Running coupling corrections to the evolution of jet-quenching

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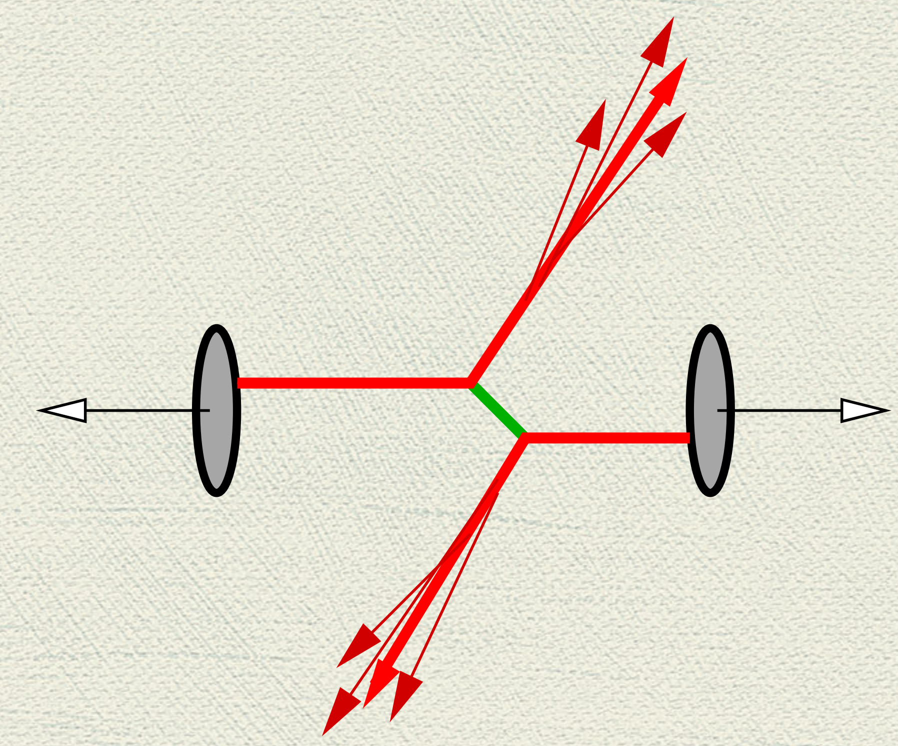
E. Iancu & DNT, submitted to PRD (no answer at all yet …) [arXiv:1405.3525]

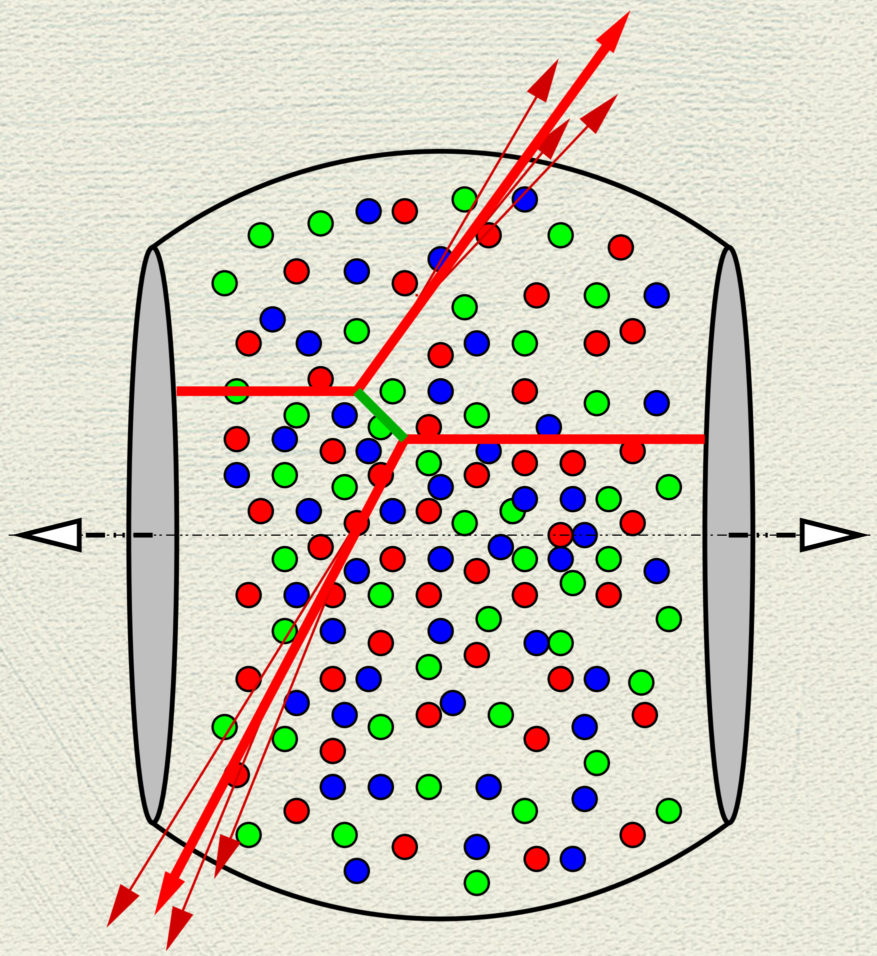
Outline

- $p_\perp$-broadening in shockwave and in medium at tree-level
- Short-lived quantum fluctuations and evolution
- Double logarithmic approximation (vs single-log in SW)
- Running coupling corrections to jet quenching parameter

Liou, Mueller, Wu : arXiv:1304.7677 (Double logs in $\boldsymbol{p}_\perp$-broadening)
Iancu : arXiv:1403.1996 (Non-linear evolution and its DLA limit)
Blaizot, Mehtar-Tani : arXiv:1403.2323 (Energy-loss and renormalization of $\hat{q}$)

Jet modification in a medium

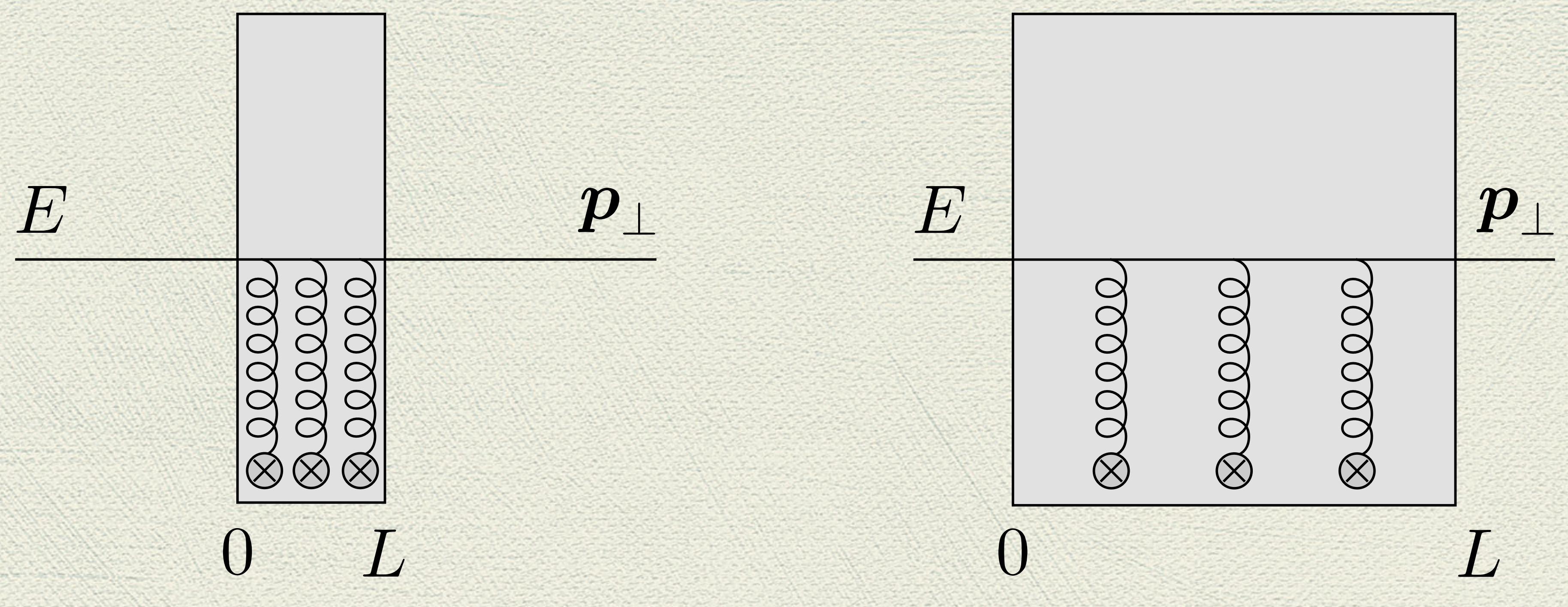




□ Partons typically produced in pairs after hard scattering

□ Parton propagation modified in medium

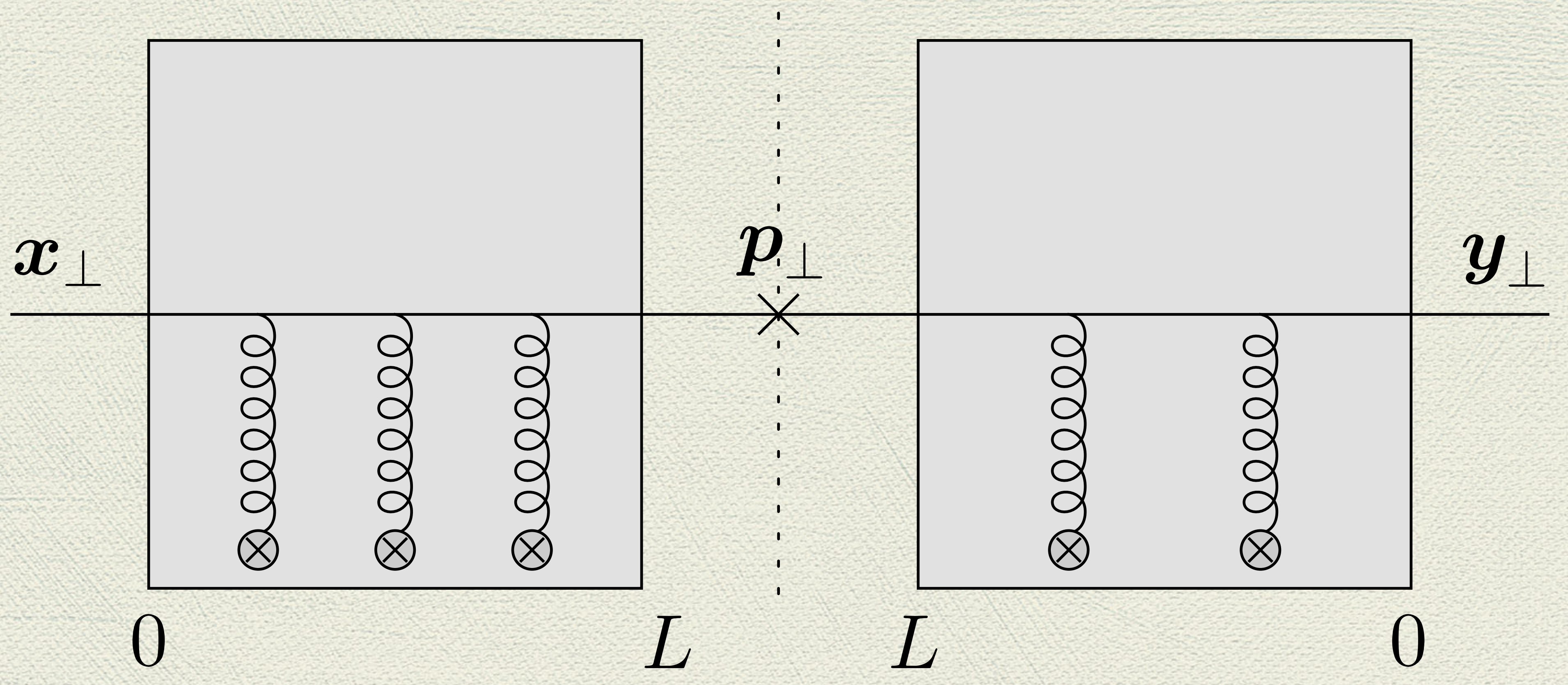
$p_\perp$-broadening in SW and medium



- High energy $\mapsto \theta \sim p_\perp/E \ll 1 \mapsto$ eikonal approximation
- Wilson line $V^\dagger(x_\perp) = \mathrm{P}\exp\left[\mathrm{i}g\int_0^L \mathrm{d}x^+ A^-(x^+, x_\perp)\right]$
- x^+ projectile LC time
- Target RF: pA $E_p \sim 10^7\mathrm{GeV}$, jet in AA $E_J \sim 10^2\mathrm{GeV}$

  $p_\perp \sim 1 \div 2\mathrm{GeV}$

Color dipoles and target average



$$\frac{\mathrm{d}N}{\mathrm{d}^2\boldsymbol{p}} = \frac{1}{(2\pi)^2}\int_{\boldsymbol{r}} \mathrm{e}^{-\mathrm{i}\boldsymbol{p}\cdot\boldsymbol{r}} \left\langle \frac{1}{N_c}\mathrm{tr}V^\dagger(\boldsymbol{x})V(\boldsymbol{y}) \right\rangle$$

- Average over target, CGC or plasma, color field
- Tree-level: Gaussian distribution (MV model)

Gaussian target average

- Independent color charges down to $m_{\rm D} \sim gT$ or $\Lambda_{\rm QCD}$

$$\left\langle A_a^-(x^+, \boldsymbol{x})\, A_b^-(y^+, \boldsymbol{y})\right\rangle = \delta_{ab}\delta(x^+ - y^+)\, n_0 \gamma(\boldsymbol{x} - \boldsymbol{y})$$

- Coulomb propagator squared $\gamma(\boldsymbol{k}) = \dfrac{g^2}{(\boldsymbol{k}^2 + \Lambda^2)^2}$

- n_0: constituents number density

- Projectile dipole

$$\mathcal{S}^{(0)}(\boldsymbol{r}) = \exp\left[-\, g^2 C_F n_0 L \int \frac{{\rm d}^2\boldsymbol{k}}{(2\pi)^2}\, \gamma(\boldsymbol{k})\left(1 - {\rm e}^{{\rm i}\boldsymbol{k}\cdot\boldsymbol{r}}\right)\right] = \exp\left[-\, T_{2{\rm g}}\right]$$

The jet quenching parameter

- Logarithmic contribution for small dipoles $\boldsymbol{r}\Lambda \ll 1$

$$\mathcal{S}^{(0)}(\boldsymbol{r}) = \exp\left[-\frac{1}{4}\,\hat{q}^{(0)}(1/\boldsymbol{r}^2)L\boldsymbol{r}^2\right]$$

with tree-level jet quenching parameter

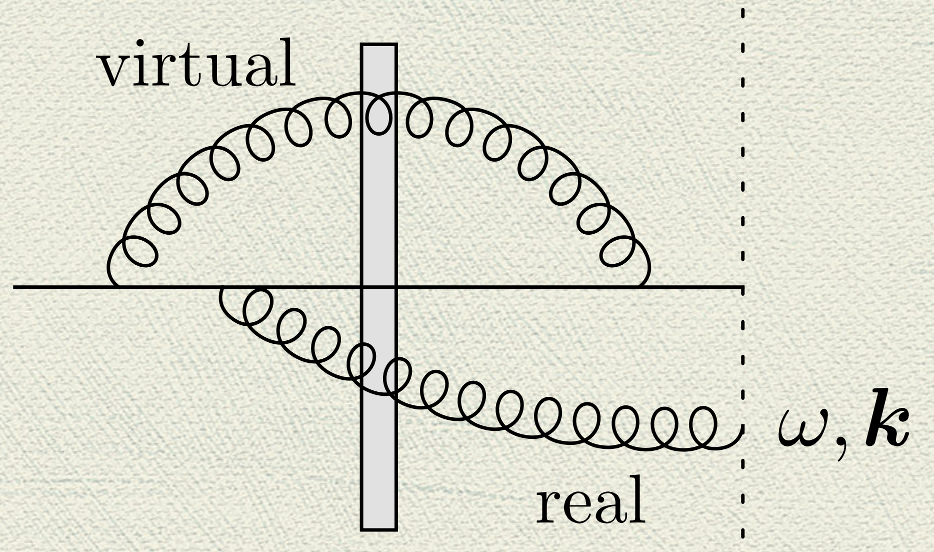
$$\hat{q}^{(0)}(Q^2) \;\simeq\; 4\pi\alpha_s^2 C_F n_0 \ln\frac{Q^2}{\Lambda^2} \sim \alpha_s \times \text{3D charge density}$$

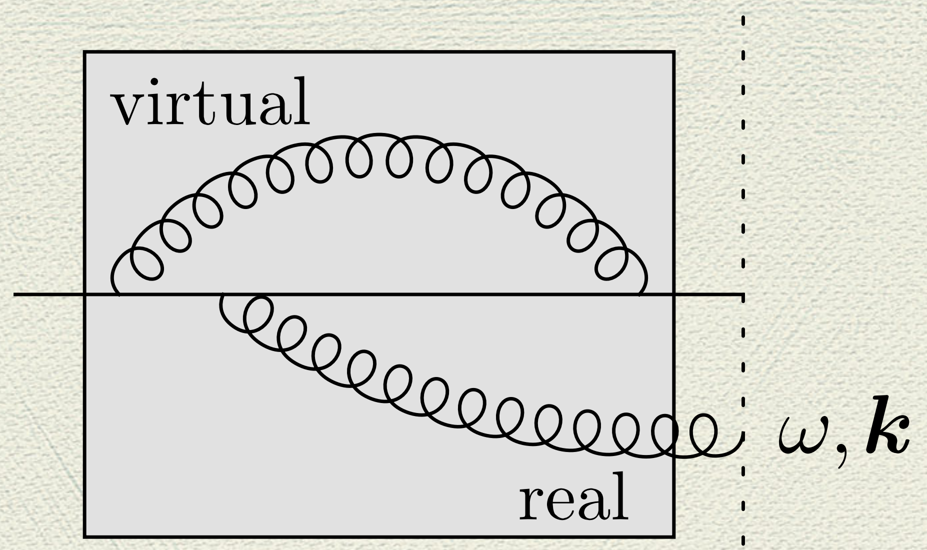
- Saturation momentum: exponent of $\mathcal{O}(1)$ when $rQ_s \sim 1$

$$Q_s^2(L) = \hat{q}^{(0)}(Q_s^2)\,L = 4\pi\alpha_s^2 C_F n_0 L \ln\frac{Q_s^2}{\Lambda^2} \sim L\ln L$$

- In CNM, this is $A^{1/3}\ln A$ dependence of $Q_s^2(A)$

Quantum evolution



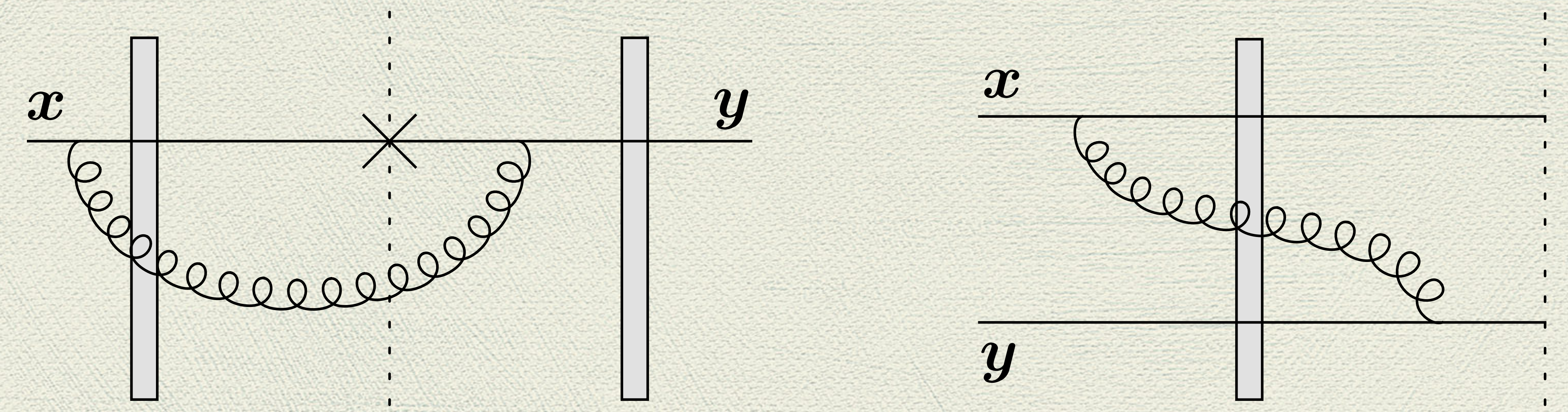


- Quantum fluctuation has formation time $\tau = 2\omega/\boldsymbol{k}^2$
- Typical case in SW $\tau \gg L$

$$\boldsymbol{k}^2 \sim Q_s^2(L) = \hat{q}L \qquad \tau_{\min} = 2\omega/\hat{q}L \qquad \omega \gtrsim \omega_c = \hat{q}L^2/2$$

- In a medium $\omega \lesssim \omega_c$ typical fluctuations live inside it

Dipole evolution in SW

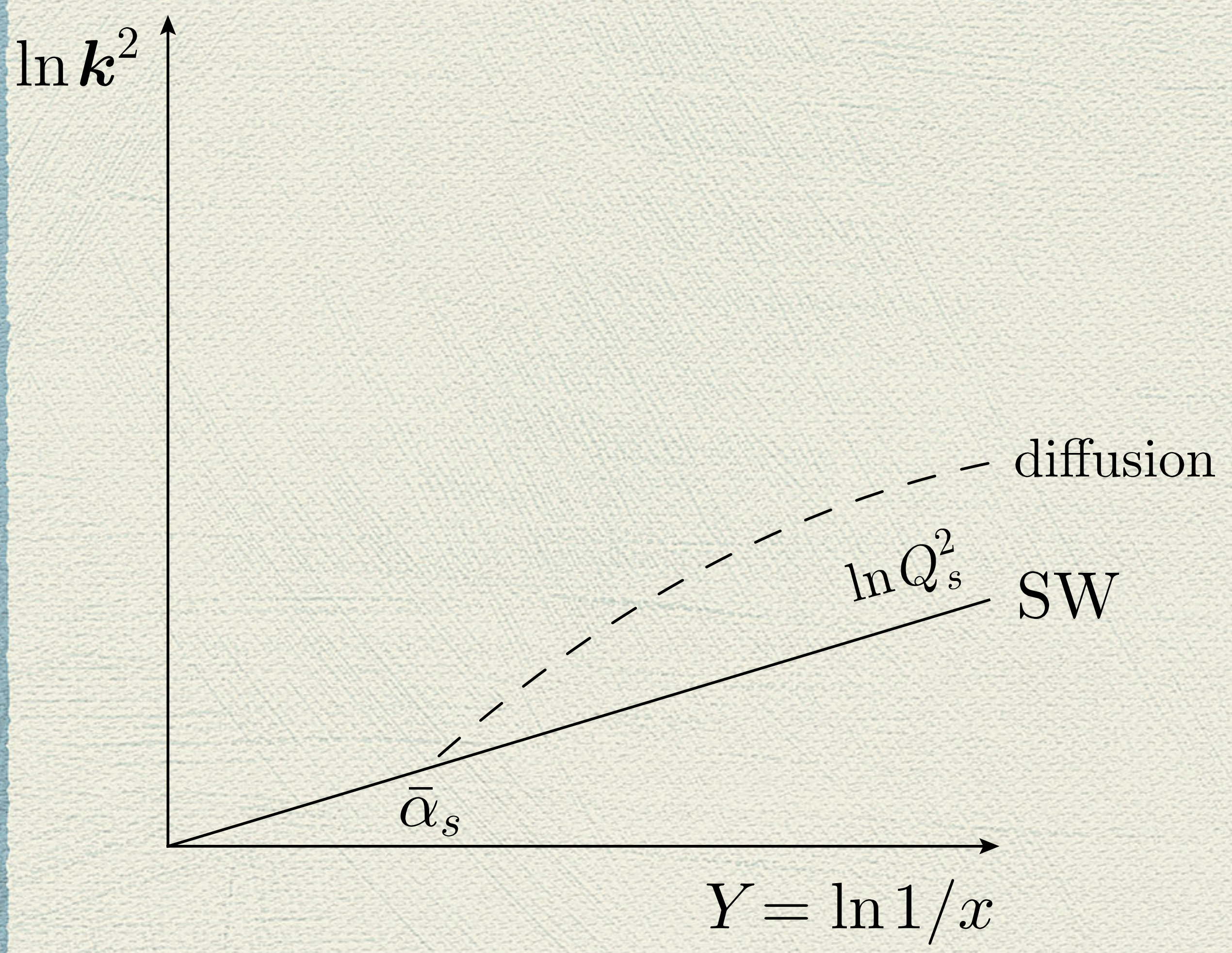


- B-JIMWLK, BK at large N_c : LL eqn in Y (rapidity separation)

$$\frac{\mathrm{d}\,\langle S_{\boldsymbol{xy}}\rangle}{\mathrm{d}Y} = \frac{\bar{\alpha}}{2\pi}\int \mathrm{d}^2\boldsymbol{z}\,\frac{(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{z}-\boldsymbol{y})^2}\,\langle S_{\boldsymbol{xz}}S_{\boldsymbol{zy}} - S_{\boldsymbol{xy}}\rangle$$

- Non-linear: target saturation/unitarity in multiple scattering

Saturation momentum in SW



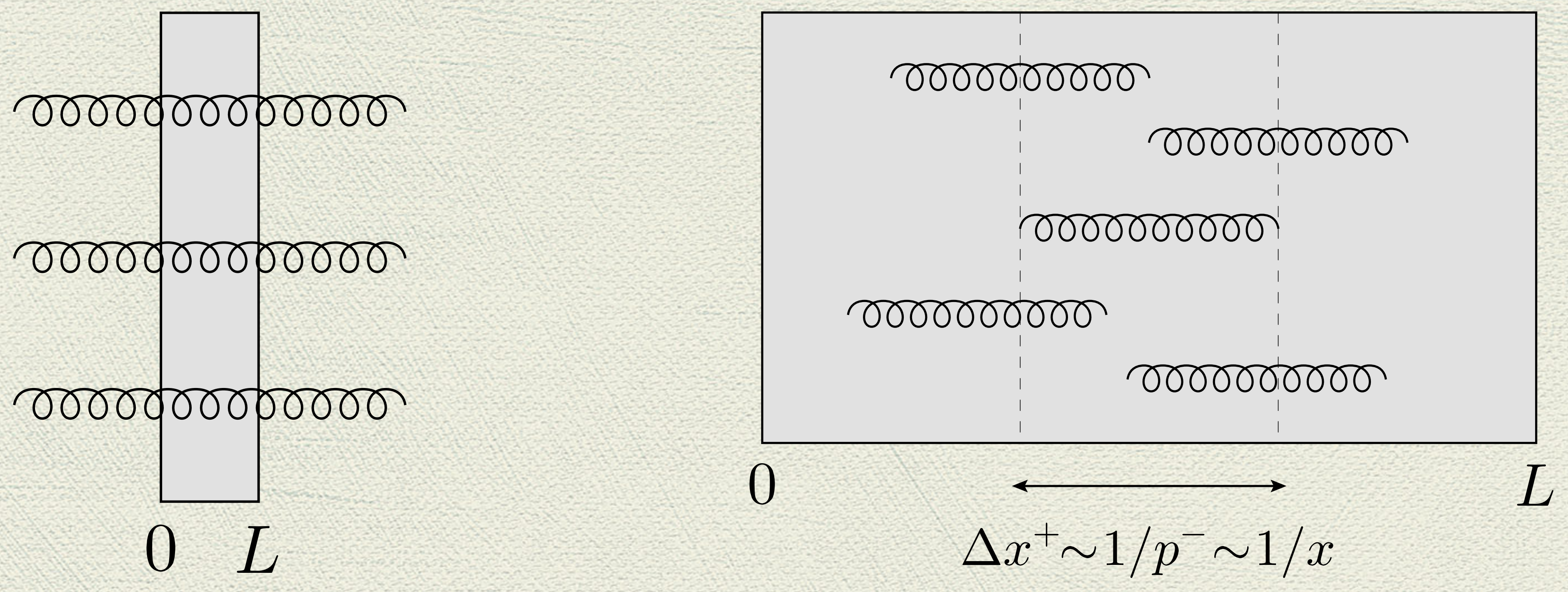
FC $\ln Q_s^2(Y) = \bar{\alpha}\dfrac{\chi(\gamma_s)}{\gamma_s} Y - \dfrac{3}{2\gamma_s}\ln(\bar{\alpha}Y)$

RC $\ln Q_s^2(Y) = c_1\sqrt{bY} - c_2|\xi_1|(bY)^{1/6}$

only FC num. confirmed

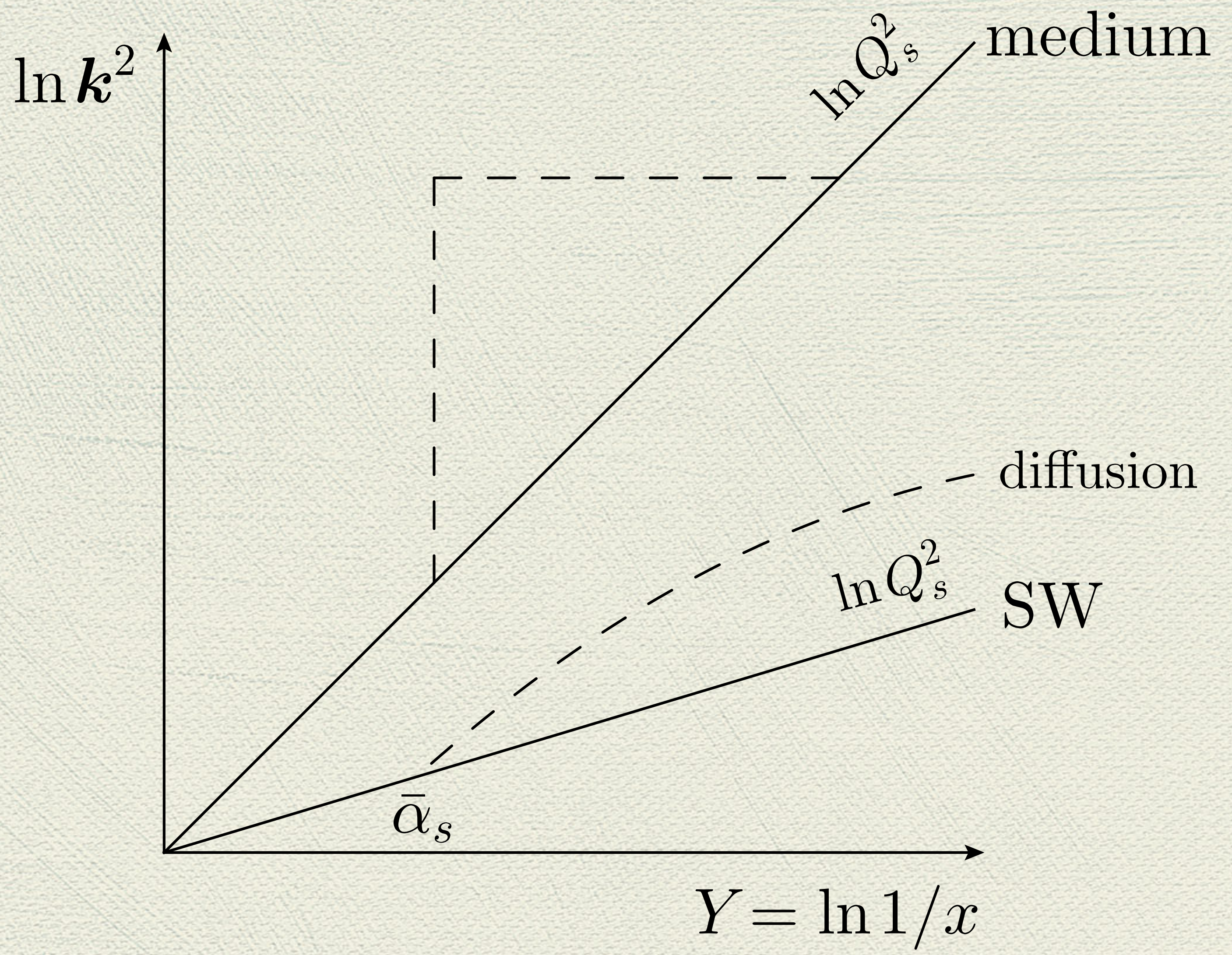
- Occupation numbers with $\boldsymbol{k} \lesssim Q_s(Y)$ saturate to $\sim 1/\bar{\alpha}_s$
- Linear evolution + saturation boundary

Initial sat. mom. in medium and SW



- Saturation momentum proportional to length over which gluon overlaps with its source
- In SW problem $\sim L$
- Δx^+: time τ for R-mover, longitudinal size for L-mover
- In medium problem $\sim 1/x$, fraction x is not that small

SW vs medium phase-space



- SW: single log problem (BFKL)
- Medium: symmetry in two types of logs $\longmapsto$ double log (DLA)

Double logarithms

$$\tau \frac{\mathrm{d}S(\boldsymbol{r})}{\mathrm{d}\tau} = \frac{\bar{\alpha}}{2\pi} \int \mathrm{d}^2\boldsymbol{z} \, \frac{\boldsymbol{r}^2}{\boldsymbol{z}^2(\boldsymbol{r}-\boldsymbol{z})^2} \exp\left[-[\boldsymbol{r}^2 + (\boldsymbol{r}-\boldsymbol{z})^2]\frac{1}{4}\hat{q}^{(0)}\tau\right]$$

- First iteration in non-linear equation, no virt. for big dipoles
- Integrate time τ from λ to L
- Integrate transverse from $\hat{q}^{(0)}\tau$ up to res.scale $1/\boldsymbol{r}^2 \sim \boldsymbol{p}^2$
- Single scattering approximation to get the second-logarithm

$$\ln(L/\lambda)\ln(\boldsymbol{p}^2/\hat{q}^{(0)}\tau)$$

- Absorb in definition of jet-quenching parameter

Fixed coupling

- Evolution equation for $\hat{q}(Y,\rho) = \hat{q}(\ln \tau/\lambda, \ln \boldsymbol{p}^2/\hat{q}^{(0)}\lambda)$

$$\hat{q}(Y,\rho) = \hat{q}^{(0)} + \bar{\alpha} \int_0^Y \mathrm{d}Y_1 \int_{Y_1}^{\rho} \mathrm{d}\rho_1 \, \hat{q}(Y_1,\rho_1)$$

- Lower limit restricts PS to single scattering

- Solution for const. IC

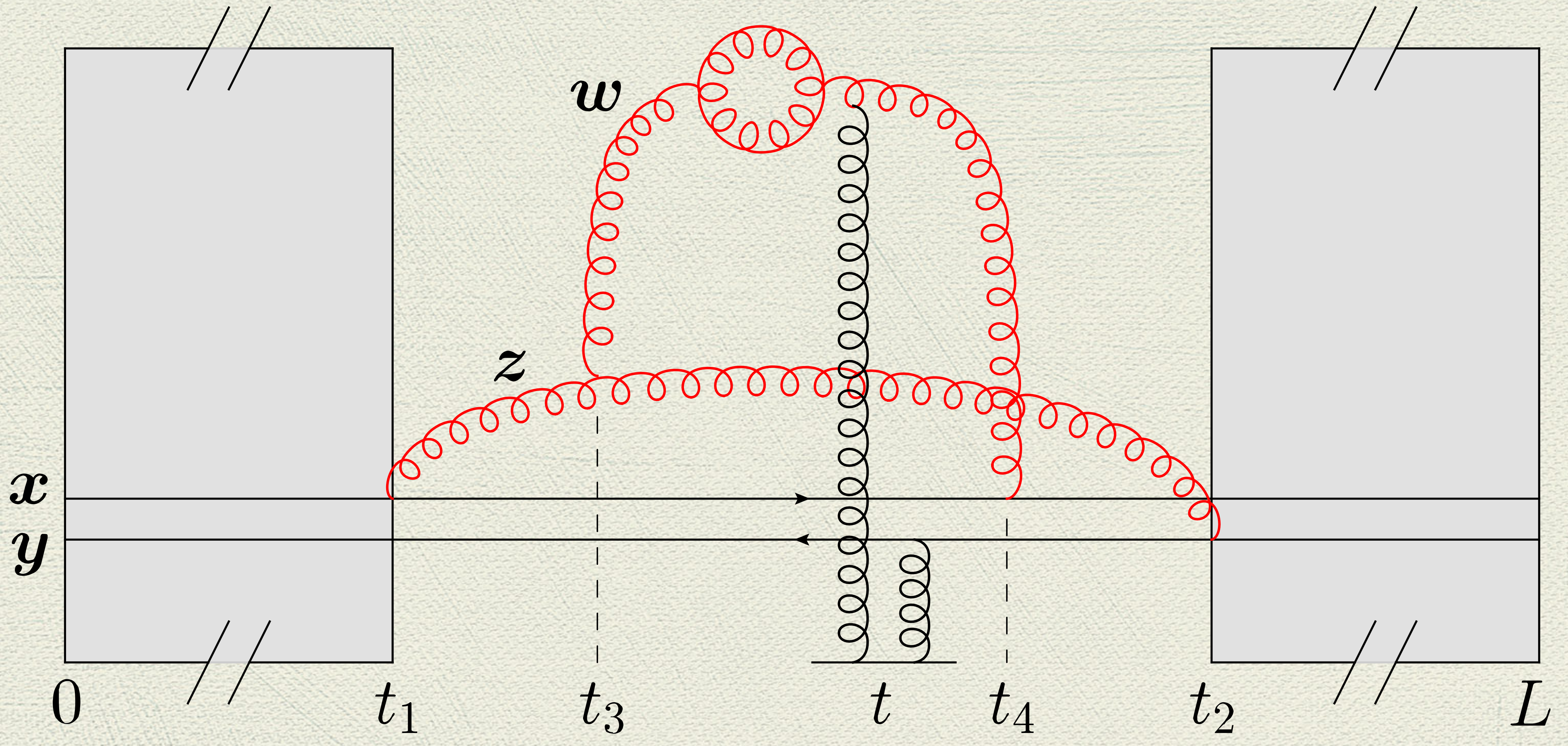
$$\hat{q}(Y,\rho) = \hat{q}^{(0)} \left[\mathrm{I}_0\big(2\sqrt{\bar{\alpha} Y \rho}\big) - \frac{Y}{\rho}\, \mathrm{I}_2\big(2\sqrt{\bar{\alpha} Y \rho}\big) \right]$$

- For $\rho = Y$ ($\tau = L$, $\boldsymbol{p}^2 = \hat{q}^{(0)} L$) leading prefactors cancel

$$\hat{q}_s(Y) = \hat{q}^{(0)} \, \frac{\mathrm{I}_1\big(2\sqrt{\bar{\alpha}}\, Y\big)}{\sqrt{\bar{\alpha}}\, Y} = \hat{q}^{(0)} \, \frac{\mathrm{e}^{2\sqrt{\bar{\alpha}}\, Y}}{\sqrt{4\pi}\, (\sqrt{\bar{\alpha}} Y)^{3/2}}$$

- Very similar to $Q_s^2(Y)$ for scattering of shockwave

Running coupling



Running coupling

- DLA → running coupling is leading order effect

$$\hat{q}(Y,\rho) = \hat{q}^{(0)} + b\int_0^Y \mathrm{d}Y_1 \int_{Y_1}^{\rho} \frac{\mathrm{d}\rho_1}{\rho_1+\rho_0}\,\hat{q}(Y_1,\rho_1)$$

- In general $\rho_0 = \ln\left(\hat{q}^{(0)}\lambda/\Lambda^2\right)$. For $\rho_0 = 0$

$$\hat{q}^{(1)}(Y,\rho) = \hat{q}^{(0)} bY \left(\ln\frac{\rho}{Y} + 1\right),$$

$$\hat{q}^{(2)}(Y,\rho) = \hat{q}^{(0)} (bY)^2 \left(\frac{1}{4}\ln^2\frac{\rho}{Y} + \frac{3}{4}\ln\frac{\rho}{Y} + \frac{3}{8}\right),$$

$$\hat{q}^{(3)}(Y,\rho) = \hat{q}^{(0)} (bY)^3 \left(\frac{1}{36}\ln^3\frac{\rho}{Y} + \frac{11}{72}\ln^2\frac{\rho}{Y} + \frac{49}{216}\ln\frac{\rho}{Y} + \frac{49}{648}\right)$$

- Series in $(bY)^n$ for $\rho = Y$, no pattern for lowest orders (Standard DLA: Only highest log-power is present)

Running coupling asymptotics

- Calculate series coefficients at very high orders
  Fit analytical data

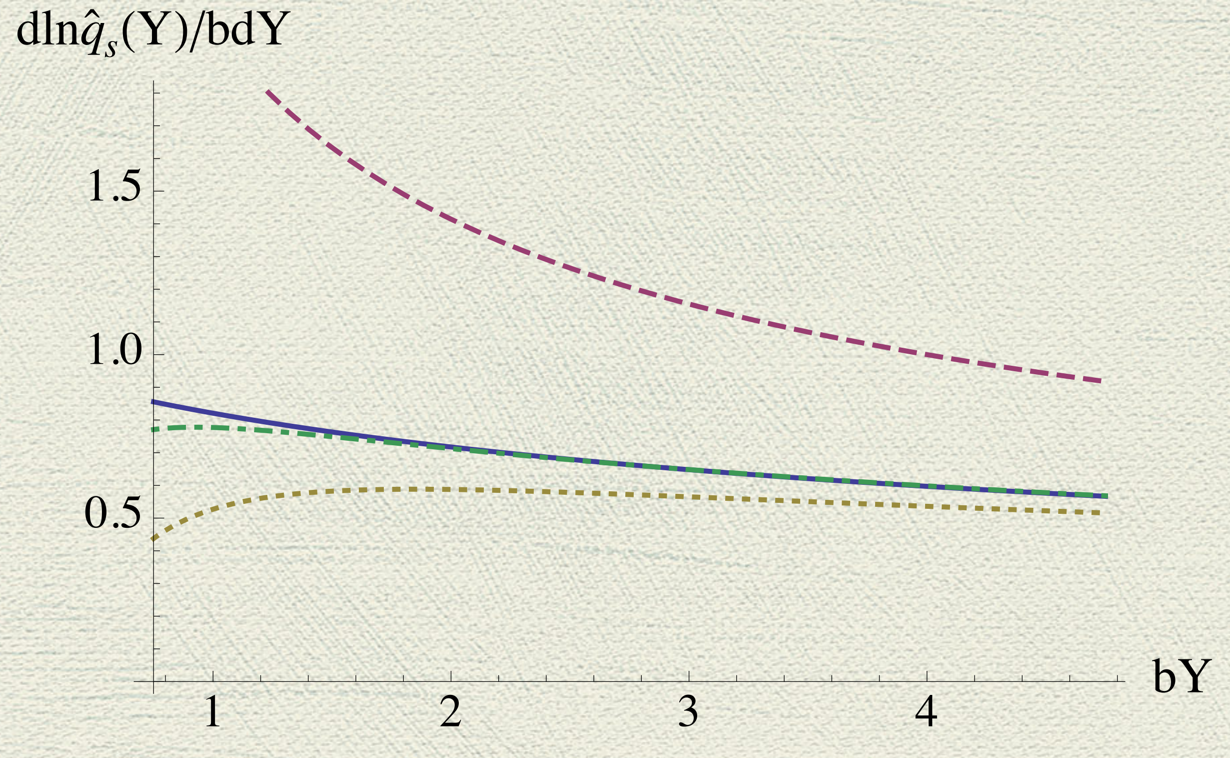
$$\ln \hat{q}_s(Y) \;=\; 4\sqrt{bY} - 3|\xi_1|(4bY)^{1/6} + \tfrac{1}{4}\ln Y + \kappa + \mathcal{O}\big(Y^{-1/6}\big)$$

- $\xi_1 \simeq -2.338,\ \kappa = \text{const}$ and recall $Y = \ln(L/\lambda)$

- Striking similarity to $\ln Q_s^2(Y)$ for scattering of shockwave
  Exactly same dependence, different coefficients

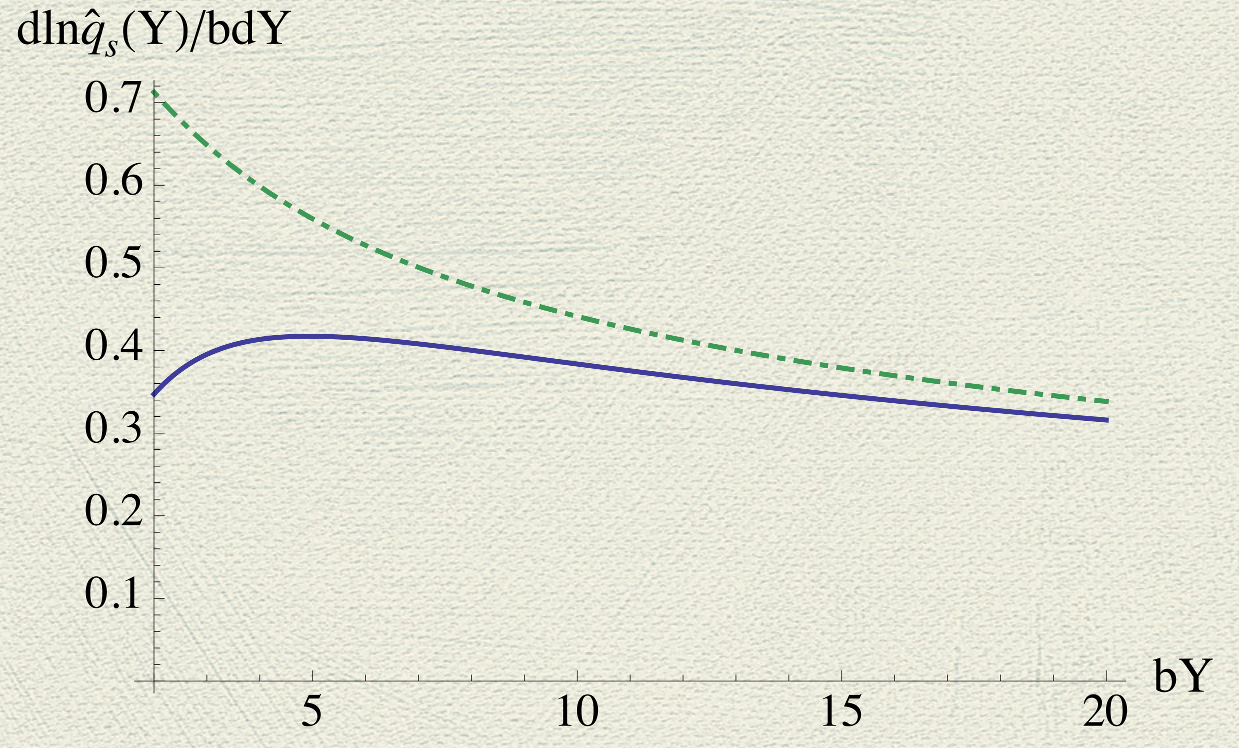
- Should exist an analytical proof …
  Should exist numerical proof for SW …

Running coupling results



Truncated vs asymptotic

$\rho_0 = 0$

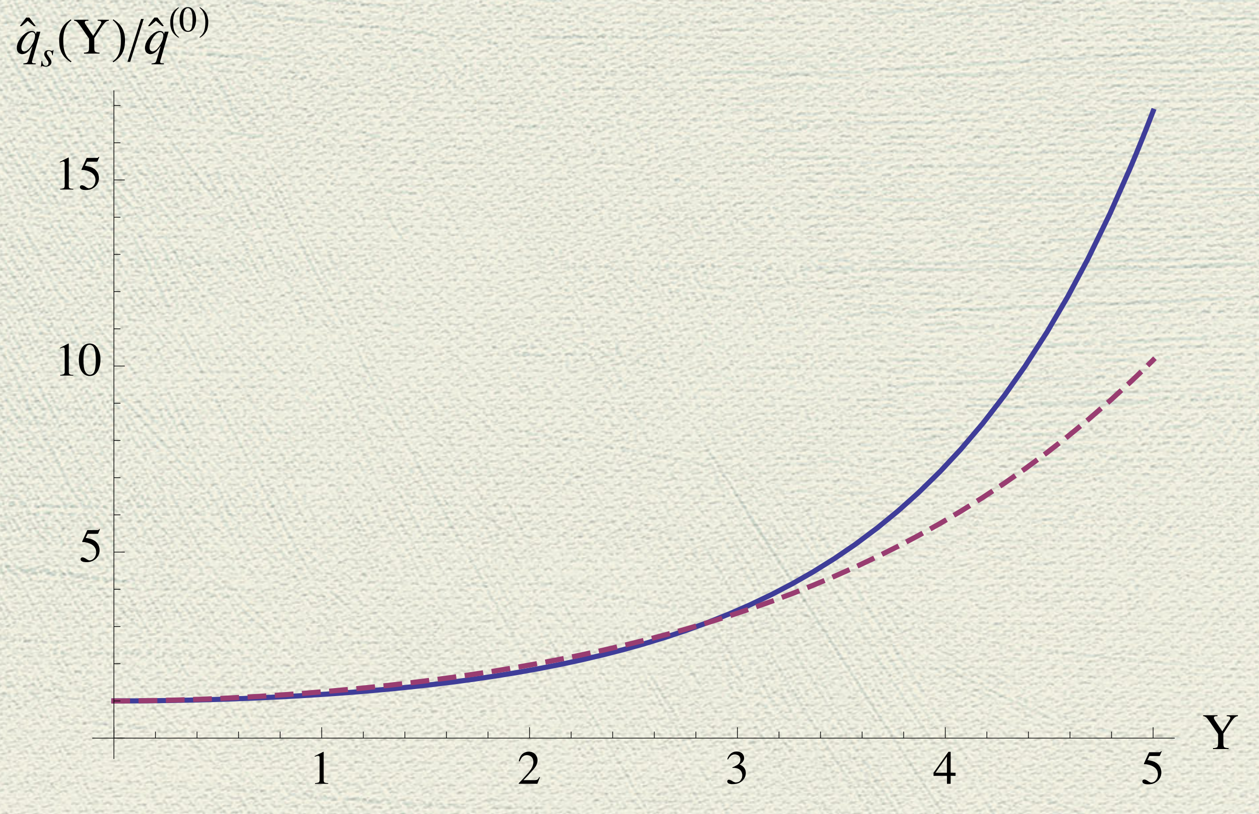


Truncated vs asymptotic

$\rho_0 = 2.3,\ b = 4/3$

- Series dominated by $n_c \sim 2\sqrt{bY}$
- IR "cutoff" does not change (very) high Y asymptotics

Fixed vs running



FC vs RC, IC const.

$$\bar{\alpha}_s = 0.33,\ b = 4/3,\ \rho_0 = 2.3$$

- For up to three units in rapidity FC~RC

$$Y = \ln(LT) \to Y_{\max} \sim \ln(8\text{fm} \times 500\text{MeV}) \sim 3$$

L-dependence of energy loss

$$E_{\text{loss}}(\text{tree}) \propto L^2$$

$$E_{\text{loss}}(\text{FC}) \propto L^{2+\gamma}, \quad \gamma = 2\sqrt{\bar{\alpha}}$$

$$E_{\text{loss}}(\text{RC}) \propto L^2 \mathrm{e}^{4\sqrt{b \ln L}}, \quad b = 12N_c/(11N_c - 2N_f)$$

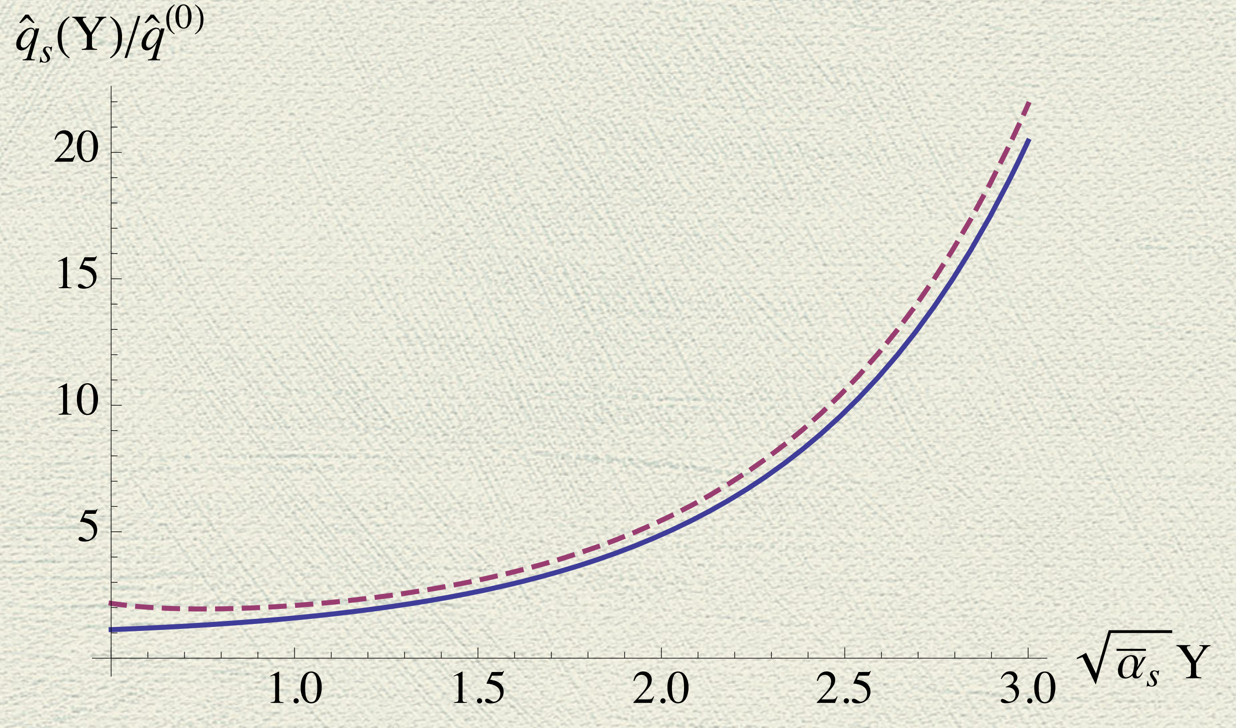
$$E_{\text{loss}}(\text{AdS/CFT}) \propto L^3$$

- FC interpolates the tree and AdS/CFT result
- RC in same direction, but different structure. Not conformal
- Leading asymptotics results not to be trusted.
  In practice: keep few terms series or fit to $E_{\text{loss}}(\text{fit}) = f(L)$

Conclusions

- Short-lived fluctuations redefine value of $\hat{q}$
- RC is leading order effect in DLA
- (Math of) Evolution similar to that of $\ln Q_s^2(Y)$ of a SW
- Universality: asymptotics independent of IC, for FC and RC
- An enhancement factor of ~3 for three rapidity units

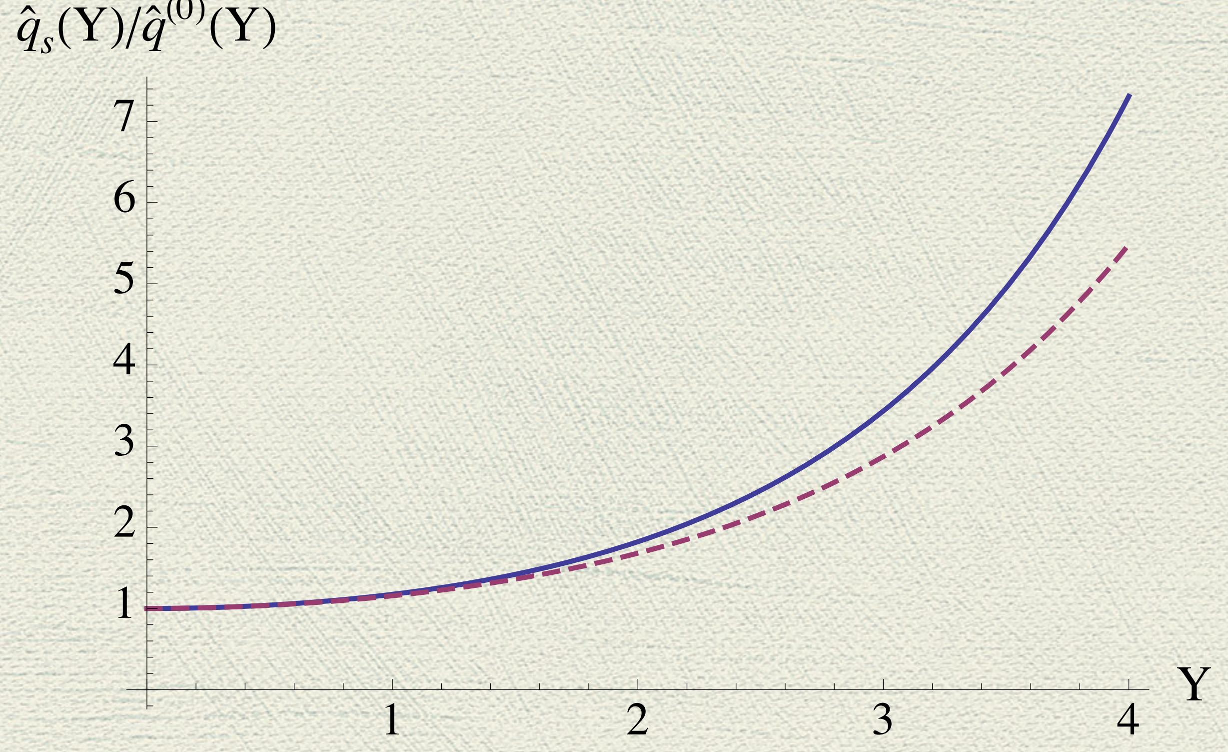
Fixed coupling results

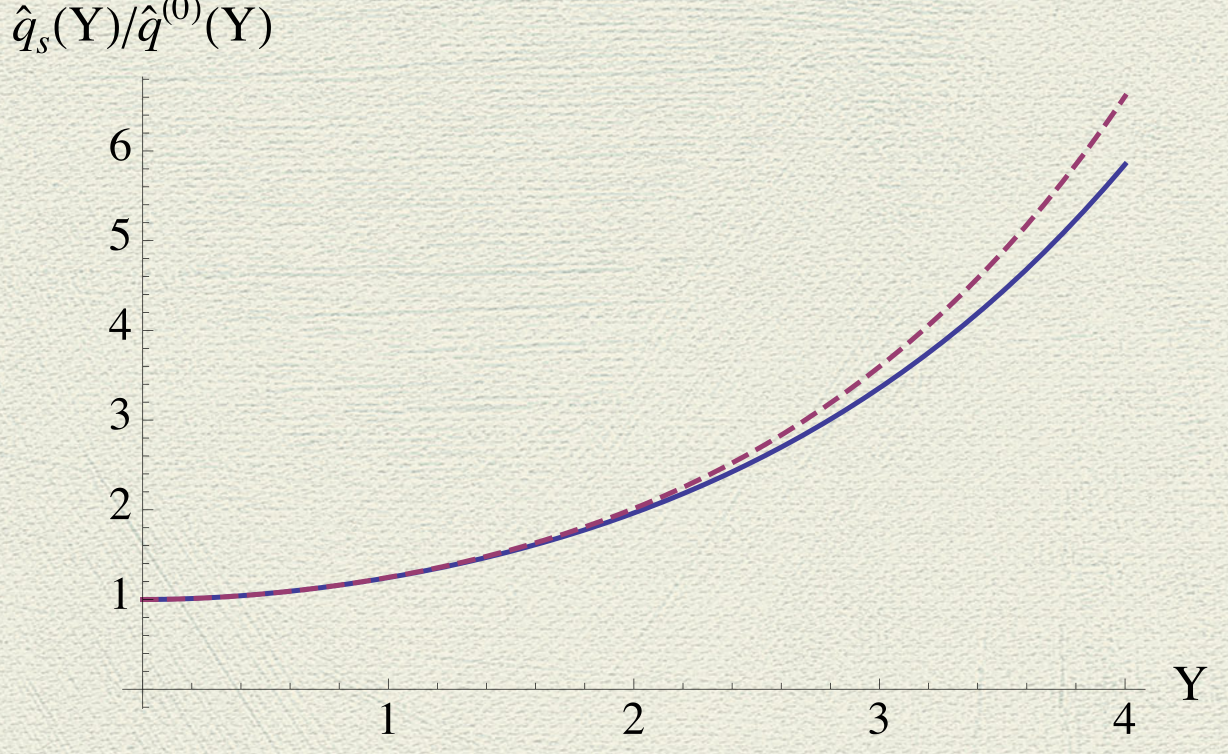


Exact vs asymptotic *Exact vs truncated (n=3,4,5)*

- □ Asymptotic correct down to $\sqrt{\bar{\alpha}_s}Y \sim 1$
- □ $n_c \sim \sqrt{\bar{\alpha}_s}Y$ terms reproduce exact result
- □ Thus: fixed order series same with asymptotic result

Initial conditions





FC, IC const. vs $\rho + \rho_0$

$\bar{\alpha}_s = 0.33,\ \rho_0 = 2.3$

RC, IC const. vs $\frac{\ln(\rho+\rho_0)}{\rho+\rho_0}$

$\rho_0 = 2.3,\ b = 4/3$

- IC ρ-dependence from gluon distribution and/or coupling