On the Perturbative Evaluation of Thermal Green's Functions in the Bulk and Shear Channels of Yang-Mills Theory

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Outline

- Motivation
- Correlators in SU(Nc) Yang-Mills theory
- Results
  - Spectral densities
  - Discussion on HTL correction
- Summary and Outlook

Linearized Viscous Hydrodynamics

- Hydrodynamic with small viscosity turns out to be a successful theory for the description of QGP in high energy HIC!

- Macroscopic Form of Energy Momentum Tensor:

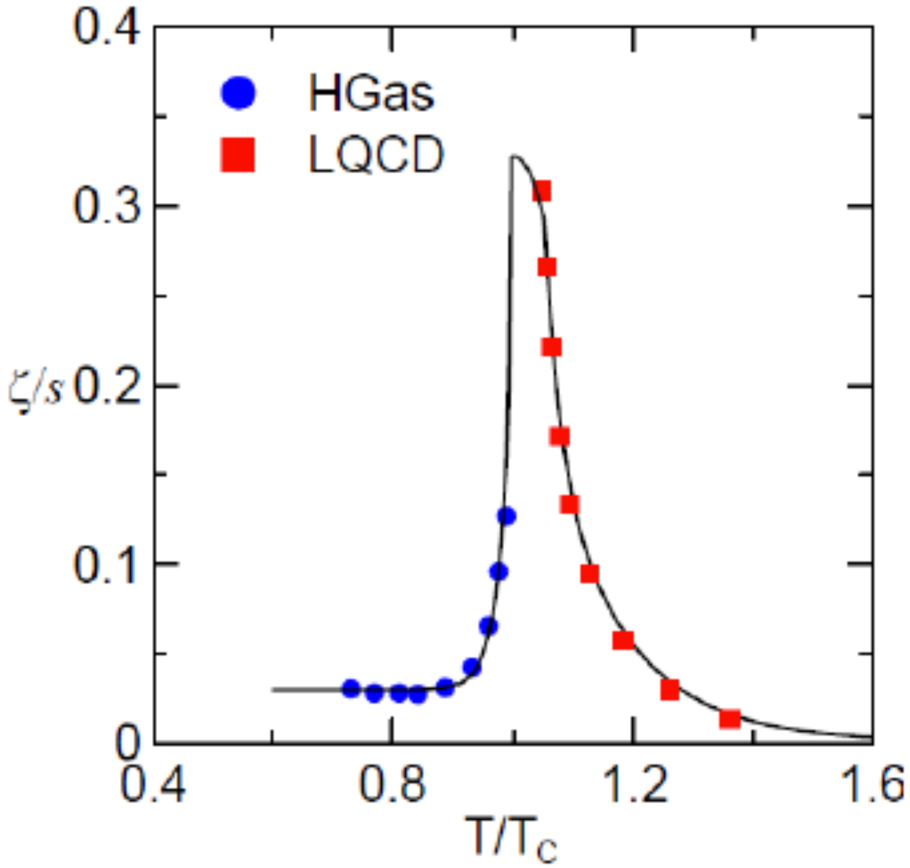
$$T^{\mu\nu} = -Pg^{\mu\nu} + (e+P)u^\mu u^\nu + \Delta T^{\mu\nu}$$

$$\Delta T^{\mu\nu} = \eta(\Delta^\mu u^\nu + \Delta^\nu u^\mu) + (\frac{2}{3}\eta - \zeta)H^{\mu\nu}\partial_\rho u^\rho$$

$$\Delta^\mu = \partial_\mu - u_\mu u^\beta \partial_\beta, \quad H^{\mu\nu} = u^\mu u^\nu - g^{\mu\nu}$$

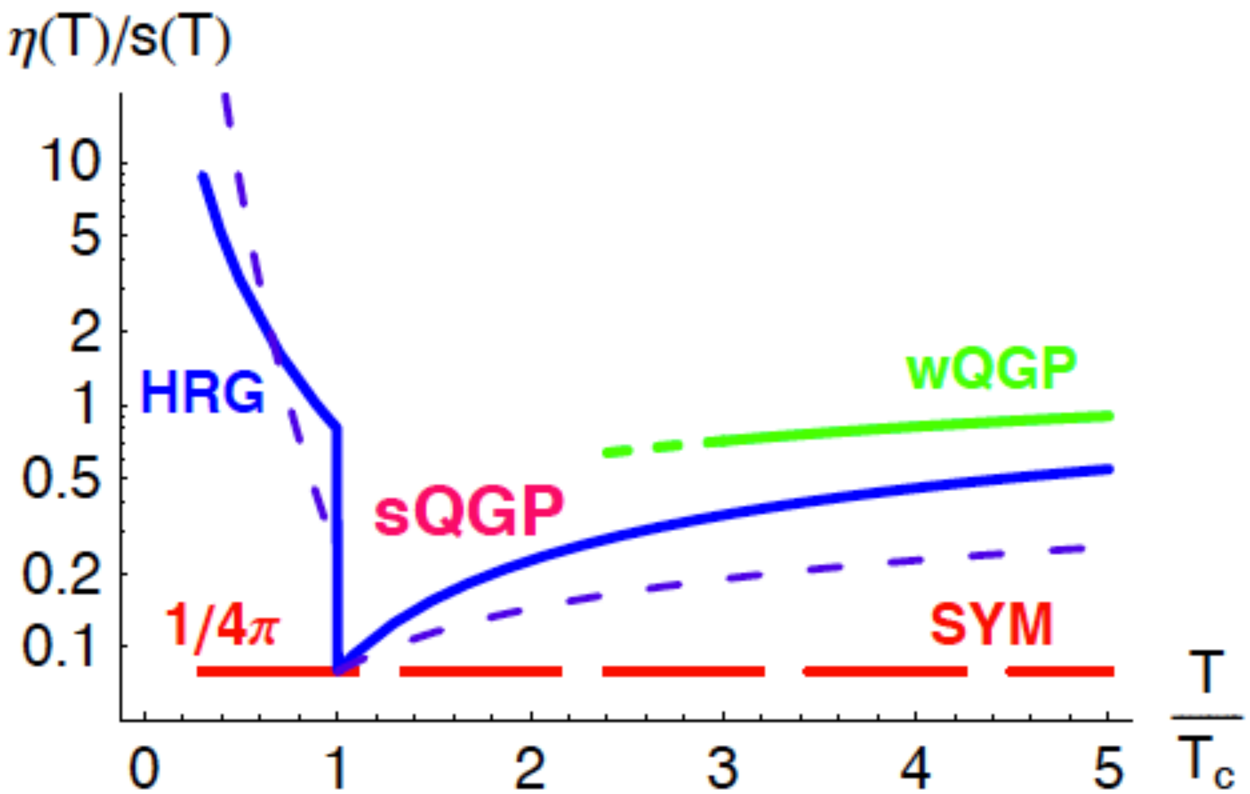
Bulk and Shear Viscosities

Bulk Viscosity



Karsh&Kharzeev&Tuchin, 0711.0914
Noronha&Noronha&Greiner, 0811.1571

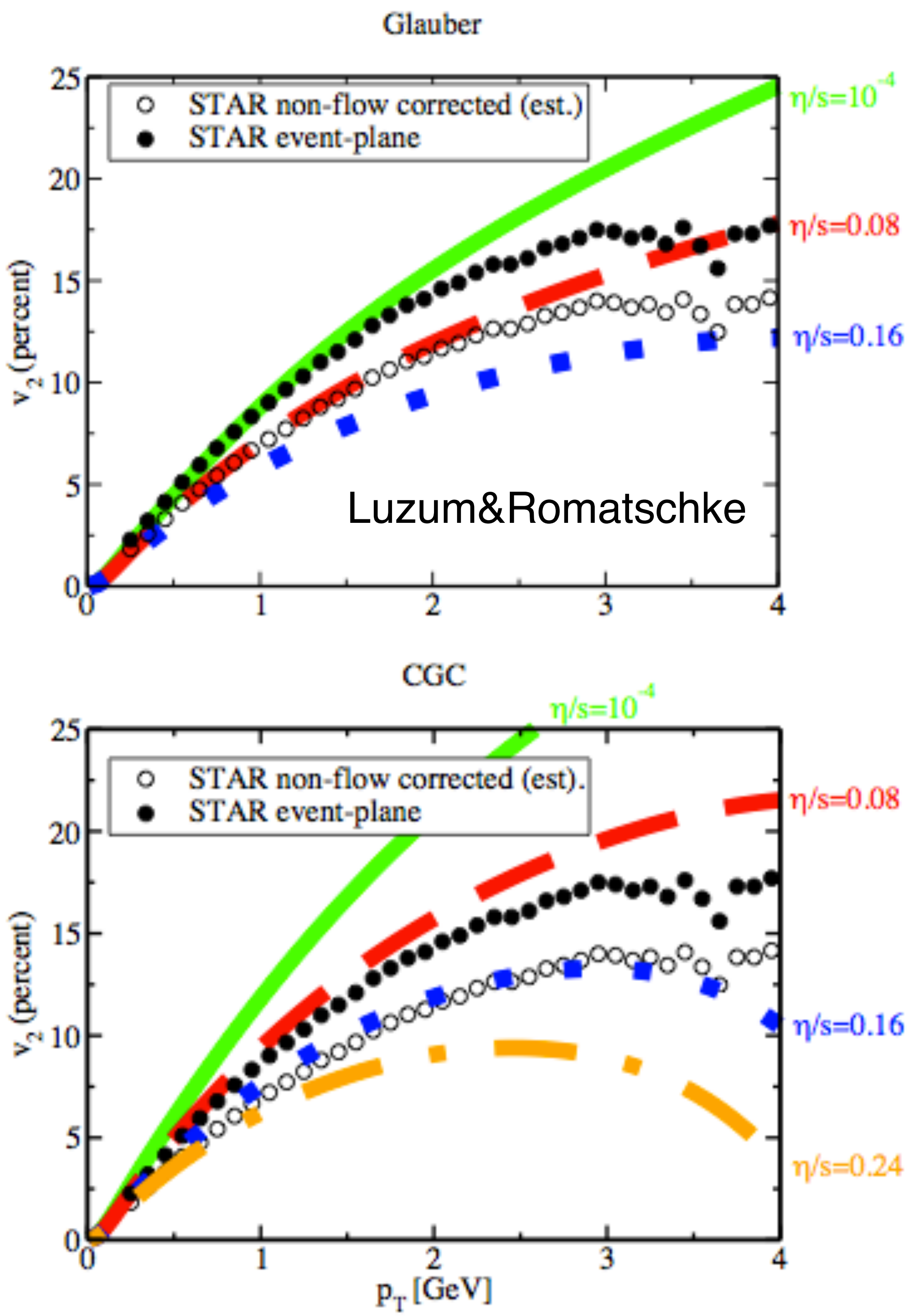
Shear Viscosity



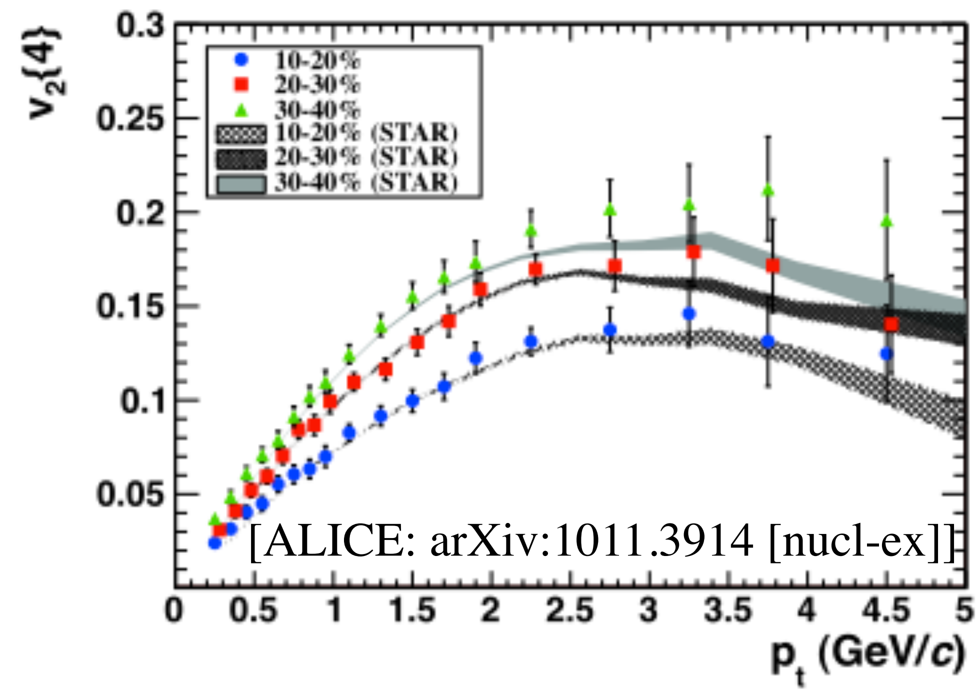
Hirano&Gyulassy, nucl-th/0506049

What about characteristics of QGP in HIC?

Puzzles from HIC



$\eta/s \sim 0.08 - 0.16$



- What are η, ζ,... in QCD? Is the plasma 'strongly coupled'? Is N = 4 SYM really a good model for QGP?
- Ultimate answer only from non-perturbative calculations in QCD!

Bulk and shear viscosities: Kubo formulae

- Matching of linearized hydrodynamic and linear response description in QFT---Kubo formulae: Viscosities and other transport coeffs. are obtainable from retarded Minkowskian correlators of energy momentum tensor

$$\eta = \pi \lim_{\omega \to 0} \frac{\rho_{12,12}(\omega, \mathbf{k} = \mathbf{0})}{\omega}$$

$$\zeta = \frac{\pi}{9} \sum_{i,j=1}^{3} \lim_{\omega \to 0} \frac{\rho_{ii,jj}(\omega, \mathbf{k} = \mathbf{0})}{\omega}$$

$$\rho_{\mu\nu\rho\sigma} = \mathrm{Im} G^R_{\mu\nu\rho\sigma}(\omega, \mathbf{0})$$

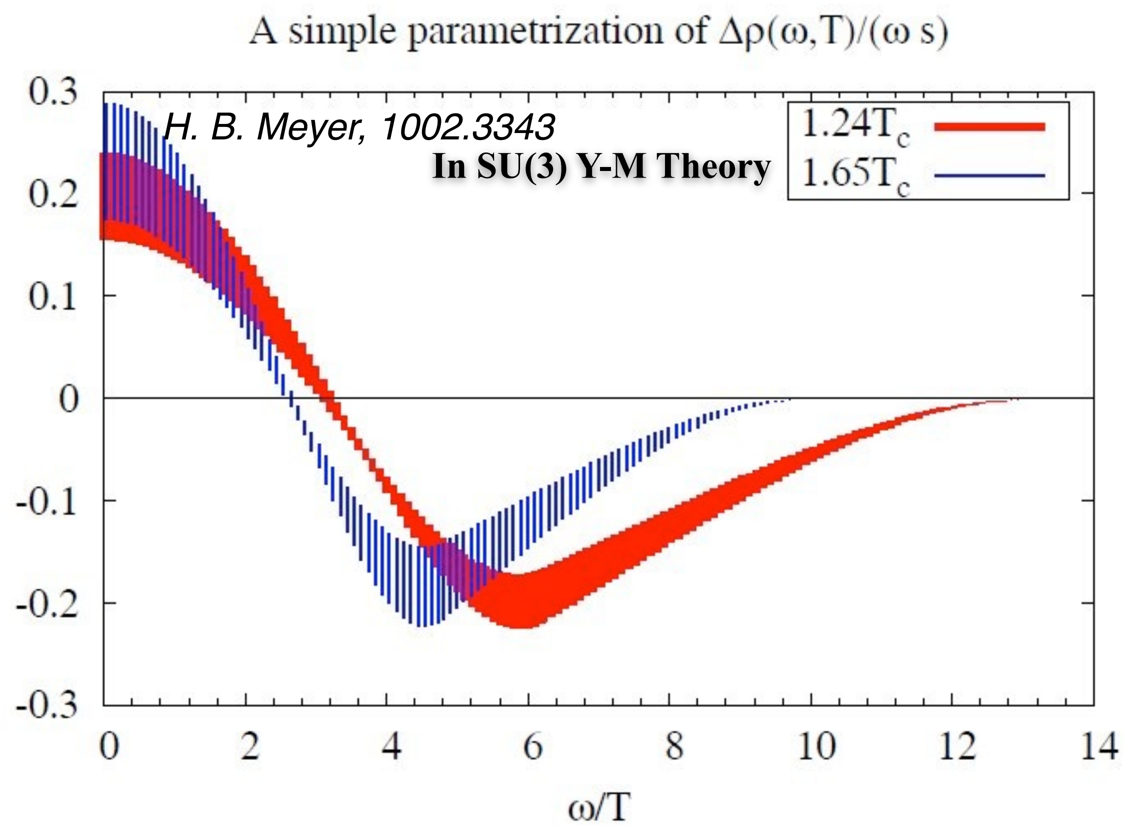
$$G^R_{\mu\nu\rho\sigma}(\omega, \mathbf{0}) \equiv i \int_0^\infty dt e^{i\omega t} \int d^3x \, \langle [T_{\mu\nu}(t, \mathbf{x}), T_{\rho\sigma}(0, \mathbf{0})] \rangle$$

$$G_R(\omega) = \tilde{G}_E(p_n \to -i[\omega + i0^+], \mathbf{0})$$

Viscosities from the lattice

- Lattice determines spectral density ρ from Euclidean correlators: Need to invert

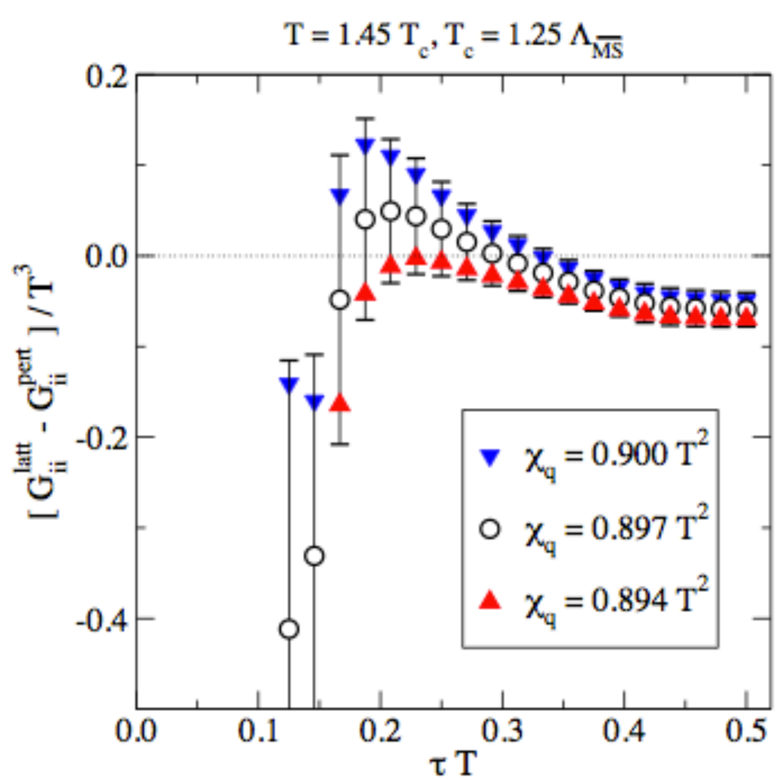
$$G(\hat{\tau}) = \int_0^\infty \frac{d\omega}{\pi} \rho(\omega) \frac{\cosh\left(\frac{1}{2} - \hat{\tau}\right) \beta\omega}{\sinh \frac{\beta\omega}{2}}$$

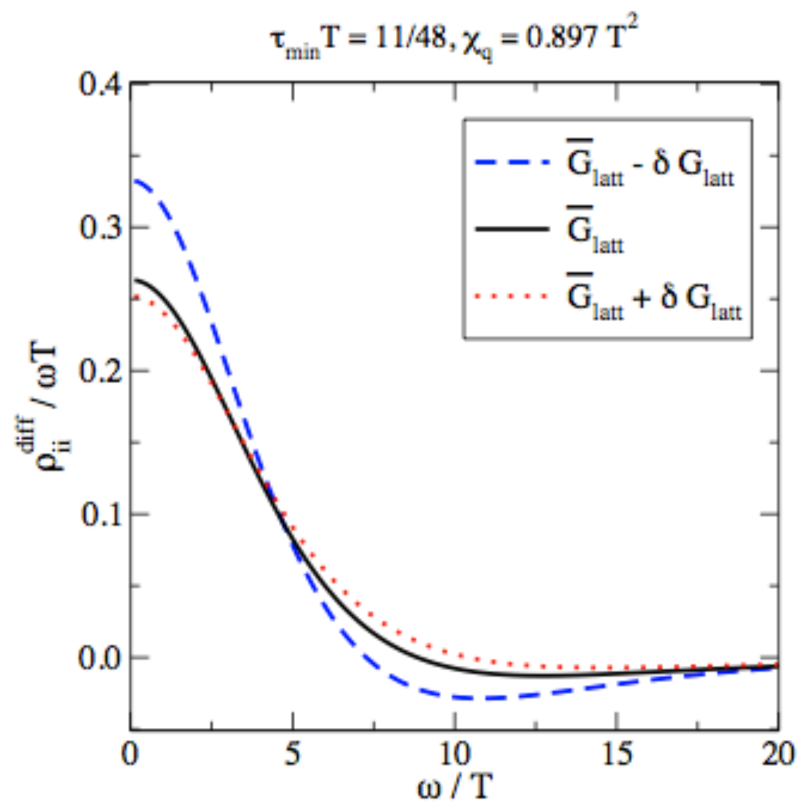


- For extracting IR limit of ρ, need to understand its behavior also at $\omega \gtrsim \pi T$ — very non-trivial challenge for lattice QCD, requiring perturbative input!

Successful application of pQCD result

- For the vector-current correlator, 5-loop vacuum limit and accurate lattice data available ⇒ Model-independent analytic continuation of Euclidean correlator à la [Burnier, Laine, Mether; EPJC 71] possible
- Result: Estimate for flavor current spectral density and flavor diffusion coefficient [Burnier, Laine; EPJC 72] $2\pi TD \gtrsim 0.8$





$$G_{ii}(\tau T) = \chi_q T + G_V(\tau T) \qquad G_V(\tau) \equiv -\sum_{\mu=0}^{3} \int_{\mathbf{x}} \left\langle (\bar\psi\gamma_\mu\psi)(\tau,\mathbf{x})\ (\bar\psi\gamma^\mu\psi)(0,\mathbf{0}) \right\rangle_T$$

Setup

- SU(Nc) YM theory

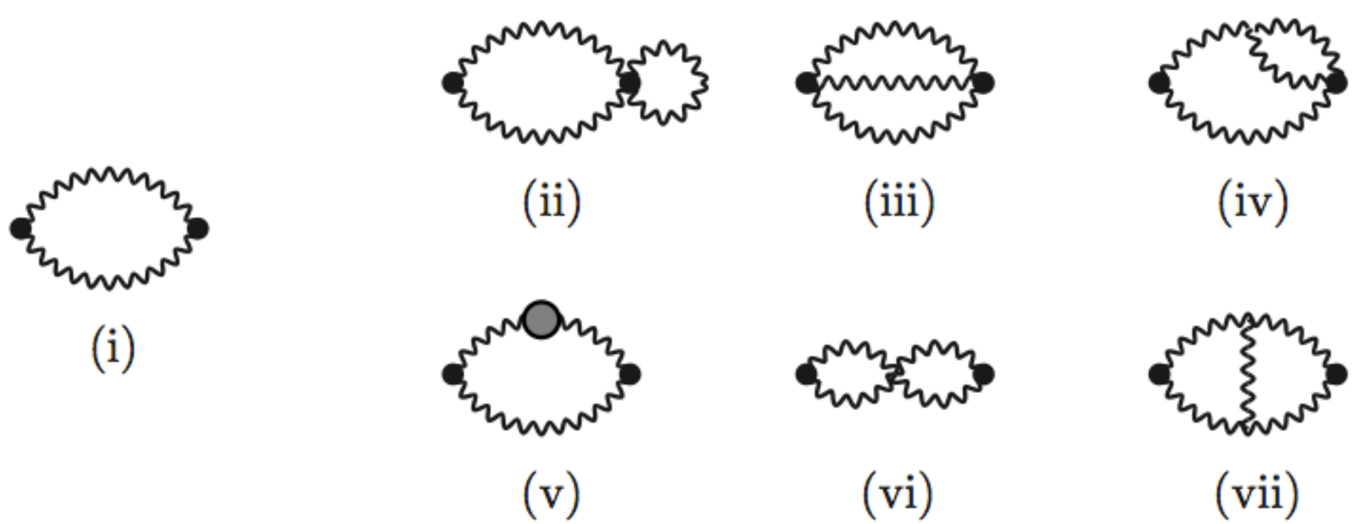
$$S_E = \int_0^\beta \mathrm{d}\tau \int \mathrm{d}^{3-2\epsilon}\mathbf{x} \left\{ \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} \right\}$$

- Define:
  - $G_\theta(x) \equiv \langle \theta(x)\theta(0)\rangle_\mathrm{c}\ , \quad \theta \equiv c_\theta\, g_\mathrm{B}^2 F^a_{\mu\nu} F^a_{\mu\nu}$
  - $G_\chi(x) \equiv \langle \chi(x)\chi(0)\rangle\ , \quad \chi \equiv c_\chi\, \epsilon_{\mu\nu\rho\sigma} g_\mathrm{B}^2 F^a_{\mu\nu} F^a_{\rho\sigma}$
  - $G_\eta(x) = -16 c_\eta^2\, \langle T_{12}(x)\, T_{12}(0)\rangle_c\, .$

  where $T_{\mu\nu} = \frac{1}{4}\delta_{\mu\nu} F^a_{\alpha\beta} F^a_{\alpha\beta} - F^a_{\mu\alpha} F^a_{\nu\alpha}\, ,$

Correlators to NLO

The LO and NLO Feynman graphs contributing to the correlators



- Write down diagrammatic expansions for Euclidean correlators in momentum space $\tilde{G}_\alpha(P) \equiv \int_x e^{-iP\cdot x}\tilde{G}_\alpha(x)$
- Carry out Matsubara sums by 'cutting' thermal lines and evaluate remaining 3d integrals at high P to get the OPE
- Extract the spectral densities with $\rho(\omega) = \mathrm{Im}\left[\tilde{G}(P)\right]_{P\to(-i[\omega+i0^+],\mathbf{0})}$.

Spectral functions

$$\rho(\omega) = \mathrm{Im}\left[\tilde{G}(P)\right]_{P\to(-i[\omega+i0^+],\mathbf{0})} \, .$$

- After Matsubara sums, the imaginary part can be extracted with

$$\frac{1}{\omega \pm i0^+} = \mathbb{P}\left(\frac{1}{\omega}\right) \mp i\pi\delta(\omega)$$

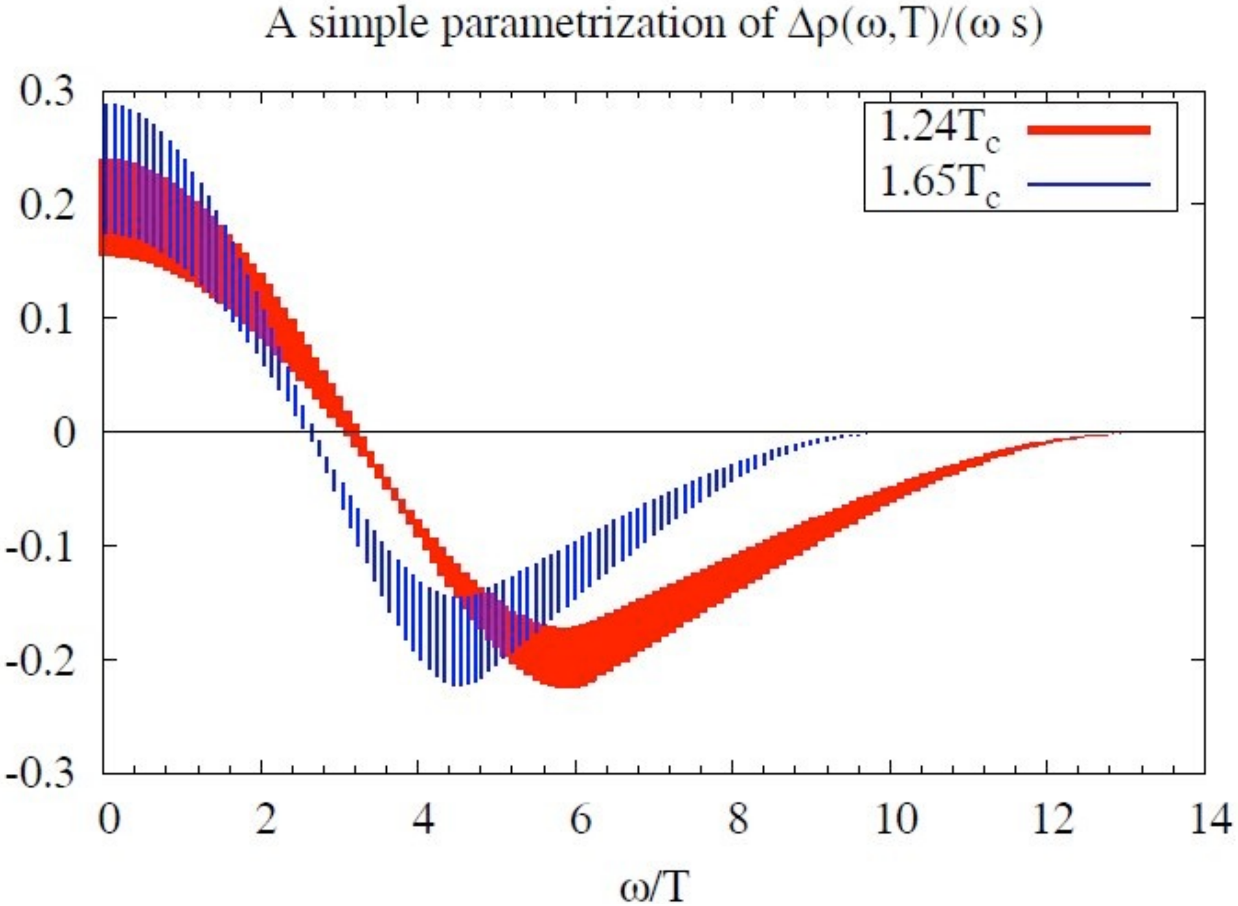
- Example:

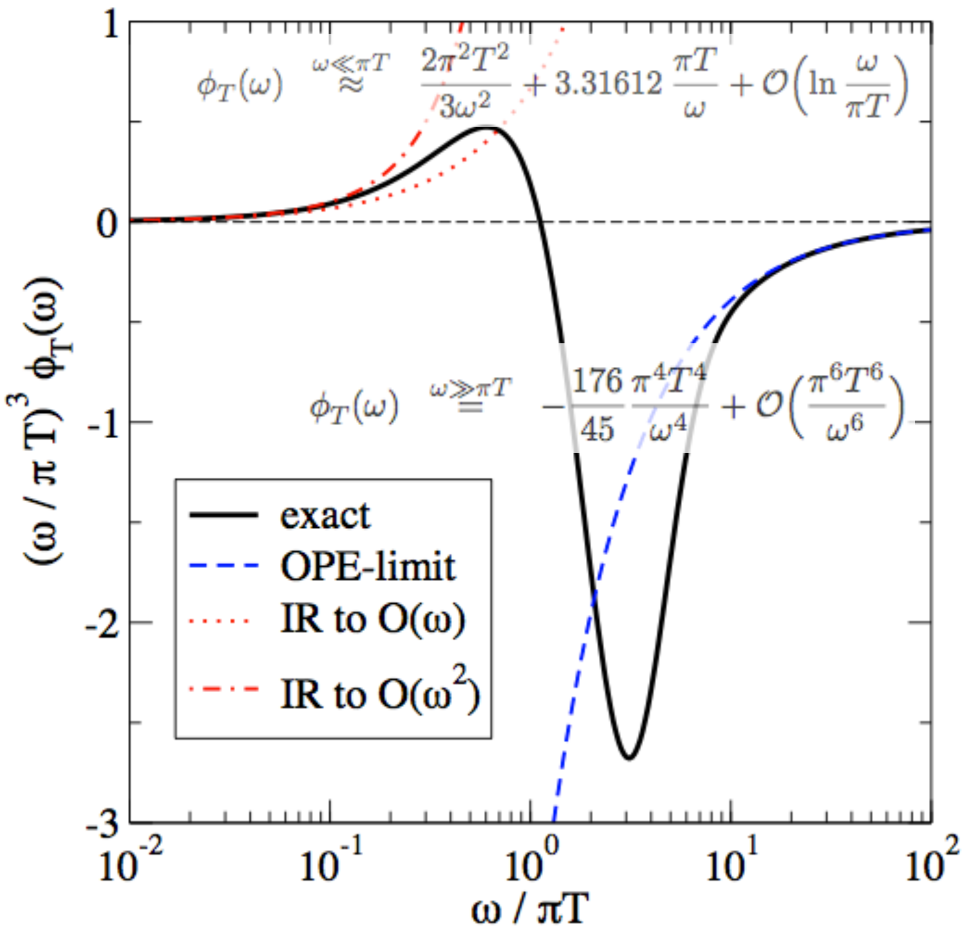
$$\mathcal{I}_{\mathrm{j}}^{0}(P) \equiv \sum\!\!\!\!\!\!\!\int_{Q,R} \frac{P^6}{Q^2R^2[(Q-R)^2+\lambda^2](Q-P)^2(R-P)^2}$$

Denoting $E_q \equiv q \,, \quad E_r \equiv r \,, \quad E_{qr} \equiv \sqrt{(\mathbf{q}-\mathbf{r})^2+\lambda^2} \,,$

Spectral functions: Bulk channel

H. B. Meyer, 1002.3343

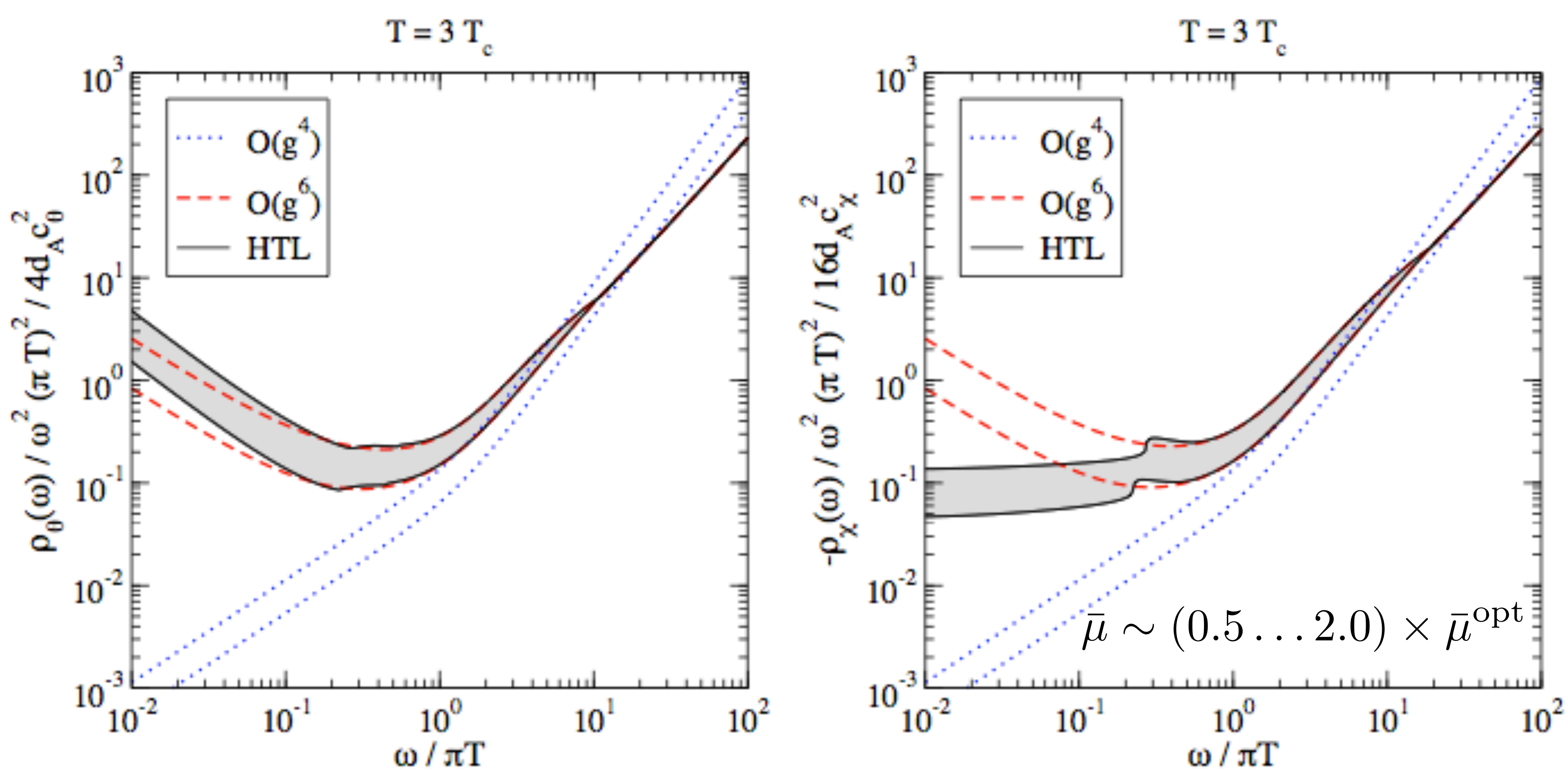




$$\frac{\rho_\theta(\omega)}{4d_Ac_\theta^2} = \frac{\pi\omega^4}{(4\pi)^2}\left(1+2n_{\frac{\omega}{2}}\right)\left\{g^4+\frac{g^6N_c}{(4\pi)^2}\left[\frac{22}{3}\ln\frac{\bar\mu^2}{\omega^2}+\frac{73}{3}+8\,\phi_T(\omega)\right]\right\}+\mathcal{O}(g^8)$$

$$\frac{-\rho_\chi(\omega)}{16d_Ac_\chi^2} = \frac{\pi\omega^4}{(4\pi)^2}\left(1+2n_{\frac{\omega}{2}}\right)\left\{g^4+\frac{g^6N_c}{(4\pi)^2}\left[\frac{22}{3}\ln\frac{\bar\mu^2}{\omega^2}+\frac{97}{3}+8\,\phi_T(\omega)\right]\right\}+\mathcal{O}(g^8)$$

Spectral functions: Bulk channel



$$\rho^{\rm QCD}_{\rm resummed} = \rho^{\rm QCD}_{\rm resummed} - \rho^{\rm HTL}_{\rm resummed} + \rho^{\rm HTL}_{\rm resummed} \approx \rho^{\rm QCD}_{\rm naive} - \rho^{\rm HTL}_{\rm naive} + \rho^{\rm HTL}_{\rm resummed} \, .$$

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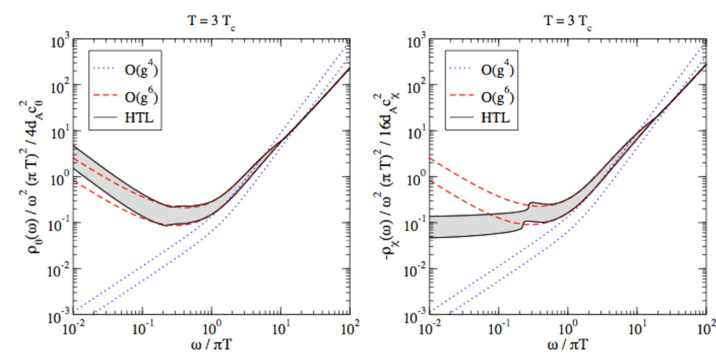
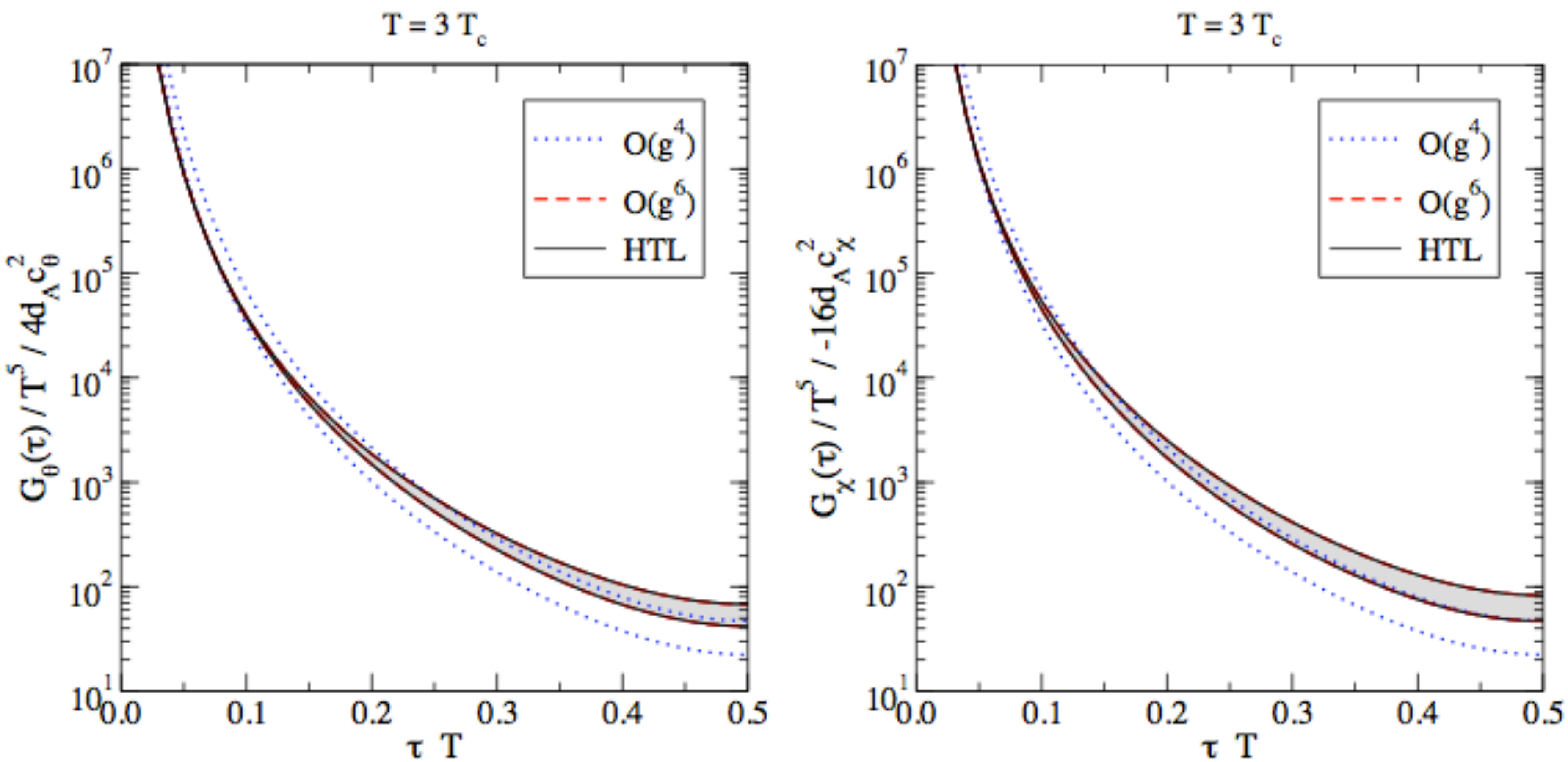
Imaginary-time correlators: Bulk channel

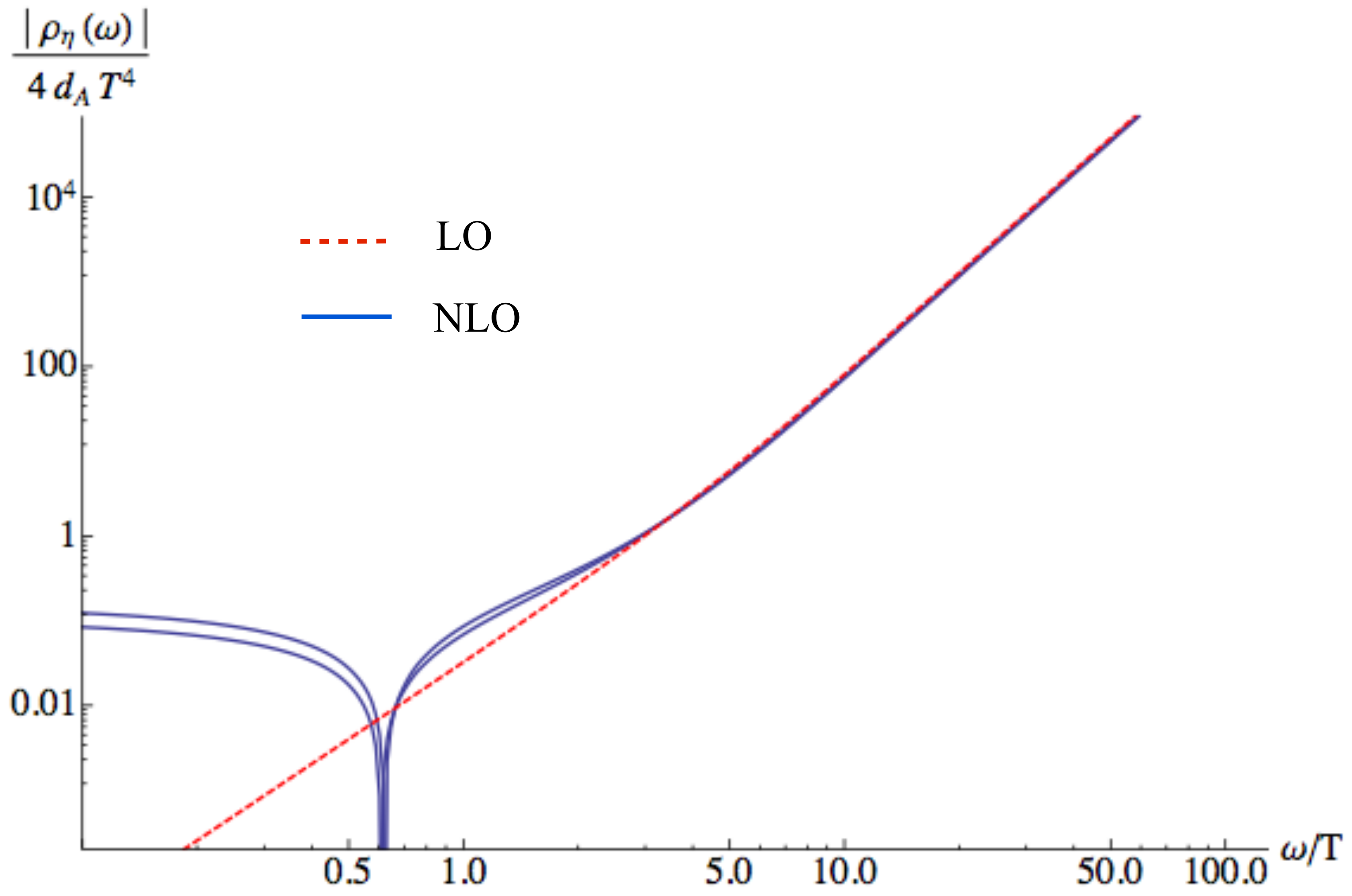
$$G(\tau) = \int_0^\infty \frac{d\omega}{\pi} \rho(\omega, \mathbf{0}) \frac{\cosh\left(\frac{\beta}{2} - \tau\right)\omega}{\sinh\frac{\beta\omega}{2}} \, .$$





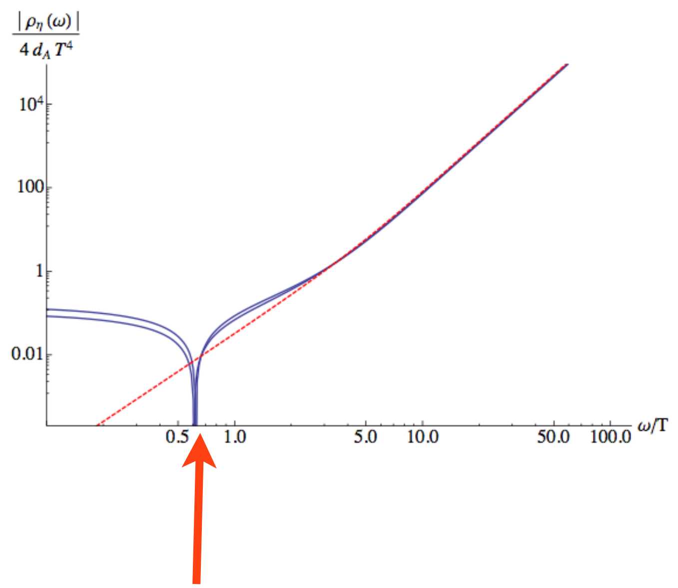
- Considerable difference between LO and NLO in spectral function leads to a small correction to the imaginary-time correlators.

Spectral functions: Shear channel



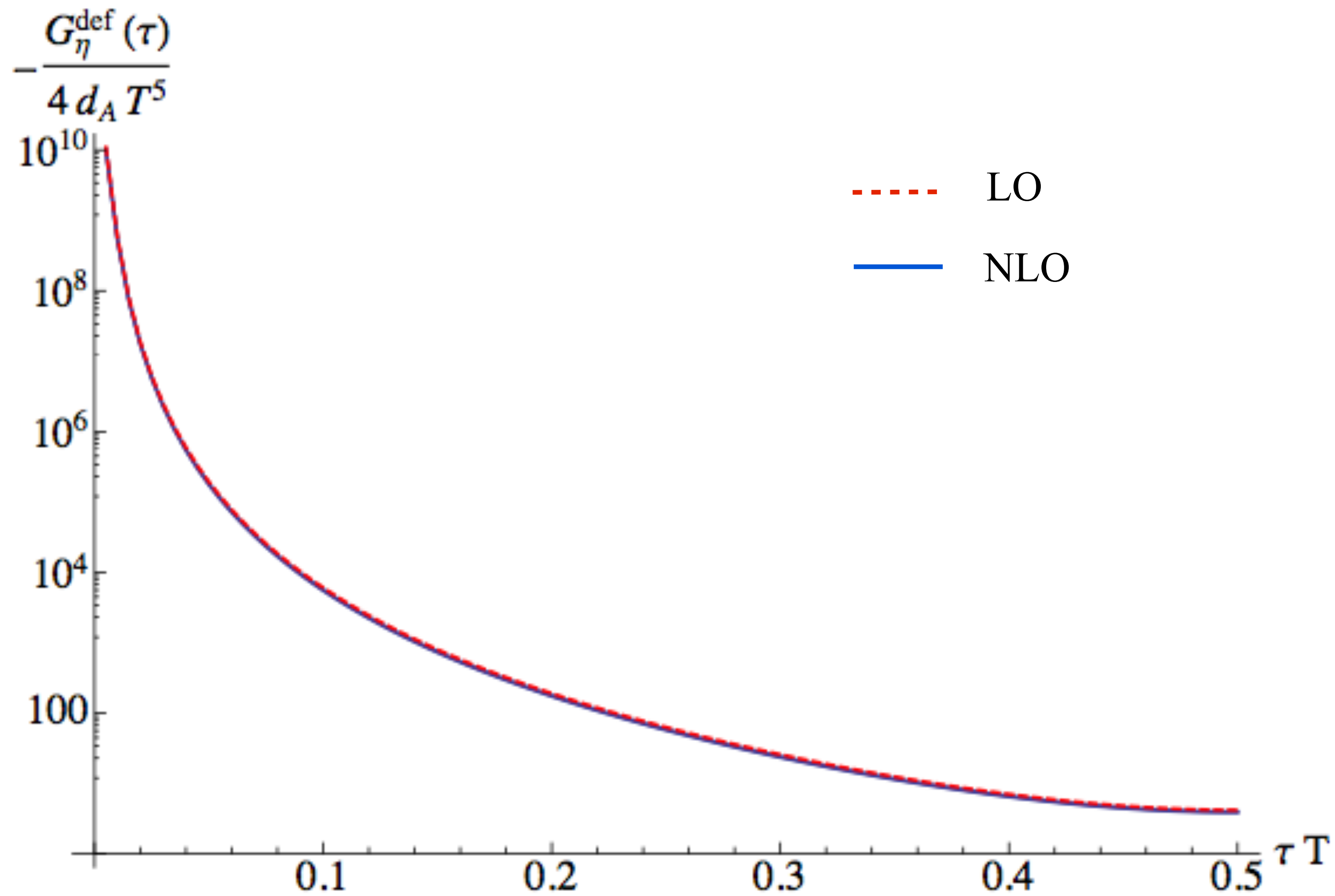
$$\frac{\rho_\eta(\omega)}{4d_A} = \frac{\omega^4}{4\pi}(1+2n_{\frac{\omega}{2}})\left\{-\frac{1}{10}+\frac{g^2N_c}{(4\pi)^2}\left(\frac{2}{9}+\phi_T^\eta(\omega/T)\right)\right\}$$

Imaginary-time correlators: Shear channel



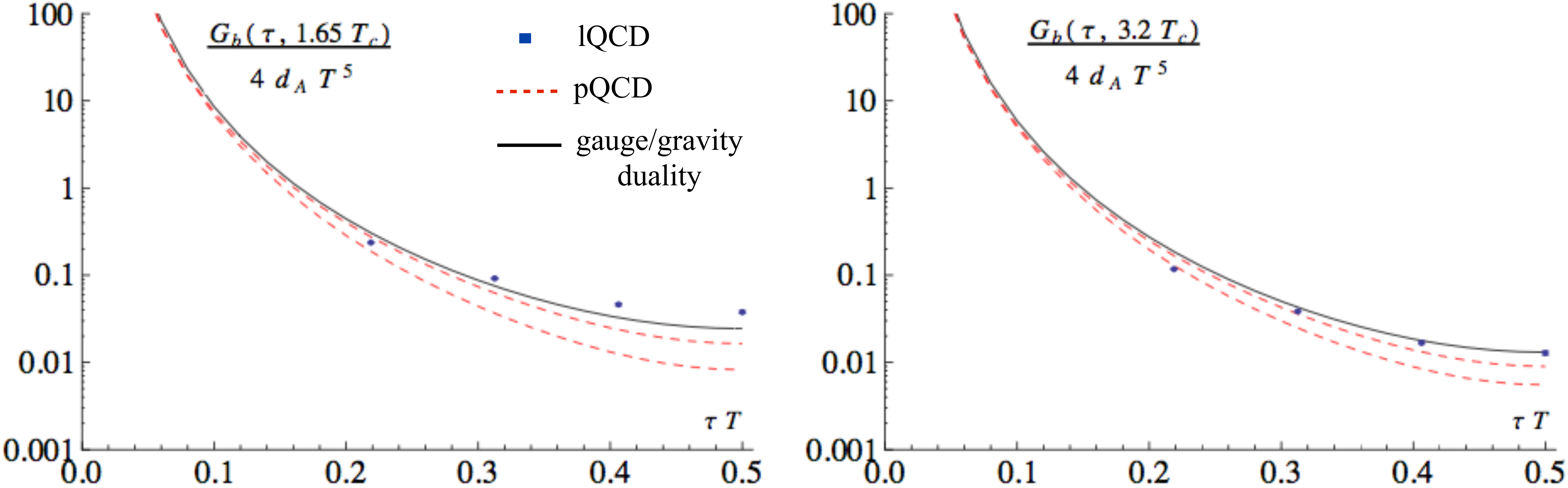
$\omega_0 \approx 0.6T$

$$G_\eta^{\rm def}(\tau) = \int_{\omega_0}^{\infty} \frac{d\omega}{\pi} \rho_\eta(\omega) \frac{\cosh\left[\left(\frac{\beta}{2} - \tau\right)\omega\right]}{\sinh \frac{\beta\omega}{2}}$$



Lattice vs. pQCD vs. gauge/gravity duality: Bulk channel

K. Kajantie, M. Krssak and A. Vuorinen, arXiv:1302.1432 [hep-ph].



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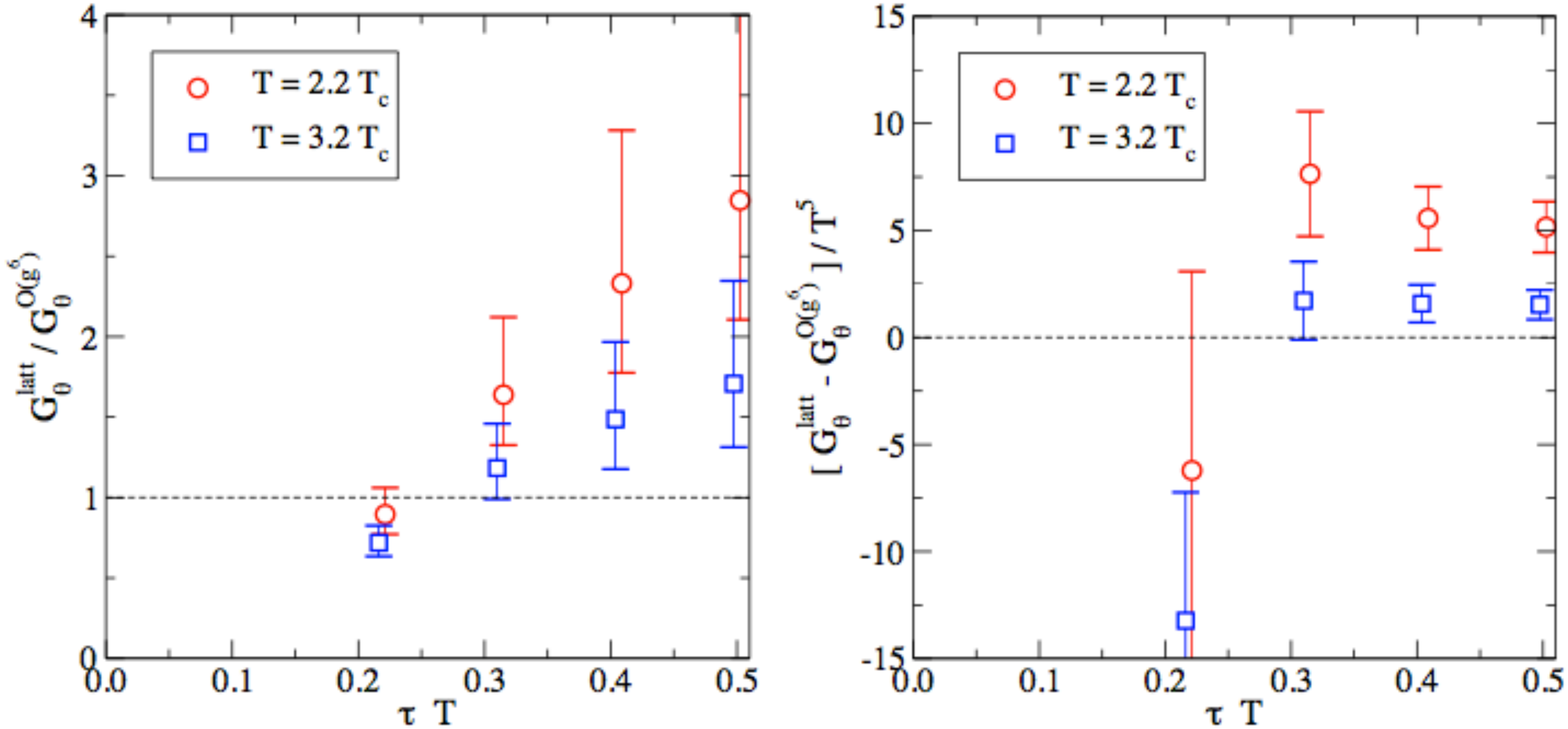
Lattice vs. pQCD: Bulk channel

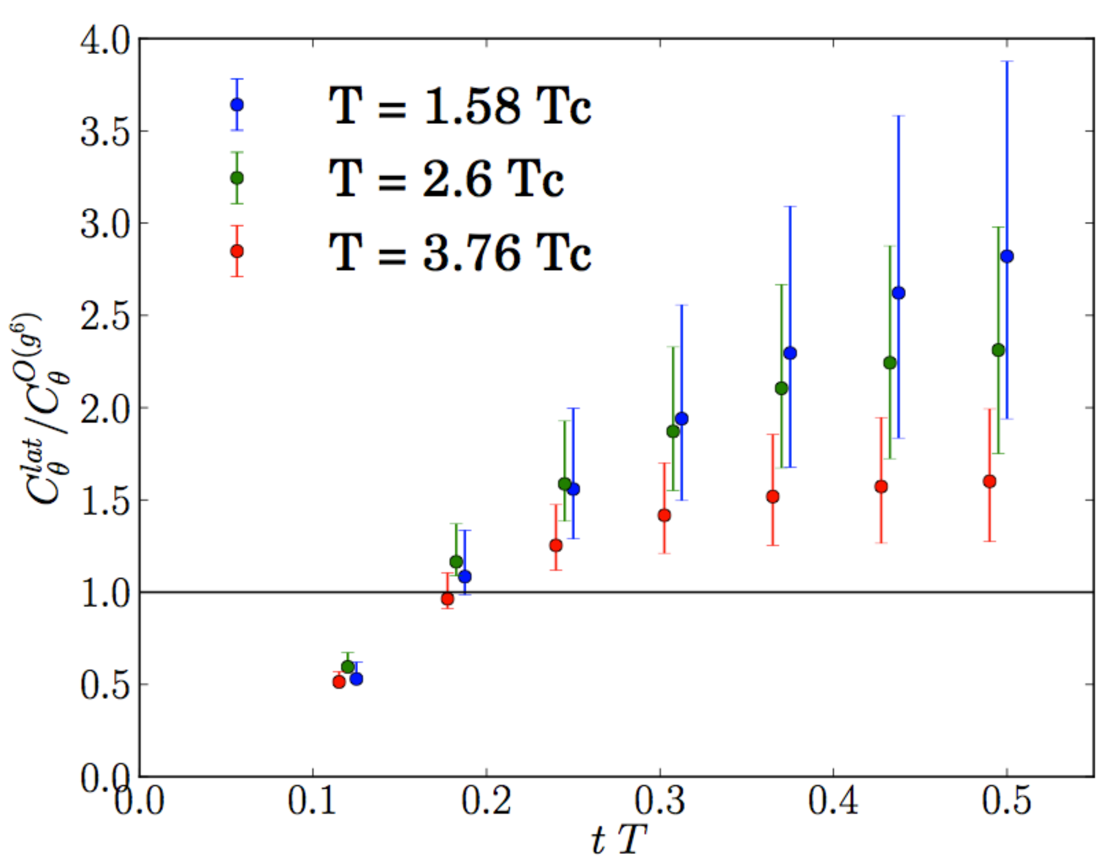
Lattice data from H.B. Meyer, JHEP 04(2010), 099 [10023344]



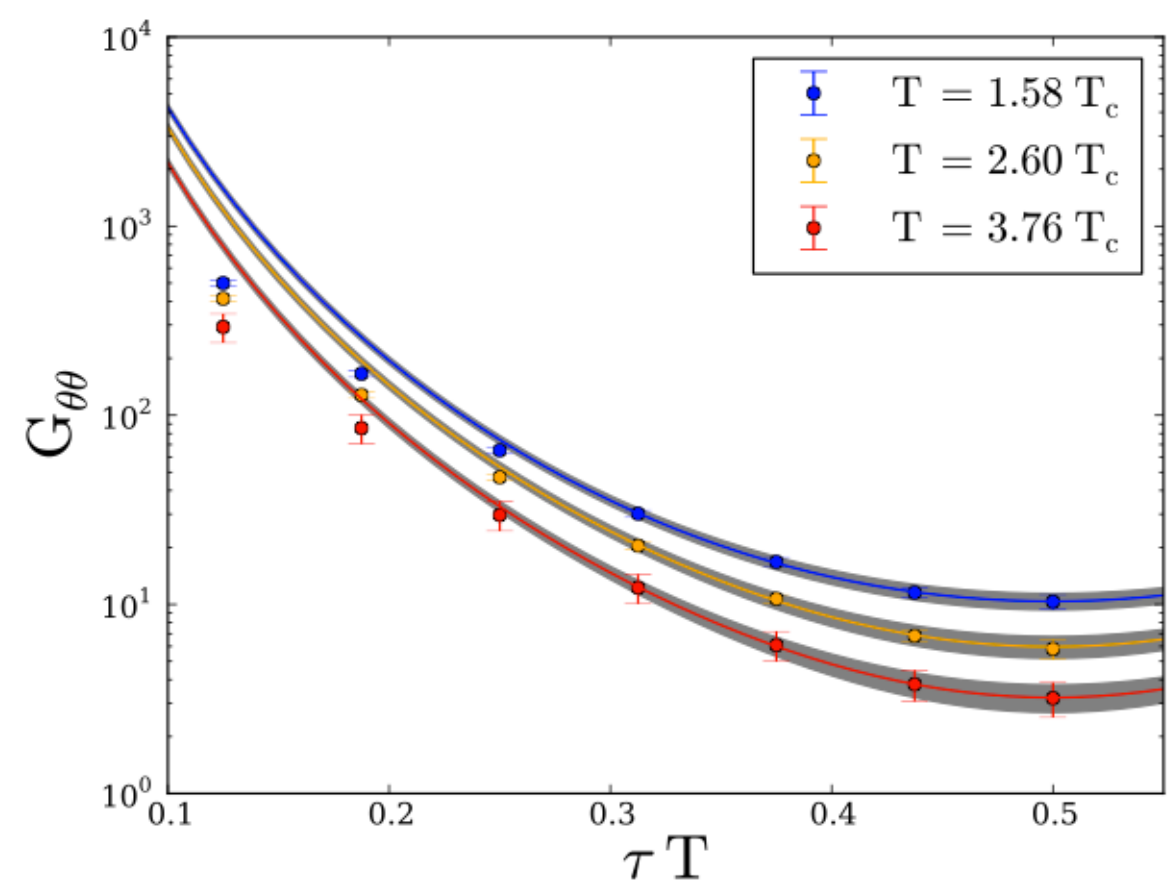
- The ratio shows good agreement at short distances.
- The difference no longer shows the short distance divergence. A model independent analytic continuation could be attempted.

Lattice vs. pQCD: Bulk channel

Chuan Miao(CPOD2011), H. B. Meyer



Chuan Miao, H.B. Meyer (Preliminary)



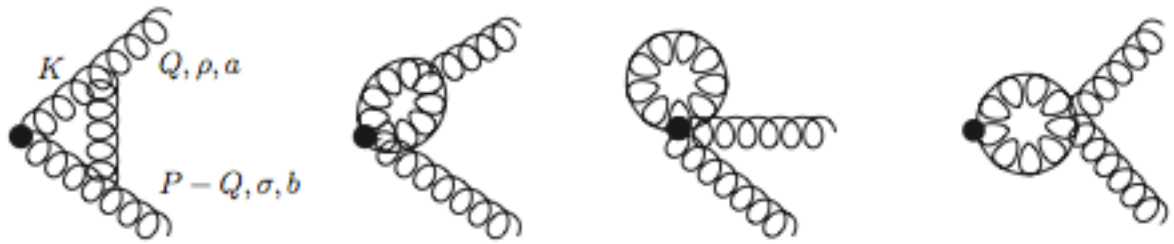
- Right panel: Fit correlators with Breit Wigner formula (low frequency) + NLO results (high frequency), the width of the B-W is fixed to 0.5πT.
- NLO perturbative input is very helpful.

HTL Propagator & Vertex

- HTL propagator:

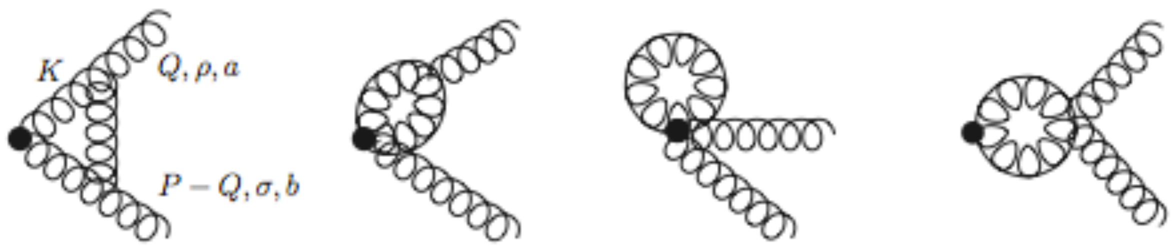
$$\langle A^a_\mu(X)\, A^b_\nu(Y)\rangle = \delta^{ab} \sum\!\!\!\!\!\!\!\int_Q e^{iQ\cdot(X-Y)}\left[\frac{\mathbb{P}^T_{\mu\nu}(Q)}{Q^2+\Pi_T(Q)} + \frac{\mathbb{P}^E_{\mu\nu}(Q)}{Q^2+\Pi_E(Q)} + \frac{\xi\, Q_\mu Q_\nu}{Q^4}\right]$$

- HTL Vertex:



$$\left[V^{\theta/\chi}_{\mathrm{HTL}}\right]^{ab}_{\rho,\sigma} = 0$$

HTL Vertex in Shear Channel



$$\tilde{G}_\eta(P) \equiv 2X_{\mu\nu,\alpha\beta}\,\tilde{G}_{\mu\nu,\alpha\beta}(P)$$

$$\begin{aligned}
\frac{[V^\eta_{\rm HTL}]^{ab}_{\mu\nu,\rho\sigma}}{g_{\rm B}^2 N_{\rm c}} &= \not{\int}_K \frac{\delta_{ab} A^a_\rho(Q) A^b_\sigma(P-Q)}{K^2(K-P)^2(K-Q)^2}\Big\{4(2-D)K_\mu K_\nu K_\rho K_\sigma \\
&- 2(2-D)K_\mu K_\nu(K_\rho Q_\sigma + Q_\rho K_\sigma + K_\rho P_\sigma) - 4\Big[(3-D)K_\mu P_\nu - P_\mu K_\nu\Big]K_\rho K_\sigma\Big\} \\
&+ \not{\int}_K \frac{1}{K^2(K-P)^2}\Big\{\delta_{\mu\rho}K_\nu K_\sigma - \delta_{\mu\sigma}K_\nu K_\rho + \delta_{\nu\sigma}K_\mu K_\rho - \delta_{\nu\rho}K_\mu K_\sigma \\
&+ (D-2)K_\mu K_\nu \delta_{\rho\sigma}\Big\} + \not{\int}_K \frac{1}{K^2(K-Q)^2}\Big\{2\delta_{\mu\nu}K_\rho K_\sigma + \delta_{\mu\rho}K_\nu K_\sigma \\
&- 4(2-D)\delta_{\mu\sigma}K_\nu K_\rho - \delta_{\nu\sigma}K_\mu K_\rho - 2\delta_{\nu\rho}K_\mu K_\sigma + K_\mu K_\nu \delta_{\rho\sigma}\Big\} \\
&+ \not{\int}_K \frac{1}{(K-P)^2(K-Q)^2}\Big\{-2\delta_{\mu\nu}K_\rho K_\sigma - \delta_{\mu\rho}K_\nu K_\sigma + \delta_{\nu\sigma}K_\mu K_\rho + 2\delta_{\nu\rho}K_\mu K_\sigma \\
&- K_\mu K_\nu \delta_{\rho\sigma}\Big\} + \not{\int}_K \frac{1}{K^2}\Big\{\delta_{\mu\sigma}\delta_{\nu\rho} + (2-D)\delta_{\mu\rho}\delta_{\nu\sigma}\Big\}
\end{aligned}$$

HTL Correction to Correlators

- With HTL propagator, naive HTL correlator in bulk channel completely matches IR limit of naive QCD.
- If $\left.\frac{\rho_\eta^{\mathrm{HTL}}(\omega)}{4d_A}\right|_{\mathrm{naive}} = \frac{1}{4\pi}(1+2n_{\frac{\omega}{2}})\left\{-\frac{\omega^4}{10}+\frac{\omega\pi^2 T}{45}m_E^2\right\}$, unfortunately, when only HTL propagator involved,

  $$\left.\frac{\rho_\eta^{\mathrm{HTL}}(\omega)}{4d_A}\right|_{\mathrm{naive}} = \frac{1}{4\pi}(1+2n_{\frac{\omega}{2}})\left\{-\frac{\omega^4}{10}-\frac{\omega\pi^2 T}{45}m_E^2\right\}$$
- Naive HTL contribution from HTL vertex should be 2 x $\frac{1}{4\pi}(1+2n_{\frac{\omega}{2}})\frac{\omega\pi^2 T}{45}m_E^2$.
- More than 4000 terms in full HTL correlator are waiting.

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Summary and Outlook

- Information on correlation functions of the energy momentum tensor crucial for disentangling the properties of the QGP
  - Wilson coefficients refined and determined in the OPE
  - Spectral densities needed in extracting transport coefficients from lattice QCD data
- NLO results in the bulk and shear channels completed, HTL for the shear channel underway
  - Results promising, but quantitative comparisons await
- If pure YM results useful, inclusion of fermions straightforward