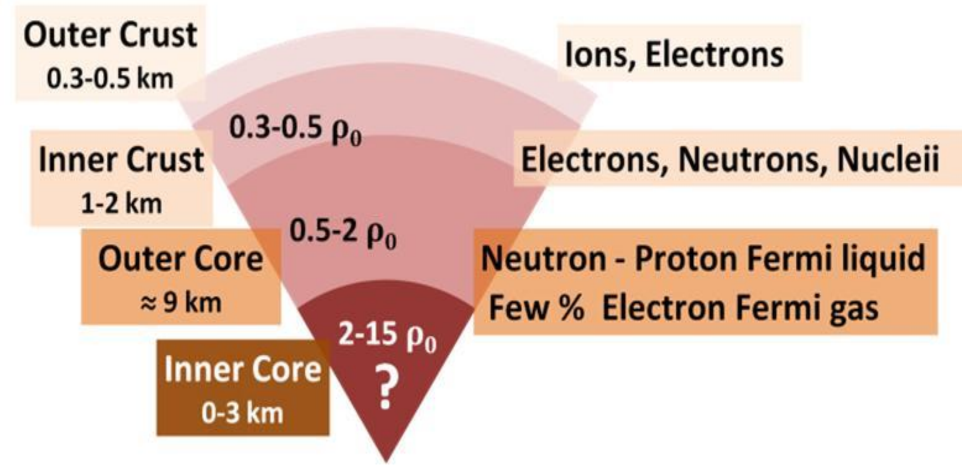
# Constraining neutron star properties with perturbative QCD

Aleksi Vuorinen
University of Helsinki

Strong and Electroweak Matter 2014
EPFL, Lausanne, 15.7.2014

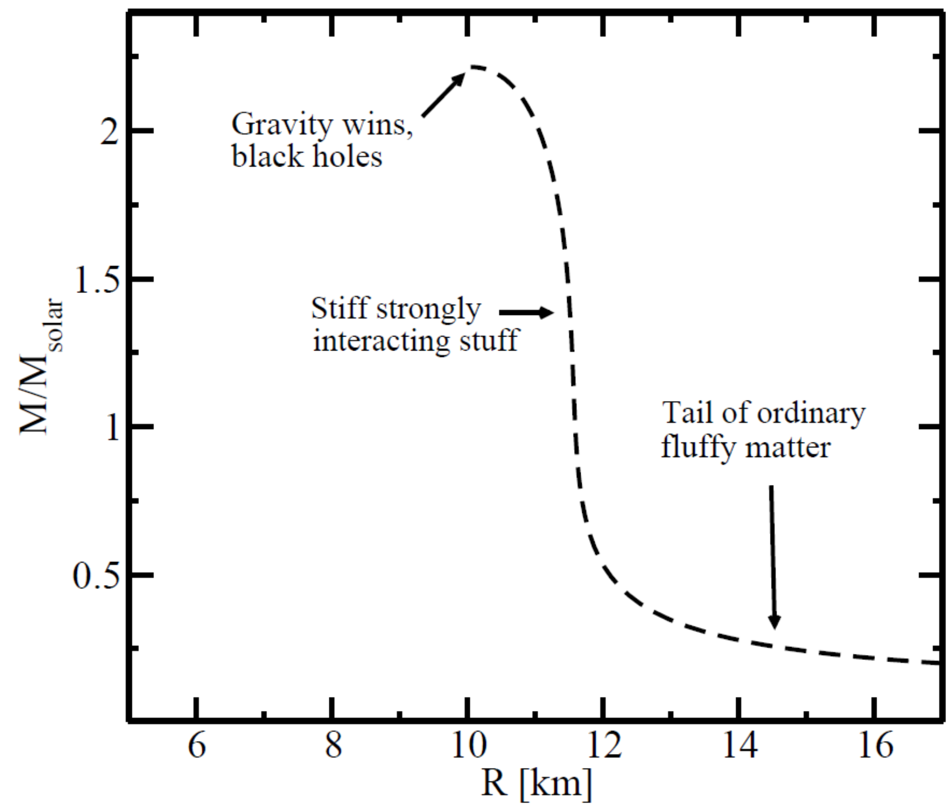
Main reference: Kurkela, Fraga, Schaffner-Bielich, AV, ApJ 789 (1402.6618)

Classic problem in nuclear astrophysics: Predict composition and main properties of neutron stars




Characteristics:

- Masses $\lesssim 2M_{\rm sun}$
- Radii $\sim 15{\rm km}$
- Densities $\lesssim 15n_{\rm s}$
- Temperatures $\lesssim {\rm keV}$
- Spin frequencies $\lesssim {\rm kHz}$



$$
\begin{aligned}
\frac{dM(r)}{dr} &= 4\pi r^2 \varepsilon(r), \\
\frac{dp(r)}{dr} &= -\frac{G\varepsilon(r)M(r)}{r^2} \frac{(1+p(r)/\varepsilon(r))\left(1+4\pi r^3 p(r)/M(r)\right)}{1-2GM(r)/r}
\end{aligned}
$$

$$
\varepsilon(p) \Rightarrow M(R)
$$

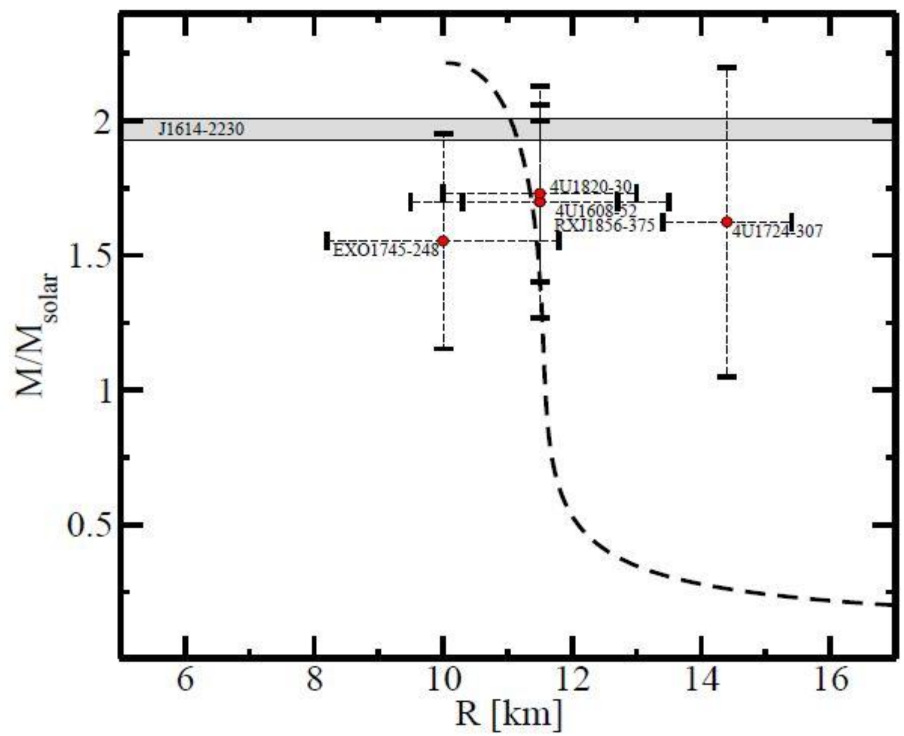
Challenge: Find EoS of nuclear/quark matter that is

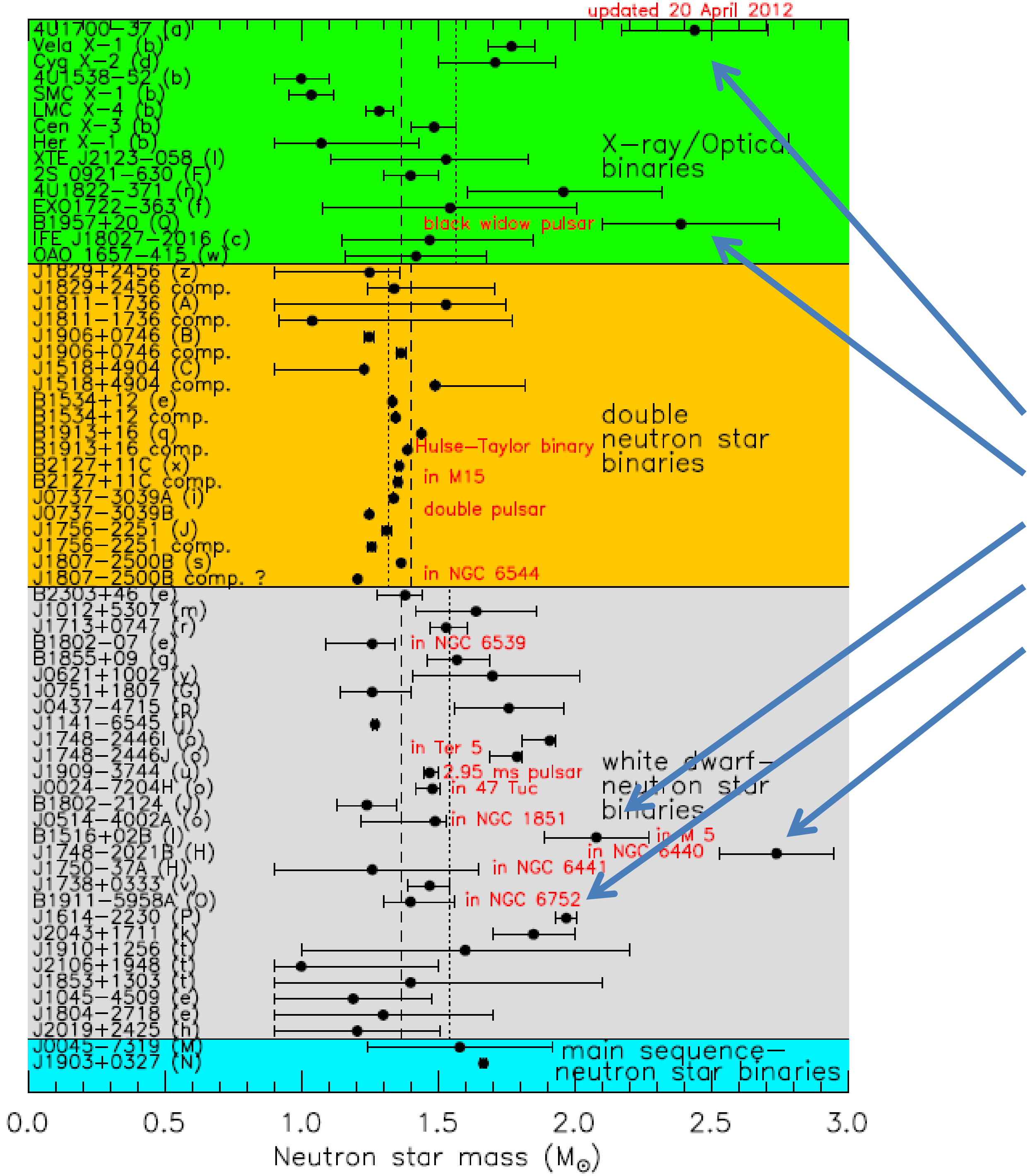
- Cold: $T \approx 0$
- Electrically neutral:

  $2/3n_u - n_d/3 - n_s/3 + n_e = 0$
- In beta equilibrium:

  $\mu_B/3 = \mu_d = \mu_s = \mu_u + \mu_e$

and compare to observations

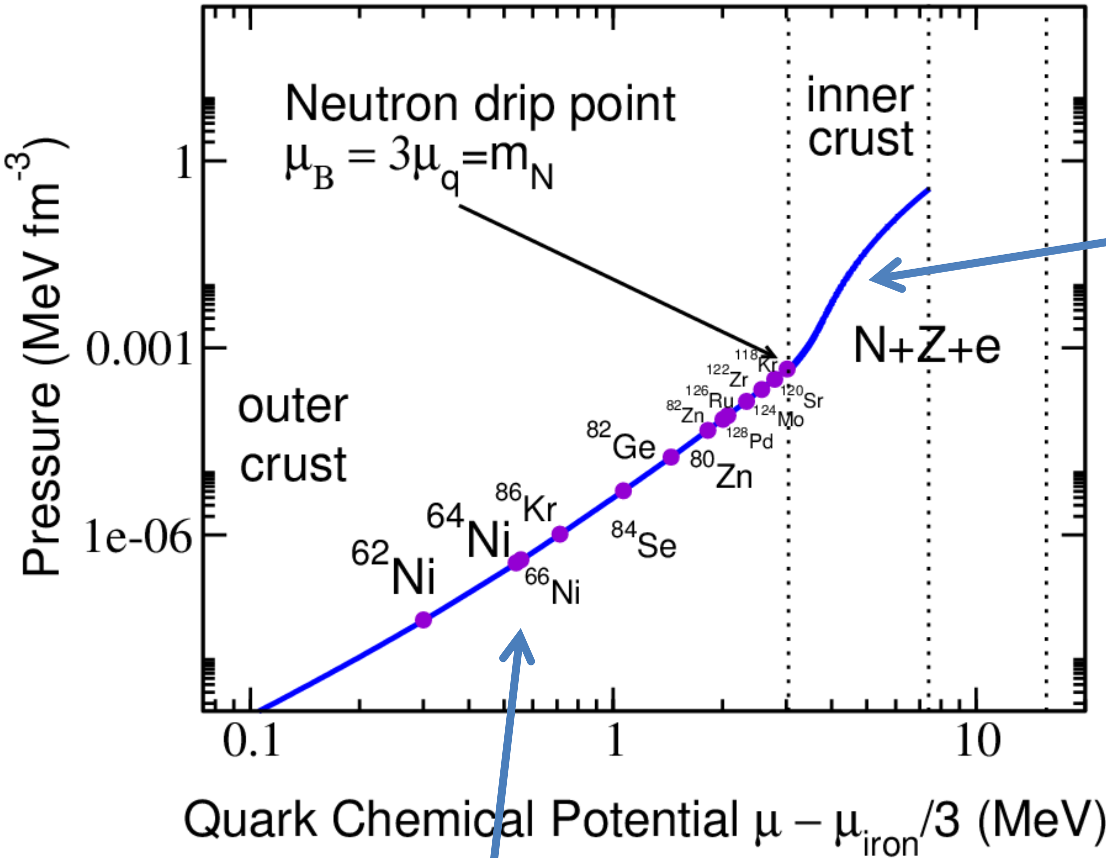



updated 20 April 2012
X−ray/Optical binaries
black widow pulsar
double neutron star binaries
Hulse−Taylor binary
in M15
double pulsar
in NGC 6544
white dwarf−neutron star binaries
in NGC 6539
in Ter 5
2.95 ms pulsar
in 47 Tuc
in NGC 1851
in M 5
in NGC 6440
in NGC 6441
in NGC 6752
main sequence−neutron star binaries
Neutron star mass (M⊙)
0.0 0.5 1.0 1.5 2.0 2.5 3.0

!!

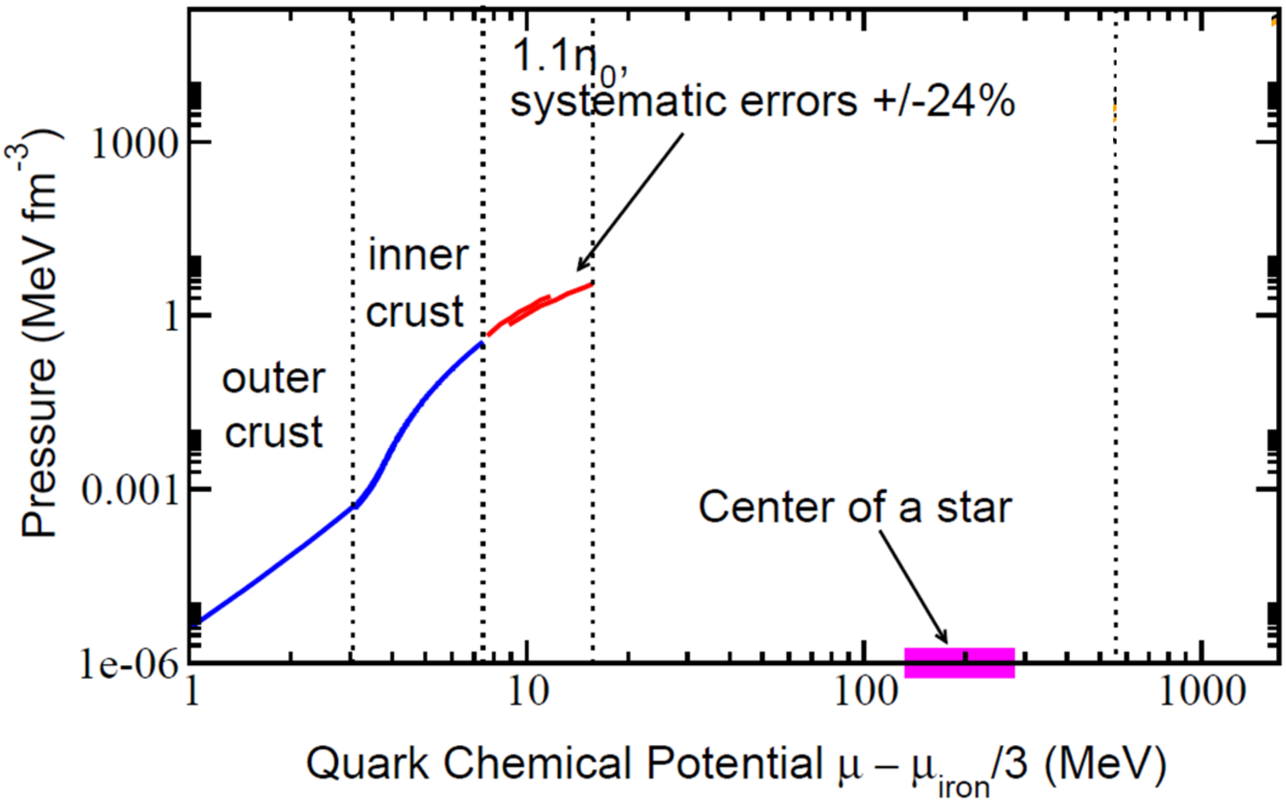
I. What is known: Limits of low and high densities
II. Entering no man’s land: Interpolating polytropes
III. Implications for neutron star physics: MR-relations and beyond

I. What is known: Limits of low and high densities
II. Entering no man’s land: Interpolating polytropes
III. Implications for neutron star physics: MR-relations and beyond

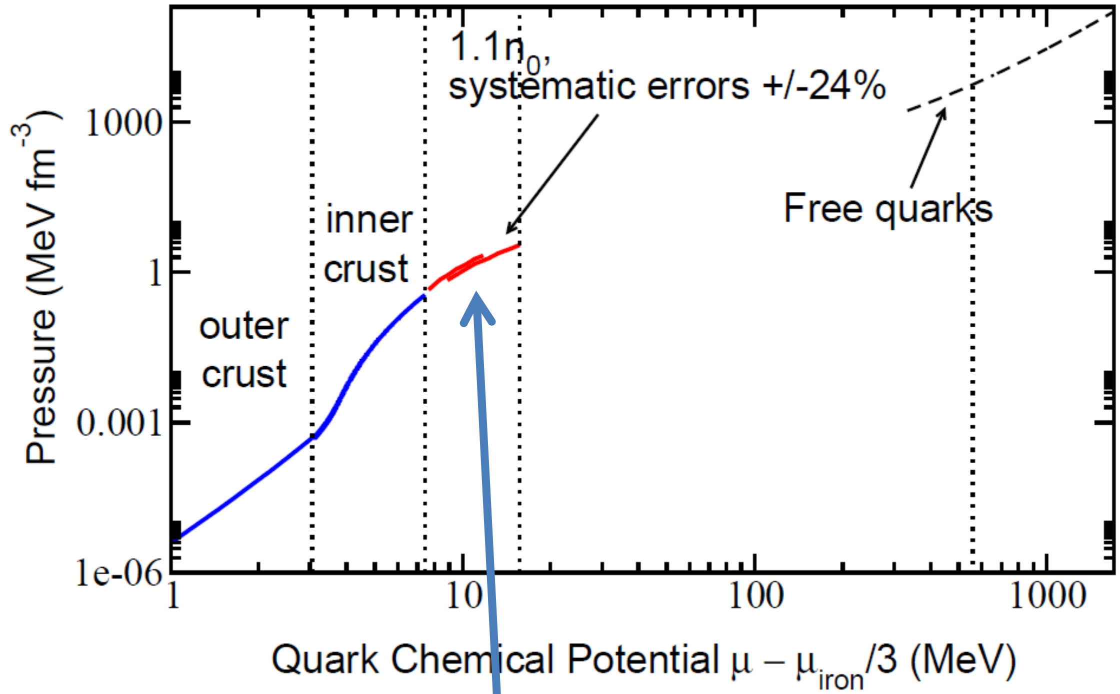



- Neutron gas with nuclei and electrons
- NN interactions important for collective properties; modeled via phenom. potential models
- Eventually need 3N interactions, boost corrections,…

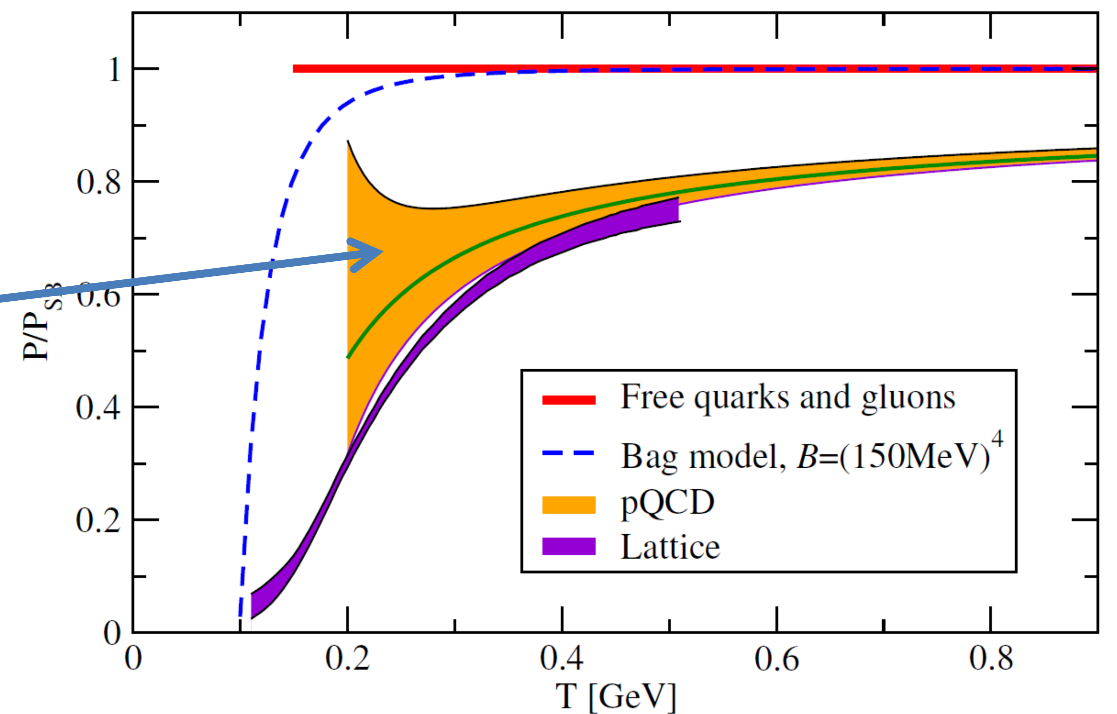
- Lattice of increasingly neutron rich nuclei in electron sea; pressure dominated by that of the electron gas
- At zero pressure nuclear ground state $^{56}\mathrm{Fe}$




- Closer to saturation density $n_{\mathrm{s}}$, need many-body calculations within Chiral Effective Theory, including 3N and 4N interactions
- At $1.1 n_{\mathrm{s}}$, errors $\pm 24\%$ - mostly due to uncertainties in effective theory parameters
- State-of-the-art NNNLO Tews et al., PRL 110 (2013), Hebeler et al., APJ 772 (2013)

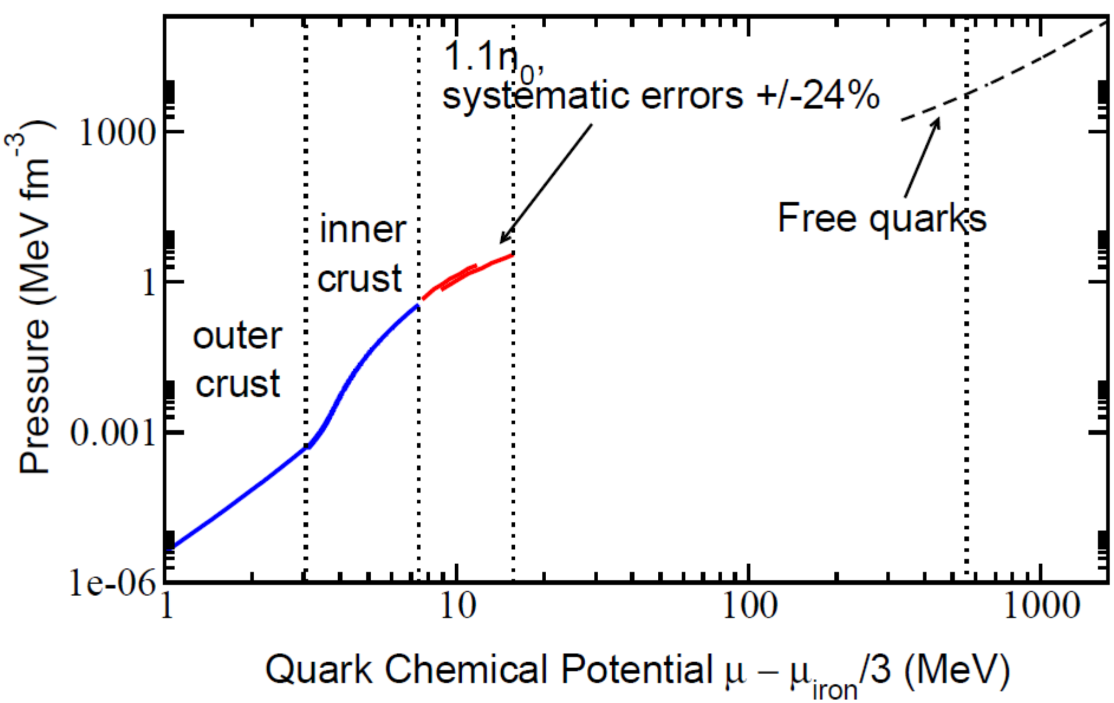



Quantitative error estimates from renormalization / resolution scale dependence



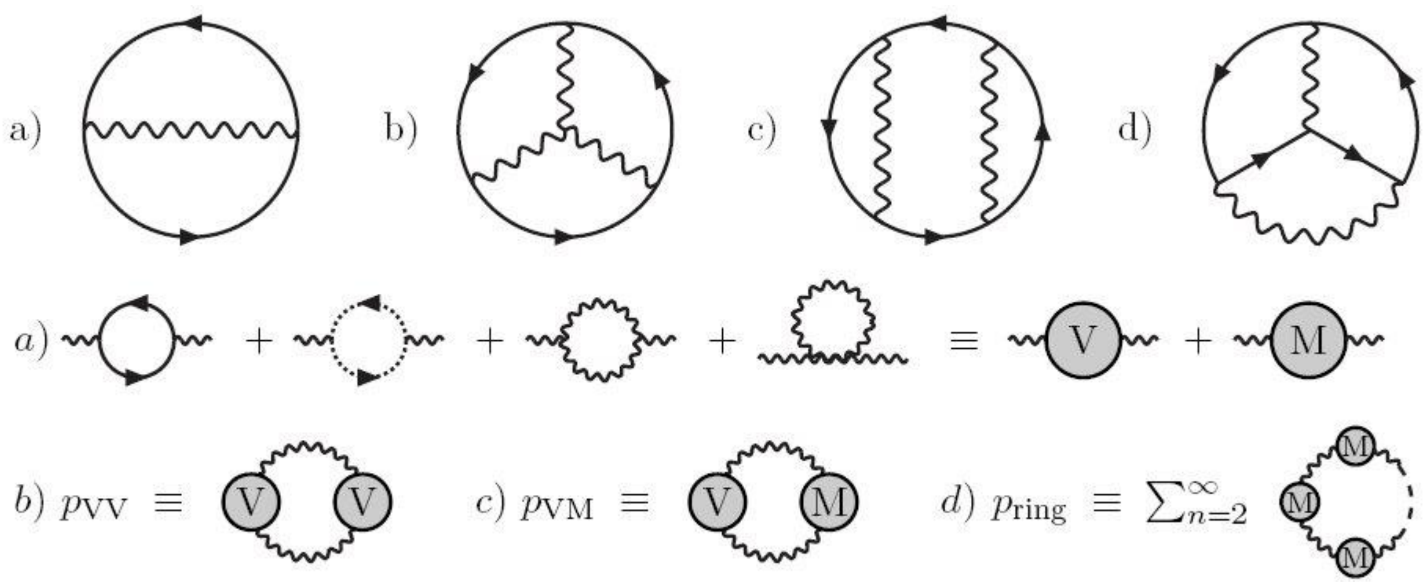
$$
\begin{aligned}
\Omega(\mu_u, \mu_d, \mu_s, m_s) &= -T \log \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A_\mu e^{-\int d^4x \mathcal{L}_{\mathrm{QCD}}}, \\
\mathcal{L}_{\mathrm{QCD}} &= \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \bar{\psi}_i(\gamma_\mu D_\mu + m_i - \mu_i\gamma_0)\psi_i
\end{aligned}
$$

Leading order at zero temperature: Gas of non-interacting quarks



$$\Omega(\mu_u, \mu_d, \mu_s, m_s) = -T \log \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A_\mu e^{-\int d^4x \mathcal{L}_{\rm QCD}},$$
$$\mathcal{L}_{\rm QCD} = \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \bar{\psi}_i(\gamma_\mu D_\mu + m_i - \mu_i\gamma_0)\psi_i$$

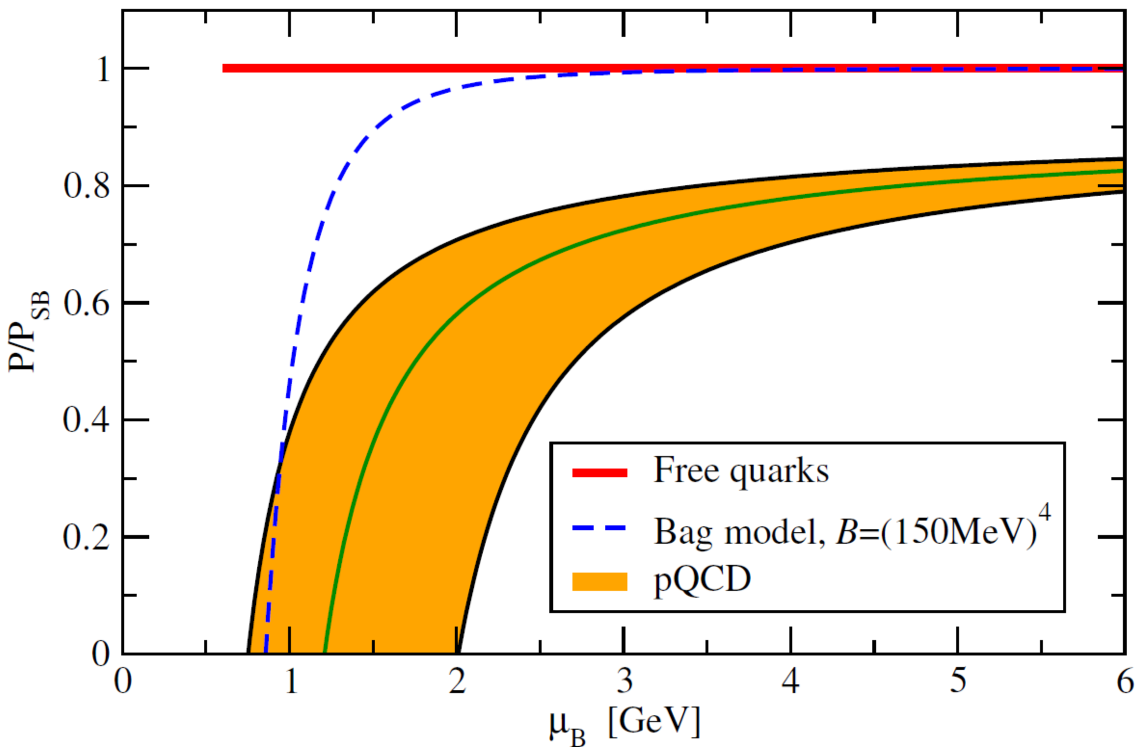
NLO and NNLO: Vacuum diagrams and ring resummation




Kurkela, Romatschke, Vuorinen, PRD 81 (2010)

$$\Omega(\mu_u, \mu_d, \mu_s, m_s) = -T \log \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A_\mu e^{-\int d^4x \mathcal{L}_{\rm QCD}},$$
$$\mathcal{L}_{\rm QCD} = \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \bar{\psi}_i(\gamma_\mu D_\mu + m_i - \mu_i\gamma_0)\psi_i$$

NLO and NNLO: Vacuum diagrams and ring resummation

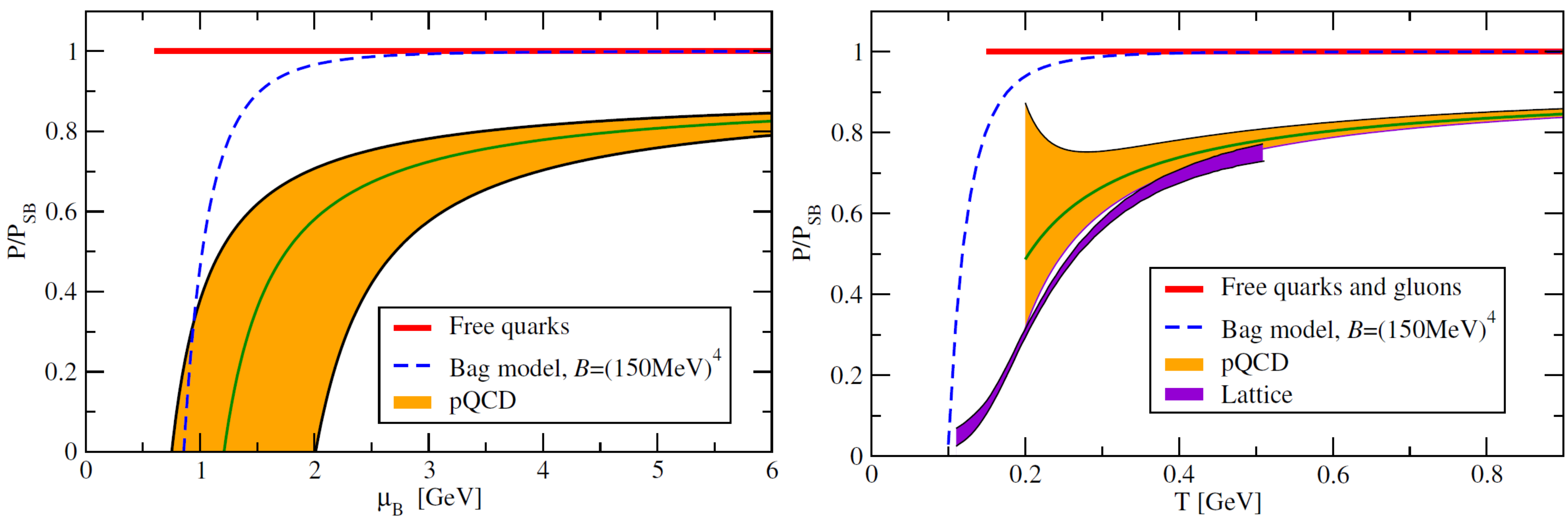


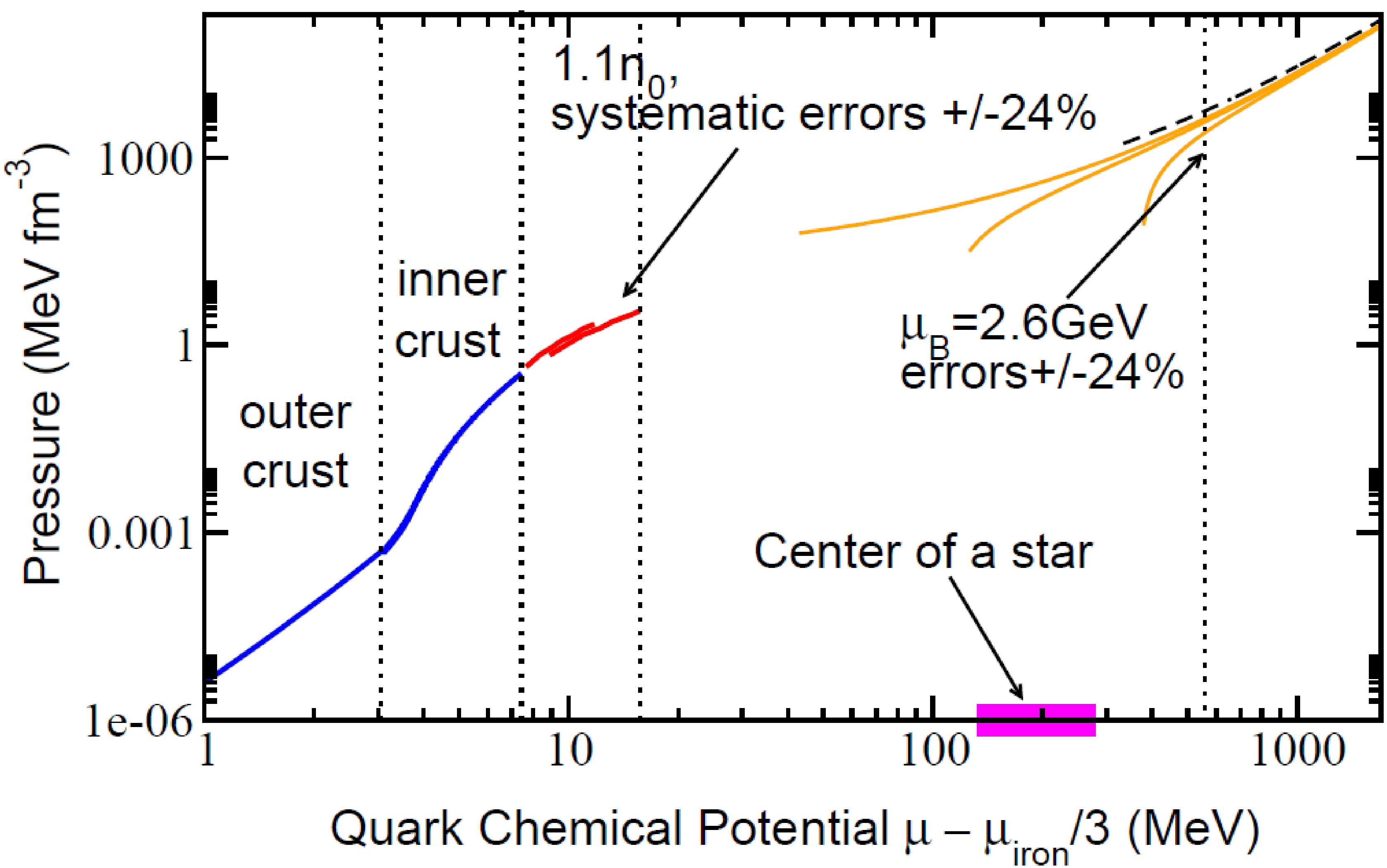

Main sources of uncertainty:

- Renormalization scale dependence
- Running of $\alpha_s$
- Value of strange quark mass

$$\Omega(\mu_u, \mu_d, \mu_s, m_s) = -T \log \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A_\mu e^{-\int d^4x \mathcal{L}_{\rm QCD}},$$
$$\mathcal{L}_{\rm QCD} = \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \bar{\psi}_i(\gamma_\mu D_\mu + m_i - \mu_i\gamma_0)\psi_i$$

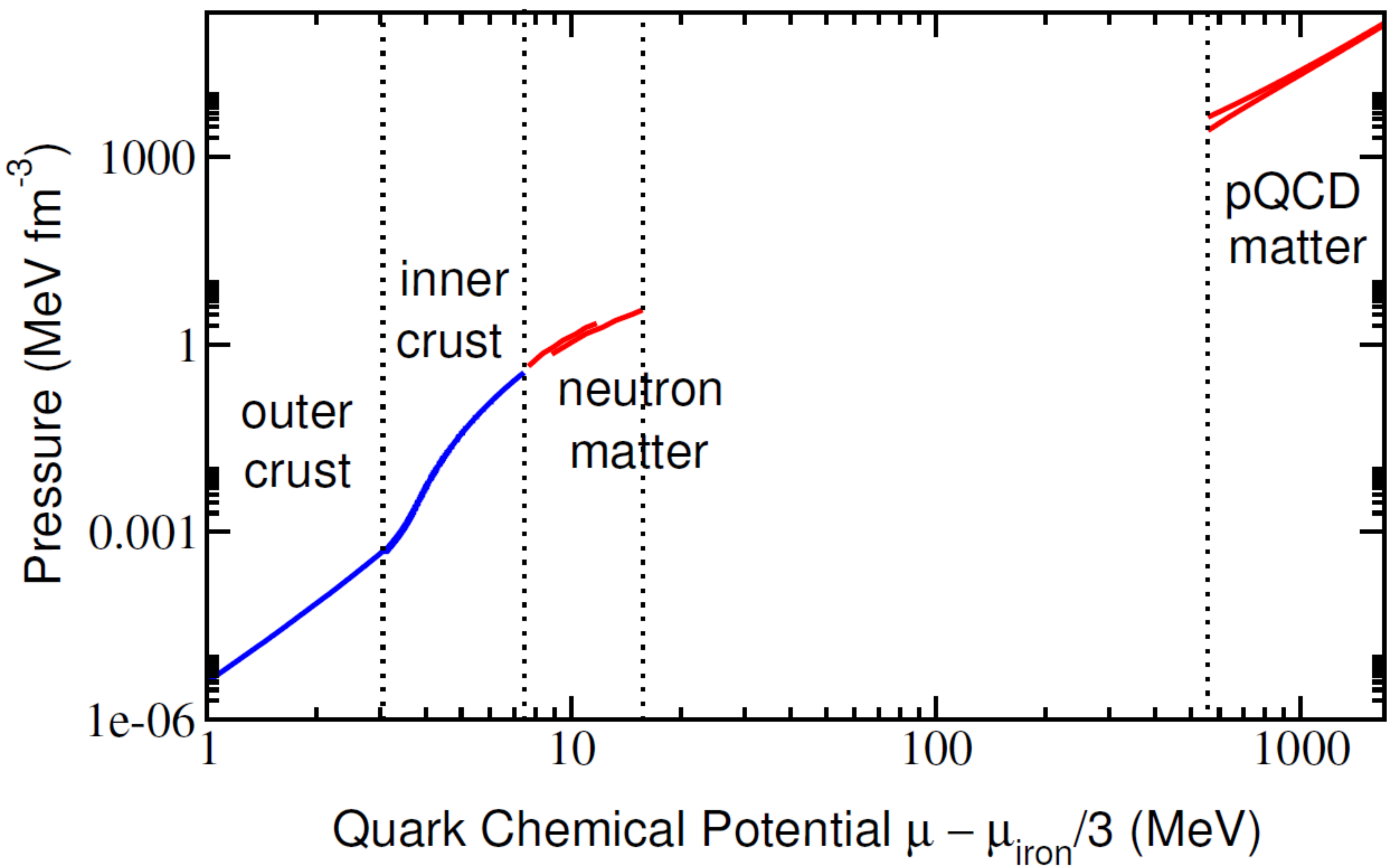
NLO and NNLO: Vacuum diagrams and ring resummation





1.1n0,
systematic errors +/-24%
inner
crust
outer
crust
μB=2.6GeV
errors+/-24%
Center of a star
1000
1
0.001
1e-06
1
10
100
1000
Pressure (MeV fm-3)
Quark Chemical Potential μ – μiron/3 (MeV)

I. What is known: Limits of low and high densities
II. Entering no man’s land: Interpolating polytropes
III. Implications for neutron star physics: MR-relations and beyond



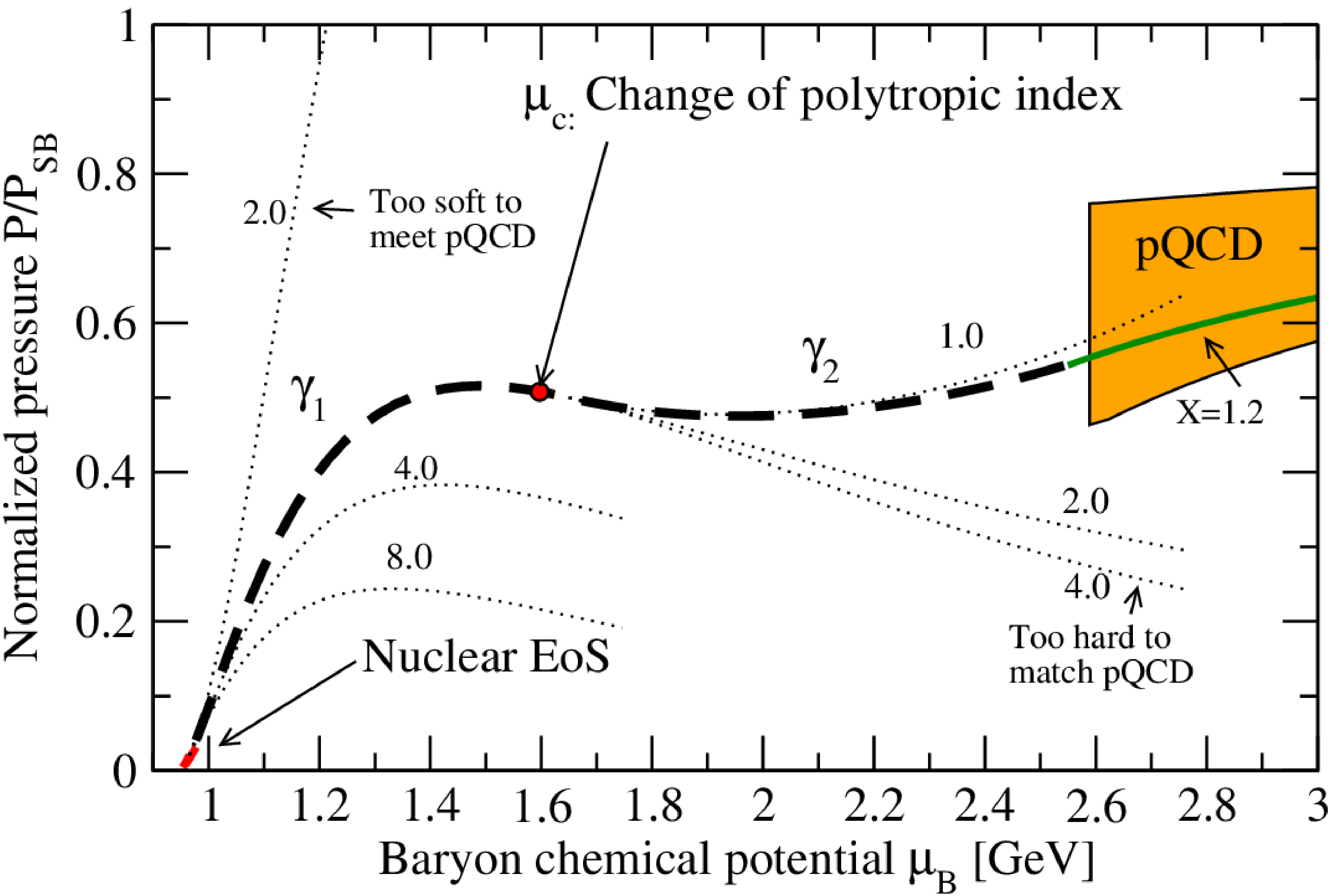

Two choices:

1) Phenomenological models
2) Controlled interpolation between known limits

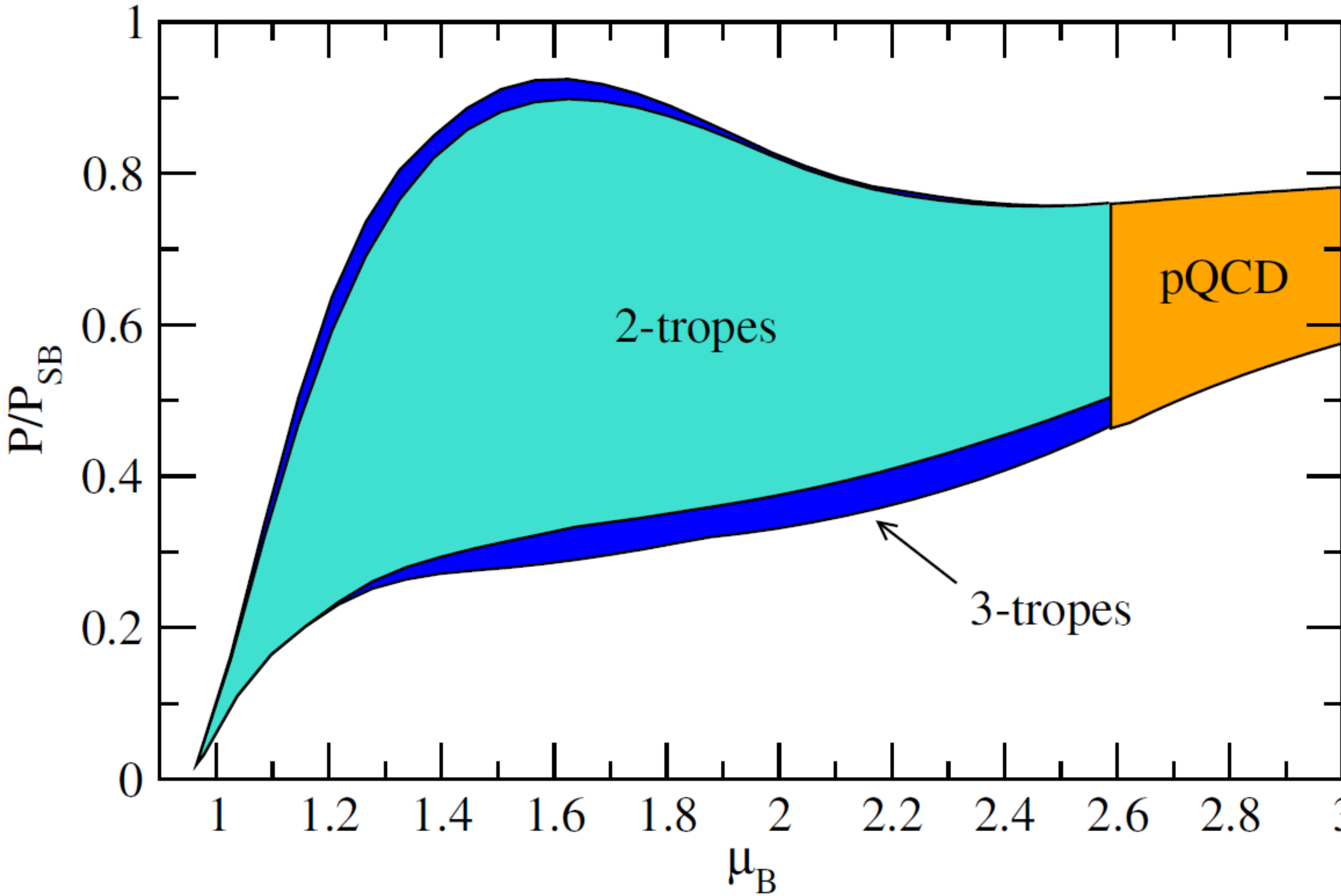
Quantify ignorance by using multiple piecewise polytropic EoSs, $p_i(n) = \kappa_i n^{\gamma_i}$, and varying all parameters. Do this requiring

1) Smooth matching to nuclear and quark matter EoSs
2) Smoothness: Continuity of $p$ and $n$ when matching monotropes (can be relaxed)
3) Subluminality $c_s \leq 1$ - asymptotically equivalent to $\gamma \leq 2$
4) Ability to support a two solar mass star

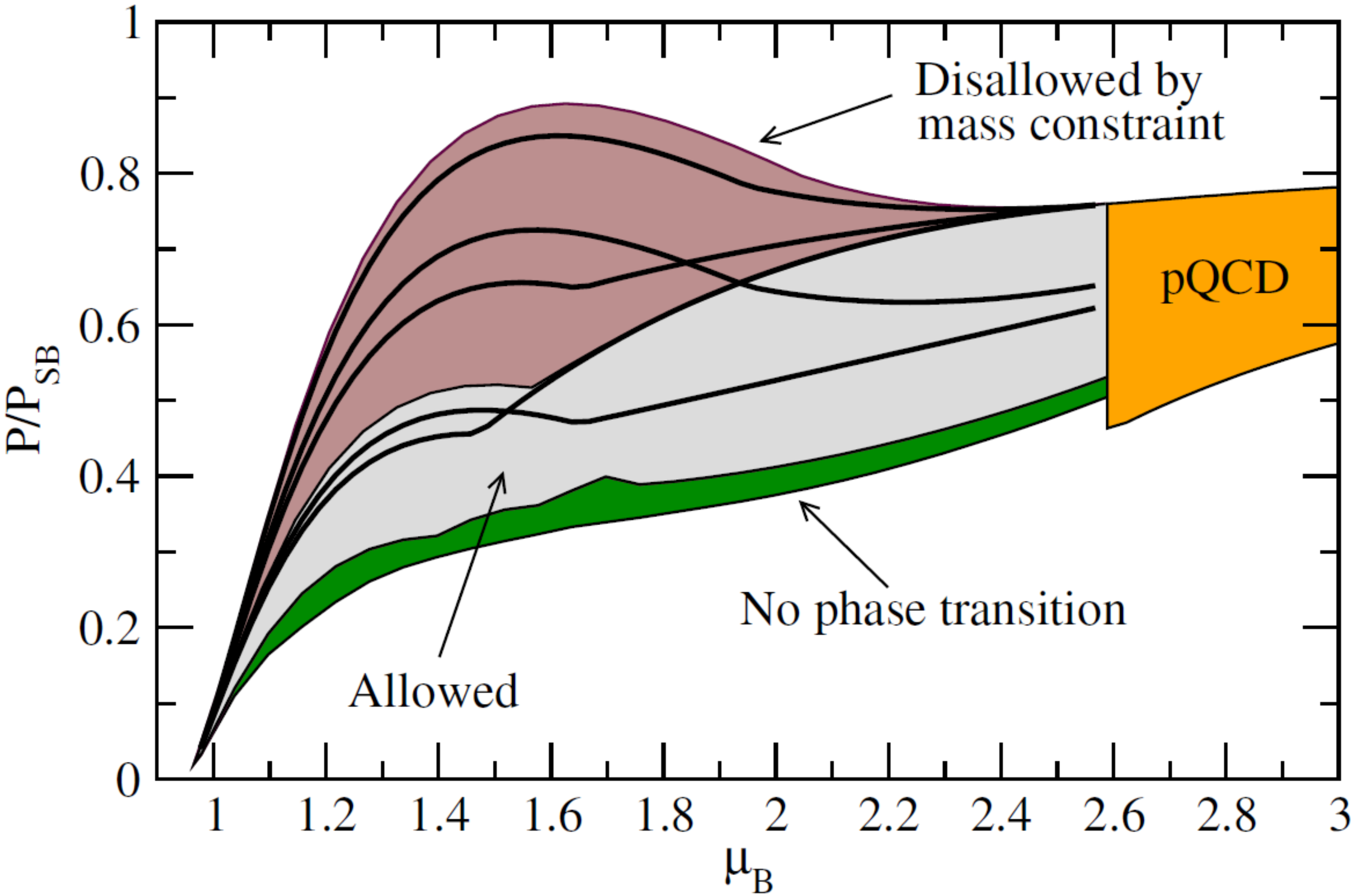
Hebeler, Lattimer, Pethick, Schwenk, APJ 773 (2013)
Kurkela, Fraga, Schaffner-Bielich, Vuorinen, APJ 789 (2014)



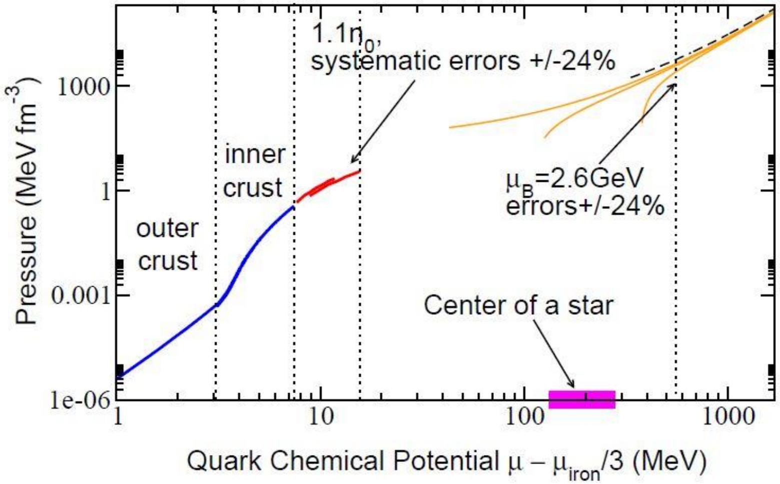

Solutions exist for $\mu_c \in [1.08, 2.05]$ GeV, $\gamma_1 \in [2.23, 9.2]$, $\gamma_2 \in [1.0, 1.5]$



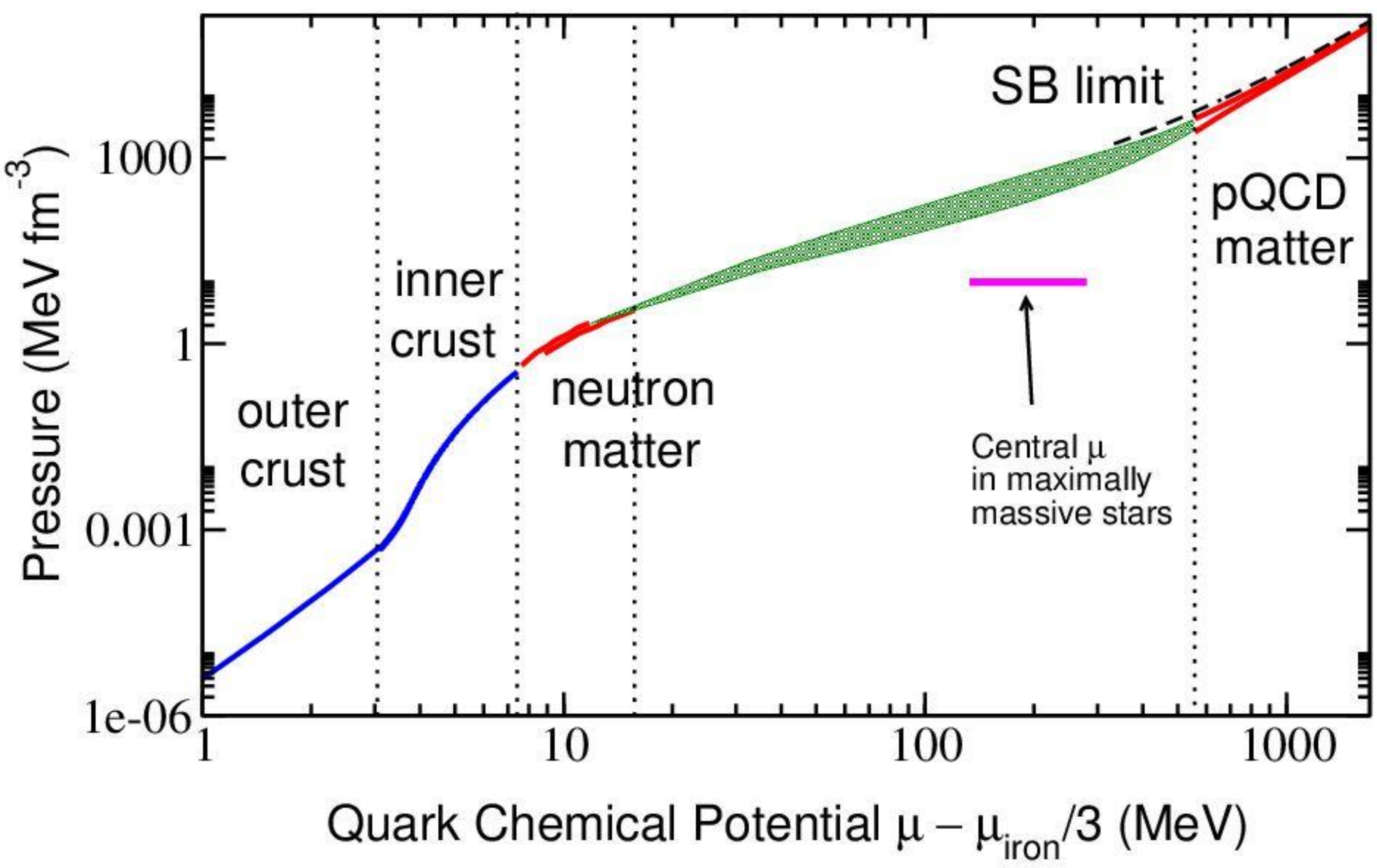

Two minimum number of monotropes – and apparently also sufficient




Two solar mass constraint significant. On the other hand, allowing for a 1st order phase transition (nonzero `latent heat') only leads to a smaller region of allowed EoSs

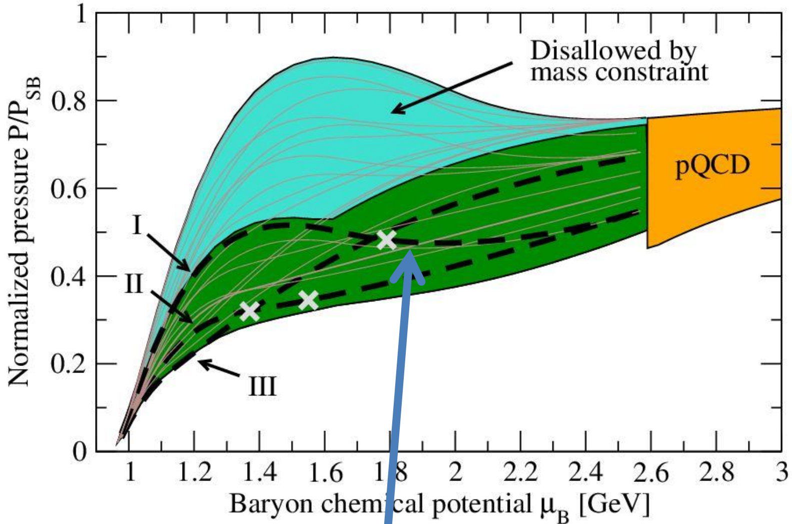


Pressure (MeV fm$^{-3}$)
1000
1
0.001
1e-06
1
10
100
1000
Quark Chemical Potential $\mu - \mu_{iron}/3$ (MeV)
outer crust
inner crust
1.1n$_0$, systematic errors +/-24%
$\mu_B$=2.6GeV errors+/-24%
Center of a star

Pressure (MeV fm$^{-3}$)
1000
1
0.001
1e-06
1
10
100
1000
Quark Chemical Potential $\mu - \mu_{iron}/3$ (MeV)
outer crust
inner crust
neutron matter
SB limit
pQCD matter
Central $\mu$ in maximally massive stars

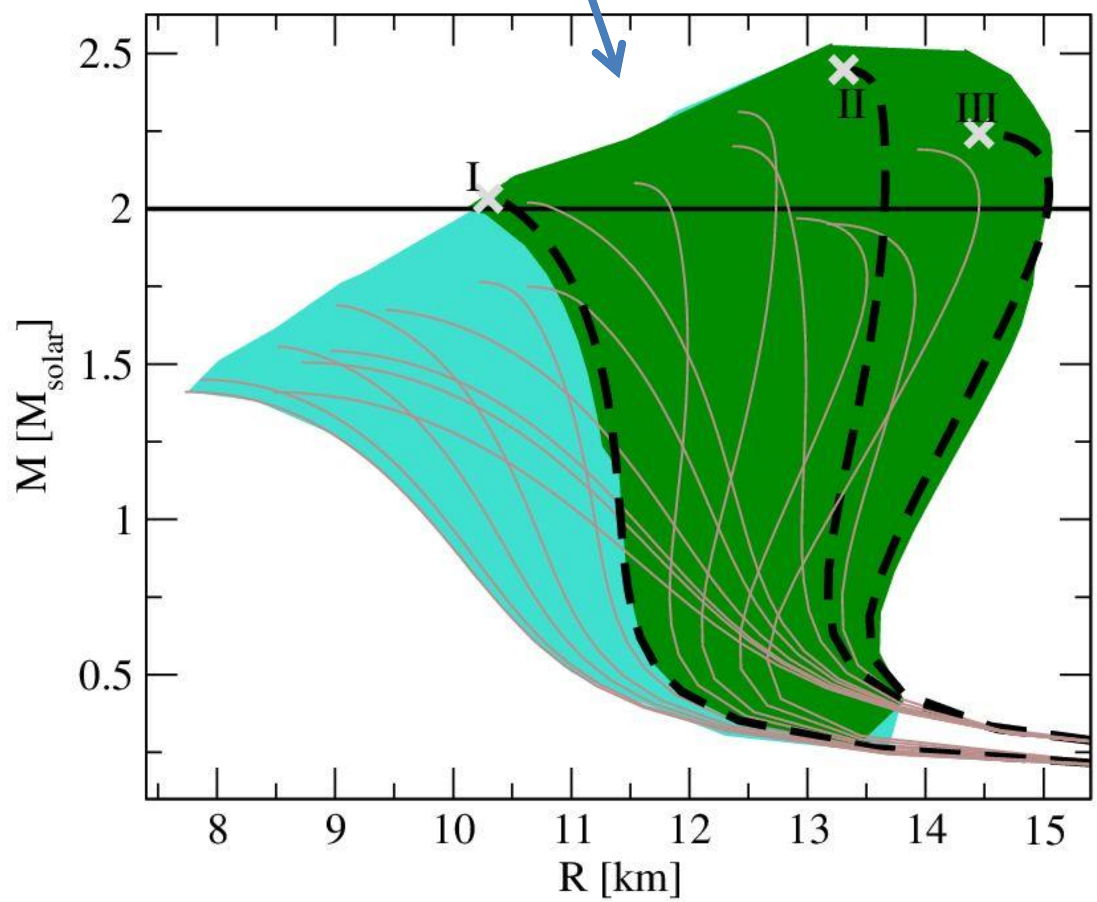
I. What is known: Limits of low and high densities
II. Entering no man’s land: Interpolating polytropes
III. Implications for neutron star physics: MR-relations and beyond

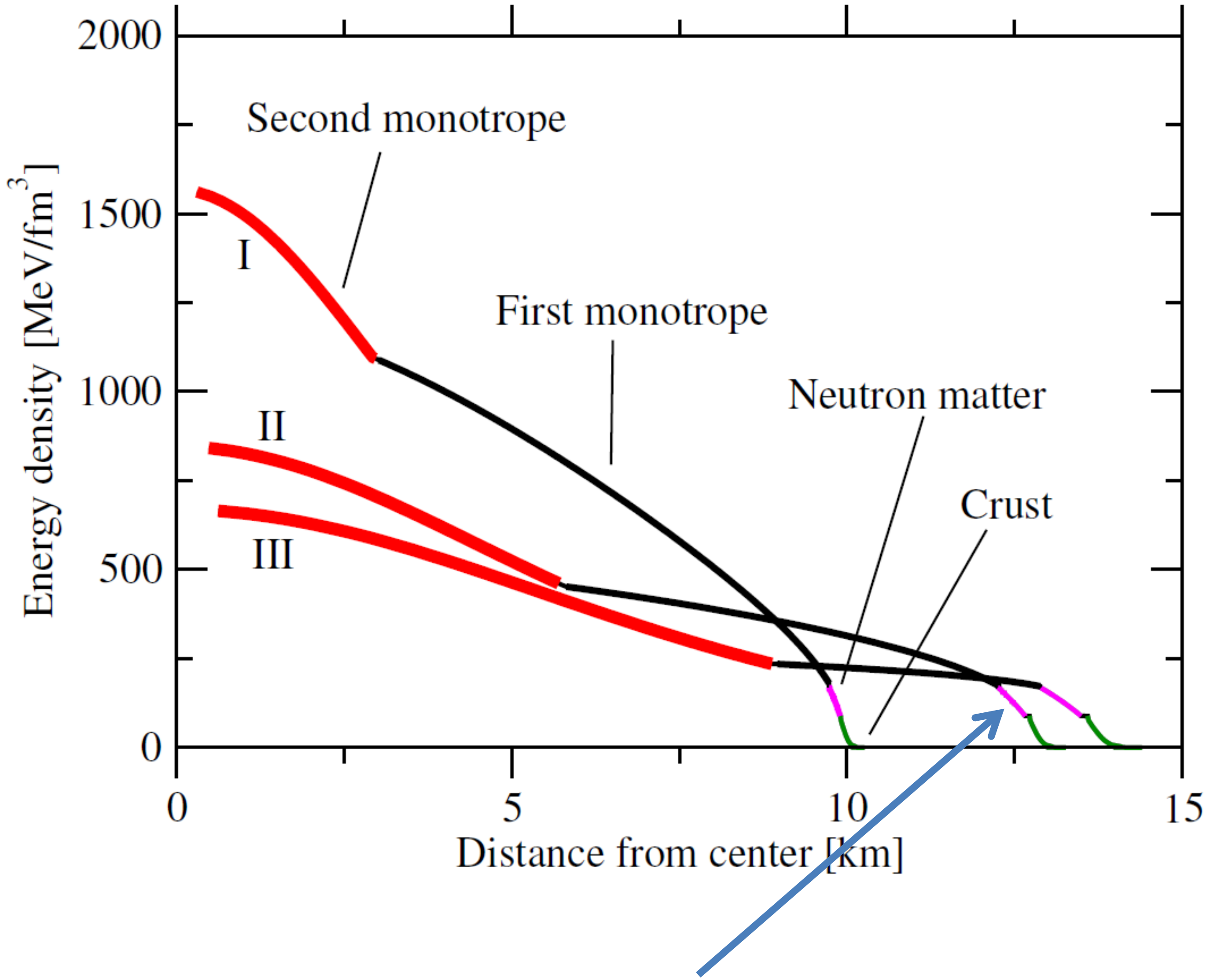



Maximal masses up to $2.5M_{\rm sun}$ !
Central densities
$n_{\rm central} \in [3.7, 14.3]n_{\rm s}$

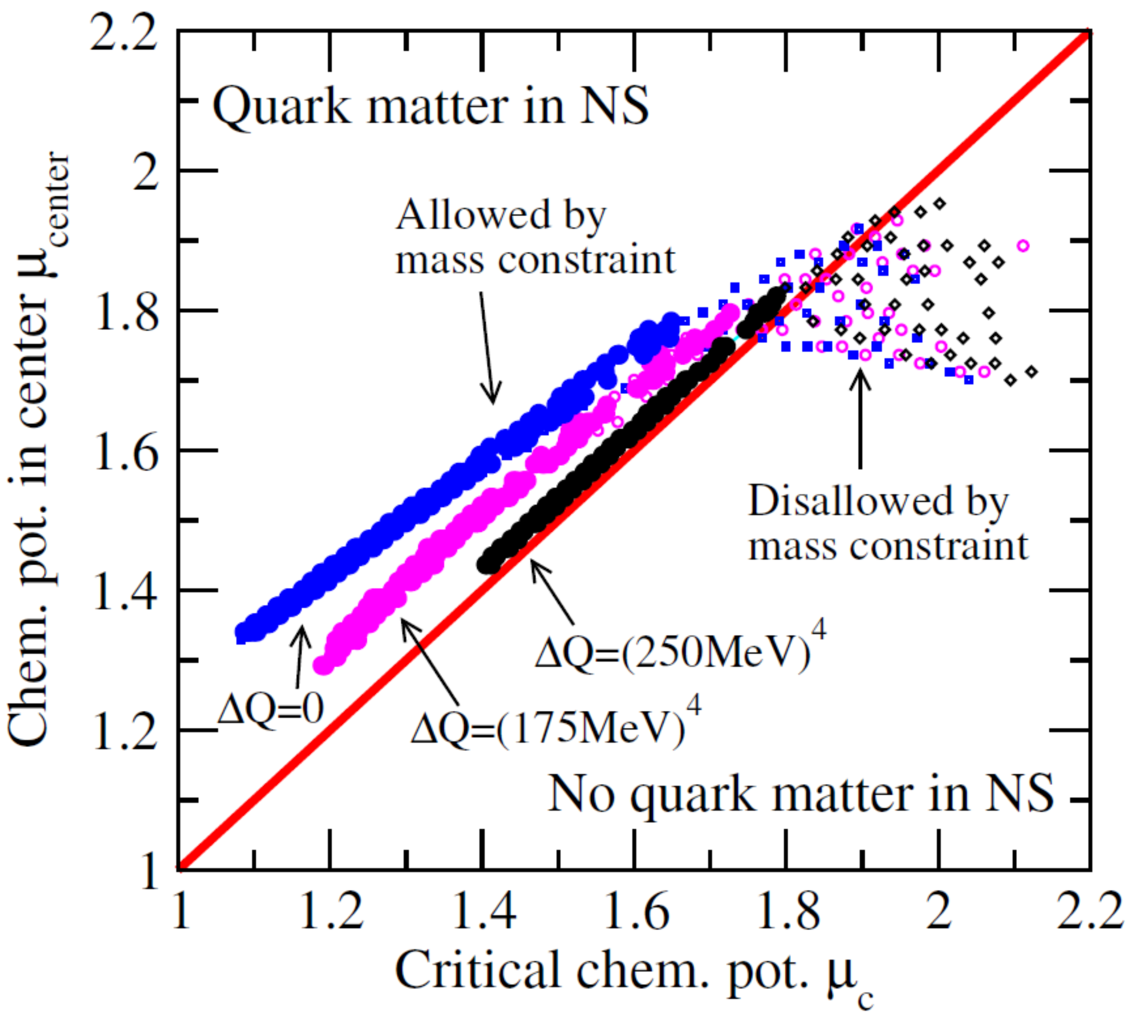
Large reduction in EoS uncertainty due to tension from mass constraint: Large stellar masses require stiff EoS, matching to pQCD soft

$\Rightarrow$ EoS uncertainty down to 30% at all densities





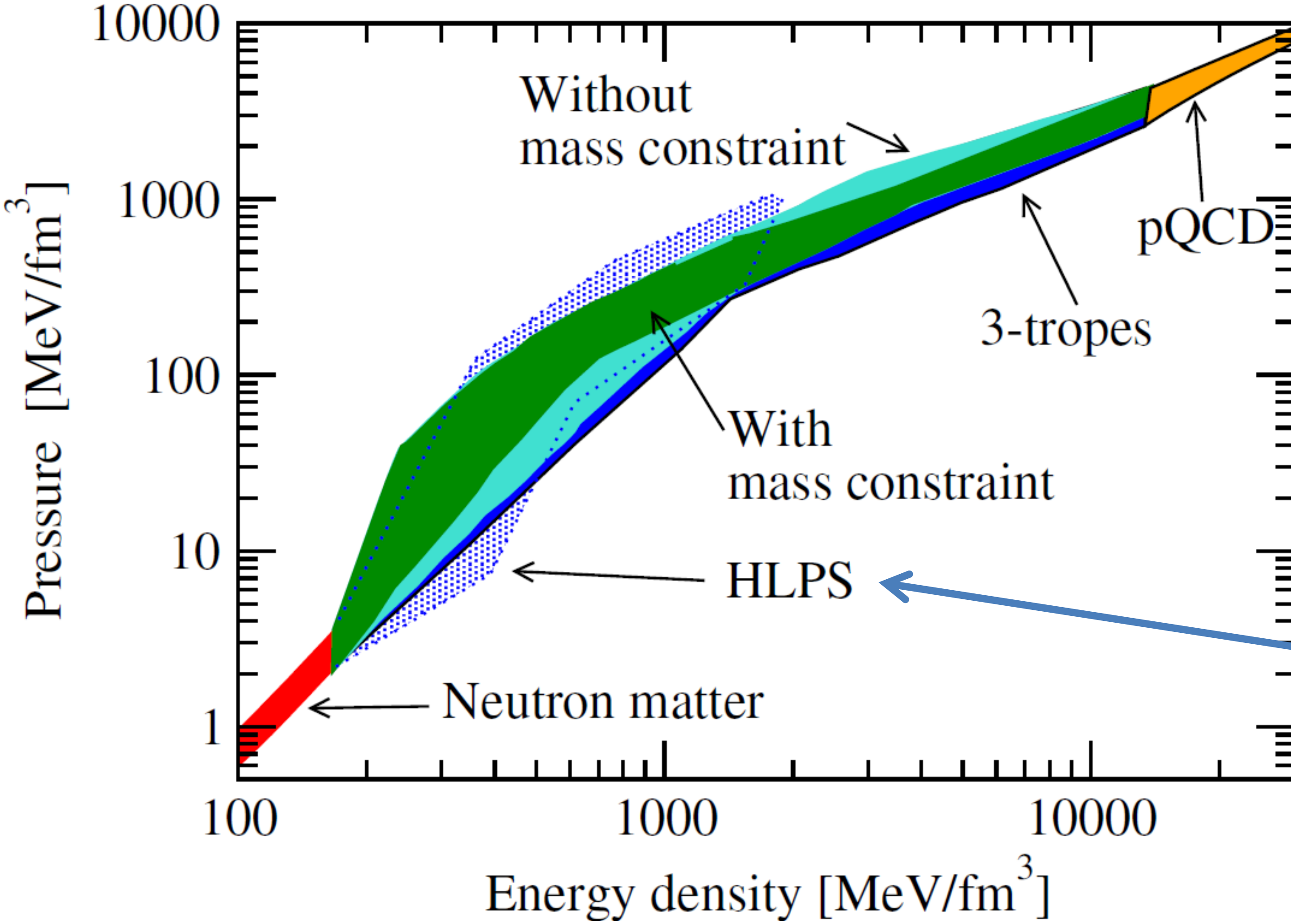

Nuclear matter EoS only used very close to star surface – yet important effects from matching



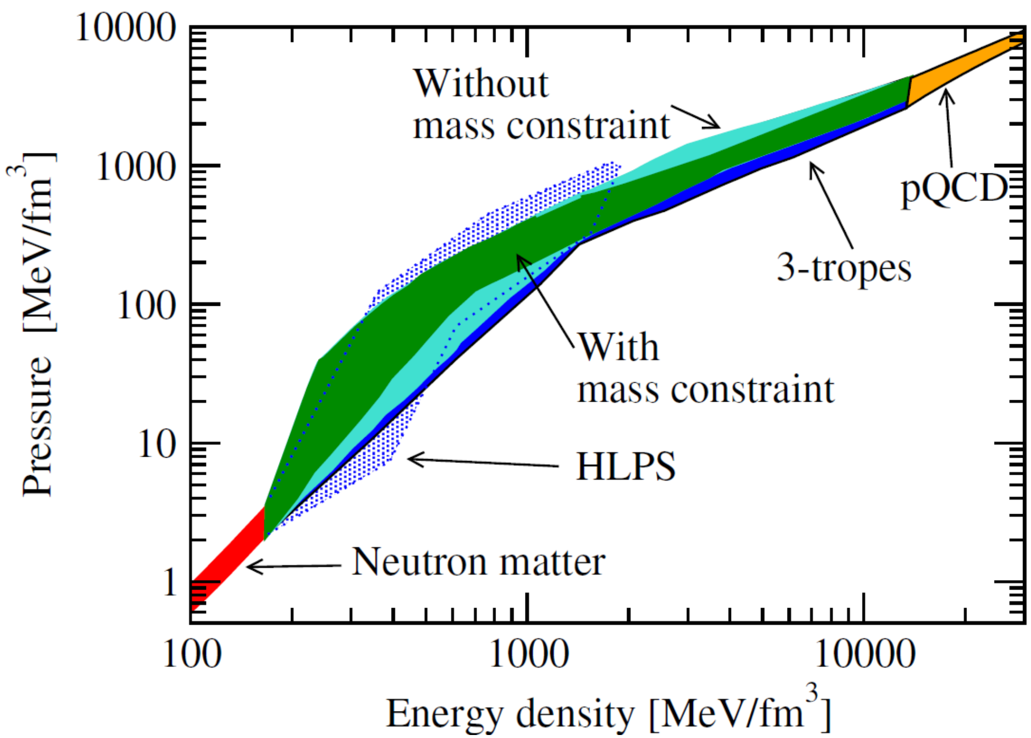

Interpreting polytrope matching point ($\mu_c$) as phase transition to quark matter, witness strong correlation between $\mu_c$ and the chemical potential at the center of a maximally massive star.

The stronger the transition, the less room for quark matter. Even then the pQCD result crucial for constraining the EoS!

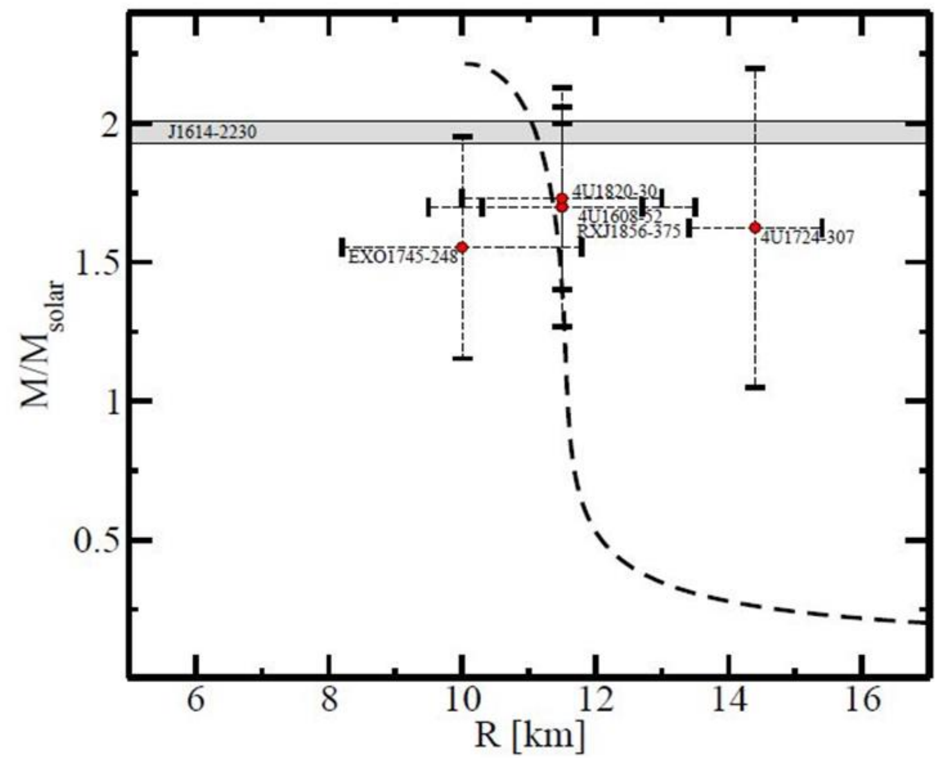
# Summary




HLPS [Hebeler, Lattimer, Pethick, Schwenk, APJ 773 (2013)] without high-density constraint



Without
mass constraint
pQCD
3-tropes
With
mass constraint
HLPS
Neutron matter
Pressure [MeV/fm^3]
Energy density [MeV/fm^3]
10000
1000
100
10
1
100
1000
10000

J1614-2230
4U1820-30
4U1608-52
RXJ1856-375
4U1724-307
EXO1745-248
M/M_solar
R [km]
2
1.5
1
0.5
6
8
10
12
14
16