'), …, AT, Markkanen.
- Expansion around H = constant (slow-roll).
  - Boyanovski, De Vega, …, Serreau, Gautier, AT, Herranen, Markkanen,
  - Also Garbecht, Prokopec, …

In general: $V^{\rm eff}[\bar\phi] \neq V^{\rm eff}[\bar\phi] \neq V^{\rm eff}[\bar\phi]$ No exact slow-roll formalism.

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# Near de Sitter: 1PI

$$\hat{\phi} = \frac{1}{\sqrt{2(2\pi)^3 a^3}} \int d^3\mathbf{k}[a_\mathbf{k} h_\mathbf{k}(t) e^{i\mathbf{kx}} + h.c.],$$

$$h_\mathbf{k}(t) = \sqrt{\frac{\pi}{2H(1-\epsilon)}} \left[ C_1(\mathbf{k}) H_\nu^{(1)}(x) + C_2(\mathbf{k}) H_\nu^{(2)}(x) \right],$$

$$x = \frac{|\mathbf{k}|}{aH(1-\epsilon)}, \qquad \nu^2 = \frac{9}{4} + 3\epsilon + 3\epsilon^2 - \delta(1 + 2\epsilon + 3\epsilon^2) - \epsilon\delta_H \qquad \delta = \frac{m^2 + \frac{\lambda}{2}\bar{\phi}^2}{H^2}$$

1PI equation of motion (1-loop):

$$\ddot{\bar{\phi}} + 3H\dot{\bar{\phi}} + (\xi R + m^2 + \frac{\lambda}{6}\bar{\phi}^2)\bar{\phi} = -\frac{\lambda\bar{\phi}H^2}{16\pi^2} \left\{ \frac{3}{\delta - 3\epsilon + 3\epsilon + \epsilon\delta_H} + (\delta + \epsilon - 2) \log\left(\frac{H}{\mu}\right) \right\}$$

Massless, de Sitter limit ⟶ IR divergence. Must resum.

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# Near de Sitter: Hartree/2PI

$$\ddot{\bar{\phi}}+3H\dot{\bar{\phi}}+\left(M_{2\mathrm{PI}}^2-\frac{\lambda}{3}\bar{\phi}^2\right)\bar{\phi}=0$$

$$\frac{M_{2\mathrm{PI}}^2}{H^2}=\frac{\tilde{\theta}}{2}+\sqrt{\frac{\tilde{\theta}^2}{4}+\frac{3\tilde{\lambda}}{16\pi^2}}+3\epsilon-3\epsilon^2-\epsilon\delta_H$$

$$\tilde{\theta}=\frac{\tilde{M}^2}{H^2}-3\epsilon+3\epsilon^2+\epsilon\delta_H,\qquad \tilde{M}^2=\tilde{m}^2+\tilde{\xi}R+\frac{\tilde{\lambda}}{2}\bar{\phi}^2$$

$$\tilde{\lambda}=\frac{\lambda}{1-\frac{\lambda}{16\pi^2}\log\frac{H}{\mu'}}\qquad \tilde{m}^2=\frac{m^2}{1-\frac{\lambda}{16\pi^2}\log\frac{H}{\mu'}}\qquad \tilde{\xi}^2=\frac{1}{6}+\frac{\xi-\frac{1}{6}}{1-\frac{\lambda}{16\pi^2}\log\frac{H}{\mu'}}$$

IR divergence gone! Self-consistent mass is generated even for ”massless” limit.
(See also Serreau, Sloth, Beneke, …)

# Resummed Friedmann equations

$$3M_{\rm pl}^2H^2 = \frac{1}{2}\dot{\bar{\phi}}^2 + 6\frac{\xi}{\lambda}(H\partial_t - H^2)M_{\rm 2PI}^2 + W_{\rm 2PI}(\bar{\phi}, H, \epsilon),$$

$$3M_{\rm pl}^2\left(H^2 + \frac{2}{3}\dot{H}\right) = \frac{1}{2}\dot{\bar{\phi}}^2 + 6\frac{\xi}{\lambda}(-\frac{1}{3}(2H\partial_t + \partial_t^2) + H^2)M_{\rm 2PI}^2 - W_{\rm 2PI}(\bar{\phi}, H, \epsilon)$$

For minimal coupling to gravity: Partial slow-roll formulation:

$$V[\varphi] \neq W_{\rm 2PI} = W_{\rm 2PI}$$

$$\epsilon H^2 = \frac{\dot{\phi}^2}{2M_{\rm pl}^2}.$$

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# Corrections to the CMB

Corrections from the dynamics of H and the mean field;
and from the self-consistent interacting spectrum

$$P_R(k) = \left(\frac{H}{\dot{\phi}}\right)^2 \left(\frac{H}{2\pi}\right)^2 |_{k=aH}$$

$$(n_s - 1) = (n_s - 1)_C + (n_s - 1)_Q, \quad (n_s - 1)_Q = -4\epsilon_Q - 2\delta_{HQ}$$

$$\frac{\epsilon_Q}{\epsilon_C} \sim \frac{\delta_{QH}}{\delta_C} \sim \frac{\lambda}{16\pi^2}\frac{(2N_C+1)^3}{3}, \quad \frac{1}{16\pi^2}\frac{m^2}{M_{\rm pl}^2}\frac{(2N_C+1)^2}{2},$$

$$\lambda \simeq 10^{-15}, \quad N_C \simeq 100, \quad m \simeq 10^{-6} M_{\rm pl}$$

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# Beyond semi-classical?

- A cosmologist would say:
  - Hang on! Scalar field fluctuations mix with scalar metric perturbations.

$$\hat{\phi}, \psi, A, E, B.$$

  - Should quantize the single physical degree of freedom (in a gauge/ gauge invariant variable).
- A particle physicist would say:
  - Hang on! In (near) Minkowski space at low energies, we can neglect metric fluctuations and just quantize.

$$\hat{\phi}$$

  - The semi-classical approximation must be some low-energy limit of something.

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# In Newtonian gauge

$$E = B = 0, \qquad A = \psi, \qquad 2M_{\rm pl}^2 \hat{\phi}\dot{\bar{\phi}} = \dot{A} + HA$$

$$\left(1 + 6\frac{1}{4M_{\rm pl}^4}F_1\right)\bar{\phi}'' + 2H\left(1 + 6\frac{1}{4M_{\rm pl}^4}(F_1+F_2)\right)\bar{\phi}' + \left(1 - 2\frac{1}{4M_{\rm pl}^4}F_1\right)a^2V_{,\phi} =$$

$$\frac{4H}{2M_{\rm pl}^2\bar{\phi}'}\left[\left(\left(\frac{H''}{H^3} + 2\frac{H'}{H^2}\right)F_1 + \left(2 + 3\frac{H'}{H^2}\right)F_2 + 3F_3 + F_4 + F_5 + F_6\right) - \frac{\bar{\phi}'''}{\bar{\phi}'H^2}(F_1+F_2)\right.$$

$$\left. -2\left(\frac{\bar{\phi}''}{\bar{\phi}'H}\right)^2(F_1-F_2) - \frac{\bar{\phi}''}{\bar{\phi}'H}\left(\left(2 + 2\frac{H'}{H^2}\right)F_1 + 5F_2 + 2F_3 + F_4\right) + \frac{1}{2}(F_1+F_2)\frac{a^2V_{,\phi\phi}}{H^2}\right]$$

$$-\frac{H^2}{2(\bar{\phi}')^2}(F_1 + 2F_2 + F_4)a^2V_{,\phi\phi\phi}.$$

$$-\frac{1}{2}(\hat{\phi})^2a^2V_{,\phi\phi\phi},$$

Preliminary

# Conclusion

- We can compute quantum dynamics in slow-roll spacetimes.
- Without resummation: ok, but massless de Sitter limit IR divergent.
- With resummation: IR divergence becomes interesting IR physics. (As for dS, see talks by Serreau, Gautier)
- Still must be cautious with SR truncation.
- Slow-roll formalism partly available (at this order).
- Difficult to generalize beyond 2PI/LO(?) (Gautier/Serreau).
- Corrections to CMB negligible for inflaton. Substantial for curvaton(?)
- Under consideration: Range of validity of semi-classical approximation as low-energy limit of quantized scalar-gravity theory.