# Numerical holography and far-from-equilibrium dynamics

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# motivation

- Use gauge/gravity duality to study, far-from-equilibrium strongly interacting dynamics
- Go beyond near-equilibrium dynamics (linear response, probe approximation)
- Honestly solve dynamics of interesting initial states

# relativistic heavy ion collisions

Relevant dynamics:

Very early: partonic, perturbative (?)

Plasma phase: strongly coupled

Evidence: screening lengths, viscosity, ...

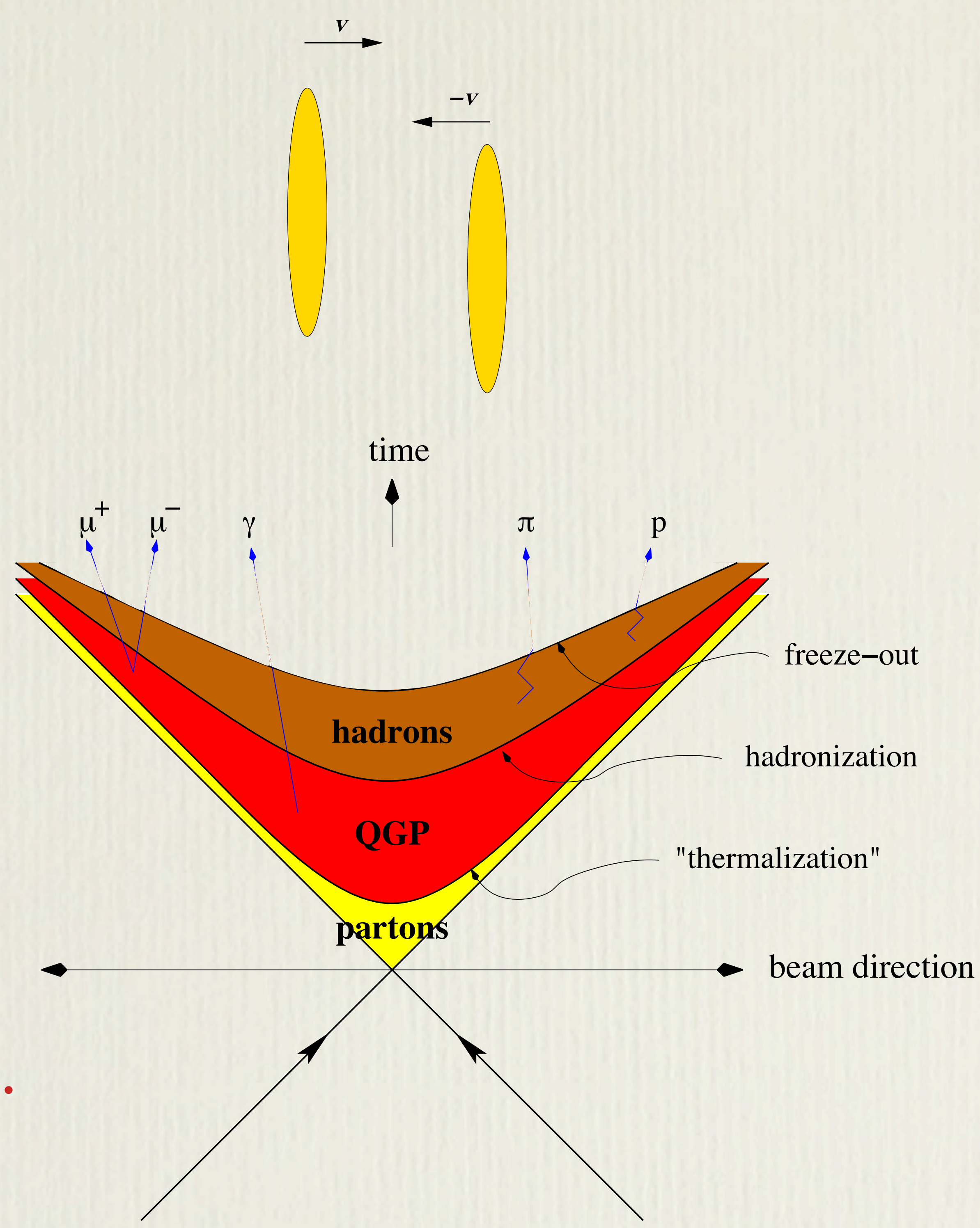
Many questions:

How fast do produced partons isotropize?

When/where is hydrodynamics valid?

Signatures of strongly coupled dynamics?

No fully controlled theoretical methods.



# relativistic heavy ion collisions

Idealize:

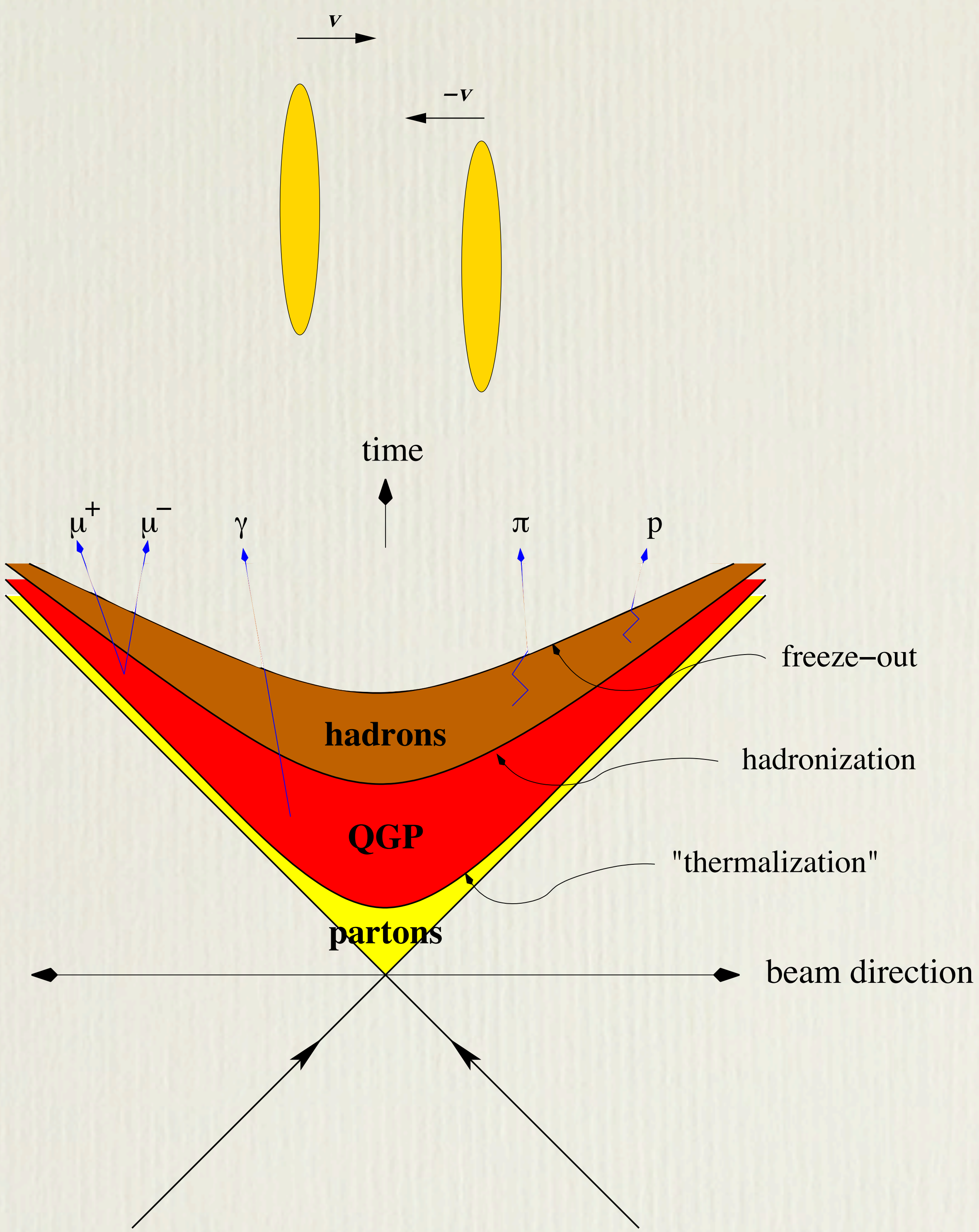
SU(3) gauge field + quarks ➡
SU($N_c$) gauge field + adjoint matter

strongly coupled QCD ➡
strongly coupled $\mathcal{N}$=4 SYM

# colors: $N_c = 3$ ➡ $N_c = \infty$

't Hooft coupling: $\lambda \approx 1$ ➡ $\lambda \gg 1$

Use holographic methods to study non-equilibrium, strongly coupled non-Abelian plasma dynamics



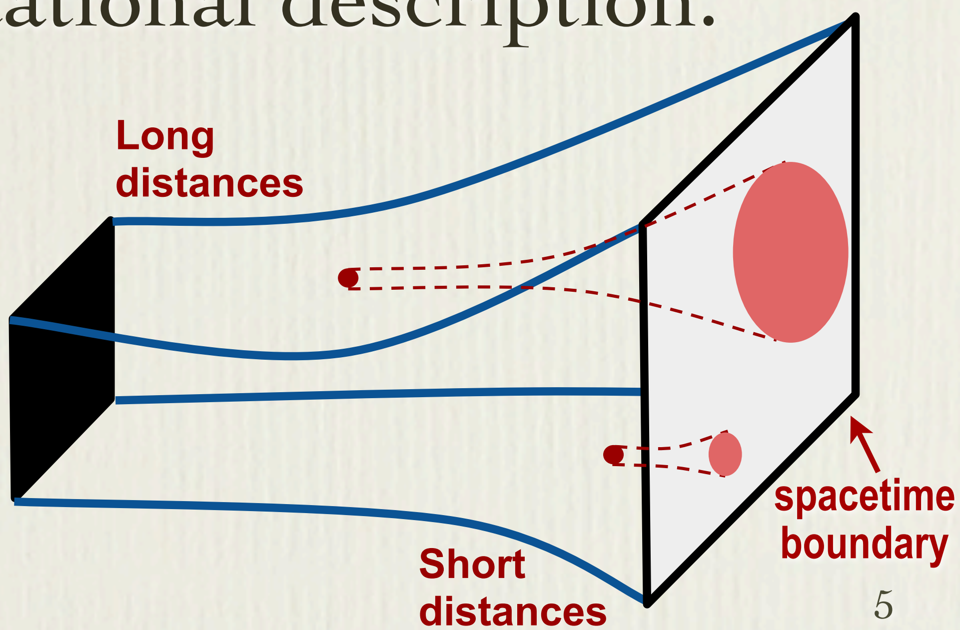
# gauge/gravity duality

- a.k.a. "AdS/CFT duality," "gauge/string duality," "holography"
- Some non-Abelian gauge theories have exact reformulation as higher dimensional gravitational (or string) theories.

  Simplest case: maximally supersymmetric SU($N_c$) Yang-Mills ($\mathcal{N}$=4 SYM)

  = string theory on $AdS_5 \times S^5$. More complicated generalizations for less supersymmetric, non-conformal theories.

- Strong coupling (and large $N_c$) limit of quantum field theory given by classical dynamics in dual gravitational description.
- Holographic description gives geometric representation of renormalization flow:



# holography: features

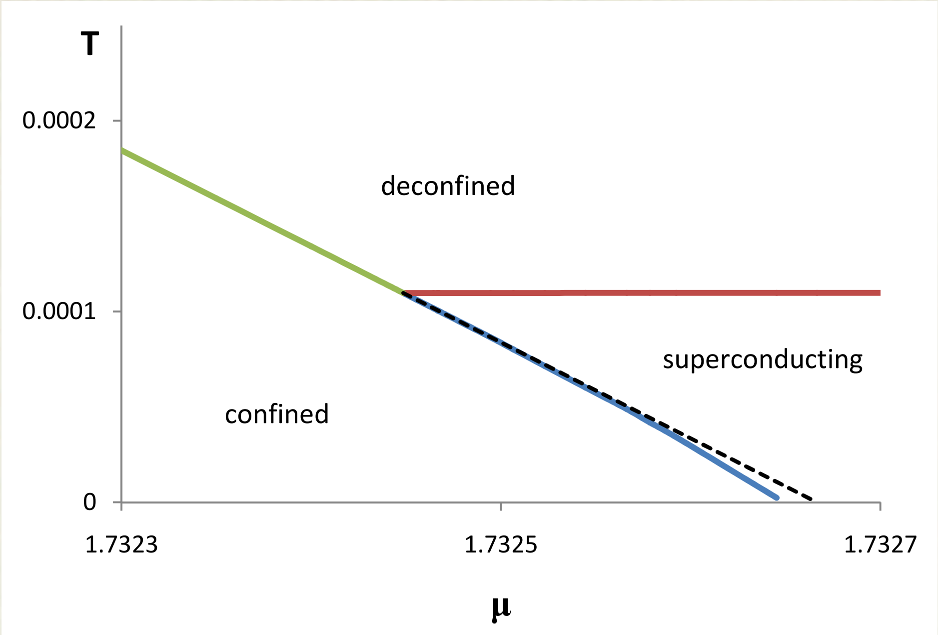
- strongly coupled, large $N$ QFT = classical (super)gravity in higher dimension
  - valid description on all scales
  - gravitational fluctuations: $1/N^2$ suppressed
  - QFT state ⇔ asymptotically AdS geometry
  - $O(N^2)$ entropy ⇔ gravitational (black brane) horizon
  - thermalization ⇔ gravitational infall, horizon formation & equilibration

# applications of holography

- Equilibrium properties of strongly coupled theories:

  static geometries ⇔ equilibrium states of QFT

  $AdS_5$ black hole ⇔ non-Abelian plasma

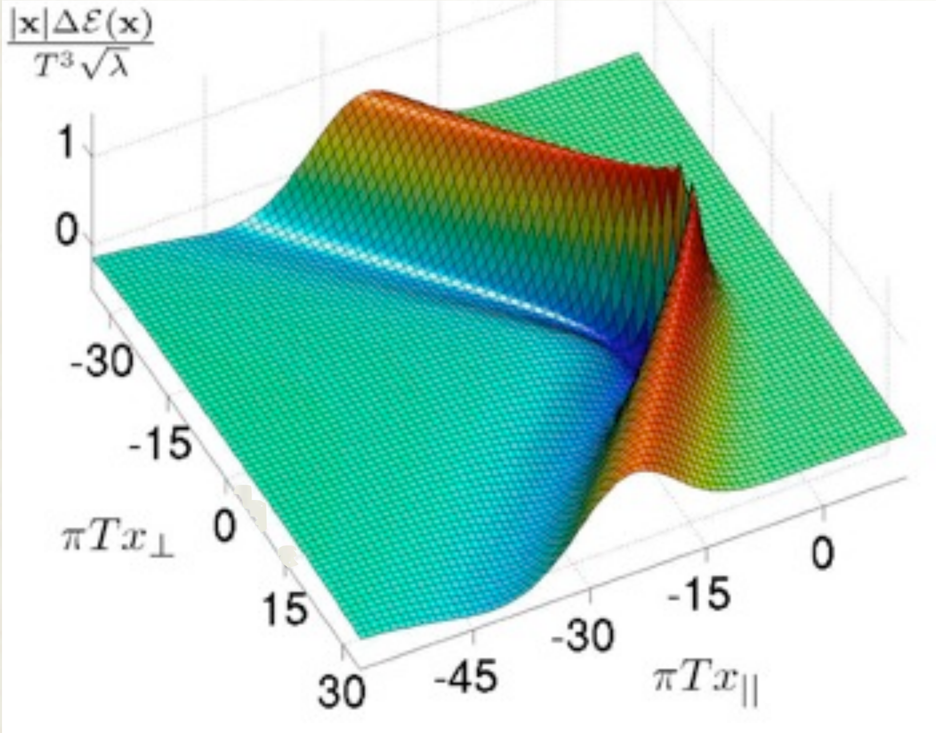



- Near-equilibrium dynamics:

  small fluctuations ⇒ linear response, spectral densities, transport coefficients, quasi-normal modes, photoemission, probe dynamics

- Far-from-equilibrium dynamics:

  gravitational initial value problems



# far-from-equilibrium dynamics

- heavy-ion collisions:
  - homogeneous isotropization — P. Chesler, L.Y., M. Heller, D. Mateos, W. van der Schee, ...
  - boost invariant flow — P. Chesler, L.Y., R. Janik, M. Heller, P. Witaszczyk, W. van der Schee
  - colliding planar shocks — P. Chesler, L.Y., J. Casalderrey Solano, M. Heller, D. Mateos, W. van der Schee
  - colliding "nuclei" — W. van der Schee, P. Romatschke, S. Pratt, P. Chesler, L.Y.
- turbulence:
  - 2D normal fluids — A. Adams, P. Chesler, H. Liu
  - 2D superfluids — A. Adams, P. Chesler, H. Liu
- other stuff:
  - dynamical quenches — M.J. Bhaseen, J. Gauntlett, J. Sonner, T. Wiseman, A. Garcia-Garcia; B. Craps, L. Lehner, K. Schalm, R. Myers
  - black hole formation/ring-down — H. Bantilan, F. Pretorius, S. Gubser

# this talk

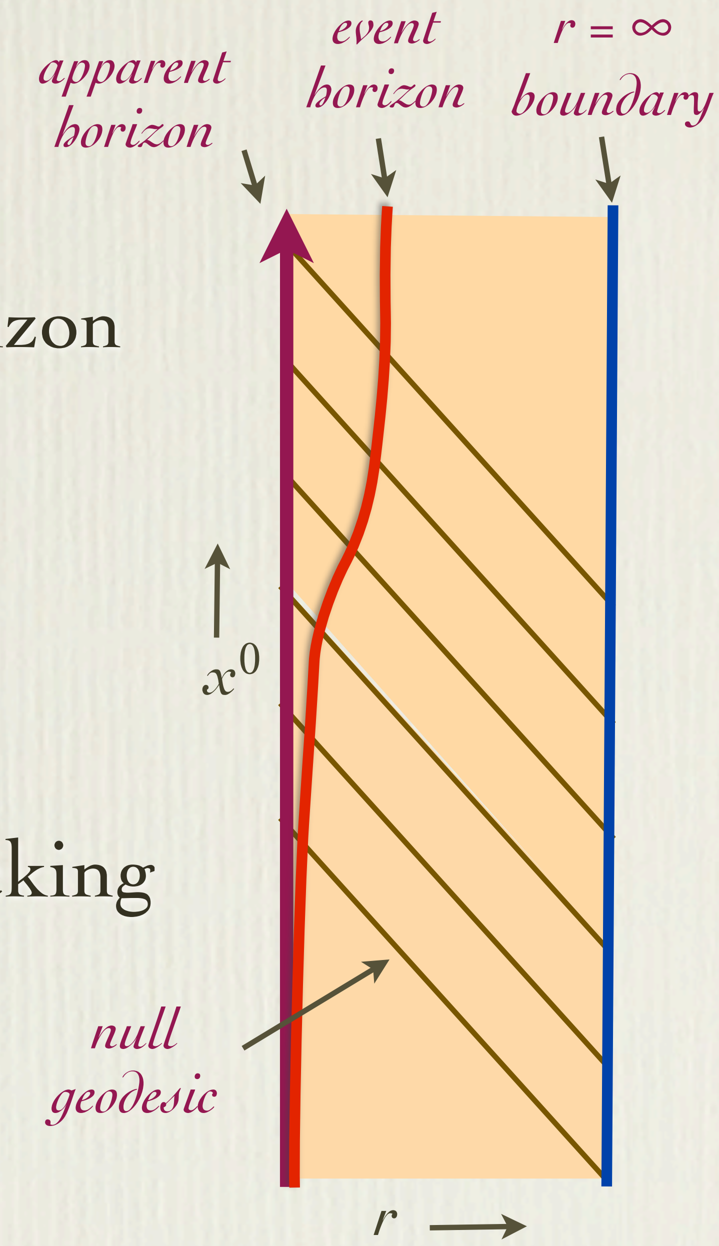
- Methods
- Colliding shocks
  - dependence on shock width?
  - surviving remnants of initial shocks?
  - approximate boost invariance?
  - finite size “nuclei”?
- Homogeneous equilibration
  - sensitivity to charge density or magnetic field?
  - degree of non-linearity?
- 2D turbulence
  - normal fluids
  - superfluids

work in progress
w. Paul Chesler

work in progress
w. John Fuini

# methods

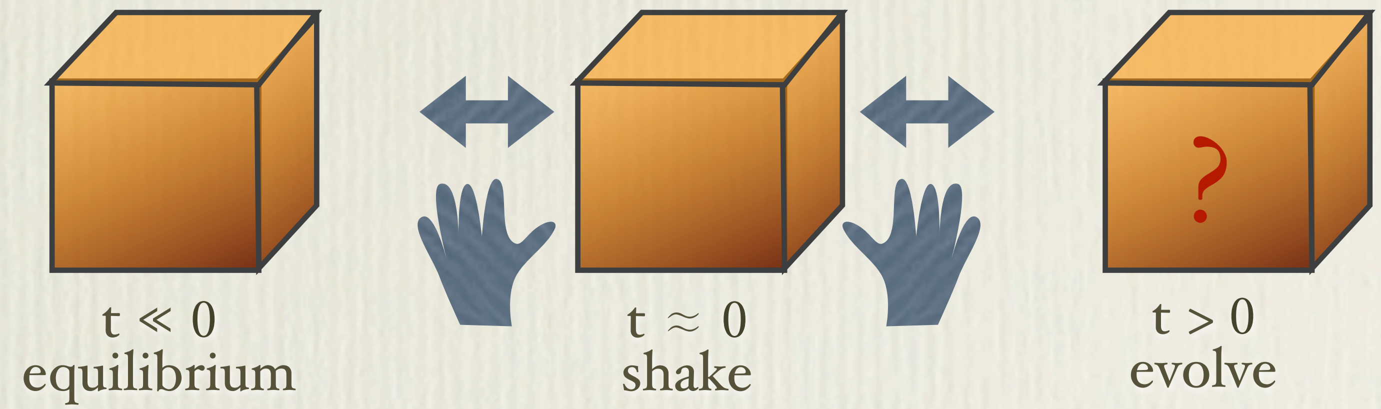
- Characteristic formulation
  - coordinates adapted to infalling null geodesics
  - fix residual diffeomorphisms: planar apparent horizon
  - Einstein equations: nested linear radial ODEs
  - discretize using pseudo-spectral derivatives
  - low-pass filtering: alleviate aliasing, spectral blocking
  - domain decomposition
- Fast, accurate, stable evolution achievable



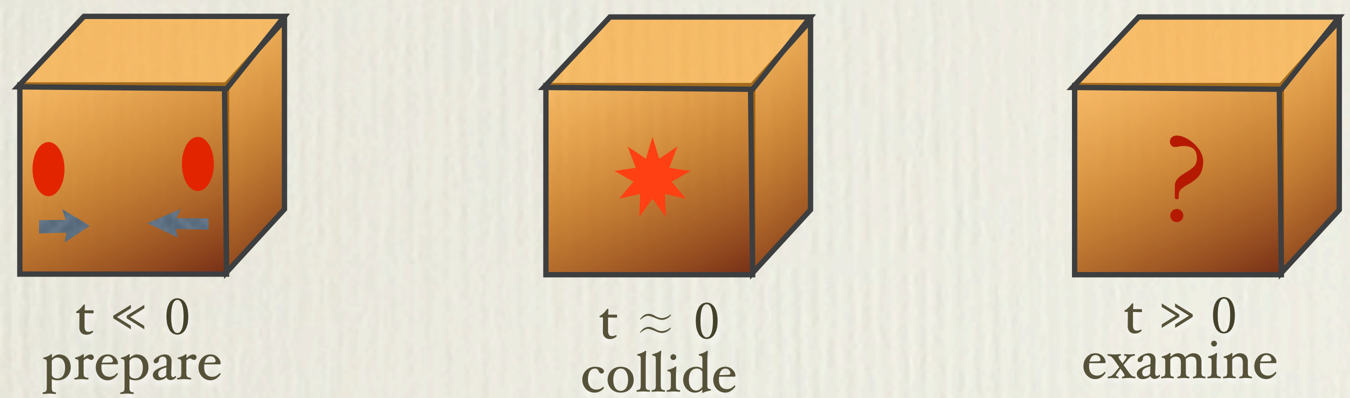
# initial data: choices

- Use time-dependent external fields:

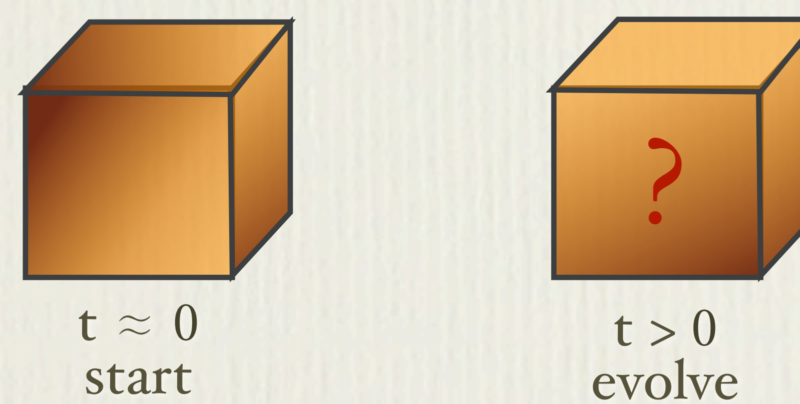
time-dependent dynamics ↔
external work done on system




- Do scattering experiment:




- Choose geometry on initial (null) surface



# colliding planar shocks

- energy density localized on infinite planar sheets
- caricature of large, Lorentz-contracted nuclei
- questions:
  - domain of validity of hydrodynamic approximation?
  - dependence on longitudinal profile?
  - surviving remnants?
  - approximate boost invariance?

# colliding planar shocks

- 2D translation invariance ➡ 2+1D PDEs

$$ds^2 = \Sigma(X)^2\, \hat{g}_{ij}(X)\, dx^i\, dx^j + 2dt\left[dr - A(X)\, dt - F_i(X)\, dx^i\right]$$

$$X = (t, z, r) \qquad\qquad \|\hat{g}_{ij}\| = \mathrm{diag}(e^B, e^B, e^{-2B})$$

- Initial conditions: superposition of counter-propagating planar shocks
  - Single shock, arbitrary longitudinal profile: known solution:

$$ds^2 = r^2[-dx_+ dx_- + d\boldsymbol{x}_\perp^2] + \frac{1}{r^2}\,[dr^2 + h(x_\pm)\, dx_\pm^2]$$

Janik & Peschanski

  - Choose Gaussian profile with width $w$, surface energy density $\mu^3$:

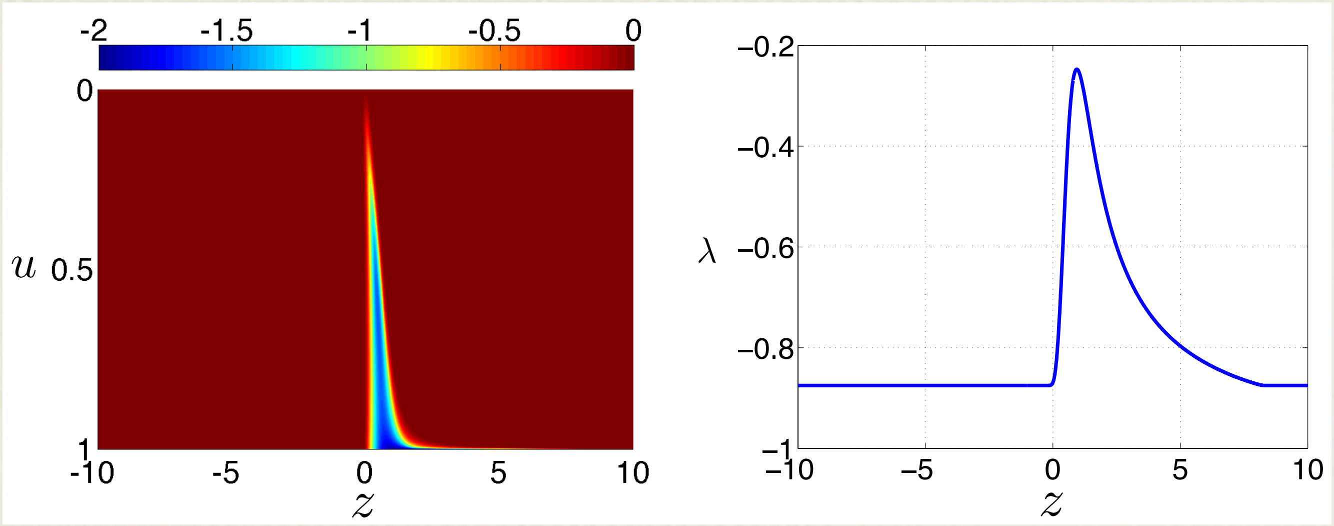
$$h(x_\pm) \equiv \mu^3\, (2\pi w^2)^{-1/2}\, e^{-\frac{1}{2}x_\pm^2/w^2}$$

    - Results depend on dimensionless width parameter $w\mu$

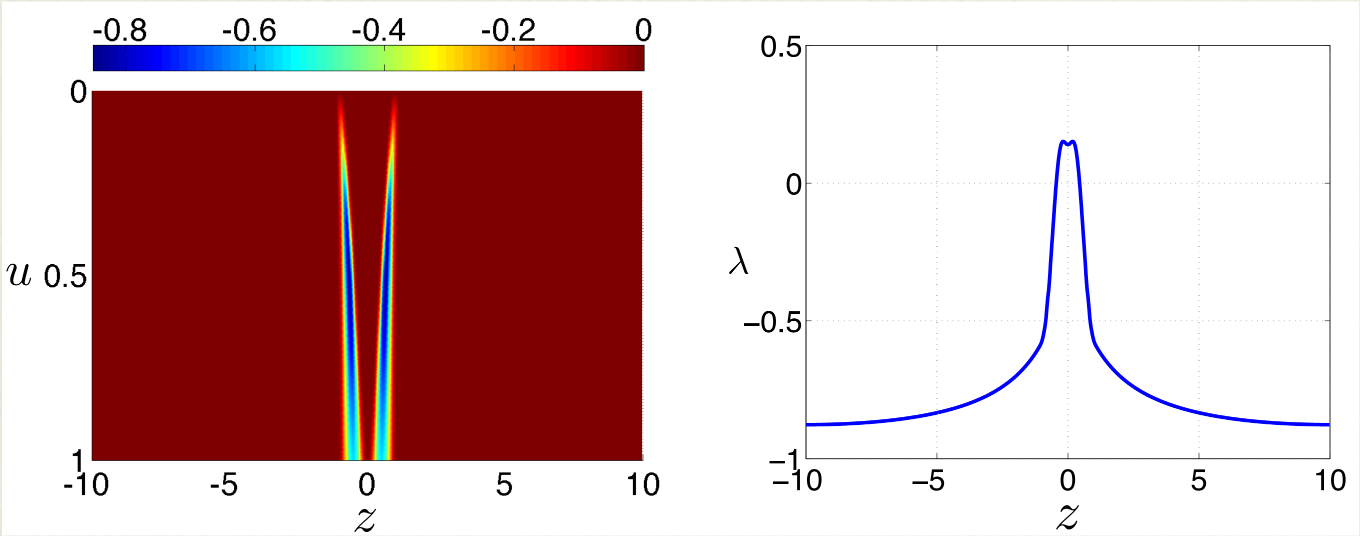
# initial data

- transformation to infalling coordinates:
    - must solve coupled 1+1D PDEs
    - shocks extend "forward" deep in bulk
    - apparent horizon exists regardless of separation

anisotropy function $B(z,r)$ radial shift $\lambda(z)$



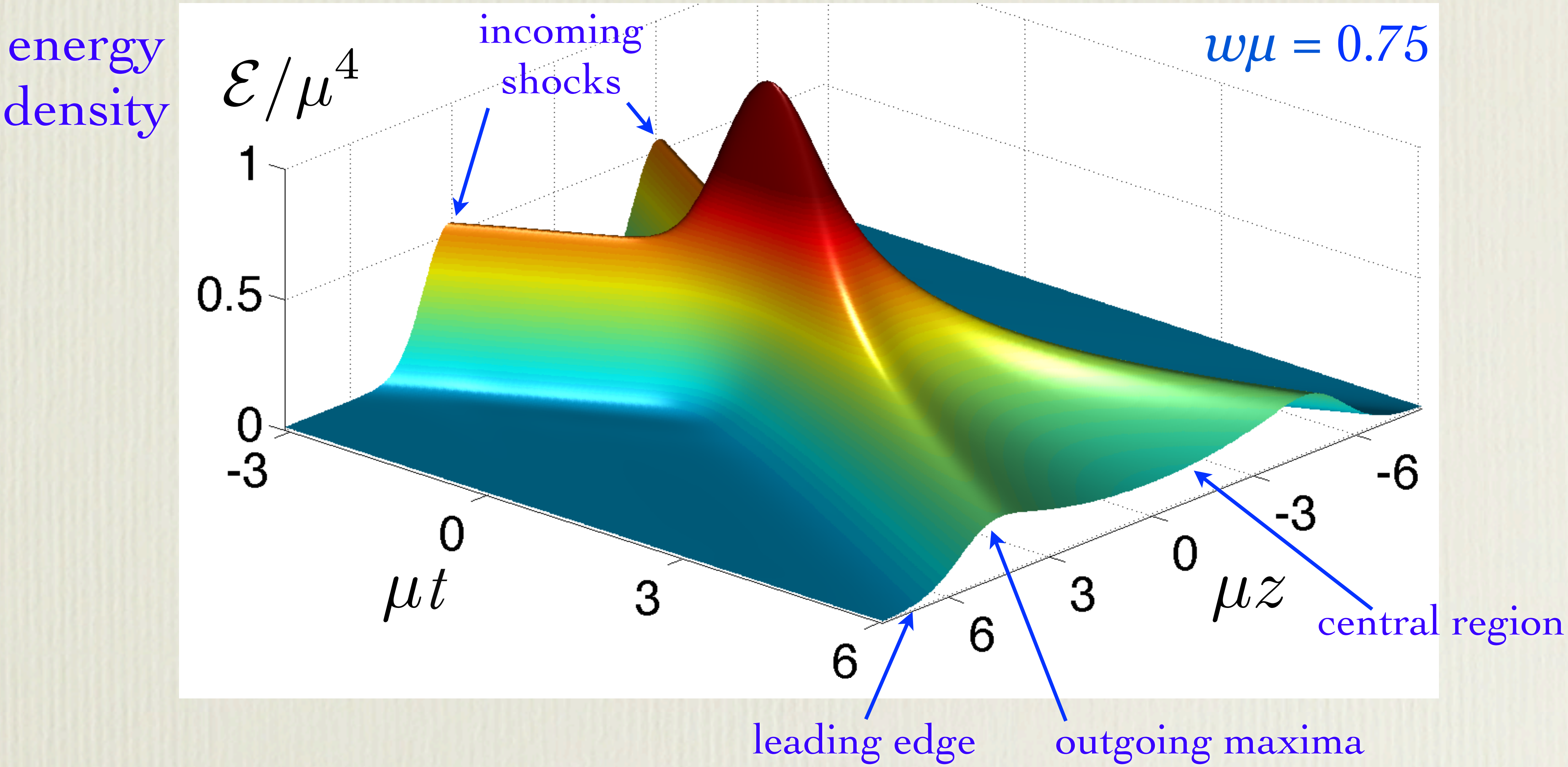

single right-moving shock




superposed shocks

# old results

C&Y: 1011.3562



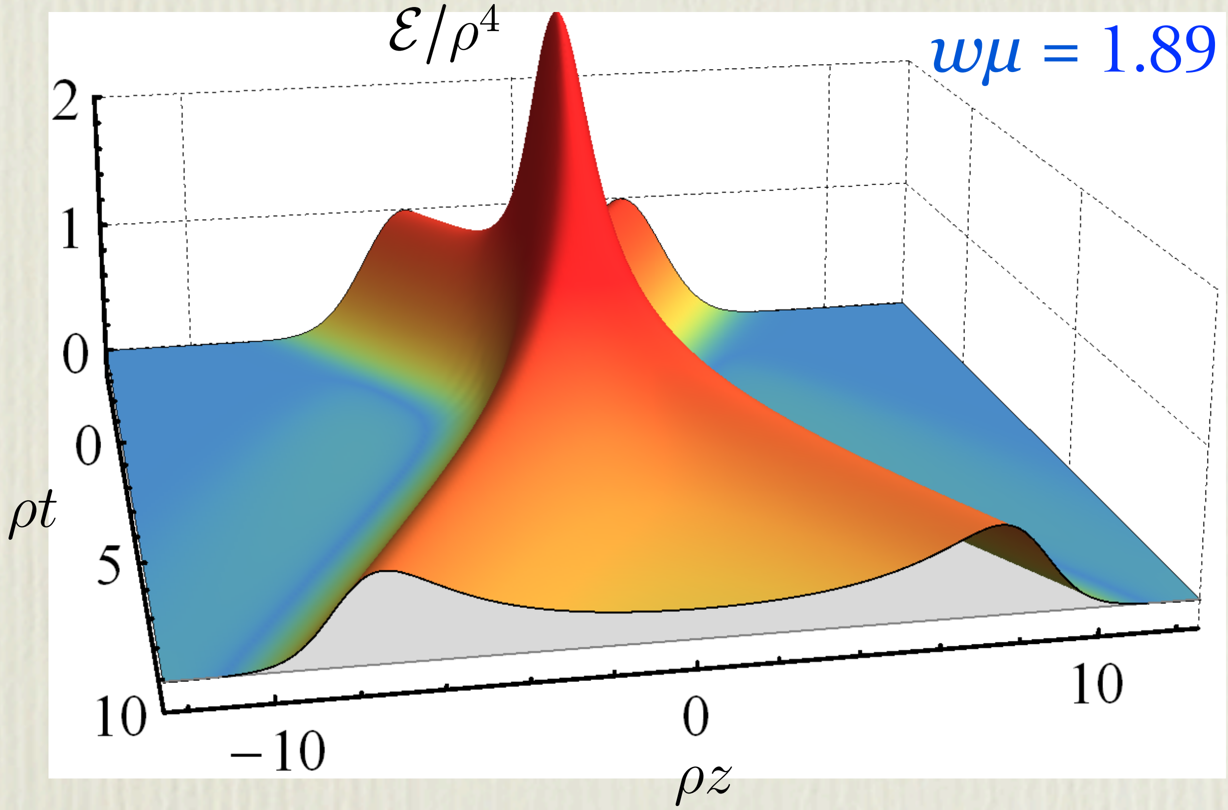

background energy density = 2% of single shock peak energy density

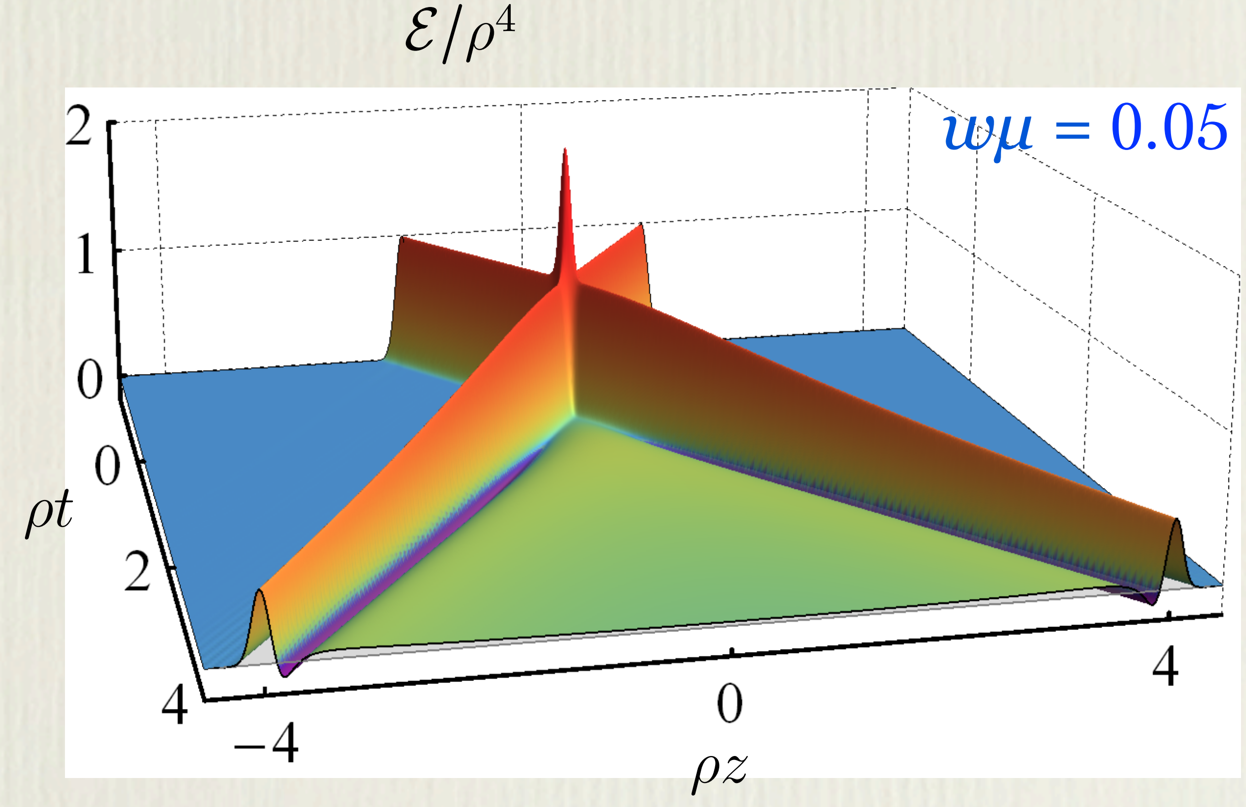
# more recent results

**From full stopping to transparency in a holographic model of heavy ion collisions**

Jorge Casalderrey-Solana,[1] Michal P. Heller,[2,*] David Mateos,[3,4] and Wilke van der Schee[5]

1305.4919



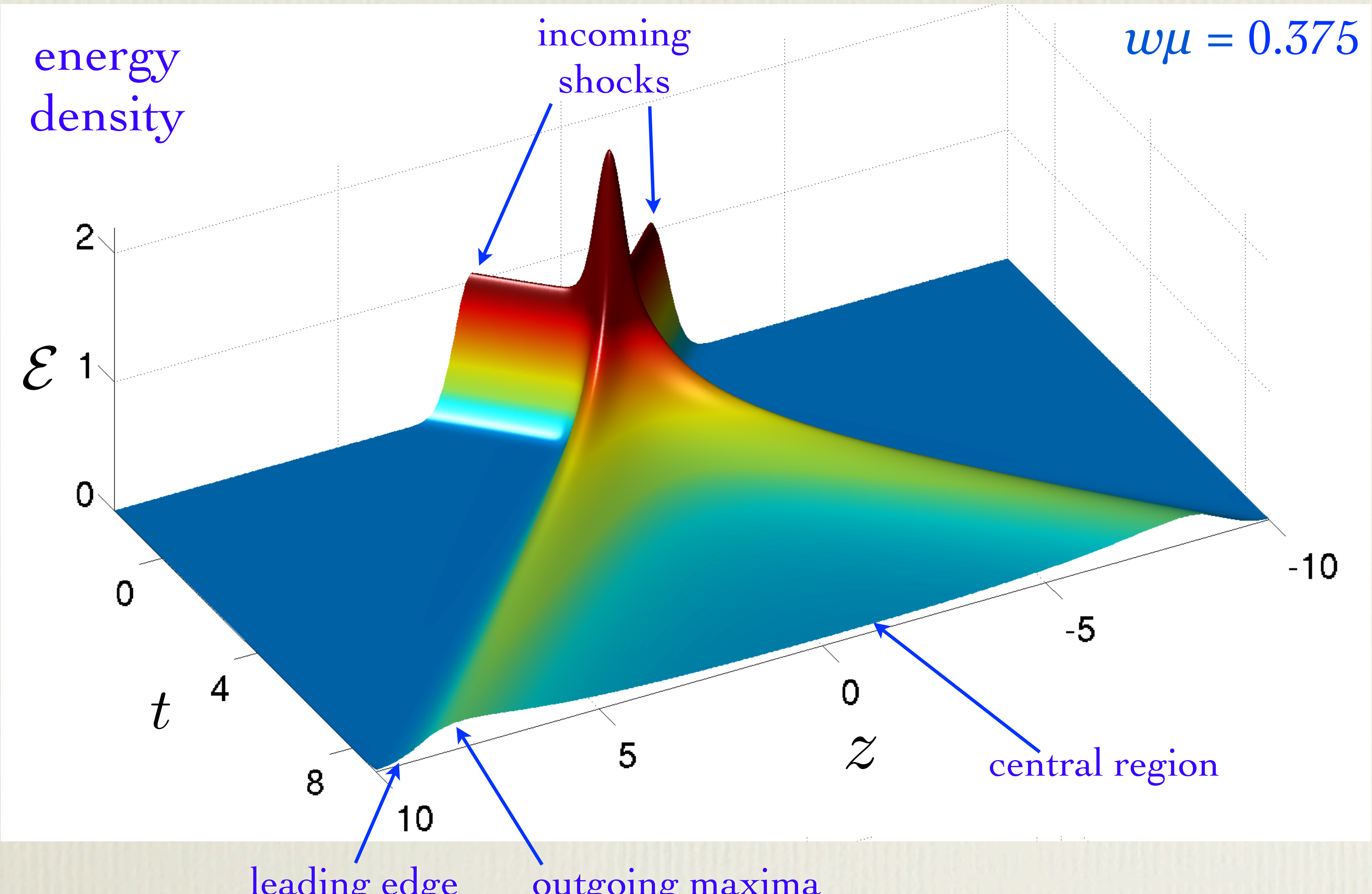

background energy density = 1.5 - 7.5% of single shock peak energy density

"We uncover a cross-over between two different dynamical regimes...
At high energies, receding fragments move outward at the speed of light."

# newer results (I)

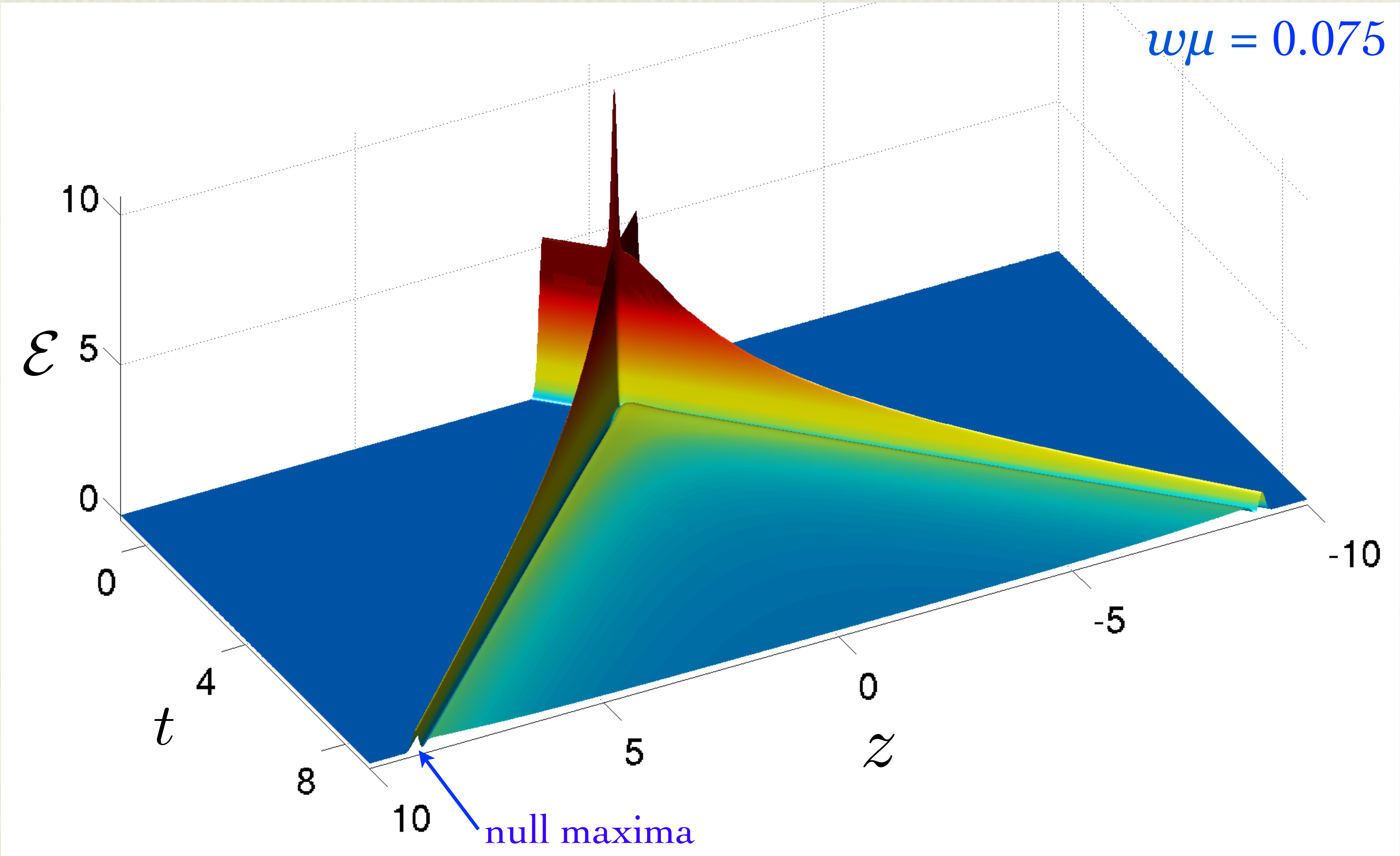
C&Y: 1309.1439

New: no background energy density

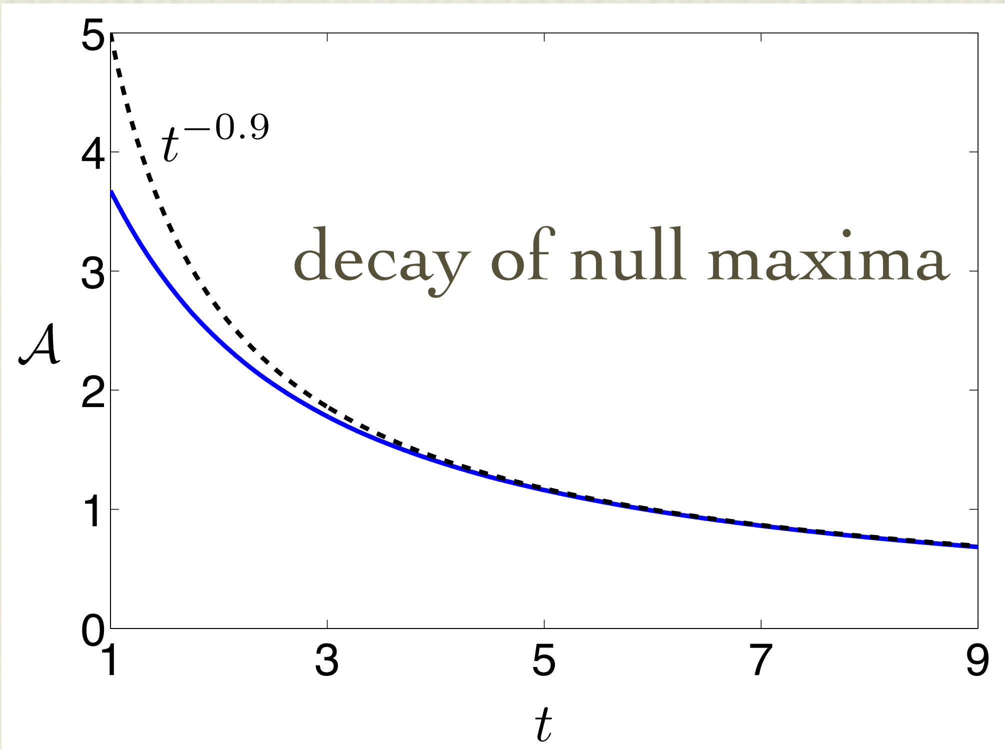


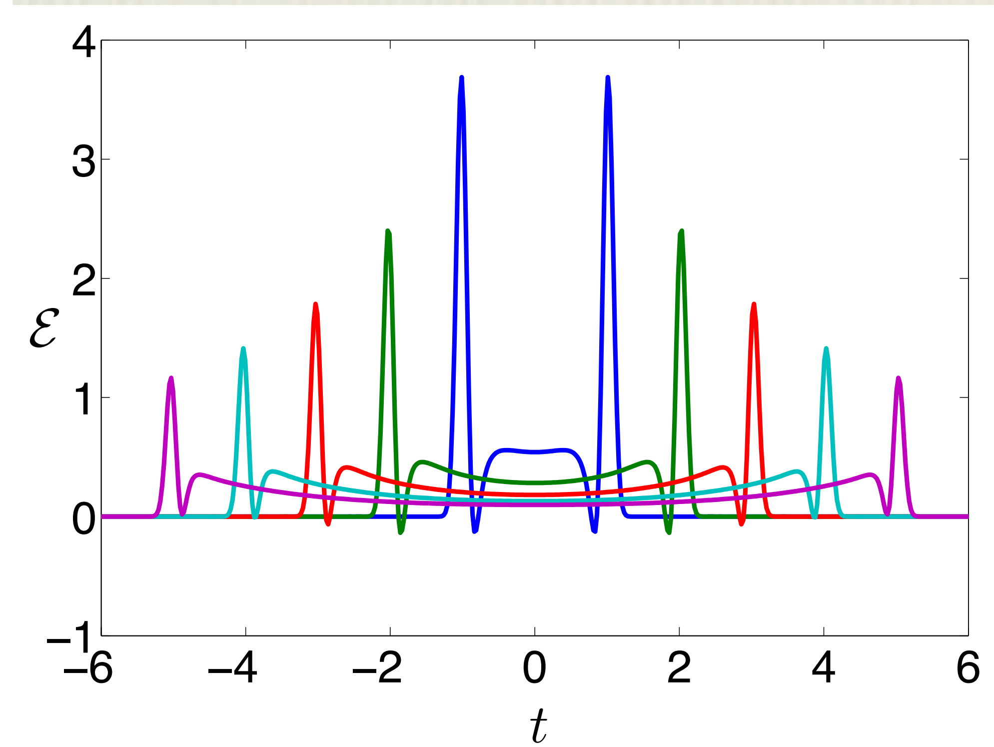
# newer results (II)

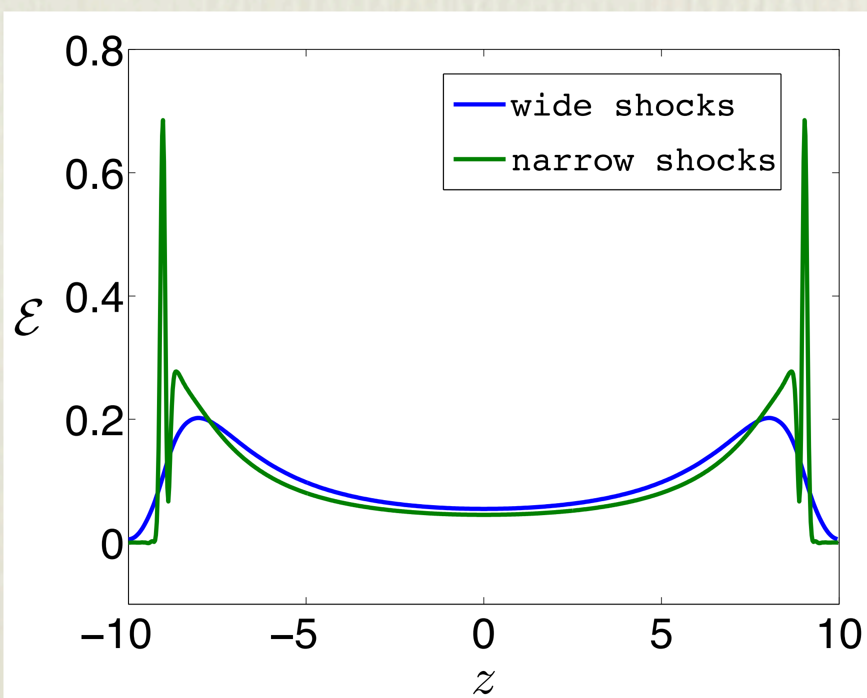
no background energy density,
longer time evolution



# qualitative features




wide vs. narrow shocks,
$t$ = 9

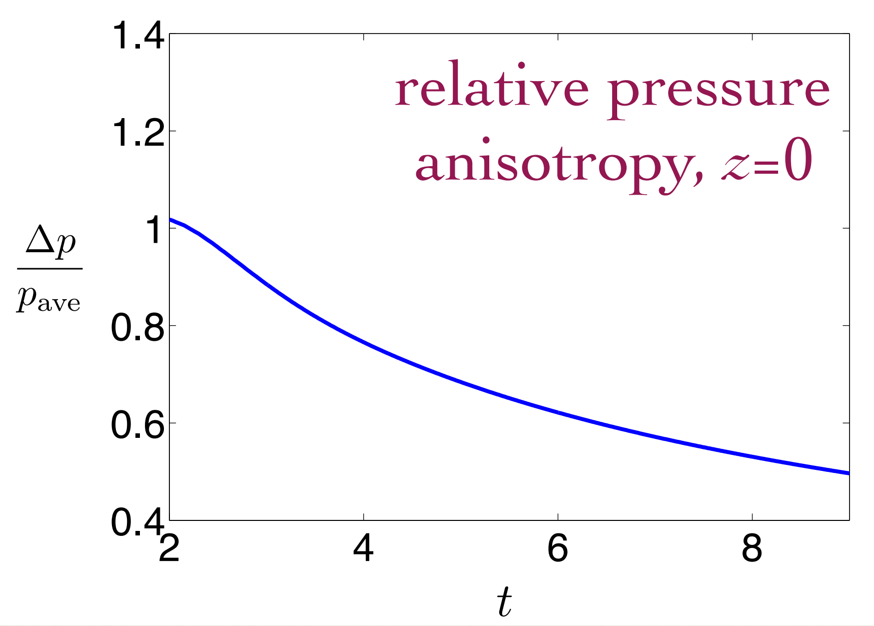
# validity of hydrodynamics

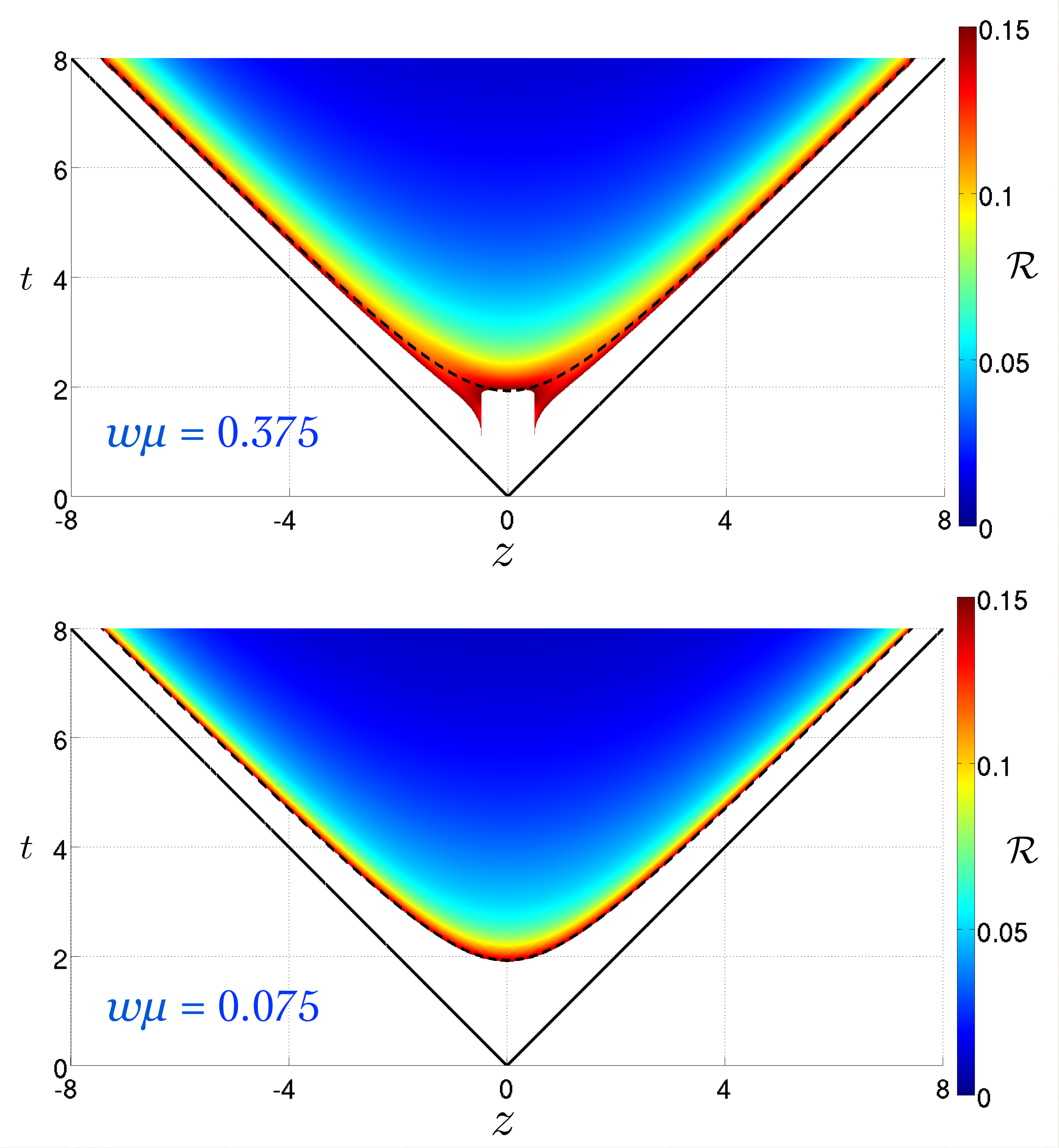
figure of merit:

$$\mathcal{R} \equiv \frac{1}{\bar{p}} \Big[ (\langle T^{xx} \rangle - T^{xx}_{\rm hydro})^2 + (\langle T^{zz} \rangle - T^{zz}_{\rm hydro})^2 \Big]^{1/2} \leq 15\%$$

first order viscous

hydro works even when viscous effects are $O(1)$:



# local boost invariance

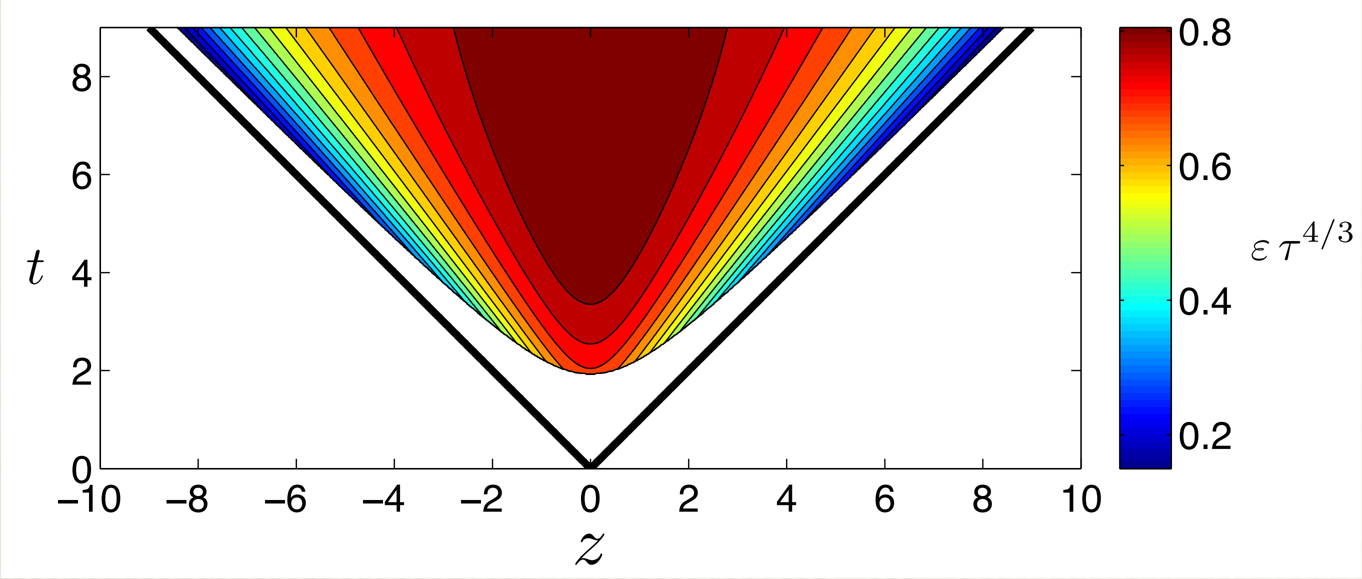
boost invariant flow:

$$u_\mu \, dx^\mu = d\tau \equiv \cosh y \, dt + \sinh y \, dz \,,$$

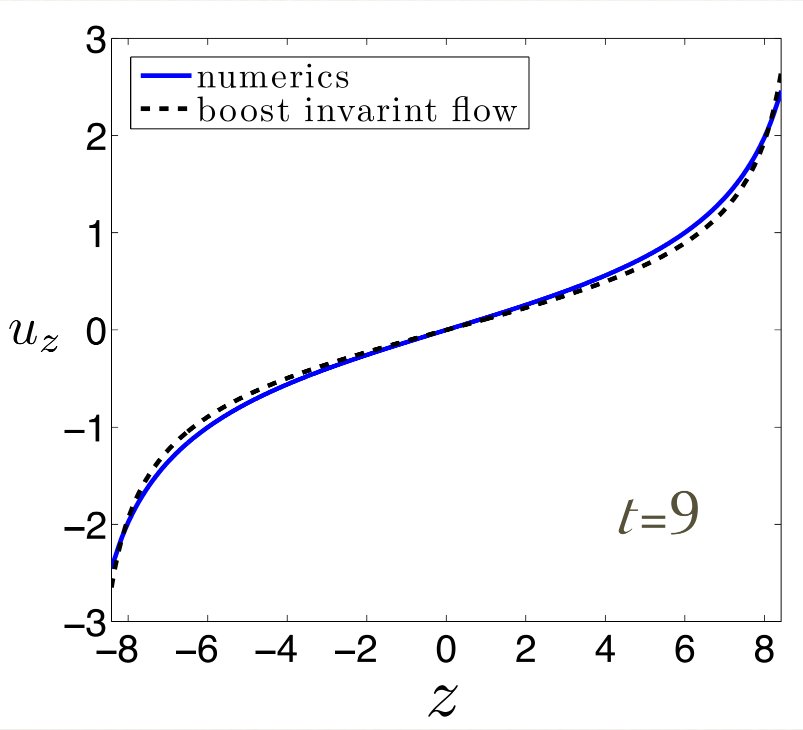
$$\varepsilon = \frac{3}{4} \frac{(\pi\Lambda)^4}{(\Lambda\tau)^{4/3}} \left[ 1 - \frac{C_1}{(\Lambda\tau)^{2/3}} + \frac{C_2}{(\Lambda\tau)^{4/3}} + O\Big(\frac{1}{(\Lambda\tau)^2}\Big) \right]$$

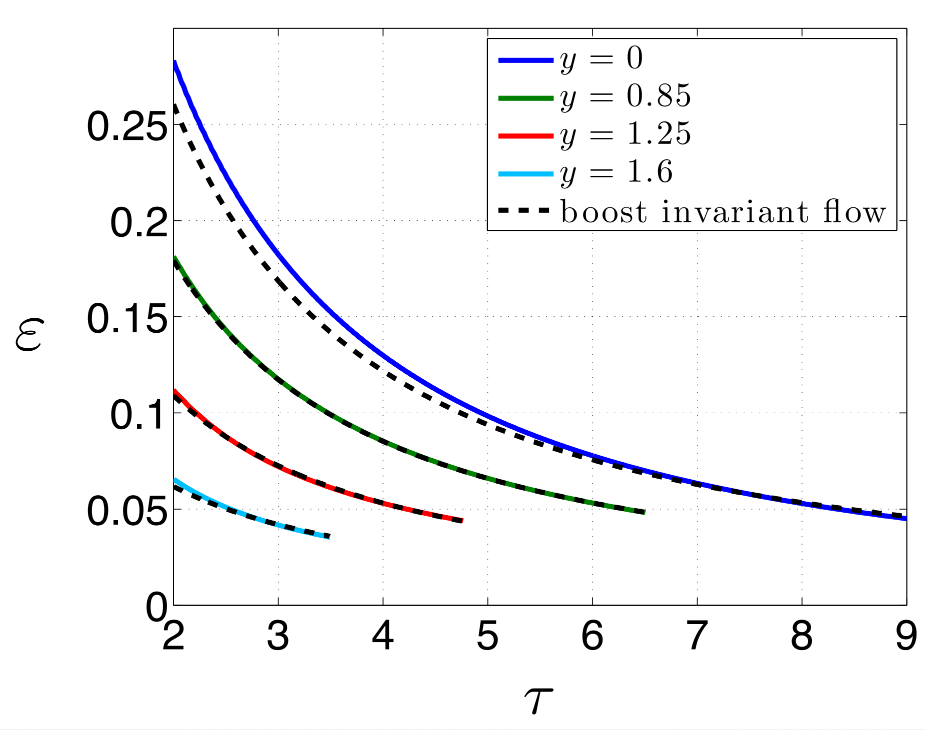
proper time $\tau \equiv \sqrt{t^2 - z^2}$

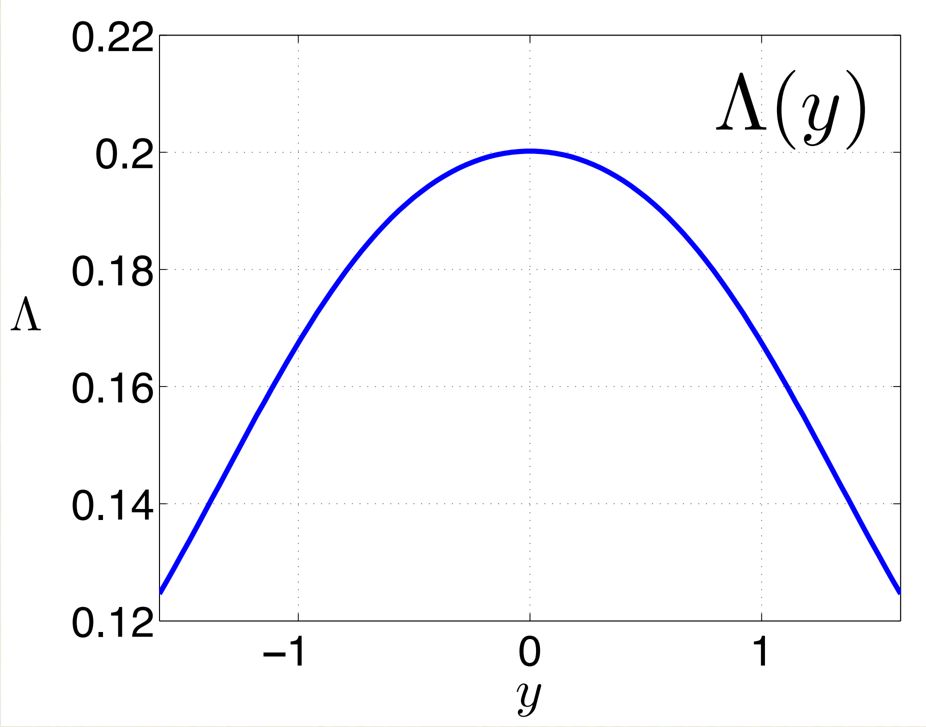
rapidity $y \equiv \tanh^{-1}(z/t)$



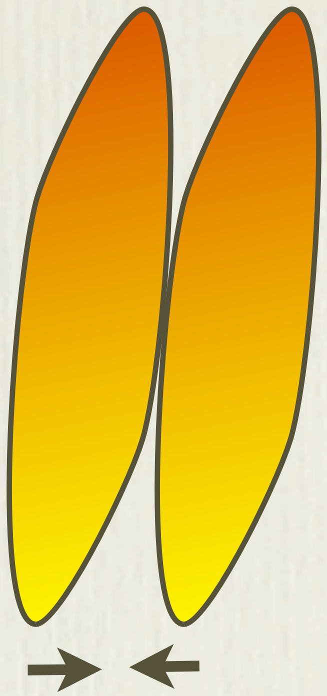

local boost invariance: $\Lambda \to \Lambda(y)$



# colliding “nuclei”



- finite transverse extent, cylindrically symmetric
- single “nucleus”: smooth, localized null “shock”
  - ✓ exact solution = linear superposition of infinitely boosted point sources — Gubser, Pufu, Yarom
  - ✓ transformation to null infalling coordinates
- implementation for general 4+1D case: in progress

# colliding shocks: lessons

- Early times: large anisotropy, far from local equilibrium
- Rapidly attenuating outgoing maxima, no surviving remnants
- Mid-rapidity: hydrodynamics quickly becomes valid, despite large viscous effects

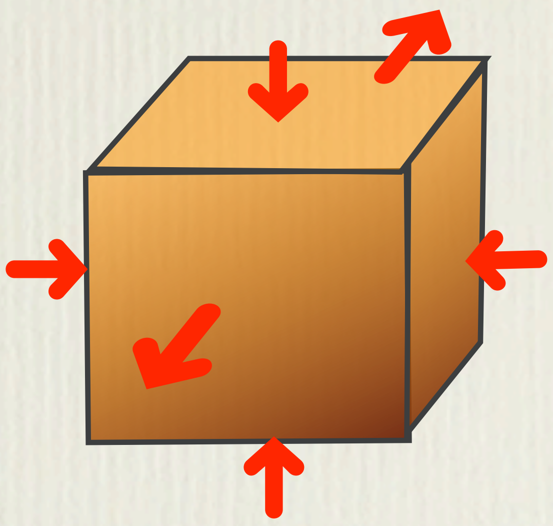
  onset of hydro validity ≈ $4/\mu$ after initial interaction

  $\mu \approx 2.3$ GeV for modeling RHIC ➡ $\tau_{hydro} \approx 0.35$ fm/c

- Near outgoing lightcone: hydrodynamics not reliable
- Central region: “local” but not global boost-invariance

# homogeneous isotropization

- caricature of early moments of QGP
- no spatial gradients
  - ➡ no hydrodynamic response
  - ➡ exponential relaxation
- questions:
  - relaxation time scale?
  - onset of linearized regime?
  - sensitivity to (baryon) charge density?
  - sensitivity to magnetic field?



# homogeneous isotropization

arXiv:0812.2053

spatial homogeneity

➡ $g_{\mu\nu} = g_{\mu\nu}(t, r)$

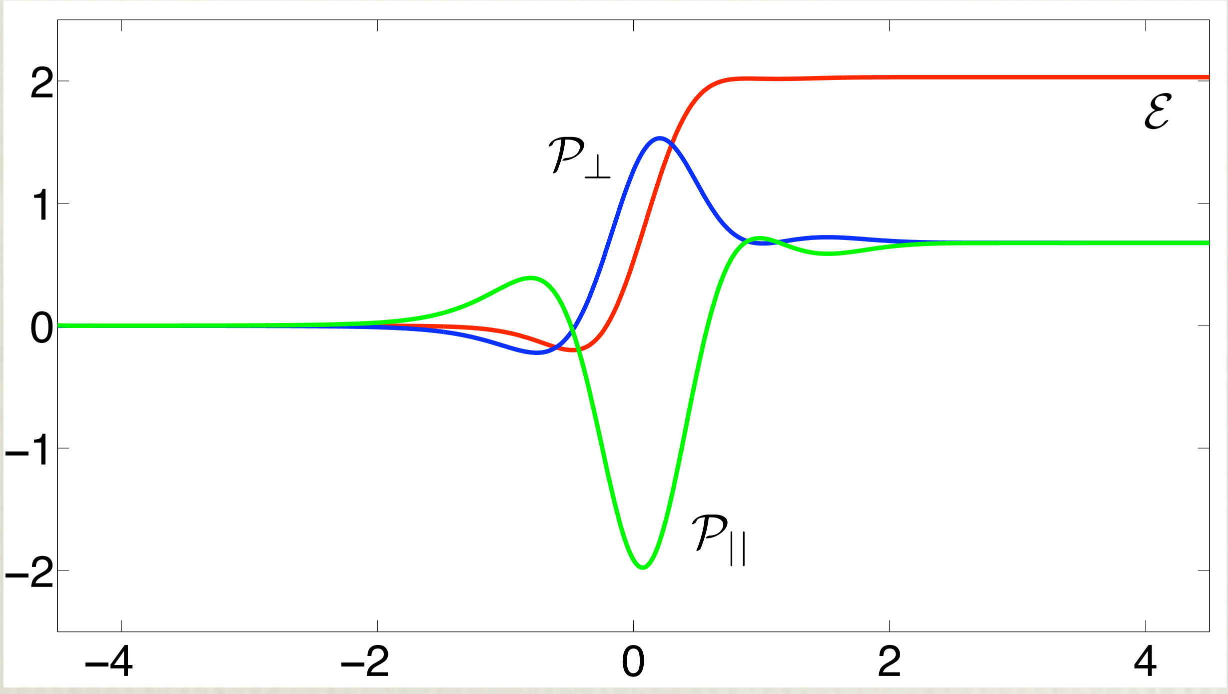
➡ 1+1D PDEs

$$\|\hat{g}_{ij}\| = \mathrm{diag}(e^{B}, e^{B}, e^{-2B})$$

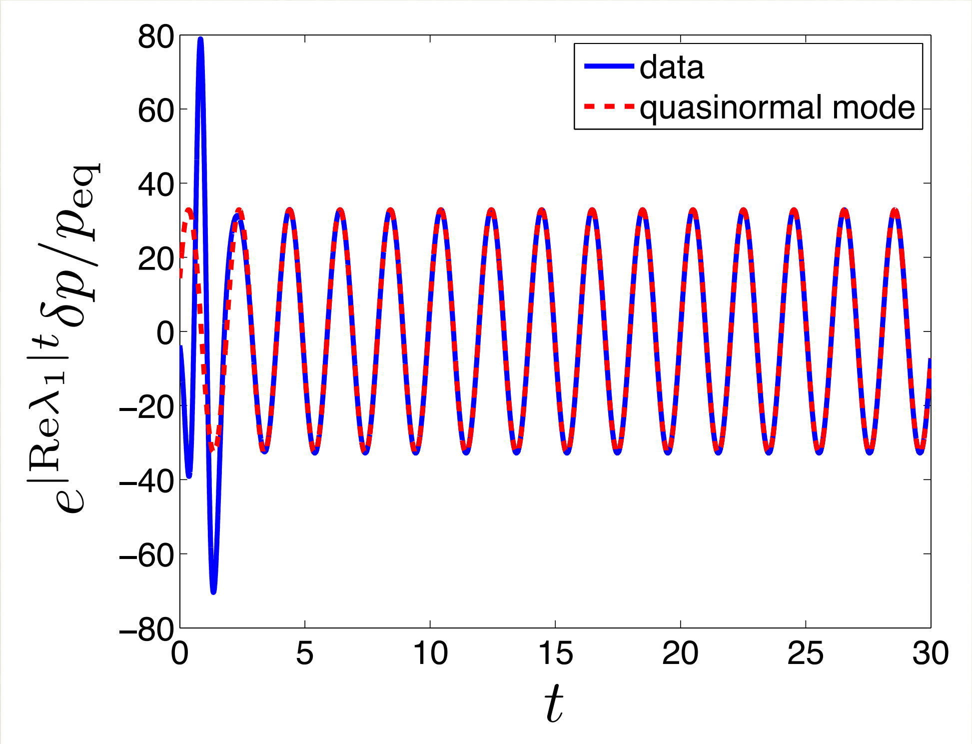
$$\begin{aligned} 0 &= \Sigma'' + \tfrac{1}{2} B'^2\, \Sigma \\ 0 &= \Sigma\,(\dot{\Sigma})' + 2\Sigma'\,\dot{\Sigma} - 2\Sigma^2\,, \\ 0 &= \Sigma\,(\dot{B})' + \tfrac{3}{2}\big(\Sigma'\dot{B} + B'\,\dot{\Sigma}\big)\,, \\ 0 &= A'' + 3B'\dot{B} - 12\Sigma'\,\dot{\Sigma}/\Sigma^2 + 4 \end{aligned}$$

$$\dot{h} \equiv d_+ h\,, \quad h' \equiv \partial_r h$$

transverse & longitudinal pressure




(rescaled) pressure anisotropy
vs. lowest quasinormal mode



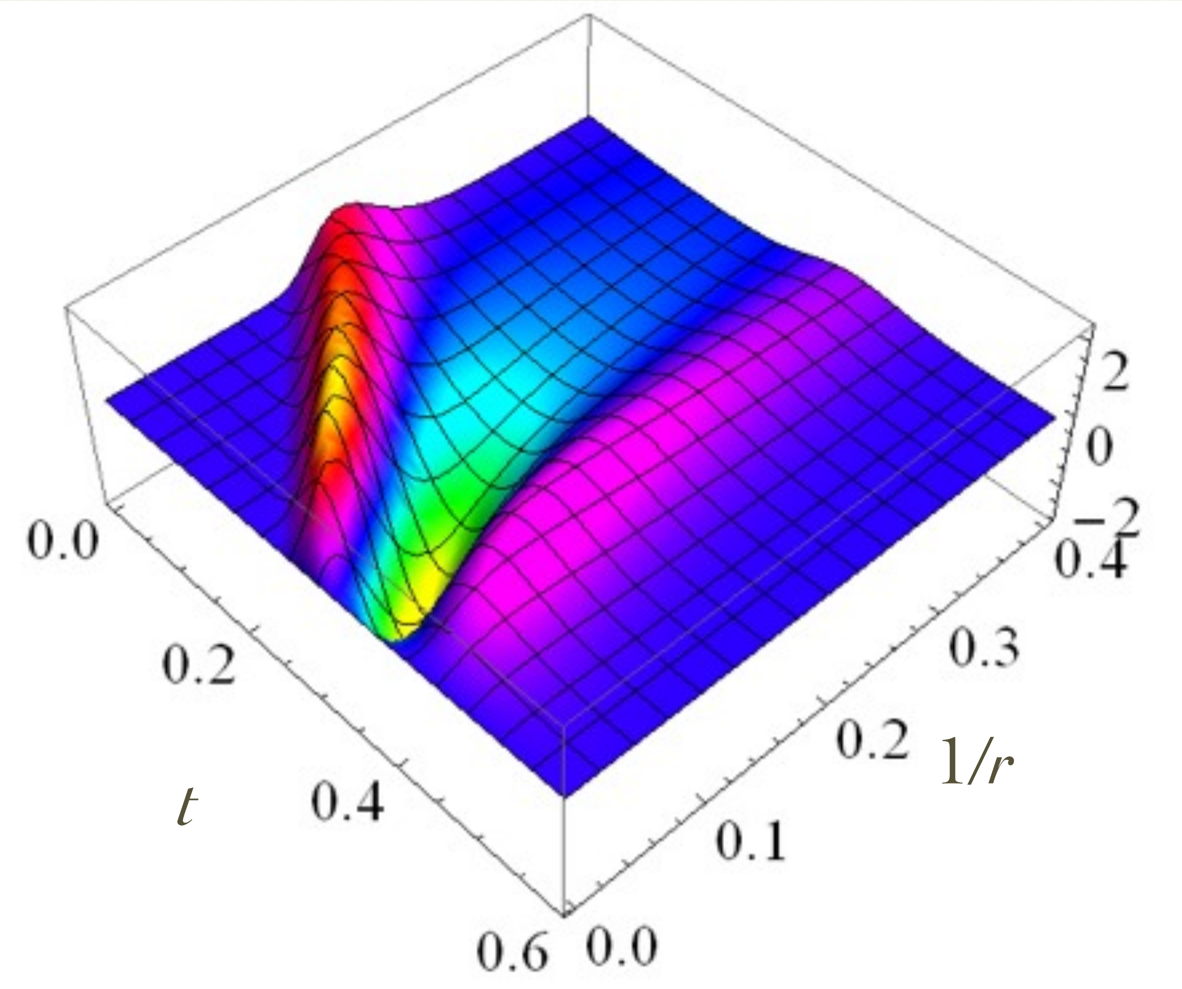
# homogeneous isotropization: recent work

- sensitivity to initial data choice
- sensitivity to non-zero charge density
- sensitivity to non-zero magnetic field
- deviation from linearized dynamics

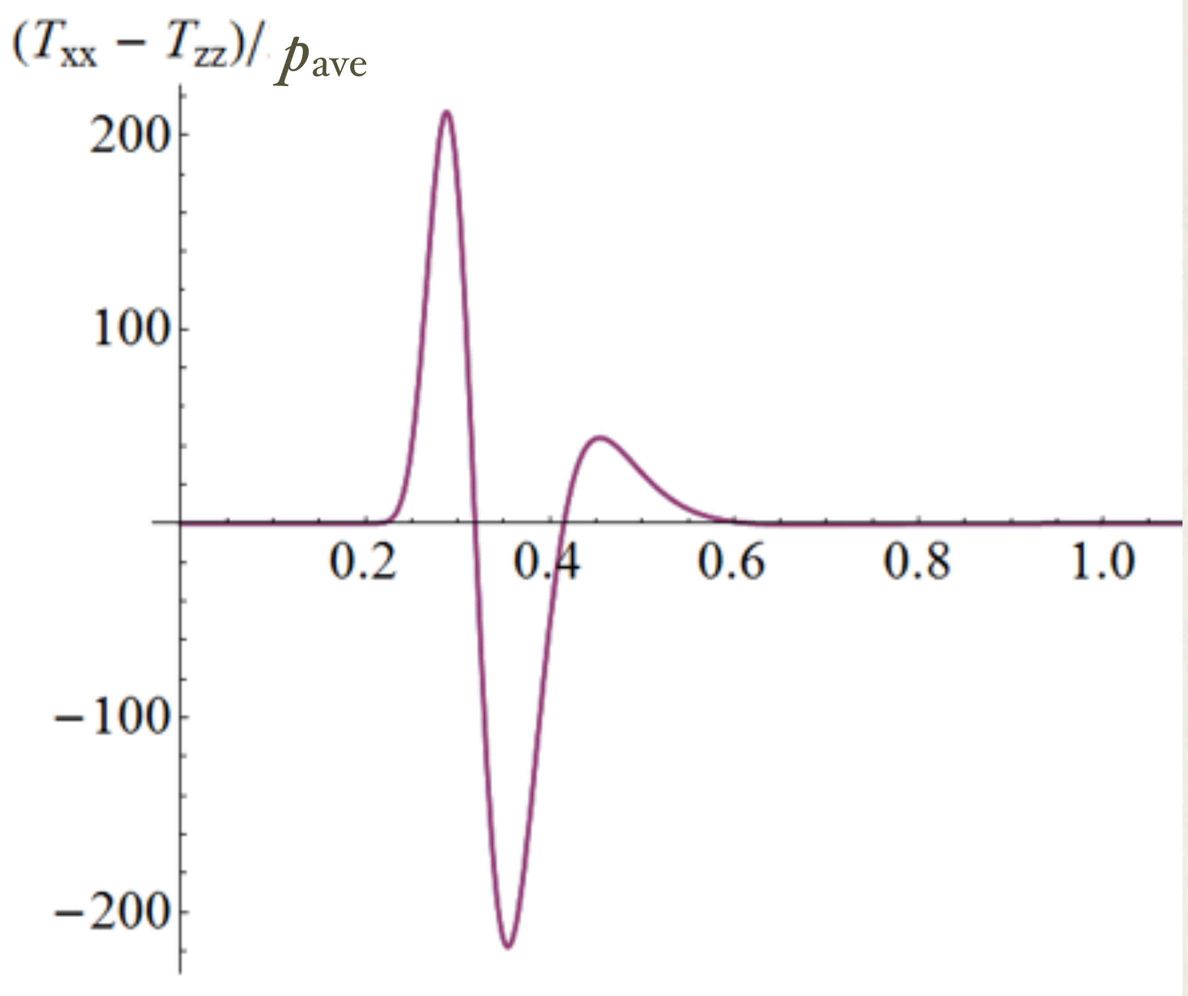
# homogeneous isotropization

initial data: Gaussian anisotropy function $B(r)$

anisotropy function $B(r)$




transverse & longitudinal pressure



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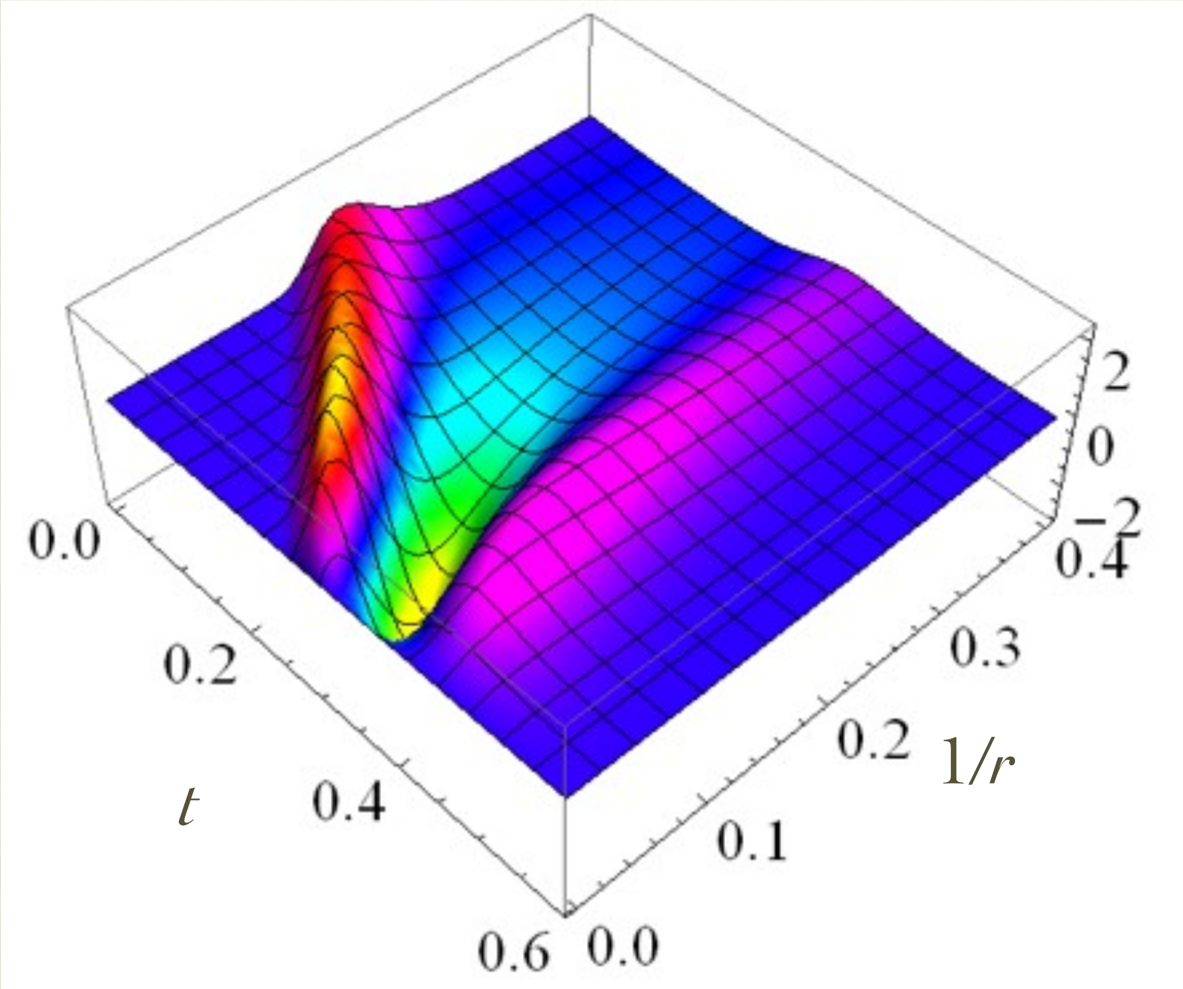
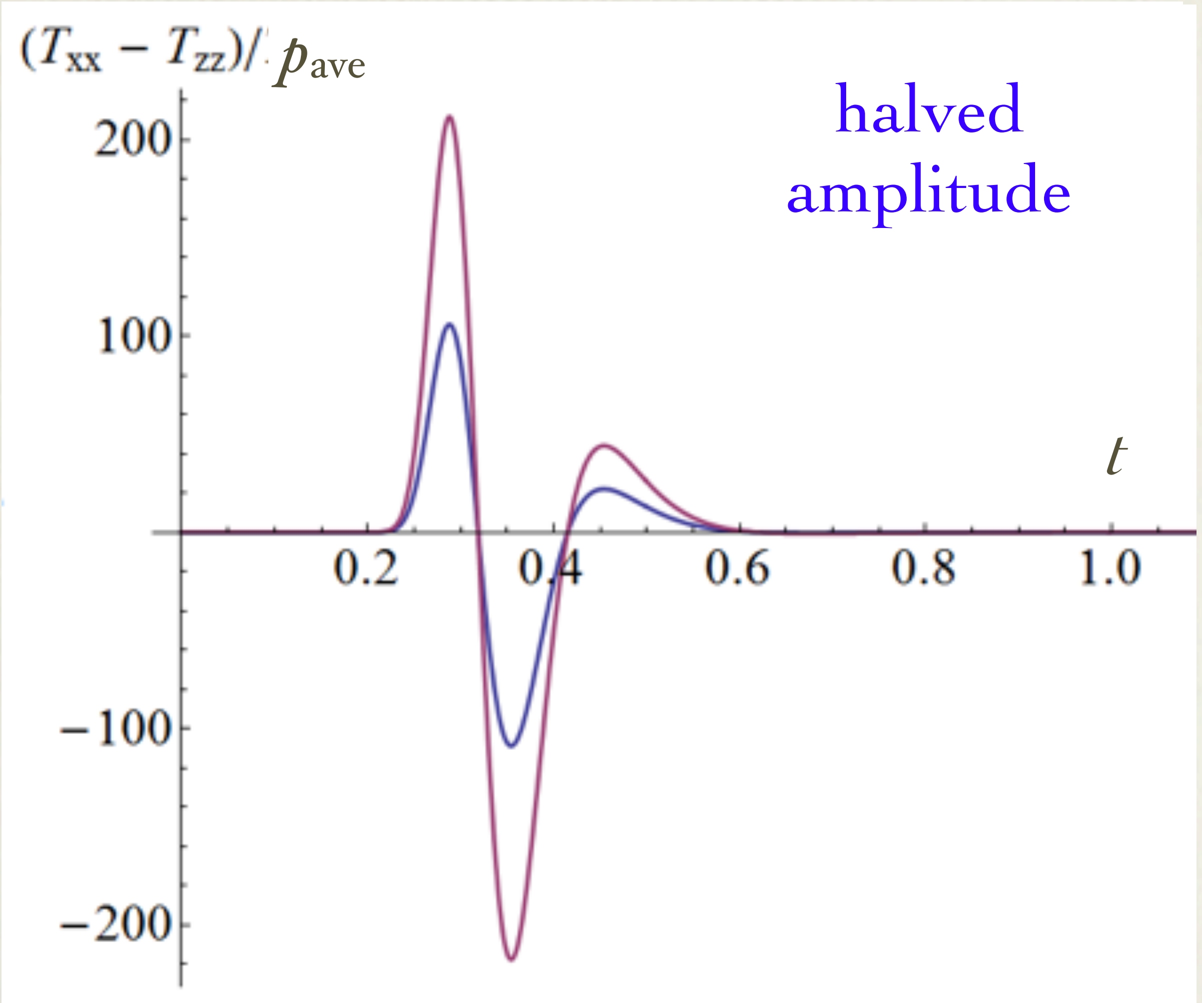



transverse & longitudinal pressure



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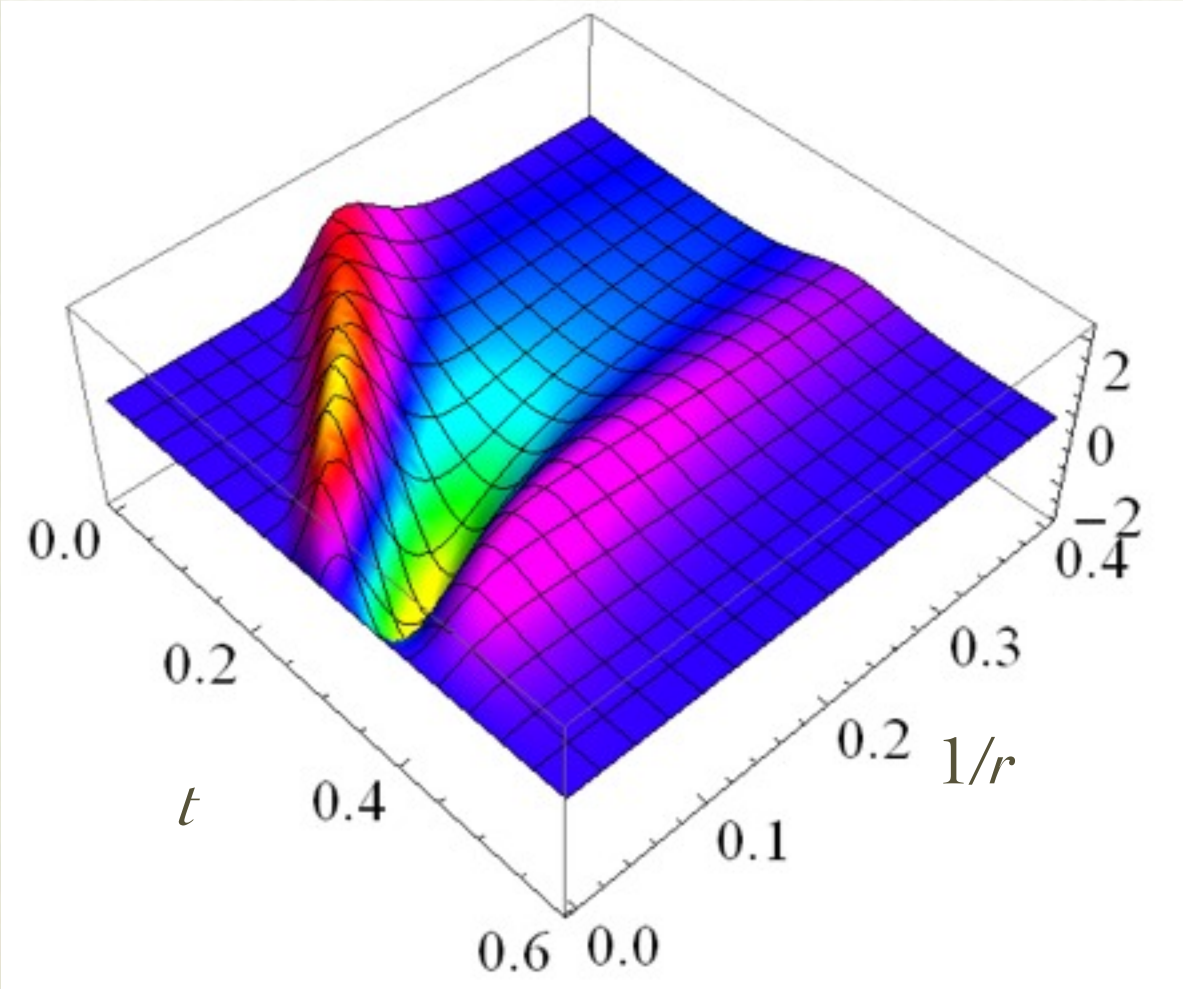
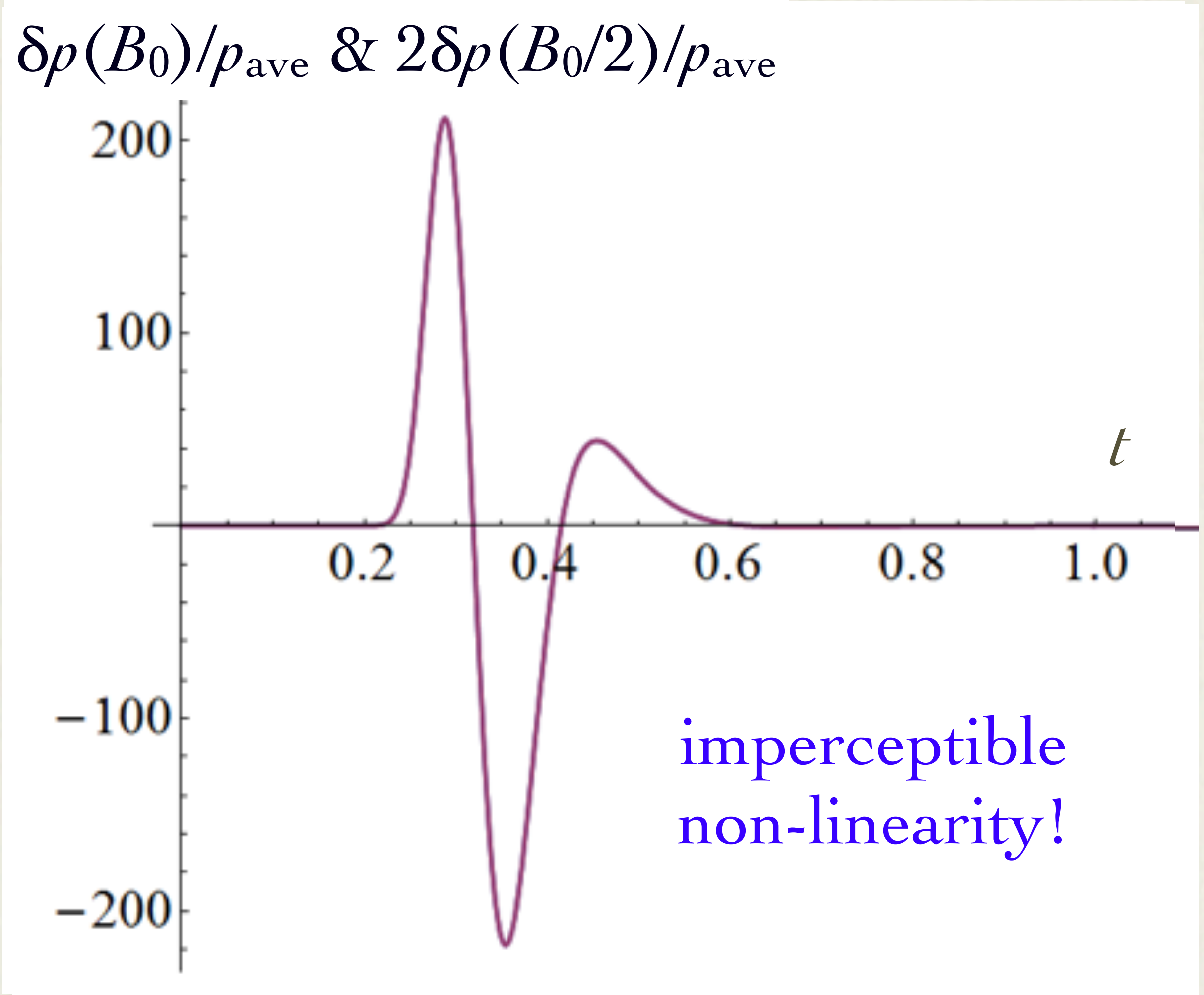



transverse & longitudinal pressure

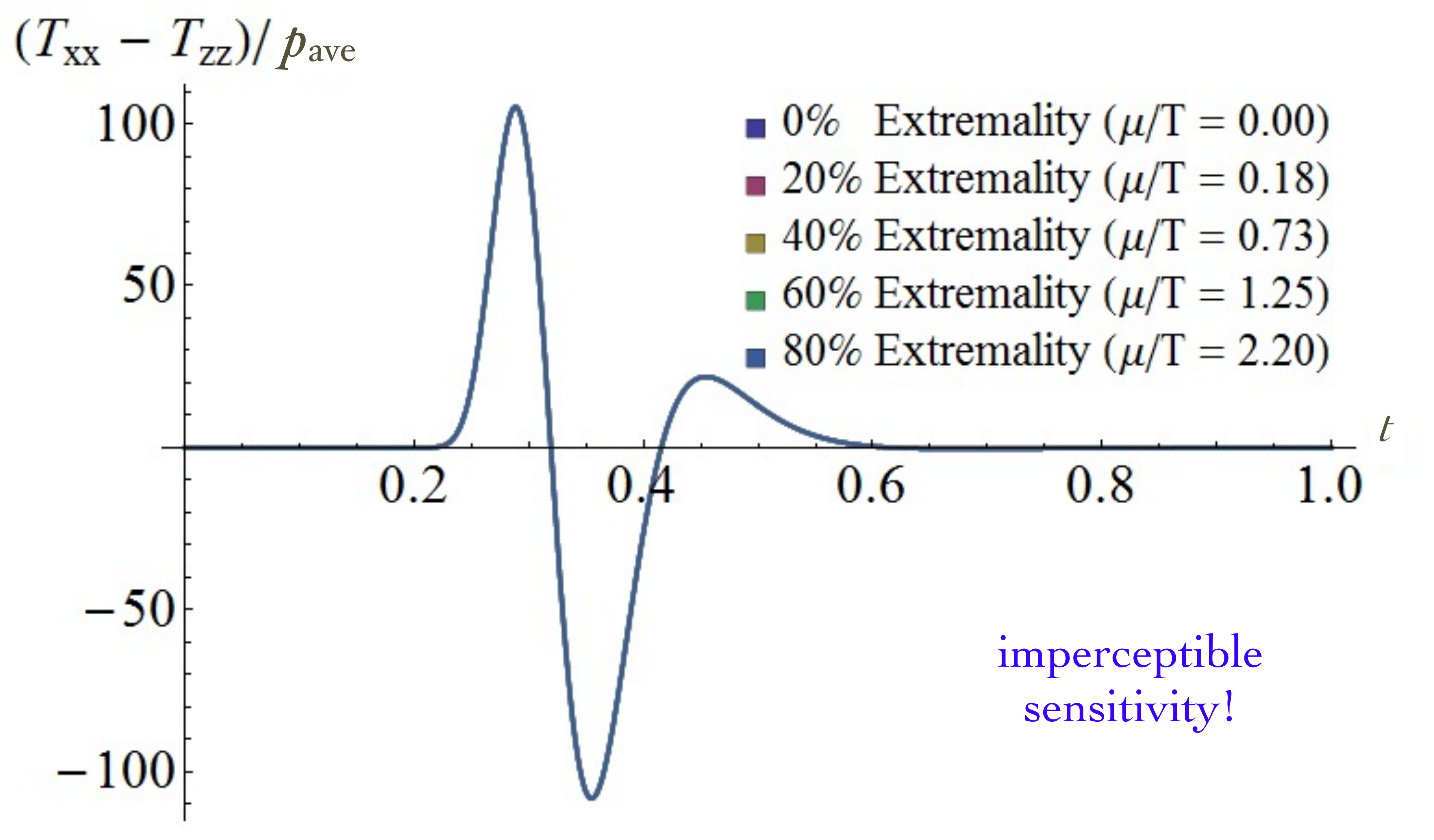



see also: Heller, Mateos, van der Schee, Triana 1304.5172

# non-zero charge density

- heavy ion collisions: $\mu/T = O(0.2)$
- sensitivity to plasma constituents?
- equilibrium state ↔ Reissner-Nordstrom black brane
  - maximal charge density ↔ extremal brane
  - $O(10\%)$ change in quasi-normal mode frequencies

# non-zero charge density

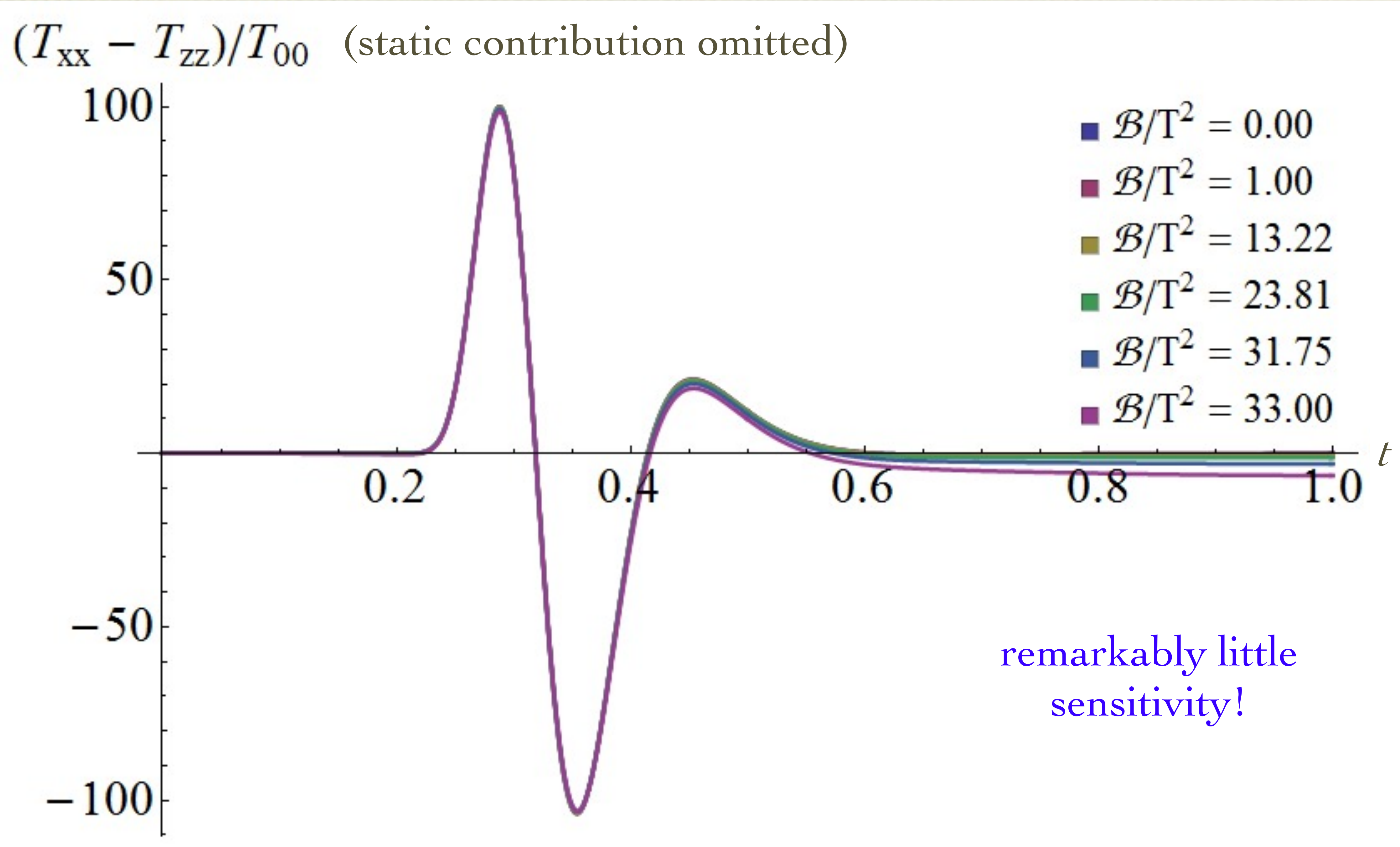


# non-zero magnetic field

- heavy ion collisions: much concern about E&M effects
- sensitivity to magnetic field?
  - externally imposed anisotropy
  - trace anomaly ➡ broken scale invariance
  - $\langle T_{\mu\nu} \rangle = \kappa \left[ \widetilde{g}^{(4)}_{\mu\nu} - \eta_{\mu\nu} \, \mathrm{tr}\,(\widetilde{g}^{(4)}) + c_0 \, \widetilde{h}^{(4)}_{\mu\nu} \right]$ $\qquad \|\widetilde{h}^{(4)}_{\alpha\beta}\| = \frac{1}{2}\mathcal{B}^2 \, \mathrm{diag}(+1,\, +1,\, +1,\, -1)$

classical E&M stress-energy

# non-zero magnetic field



# homogeneous isotropization: lessons

- relaxation time scale = gravitational infall time
- remarkably little sensitivity to plasma constituents
- remarkably little sensitivity to added magnetic field
- dynamics, as probed by boundary observables, is nearly linear even very, very far from equilibrium!

# remarks (I)

- using gauge/gravity duality to study strongly coupled far-from-equilibrium dynamics works for interesting variety of problems
  - characteristic formulation, adapted to gravitational infall ➡ remarkably simple equations allowing efficient integration
  - can achieve stable evolution
  - desktop resources suffice for 1+1D, 2+1D, and even 3+1D problems
- no need to be professional numerical relativist!

# remarks (II)

- work to date has only scratched the surface; many interesting generalizations await:
  - collisions:
    - asymmetric planar shocks
    - "nuclei" with finite transverse extent
  - turbulence in three spatial dimensions:
    - normal fluids
    - superfluids
  - dynamics in non-conformal theories with more complicated dual gravitational descriptions